The role of causality in quantum gravity

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The Time Machine Factory, Sept 2019

Outline

- The causal structure poset $(M, \prec)$
- The Hawking-King-McCarthy-Malament theorem
- Quantum Gravity and Causality: the different approaches
- Discreteness+ Order : the causal set approach to quantum gravity
- Kinematic and Dynamical Causality in CST
- Afterword

The causal structure poset $(M, \prec)$

- (M, g) has local lightcones $\Rightarrow$ Local Causality:
  - $\prec$: causality relation (causal, $J^{\pm}(x)$)
  - $\prec\!\!\prec$: chronology relation (timelike, $I^{\pm}(x)$)
  - $\rightarrow$: horismos relation (null, $J^{\pm}(x) \backslash I^{\pm}(x)$)
- In any causal spacetime $(M, \prec)$ is a poset.
  - Acyclic: $x \prec y$ and $y \prec x \Rightarrow x = y$
  - Transitive: $x \prec y$ and $y \prec z \;\Rightarrow x \prec z$

How fundamental is $(M, \prec)$?

Zeeman, 1964, Penrose and Kronheimer, 1966

Spacetime geometry from $(M, \prec)$

- For Minkowski spacetime, group of chronological automorphisms is isomorphic to the group of inhomogeneous Lorentz transformations and dilations.

  Zeeman, 1964

- The Hawking-King-McCarthy-Malament Theorem:

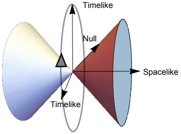
  Let $f : (M_1, g_1) \to (M_2, g_2)$ be a causal bijection between two future and past distinguishing spacetimes, i.e., $x_1 \prec_1 y_1 \Leftrightarrow f(x_1) \prec_2 f(x_2)$. Then f is a smooth conformal isometry: f and f^{-1} are smooth and $f_* g_1 = \Omega^2 g_2$.

  S. W. Hawking, A.R. King, P.J. McCarthy (1976); D. Malament (1977)

  O. Parrikar, S. Surya (2011)

- $(M, \prec)$ contains all but one of the $n(n+1)/2$ independent components of (M, g)

- $(M, \prec)$ is a poset only for Lorentzian spacetime: $(-, +, +, +)$



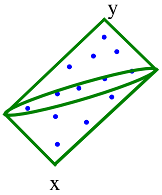
Order is most of geometry

Spacetime geometry = Causal Structure + Volume

- "Causal structure is $9/10^{\text{th}}$ of the spacetime geometry."
- Remaining $1/10^{\text{th}}$ is the volume element

$$\epsilon = \Omega^n \times \sqrt{g} dx^1 \wedge \ldots \wedge dx^n$$

- A route to quantisation:
  - Poset structure is fundamental
  - Volume from discreteness $N \sim V/V_p$



Causality in Different Approaches to Quantum Gravity

- Canonical Quantum Gravity (LQG, LQC, etc)
  - Initial Value Formulation
  - Hamiltonian/Unitary Evolution
- Causal Dynamical Triangulation
  - Allow only causal evolution of simplices
  - Foliations into spacelike hypersurfaces
  - Causality helps solve the entropy problem
- Spin Foams, Asymptotic Safety,: use of Euclidean geometry
- String Theory: Euclidean or Lorentzian background

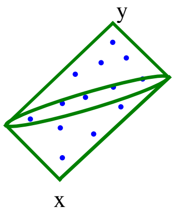
Geometry $= g_{ab}$.

Causality emergent (or can be sacrificed?)

The Causal Set Theory (CST) Route to Quantum Gravity

– L.Bombelli, J.Lee, D. Meyer and R. Sorkin (1987)

- (Discrete) casual structure poset $(M, \prec)$ *is* spacetime,
- Spacetime Continuum $(M, g) \rightarrow$ *locally finite* partially ordered set or **causal set** C
  - Acyclic: $x \prec y$ and $y \prec x \Rightarrow x = y$
  - Transitive: $x \prec y$ and $y \prec z \;\; \Rightarrow x \prec z$
  - $|Fut(x) \cap Past(y)| < \infty$
- Finite spacetime volume has finite number of "elements" of the causal set.



