Remote Time Manipulation

David Trillo (Joint work with Ben Dive and Miguel Navascués)
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Time Machine Factory, September 2019

Motivation

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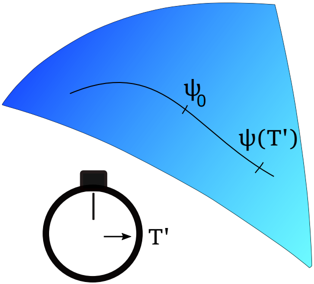

Figure: Time evolution of a quantum system

Motivation

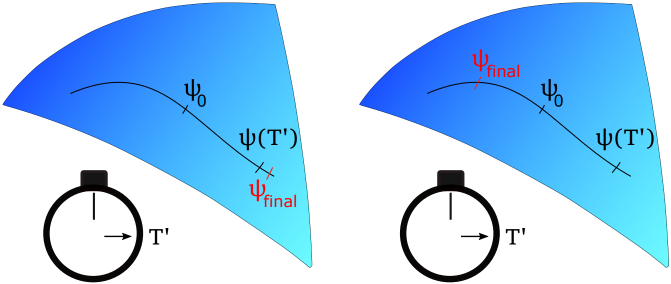

Figure: Fast forward and rewind of a quantum system.

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• With universal protocols.

We allow for probabilistic protocols, as long as they are also heralded (that is, we must know if the protocol succeeds).

Context

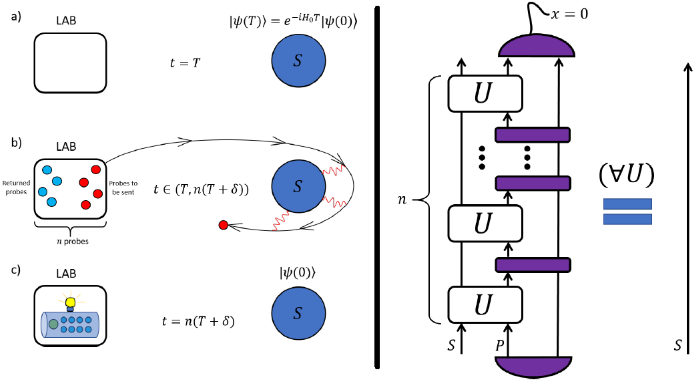

Figure: One way to influence an unknown system. [1]

[1] Miguel Navascues, Phys. Rev. X 8, 031008 (2018)

Matrix Polynomials

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If, after the interaction, we post-select on the probes being in the state $|0...0\rangle_P$, the final state of the system will be

$$\langle 0...0|_P \, W(V \otimes 1) \, |\phi\rangle_S \, |\psi\rangle_P$$

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We get

$$|\phi_{final}\rangle_S = \sum_{\vec{k}} c_{k_1 \cdots k_n} U_{k_1} \cdots U_{k_n} |\phi\rangle_S \, .$$

Central polynomials

Definition

A polynomial $f(x_1, \cdots, x_n) \in K\langle X\rangle$ is a *central polynomial* for a ring R if

1. for any $r_1, \cdots, r_n \in R$, $f(r_1, ..., r_n)$ lies in the center of R.
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Theorem (Formanek, Razmyslov)

$M_n(K)$ has a central polynomial.

Remark

Formanek's polynomial is of the form $F(x, y_1, \cdots, y_n)$, homogeneous of degree $n^2 - n$ in x and linear in y_i.

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Therefore,

$$[A, B]^2 \propto 1.$$

Our setup

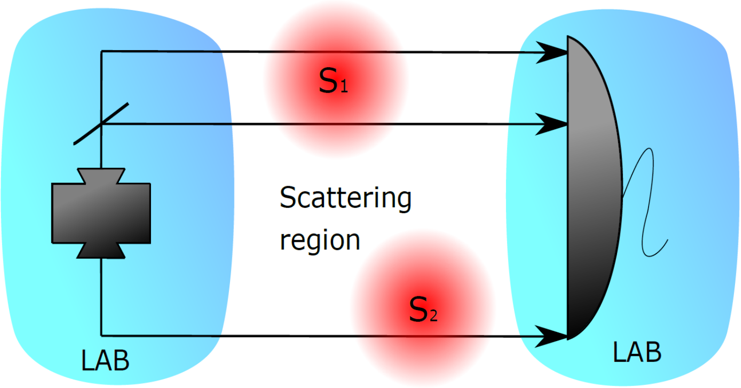

Figure: A more general interaction between systems and probes, with the targeting assumption.

Tensor polynomials

These protocols correspond to polynomials of the form

$$f(V, U_1, \cdots, U_n) = \sum_k c_k p_k(V, U_1, \cdots, U_n) \otimes \cdots \otimes q_k(V, U_1, \cdots, U_n).$$

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Theorem

There exist polynomials in $M_n \otimes \cdots \otimes M_n$ which are proportional to P_S, the projector onto the symmetric subspace; to P_A, the projector onto the antisymmetric subspace and to permutations of the tensor factors (i.e., SWAPs).

Main result

Theorem

Let P be a protocol on n copies of a system of dimension d with the targeting assumption. If at the end of a heralded success, system i is in state $\psi(T_i)$ and the protocol took time T', then it must be that

$$\sum_{i:T_i<0} (d-1)|T_i| + \sum_{j:T_j>0} T_j \leq nT'.$$

Moreover, this inequality is optimal.

Take-home messages

1. Control of a system can be used to get some heralded control of another system
2. Evolution time behaves a bit like energy: it cannot be created or destroyed, but it can be transferred between systems or wasted.
3. Evolution time can be inverted at a cost depending on dimension.