Gravitational lensing 100 years after the Eddington expedition

Volker Perlick

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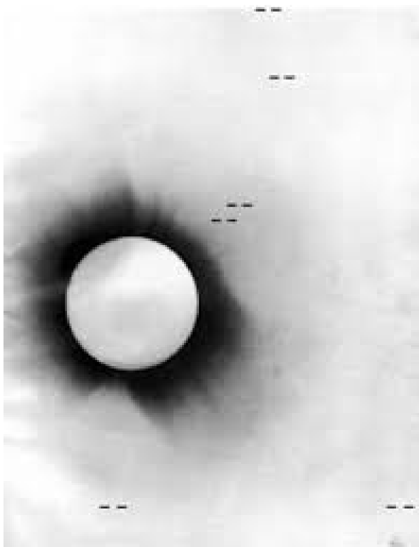
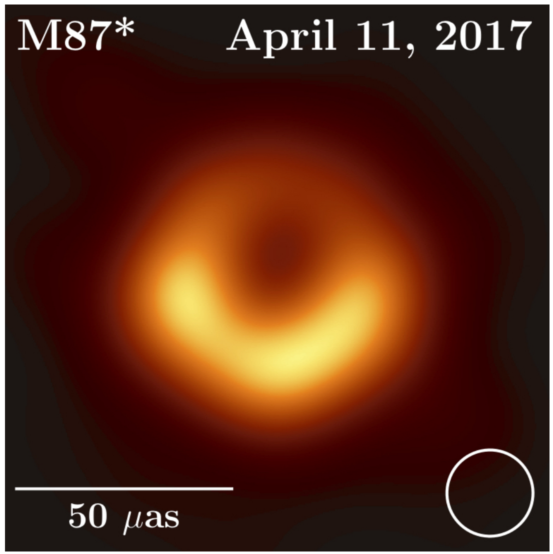




The Time Machine Factory, Torino, Italy, 23 September 2019

1. **Historic overview on lensing**
   - **The 1919 Eddington expedition**
   - **Phenomenology of lensing**
2. **Black-hole lensing**
   - **Lensing by Schwarzschild black holes**
   - **Lensing by Kerr black holes**
   - **The shadow of the object at the centre of M87**

Historic overview on lensing

Newtonian light deflection:

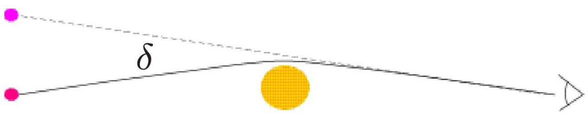
Henry Cavendish 1786, Johann v. Soldner 1801

Henry Cavendish

(1731 – 1810)

Johann v. Soldner

(1776 –1833)



$$\delta = \frac{2\,G\,M}{c^2\,R}, \quad \delta_\odot = 0.87''$$

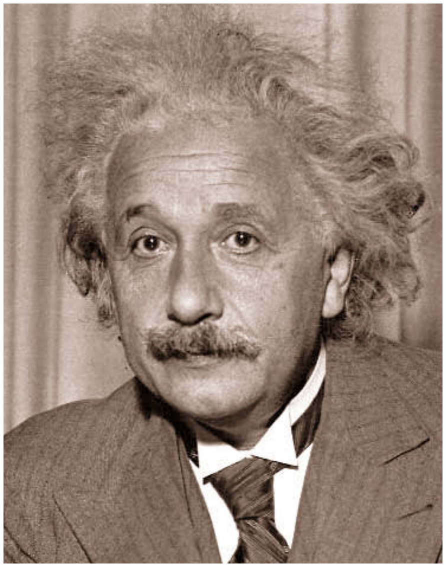
Light deflection according to (linearised) general relativity:

Albert Einstein 1915

$$\delta = \frac{4\,G\,M}{c^2\,R}, \quad \delta_\odot = 1.73''$$

Verification:

Arthur Eddington 1919



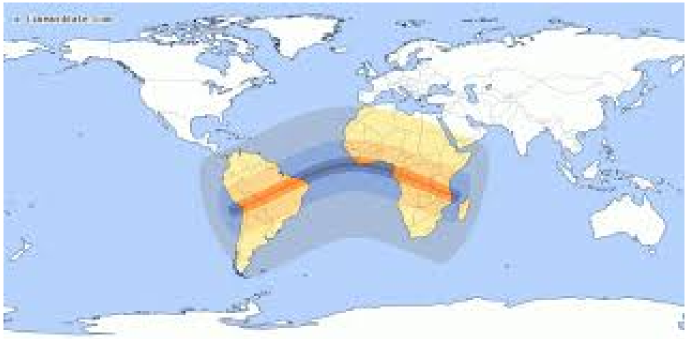
Albert Einstein

(1879-1955)

Arthur S. Eddington

(1882 – 1944)

The 1919 expedition:



Solar eclipse 29 May 1919

Observers at Principe: A. Eddington and E. Cottingham

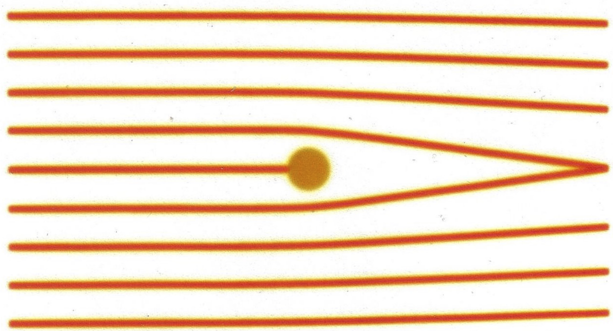
Evaluation: A. Eddington, $\delta = 1.61'' \pm 0.40''$,

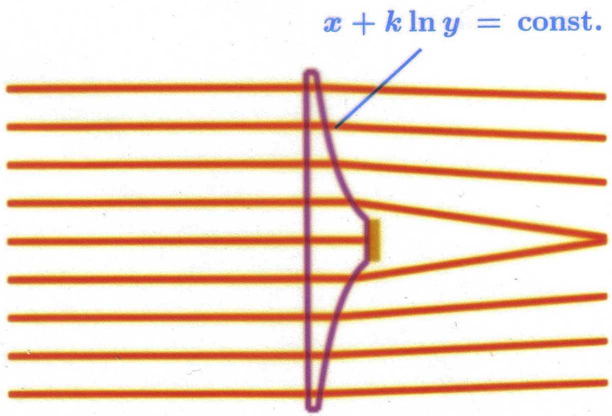
Observers at Sobral: A. Crommelin and C. Davidson

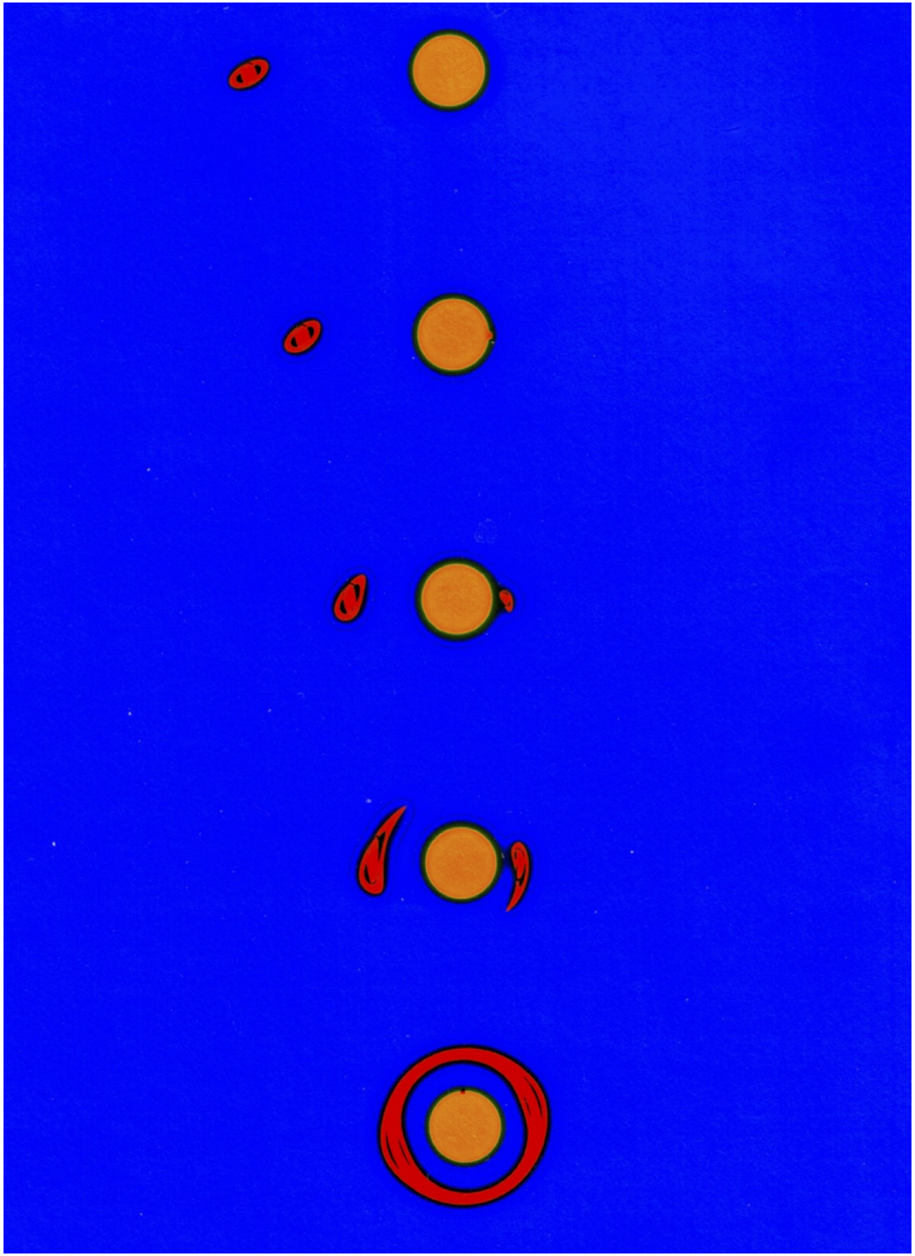
Evaluation: F. Dyson, $\delta = 1.98'' \pm 0.16''$