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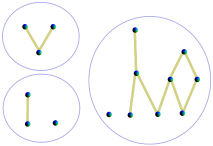
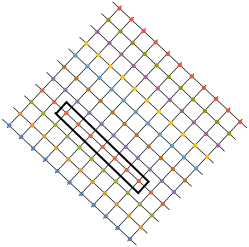
- Finite spacetime volume has finite number of "elements" of the causal set.
- Continuum Approximation $C \sim (M, g)$
  - Order $\rightarrow (M, \prec)$
  - Number $\rightarrow$ Volume

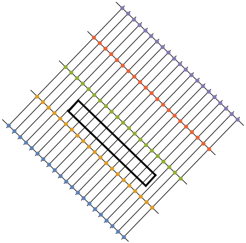


The Continuum Approximation

Order + Number ∼ Spacetime

- Causal Order ⇔ Order
- Number ⇔ Volume

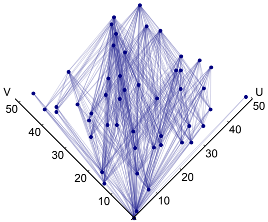




The Continuum Approximation

Order + Number ∼ Spacetime

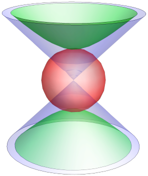
- Causal Order ⇔ Order
- Number ⇔ Volume
- Random Lattice
  - Poisson process: $P_V(n) \equiv \frac{1}{n!} \exp^{-\rho V} (\rho V)^n$
  - $\langle n \rangle = \rho V$



Discreteness + Lorentz invariance ⇒ Non-locality

- Local Lorentz invariance: there are no preferred directions

– L.Bombelli, J.Henson, R. Sorkin, Mod.Phys.Lett. 2009



- Non-locality: A causal set need not be a fixed valency graph.
- No Cauchy evolution

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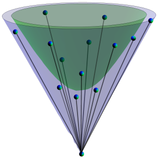

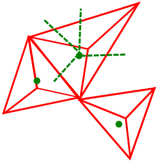

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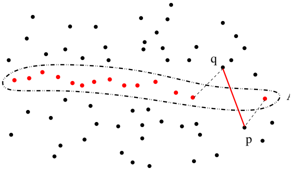
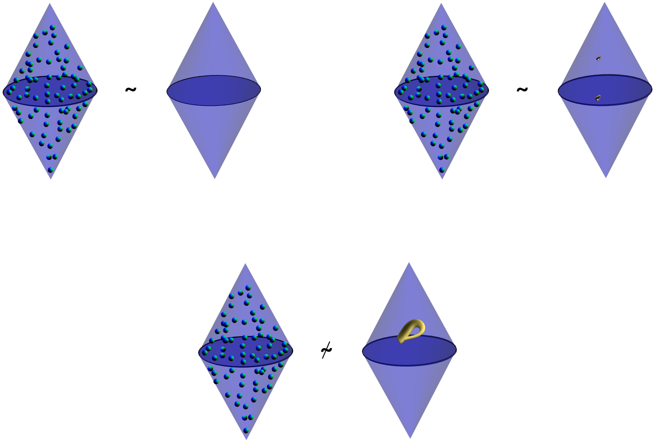


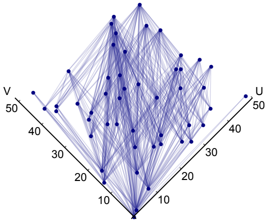
The Fundamental Conjecture of CST

- $C \sim (M_1, g_1)$, $C \sim (M_2, g_2) \Rightarrow (M_1, g_1) \sim (M_2, g_2)$.



Geometric Reconstruction/Covariant Observables

When does a causal set look like a spacetime?



Order is geometry

Geometric Reconstruction/Covariant Observables

- Dimension Estimators – Myrheim, Myer, Glaser & Surya
- Timelike Distance –Brightwell & Gregory
- Spatial Homology –Major, Rideout & Surya
- Spatial and Spacelike Distance –Rideout & Wallden
  –Eichhorn, Mizera & Surya, Eichhorn, Surya & Versteegen
- D'Alembertian –Sorkin, Henson, Benincasa & Dowker, Dowker & Glaser
- Benincasa-Dowker Action –Benincasa & Dowker, Dowker & Glaser
- GHY terms in the Action – Buck, Dowker, Jubb & Surya
- Locality and Interval Abundance –Glaser & Surya
- Scalar Field Greens functions –Johnston, Dowker, Surya & Nomaan X

Surya, S, Living Reviews in Relativity (2019)

The CST Postulate

- Spacetime is replaced by a set of locally finite posets or causal sets

$$Z = \int Dg \exp[iS[g]/\hbar] \quad \rightarrow \quad Z = \sum_{C \in \Omega} \mu(C) \tag{1}$$

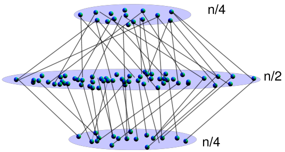
  - Sample Space: Ω
    - Ω_n : Finite/Fixed cardinality element causal sets
    - Ω_{pf}: Countable, past finite causal sets
    - Ω_d: "Dimensionally" restricted causal sets
  - "Quantum measure": $\mu(c)$. Eg: $\mu(c) = \exp(iS[c]/\hbar)$

The CST Postulate

- Spacetime is replaced by a set of locally finite posets or causal sets

$$Z = \int Dg \exp[iS[g]/\hbar] \quad \rightarrow \quad Z = \sum_{C \in \Omega} \mu(C) \tag{1}$$

- What does a typical causal set in Ω_n look like?

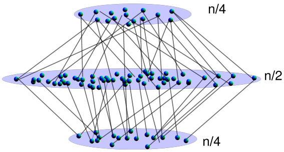


- Ω_n grows very rapidly: $|\Omega_n| \sim 2^{n^2/4}$

–Kleitman and Rothschild, Trans AMS, 1975
– J. Henson, D. Rideout, R. Sorkin and S.Surya, JEM, 2015

Proto-Spacetime and Proto-causality

- When $C \sim (M, g)$, $\prec$ is spacetime causality
- More genereally, $\prec$: "Proto-causality" relation.

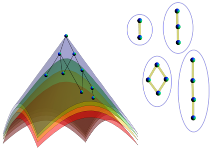


- No lightcones for generic causal sets – no simple geometric representation.
- "Order" replaces "time".

Causality in Dynamics

- Continuum inspired dynamics: $Z = \sum_{C \in \Omega} \exp(iS[c]/\hbar)$
  - Benincasa Dowker Action: $\frac{1}{\hbar} S_\epsilon(c) = 4\epsilon \left(N - 2\epsilon \sum_{n=0}^{N-2} N_n\, f(n,\epsilon) \right)$

    –Benincasa and Dowker, Dowker and Glaser
    - Weighted sum over number of neighbour pairs, next to neighbour pairs, etc.



- Mesoscale $l_k >> l_p$, $\epsilon = \left(\frac{l_p}{l_k}\right)^2 \in (0,1]$,
  $f(n,\epsilon) = (1-\epsilon)^n - 2\epsilon n(1-\epsilon)^{n-1} + \frac{1}{2}\epsilon^2 n(n-1)(1-\epsilon)^{n-2}$

Causality in Dynamics

- Continuum inspired dynamics: $Z = \sum_{C \in \Omega} \exp(iS[c]/\hbar)$
- Dynamics: "block universe"
- Causality is *kinematic* and defined intrinsically by the choice of Ω.
- Entropy versus Action

Carlip and Loomis (2017)

- Use analytic continuation to study Z via MCMC *without losing causality!*

Surya 2011
Glaser and Surya 2014, O'Connor, Glaser and Surya, 2017, Glaser, 2018
Cunningham and Surya, 2019

Dynamics and $\prec$: Sequential Growth Models

Grow the causal sets element by element, while satisfying causality.

Cardinality $\sim$ "Unimodular" time

Dynamics and $\prec$: Sequential Growth Models

- Classical Sequential Growth Dynamics – D.P. Rideout, R.D. Sorkin, Phys. Rev D (2000)

The most natural initial condition:

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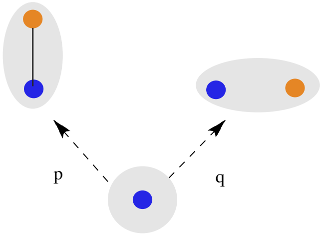
New element is to the future or unrelated to existing element



$p + q = 1$

It can never be to its past

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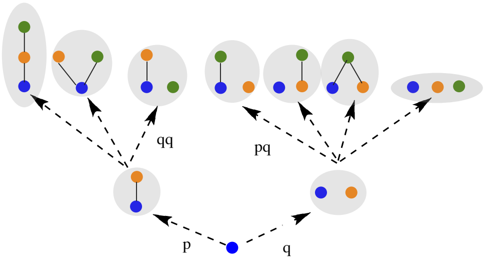

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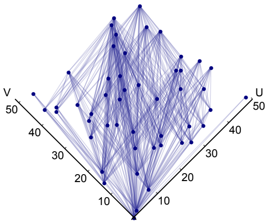
Until..