D. Lebach et al. (1995): $\left|\frac{\delta - \delta_{\text{Einstein}}}{\delta_{\text{Einstein}}}\right| \leq 0.02\,\%$

Simulation with a plastic lens:

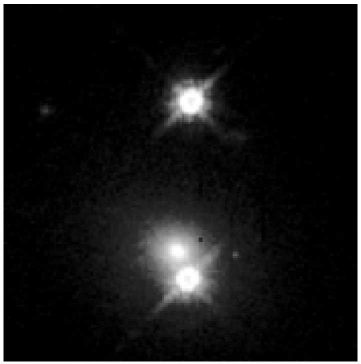


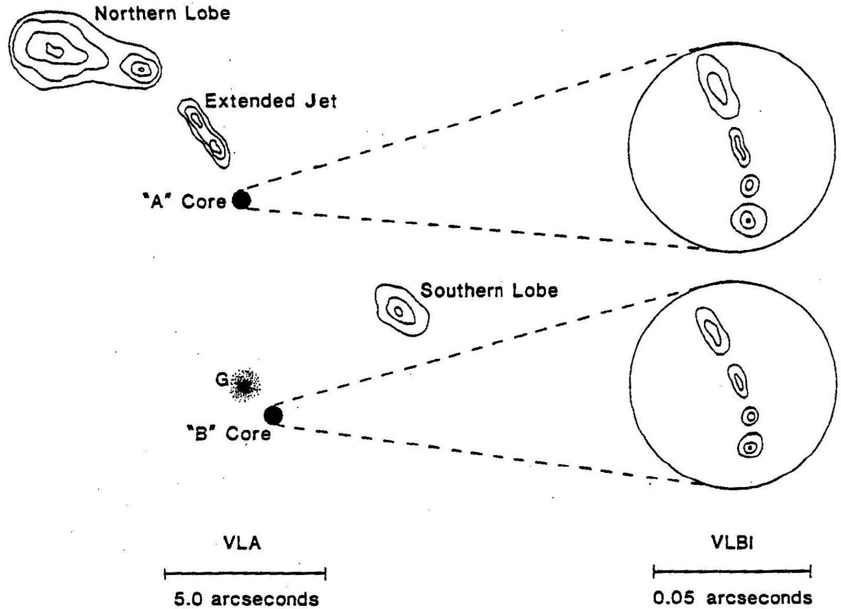




First example of multiple imaging: double quasar QSO 0957+561

D. Walsh, R. Carswell and R. Weymann 1979



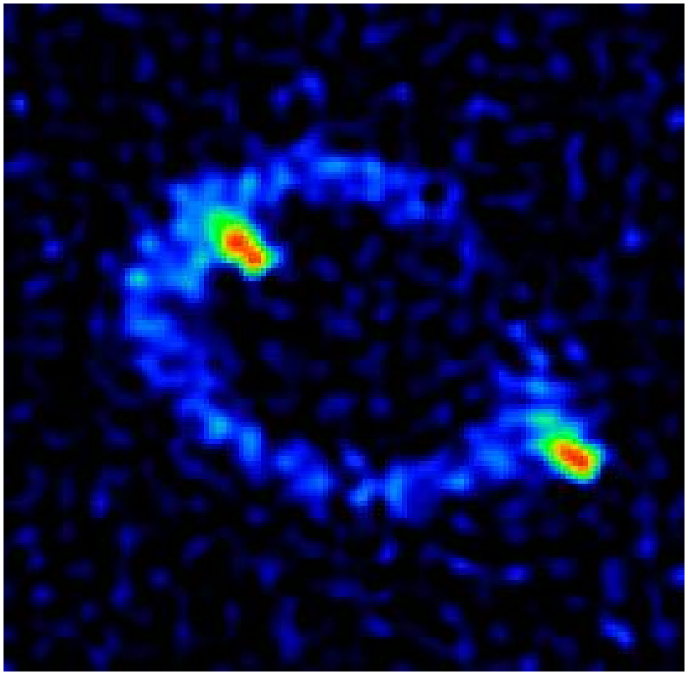


One of the first examples of a giant luminous arc: Abell 370

R. Lynds and V. Petrosian 1986, G. Soucail et al. 1986

First example of an Einstein ring: MG1131+0456

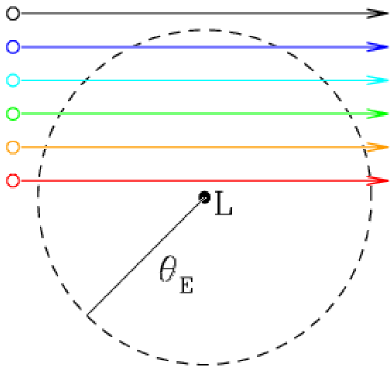
J. Hewitt et al. 1988

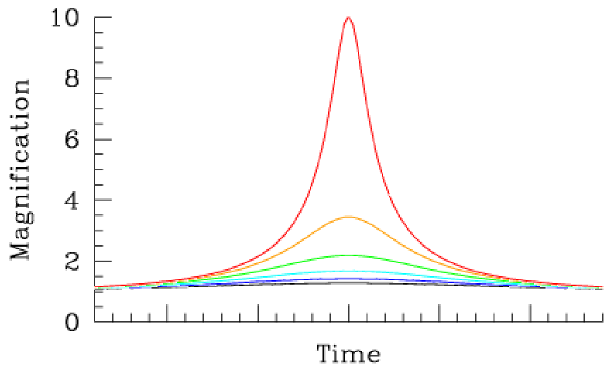


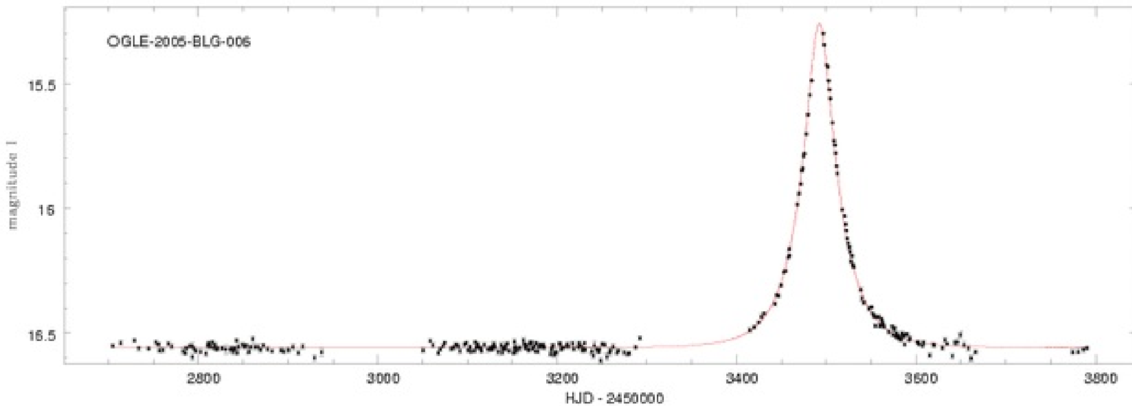
Detecting dark matter I: Weak lensing

The Bullet Cluster 1E 0657-558

Detecting dark matter II: Microlensing

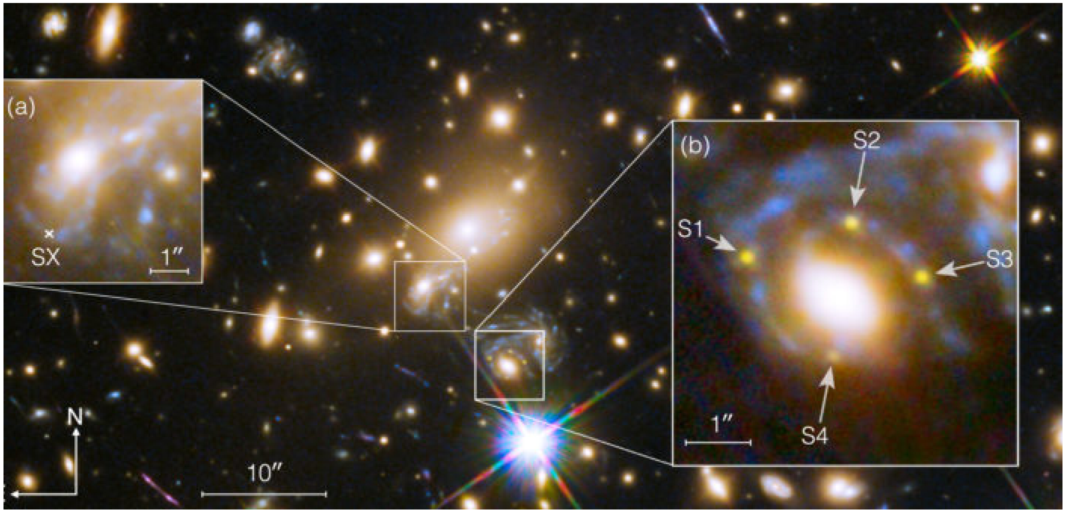






Lensing of the Supernova Refsdal

Four images discovered in 2014, fifth image correctly predicted for 2015

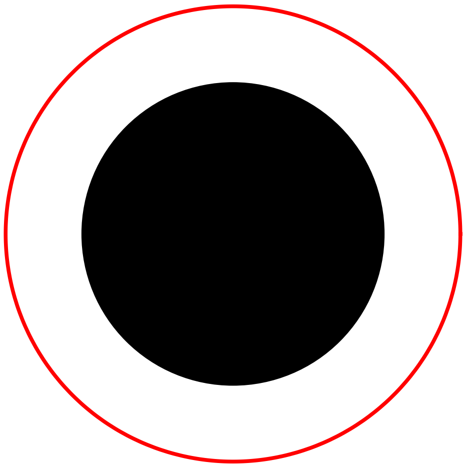


2. Black-hole lensing

Schwarzschild(-Droste) metric:

$$g_{\mu\nu}dx^\mu dx^\nu = -\left(1 - \frac{2m}{r}\right)c^2 dt^2 + \frac{dr^2}{\left(1 - \frac{2m}{r}\right)} + r^2\left(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2\right)$$

where $m = \dfrac{GM}{c^2}$



Horizon:

$r_S = 2m =$ **Schwarzschild radius**

Light sphere (photon sphere)

$\dfrac{3}{2} r_S = 3m$

2. Black-hole lensing

Schwarzschild(-Droste) metric:

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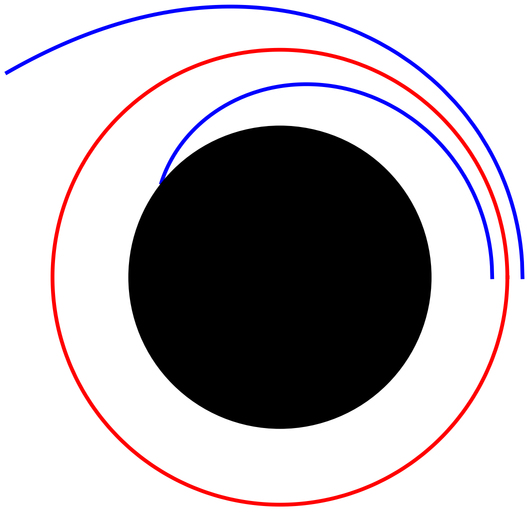

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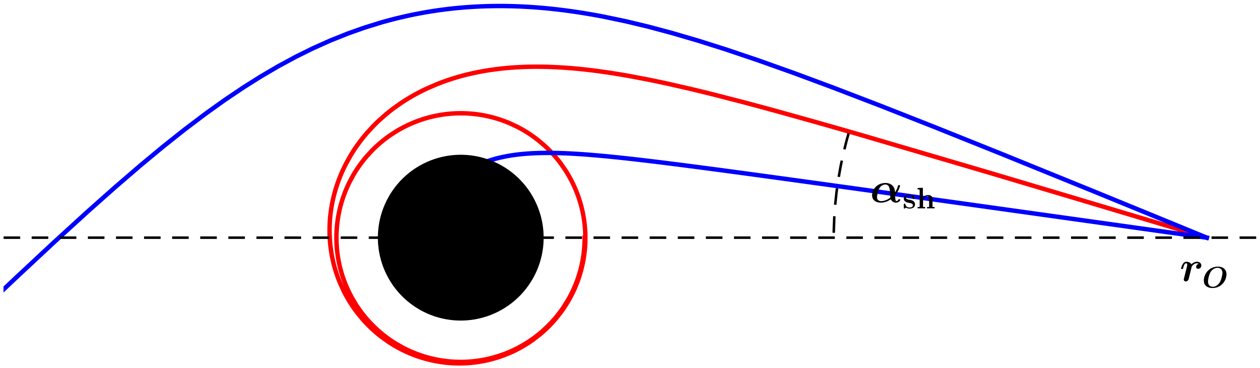
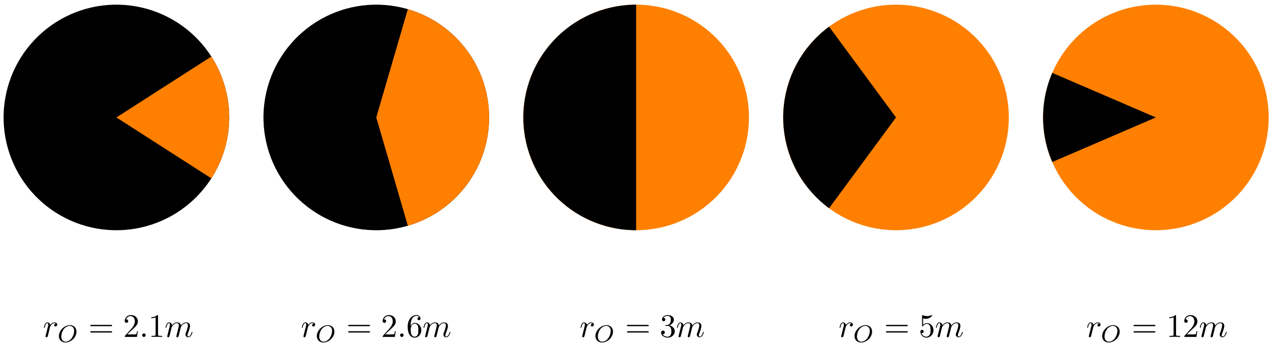


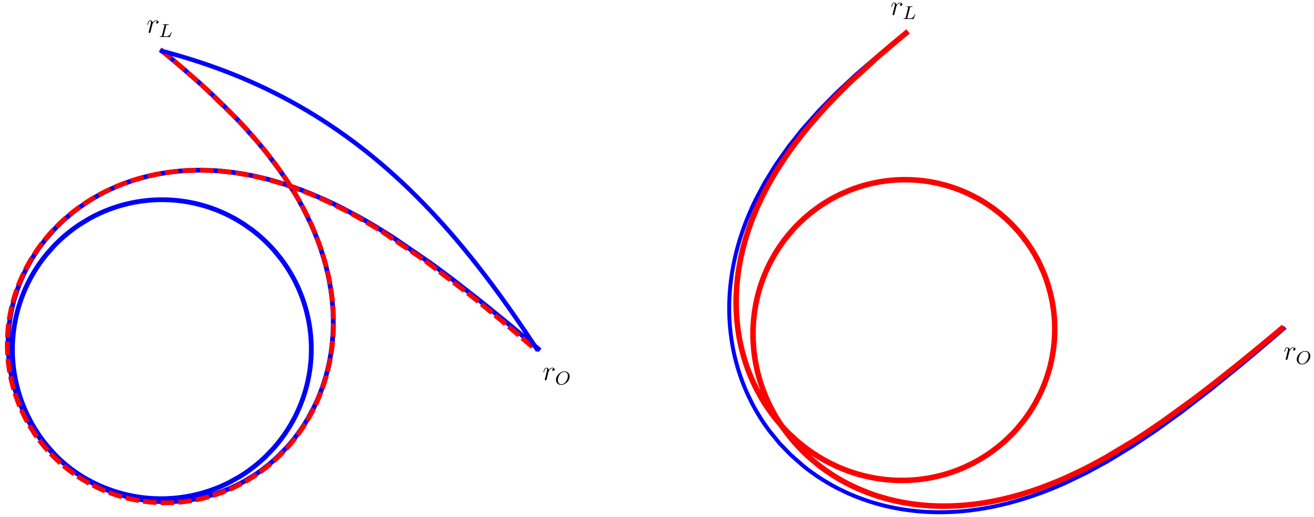
Angular radius $\alpha_{\rm sh}$ of the shadow of a Schwarzschild black hole:

$$\sin^2\alpha_{\rm sh} = \frac{27\, r_S^2\, (r_O - r_S)}{4\, r_O^3} = \frac{27\, m^2}{r_O^2}\Big(1 - \frac{2m}{r_O}\Big)$$

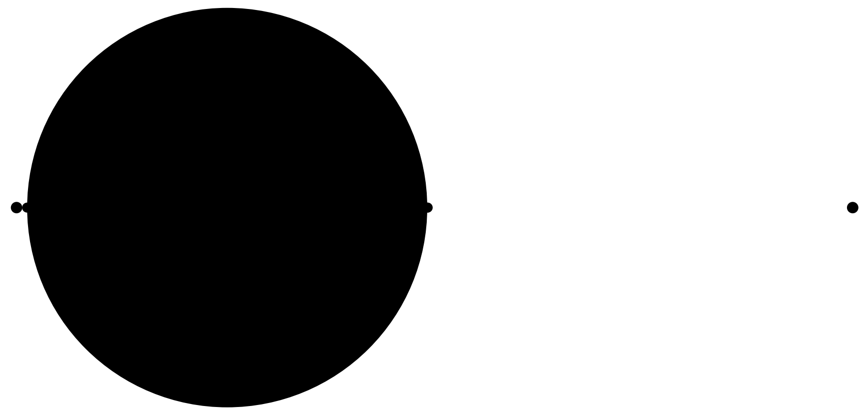
J. L. Synge, Mon. Not. R. Astr. Soc. 131, 463 (1966)

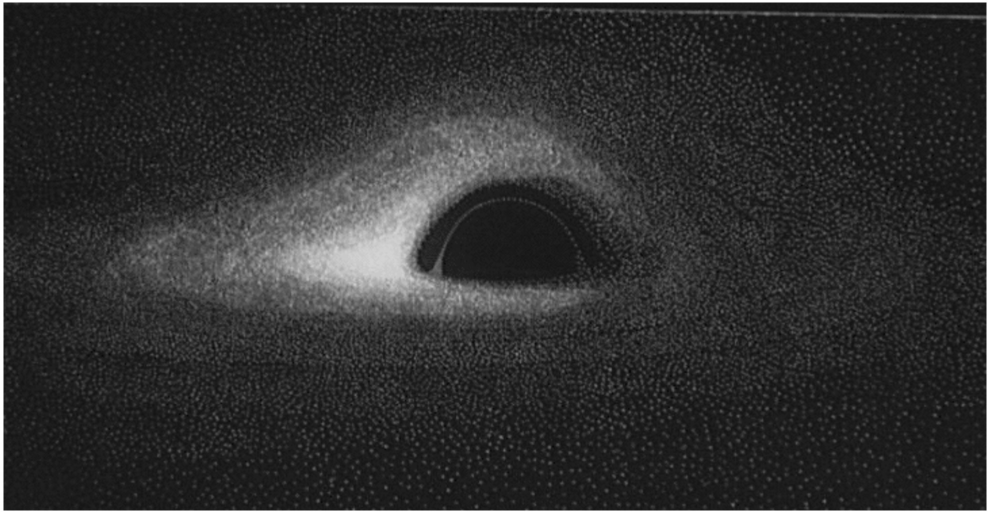


Schwarzschild black hole produces infinitely many images:

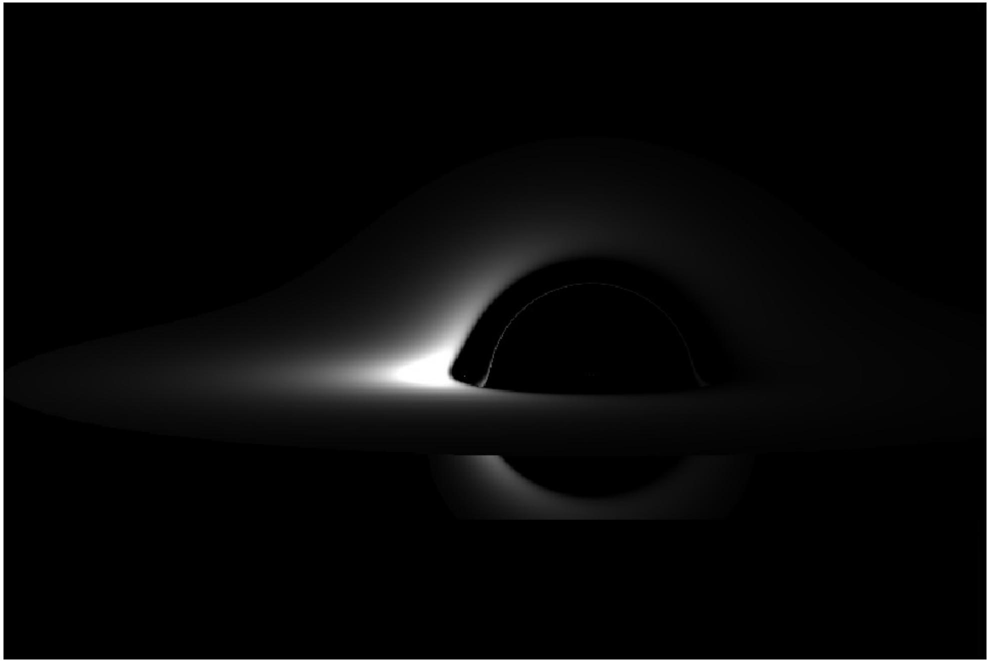


Imaging of a point source by a Schwarzschild black hole

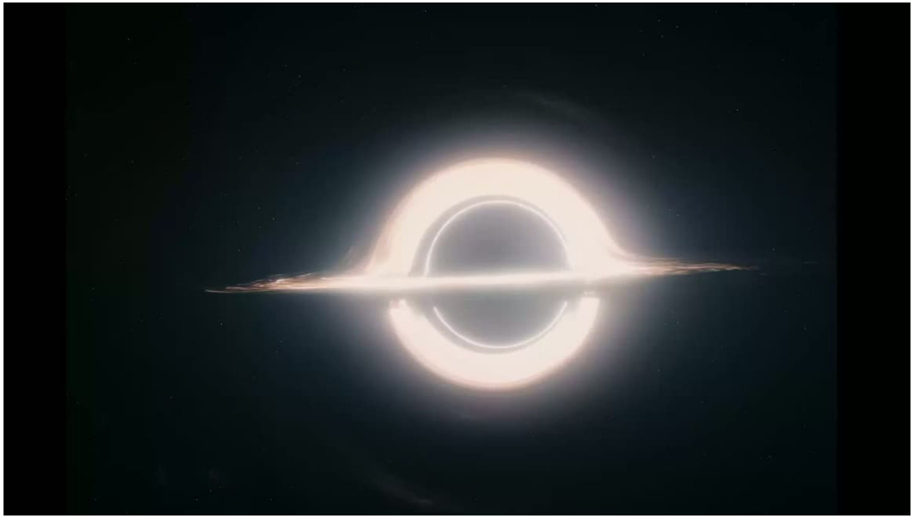




J.-P. Luminet (1979)



T. Müller (2012)



Interstellar (2014)

Shadow size of black-hole candidates

Object at the centre of our galaxy:

Mass $= 4.1 \times 10^6 \, M_\odot$

Distance $= 8.1 \, \mathrm{kpc}$

Angular diameter of the shadow by Synge's formula $\approx 54\mu$**as**

Corresponds to a grapefruit on the moon; shadow not yet observed

Object at the centre of M87:

Mass $= 6.5 \times 10^9 \, M_\odot$

Distance $= 16.4 \, \mathrm{Mpc}$

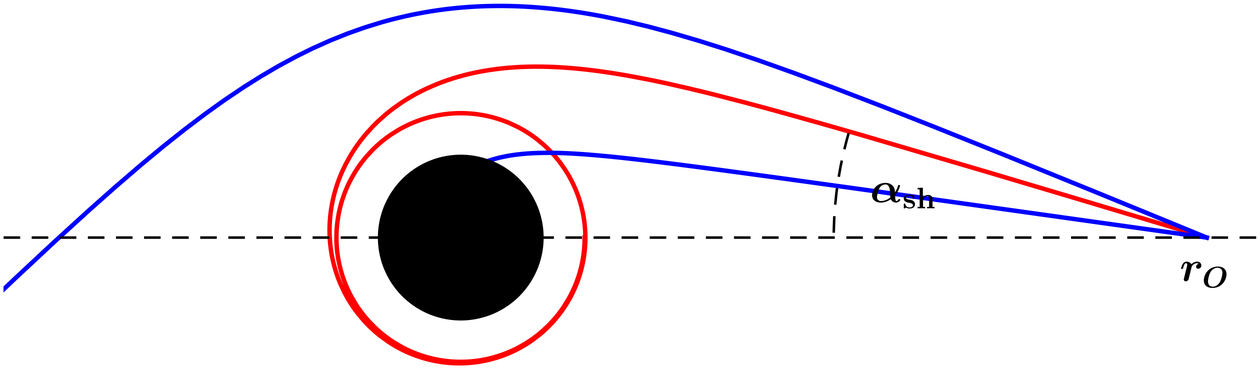
Angular diameter of the shadow by Synge's formula $\approx 42\mu$**as**

Observed by the Event Horizon Telescope Collaboration

Data taken in April 2017, released to the public in April 2019

Black hole impostor 1: Ultracompact star

Dark star with radius between $2m$ and $3m$



Shadow indistinguishable from Schwarzschild black hole

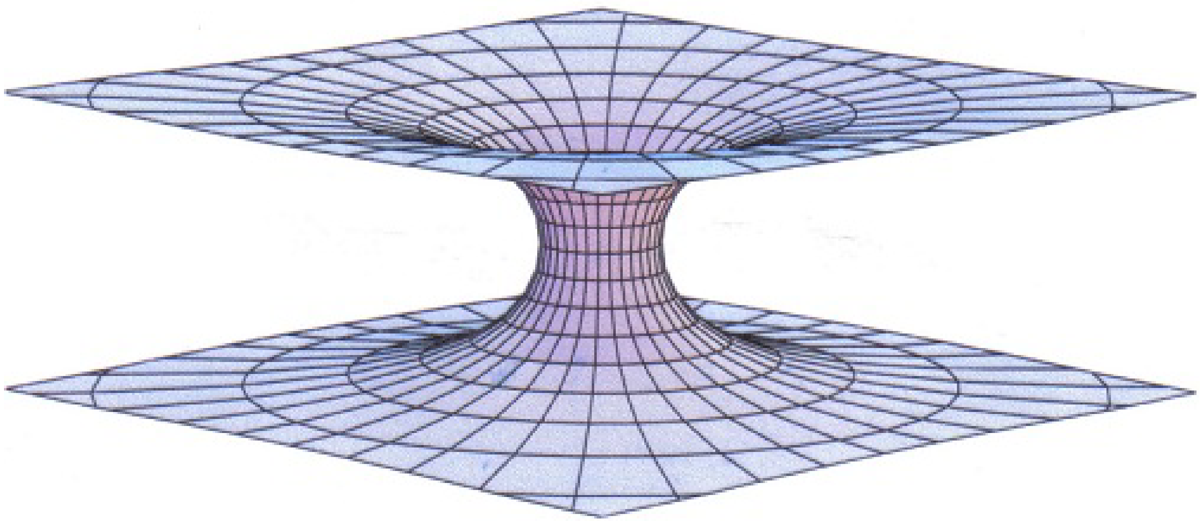
Ultracompact objects are unstable, see

V. Cardoso, L. Crispino, C. Macedo, H. Okawa, P. Pani: Phys. Rev. D 90, 044069 (2014)

Black hole impostor 2: Ellis wormhole

H. Ellis: J. Math. Phys. 14, 104 (1973)

$$g_{\mu\nu}dx^\mu dx^\nu = -c^2dt^2 + dr^2 + (r^2 + a^2)\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)$$



Angular radius α of shadow: $\sin^2\alpha = \dfrac{a^2}{r_O^2 + a^2}$

Kerr metric

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = \varrho(r,\vartheta)^2\left(\frac{dr^2}{\Delta(r)} + d\vartheta^2\right) + \frac{\sin^2\vartheta}{\varrho(r,\vartheta)^2}\Big(a\,dt - (r^2 + a^2)d\varphi\Big)^2$$

$$-\frac{\Delta(r)}{\varrho(r,\vartheta)^2}\Big(dt - a\sin^2\vartheta\,d\varphi\Big)^2$$

$$\varrho(r,\vartheta)^2 = r^2 + a^2\cos^2\vartheta, \qquad \Delta(r) = r^2 - 2mr + a^2\,.$$

$m = \dfrac{GM}{c^2}$ where $M =$ mass, $\quad a = \dfrac{J}{Mc}$ where $J =$ spin

Black hole: $0 \le a^2 \le m^2$

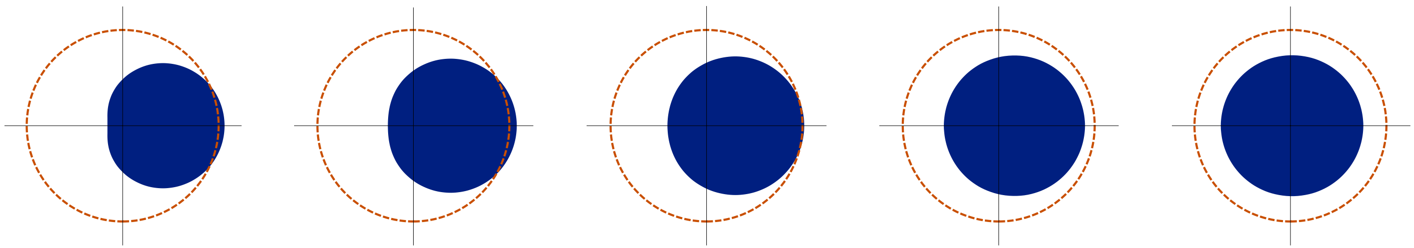
Shadow is no longer circular

Shape of the shadow of a Kerr black hole for observer at infinity:

J. Bardeen in C. DeWitt and B. DeWitt (eds.): "Black holes" Gordon & Breach (1973)

Shape and size of the shadow of Kerr black holes (and other black holes) for observer at coordinates (r_O, ϑ_O) (analytical formulas):

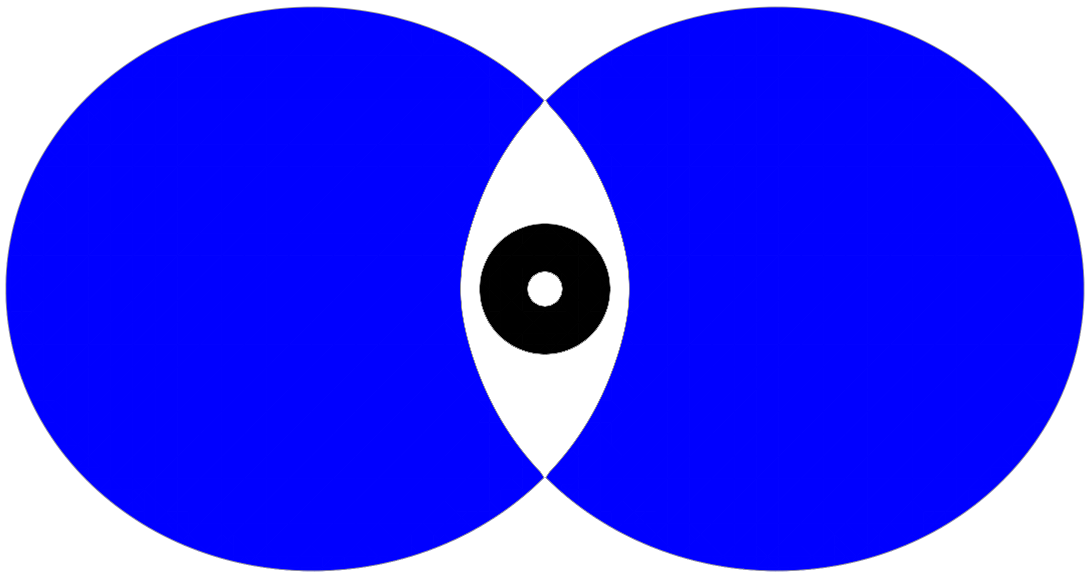
A. Grenzebach, VP, C. Lämmerzahl: Phys. Rev. D 89, 124004 (2014), Int. J. Mod. Phys. D 24, 1542024 (2015)



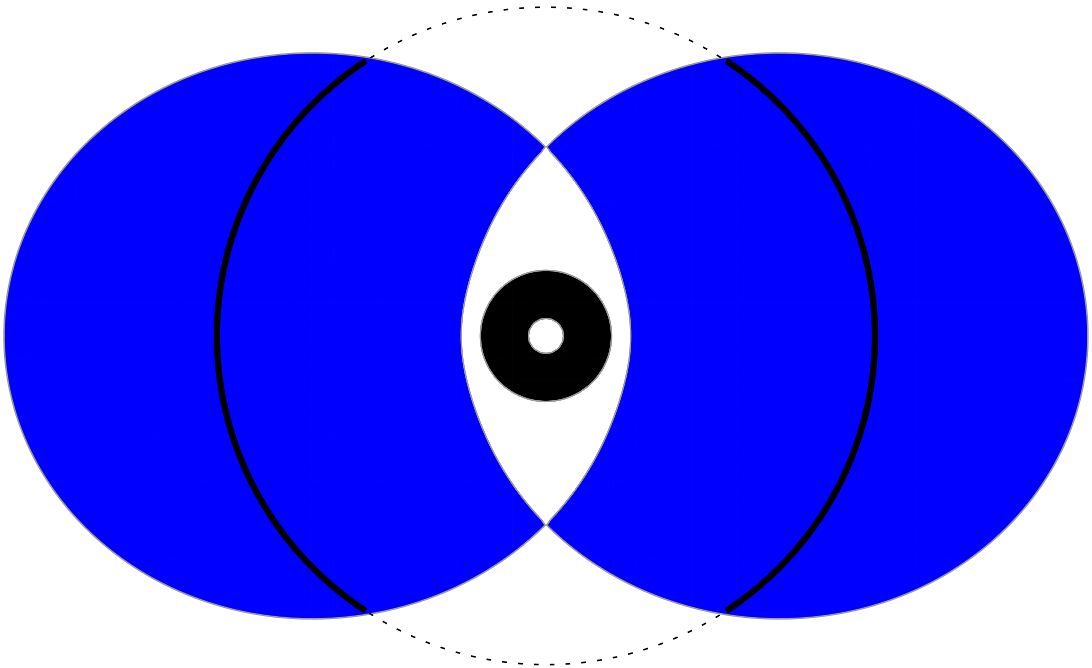
$\vartheta_O = \frac{\pi}{2}$ $\vartheta_O = \frac{3\pi}{8}$ $\vartheta_O = \frac{\pi}{4}$ $\vartheta_O = \frac{\pi}{8}$ $\vartheta_O = 0$

Shadow of Kerr black hole with $a = m$ for observer at $r_O = 5m$

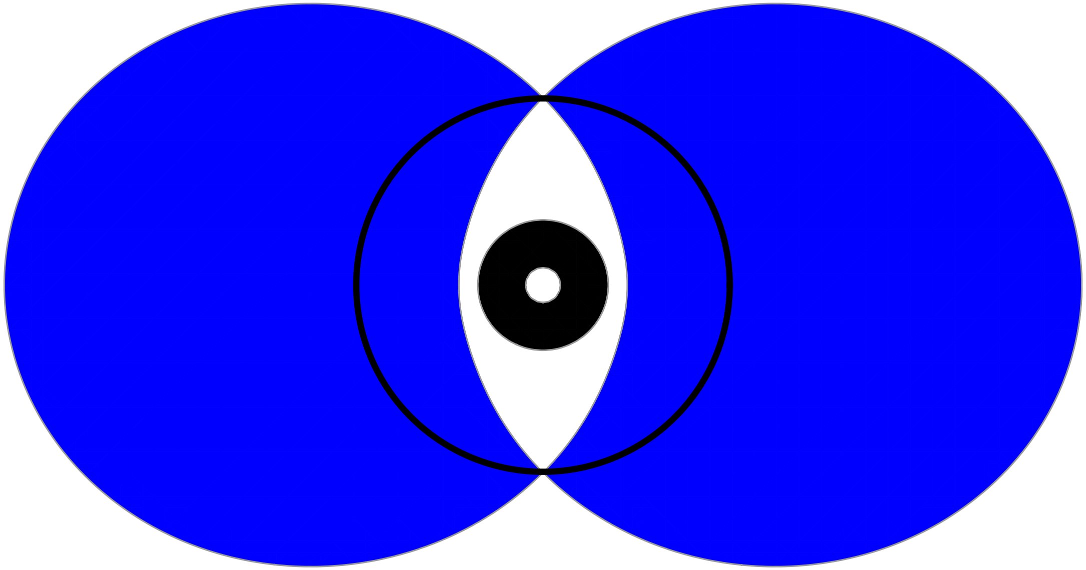
Photon region for Kerr black hole with $a = 0.75\,m$



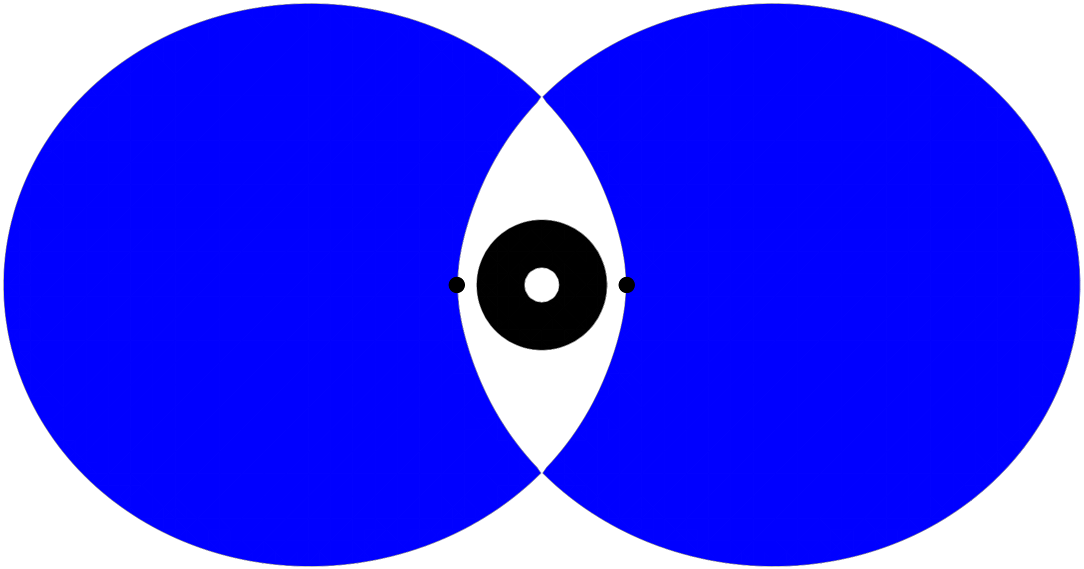
Photon region for Kerr black hole with $a = 0.75\, m$