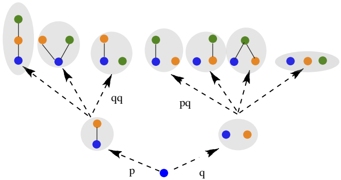


Evolution is Causal/Ordered

Dynamics and $\prec$: Sequential Growth Models

- Classical Sequential Growth Dynamics — D.P. Rideout, R.D. Sorkin, Phys. Rev D (2000)

  - Example: Transitive percolation
    - p: probability of adding in a link
    - $q = 1 - p$: probability for being unrelated



Dynamics and $\prec$: Sequential Growth Models

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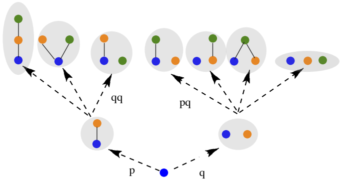
  - Example: Transitive percolation

  - Principles:
    - General Covariance or Label Independence,
    - Bell-causality condition – Spectator independence



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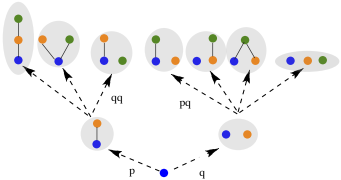
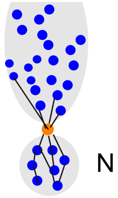


Causality/Order is Implemented Dynamically

Covariant Observables

- Framework: Measure theory $(\Omega, \mathfrak{A}, \mu)$ Classical μ "extends" to σ-algebra
- Ω : Sample space of countable past finite labelled causal sets
- Labels give a "fake" causality even though dynamics is covariant
- To restore coordinate invariance, ask covariant questions (possible because of extension)
  - Example, The Post Event



Quantum Sequential Growth

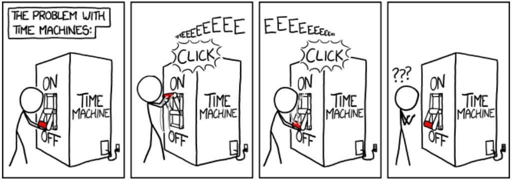
- Framework: Quantum Measure theory $(\Omega, \mathfrak{A}, \mu)$
- Decoherence functional $D(c, c')$: Strongly Positive, Hermitian and Biadditive
  - , $\mu(c) = D(c, c)$
  - Quantum Sum Rule: $\mu(A \cup B) \neq \mu(A) + \mu(B)$
  - $\mu(A \cup B \cup C) = \mu(A \cup B) + \mu(B \cup C) + \mu(A \cup C) - \mu(A) - \mu(B) - \mu(C)$.
  - μ is a **vector measure**: $\mu : \mathfrak{A} \to \mathcal{H}$, Histories Hilbert space
  - Does μ extend to the σ-algebra?
- $D(c, c') = A^*(c)A(c')$ class of Dynamics

Dowker, Johnston, Surya (2010), Surya and Zalel

- Implementing Spectator independence or Bell causality condition: open question

Afterword

- Causality is not exclusive to causal set theory :
  - Causal Dynamical Triangulation
  - Hamiltonian approaches like LQG
- Violations of causality and fuzzing of lightcones:
  - What are the covariant observables?
  - Quantum interpretation of closed systems?



Cueball activates a time machine to go back into the past. The time machine rewinds time, but in the process rewinds the event where the time machine itself was turned on, turning the time machine off in the process. He is now a few seconds in the past, prior to having activated the time machine, but he is baffled that he does not seem to have accomplished anything and turned off the time machine unintentionally.

https://www.explainxkcd.com/wiki/index.php/1203:_Time_Machines