Photon region for Kerr black hole with $a = 0.75\,m$



Photon region for Kerr black hole with $a = 0.75\, m$



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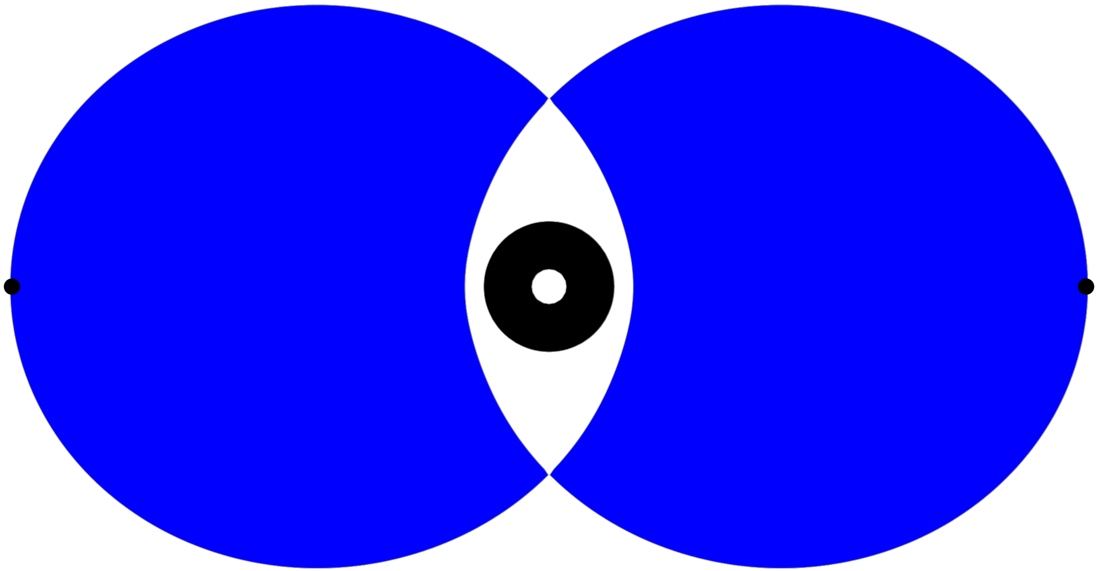


Boundary curve of shadow corresponds to light rays that approach a spherical lightlike geodesic

Analytical formula for the shadow follows from identifying constants of motion of a light ray with the constants of motion of its limit curve

This gives, in particular, a formula for the vertical angular radius α_v of the shadow ($\vartheta_O = \pi/2$):

$$\sin^2 \alpha_v = \frac{27 m^2 r_O^2 \big(a^2 + r_O (r_O - 2m) \big)}{r_O^6 + 6 a^2 r_O^4 + 3 a^2 (4a^2 - 9m^2) r_O^2 + 8 a^6} = \frac{27 m^2}{r_O^2} \Big(1 + O(m/r_O) \Big)$$

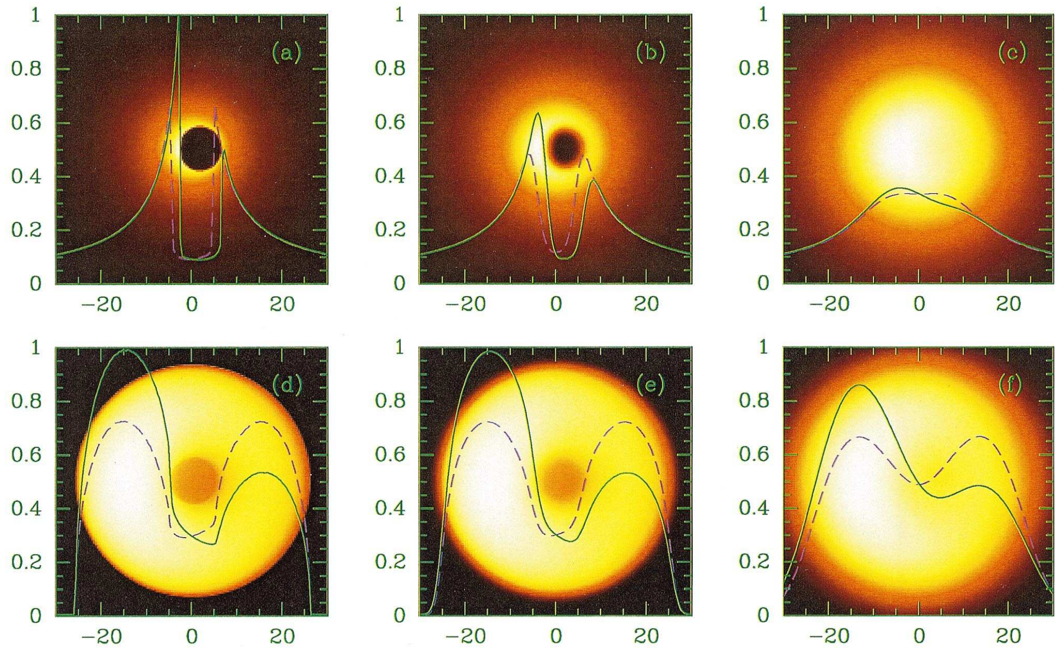
A. Grenzebach, VP, C. Lämmerzahl: Int. J. Mod. Phys. D 24, 1542024 (2015)

Up to terms of order $O(m/r_O)$, Synge's formula is still correct for the vertical diameter of the shadow

Observability of the shadow:

Kerr shadow with emission region and scattering taken into account:

no scattering $\lambda = 0.6$ mm $\lambda = 1.3$ mm

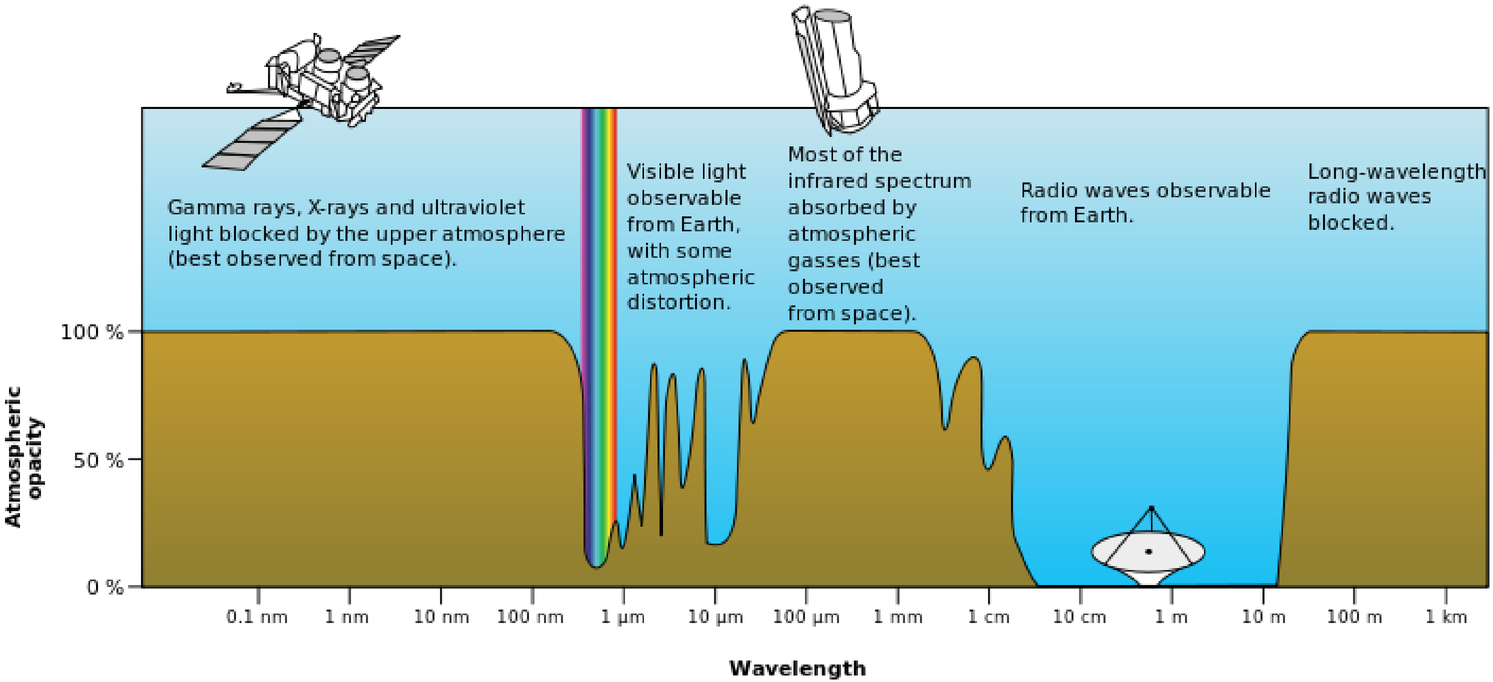


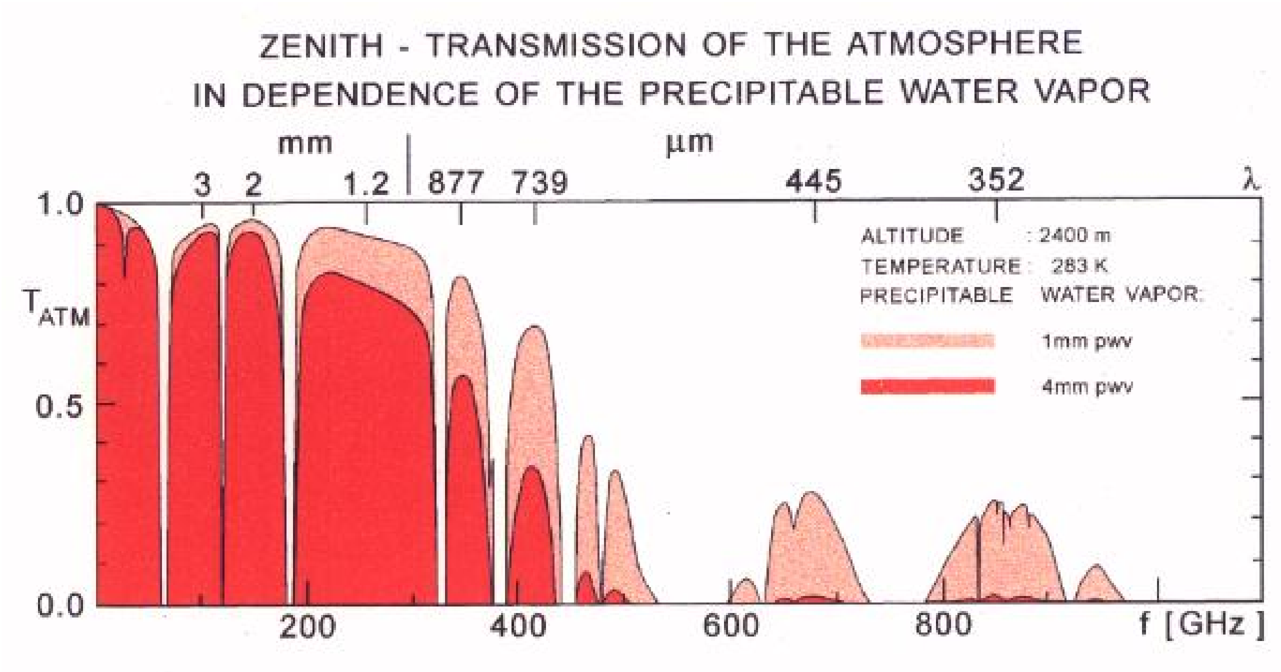
H. Falcke, F. Melia, E. Agol: Astrophys. J. 528, L13 (2000)

Observations should be done at sub-millimeter wavelengths

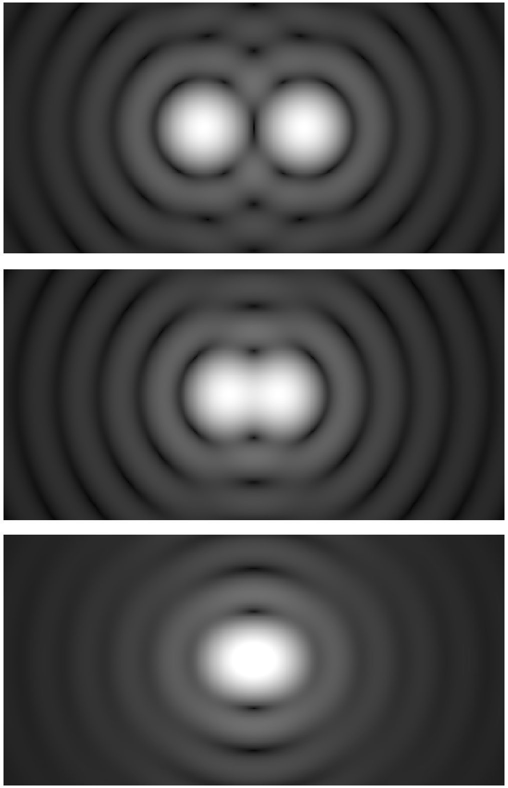
Transmission of the atmosphere

(at sea level under average atmospheric conditions)





Groundbased observations at ≈ 1 mm wavelength are possible with telescopes at high altitude (in dry areas)



Rayleigh criterion

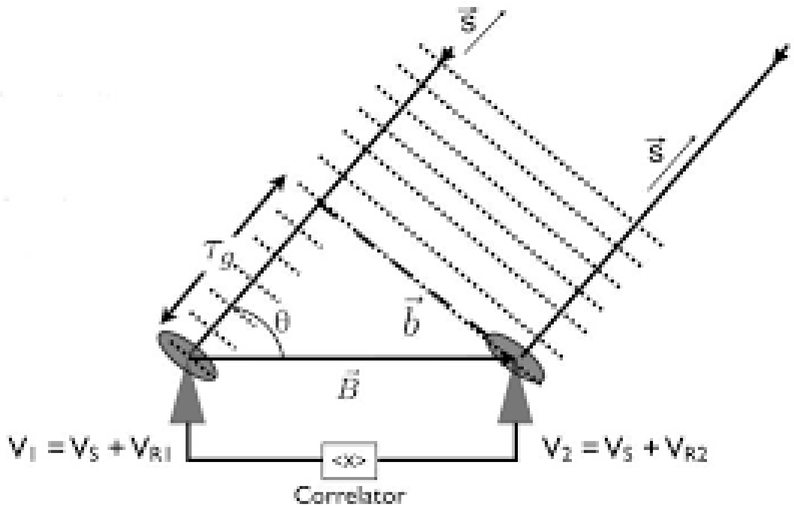
Angle θ that can be resolved by a telescope with circular aperture of diameter D at wavelength λ:

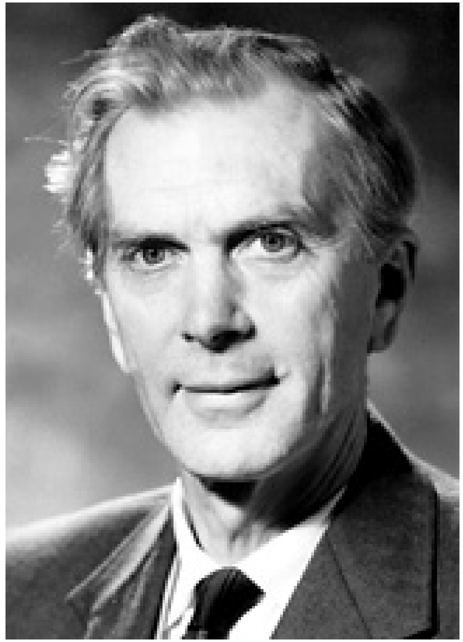
$$\theta = 1.22\,\frac{\lambda}{D}$$

$\lambda \approx 1\,$mm, $\theta \approx 20\mu$as

$D \approx$ diameter of Earth

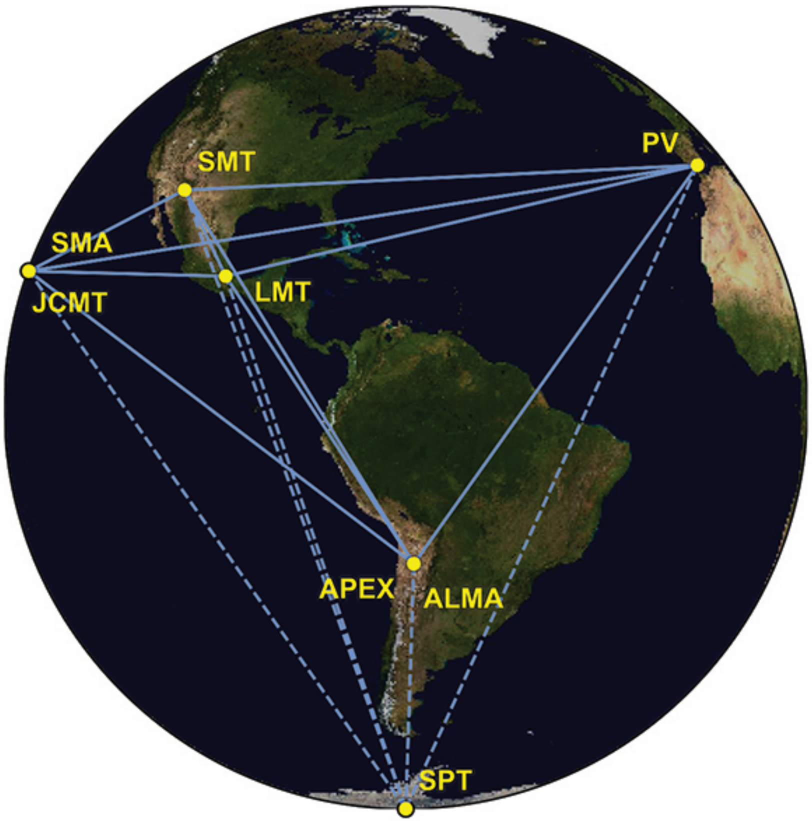
Aperture Synthesis

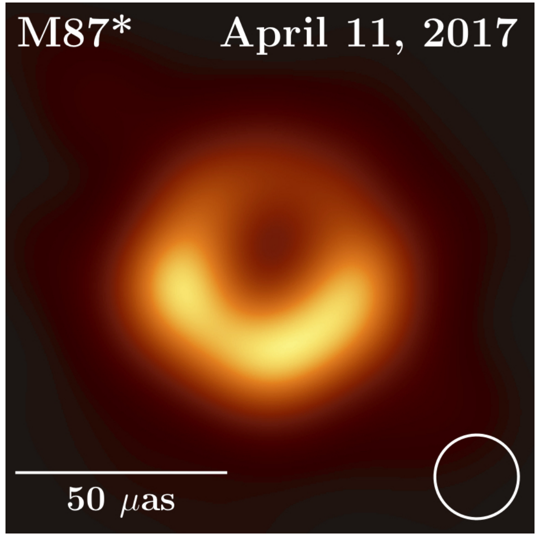




Martin Ryle

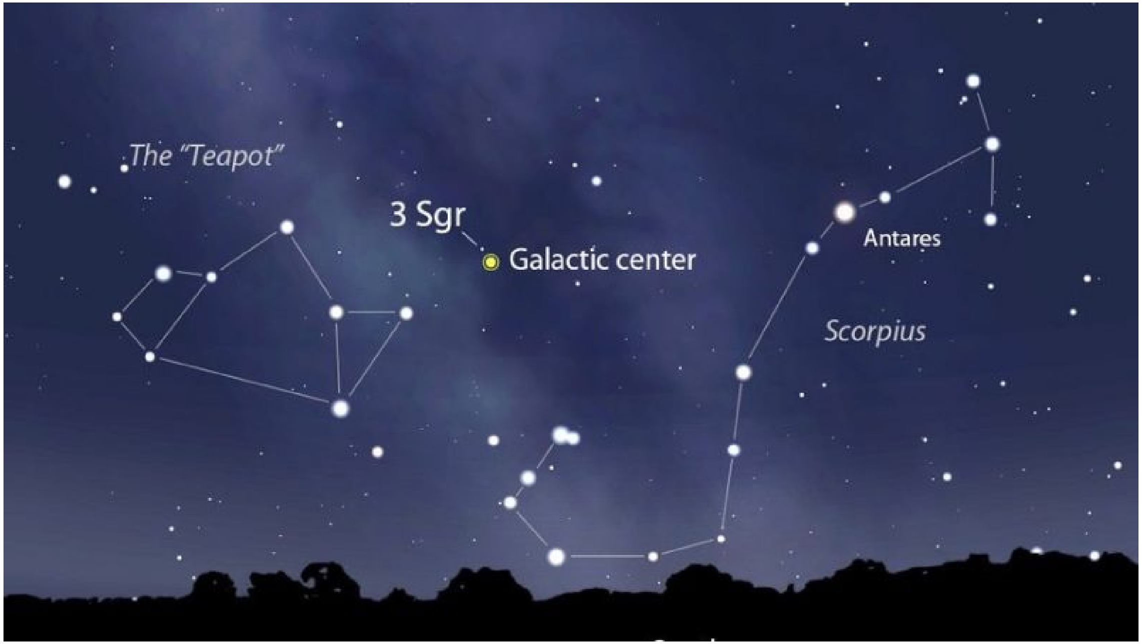
(1918-1984)



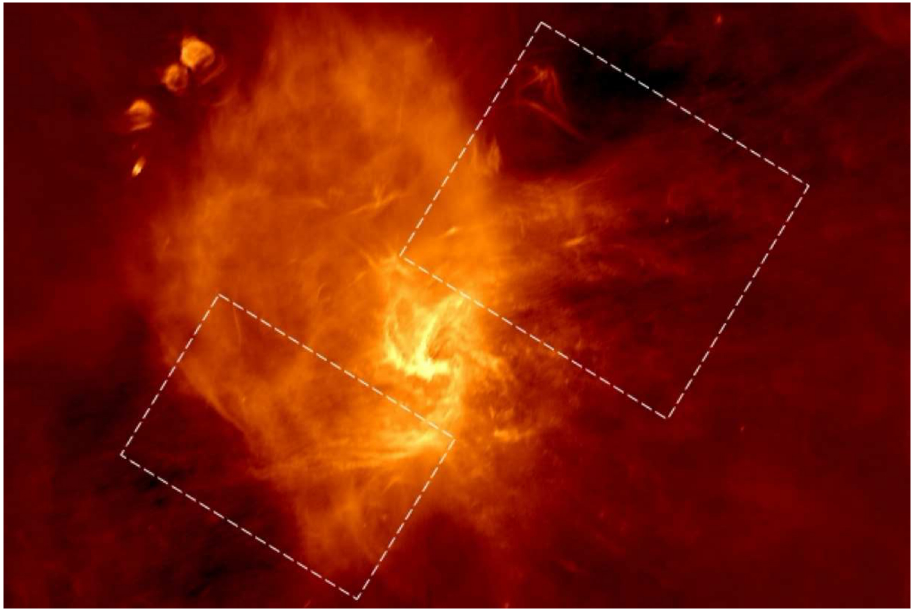


Event Horizon Telescope

Candidate 1: The centre of our Galaxy

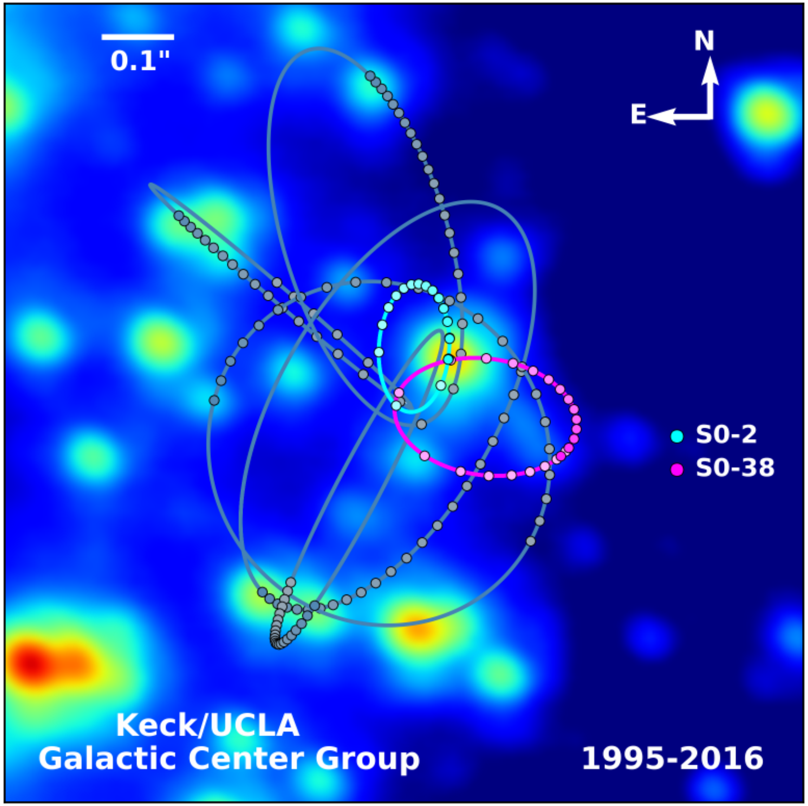


In the optical, the centre of our galaxy is hidden behind dust



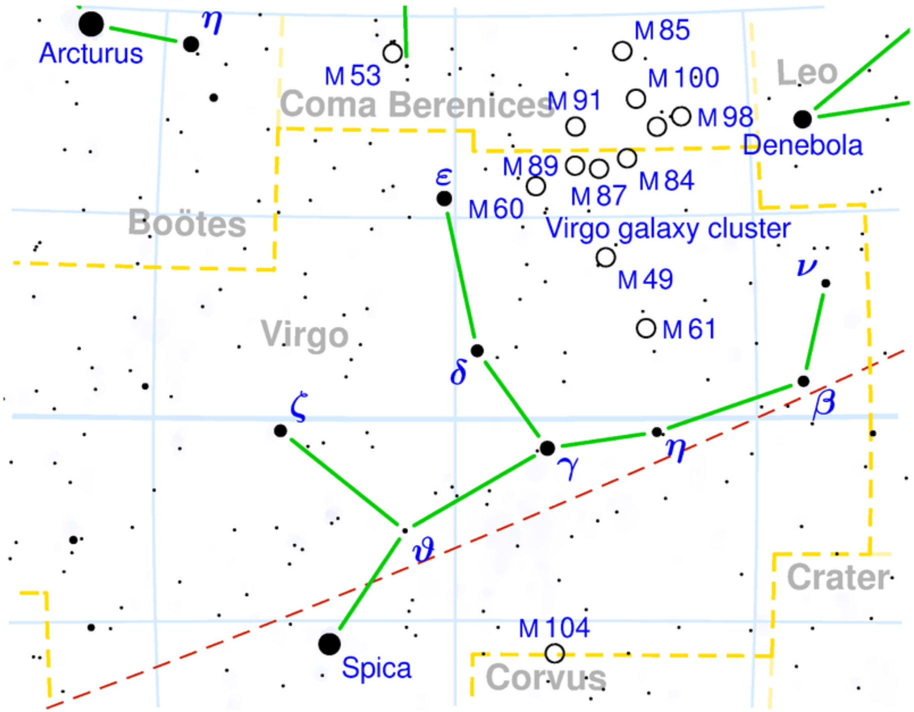
In the radio one sees at the location of the centre of our galaxy a bright radio source, called Sgr A. The brightest spot, called Sgr A*, is located near the centre of the “minispiral” Sgr A West.
Picture taken with the Very Large Array at 5.5 GHz, diameter 13’

J.-H. Zhao, M. Morris and M. Goss, Astrophys. J. 817, 171 (2016)

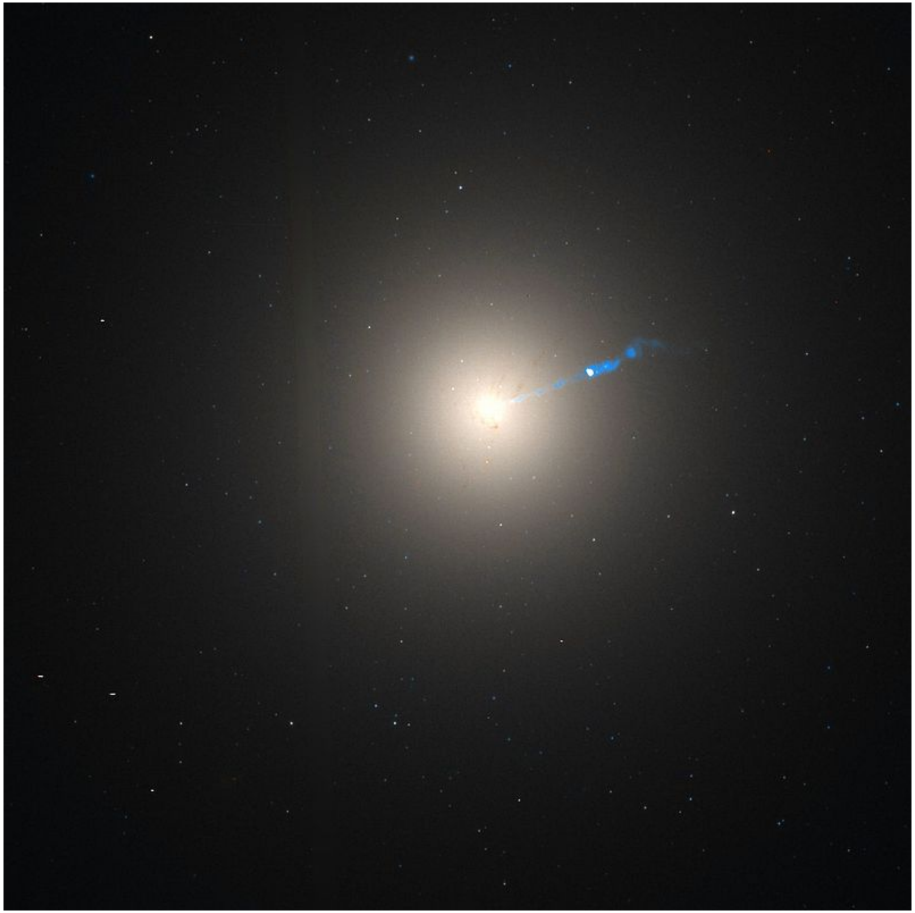


In the near infrared, one sees several stars (S0-2, SO-38 etc.) orbiting the centre of our galaxy. This allows to estimate the size and the mass of the central object
Picture: Andrea Ghez et al. (UCLA)

Candidate 2: The centre of M87

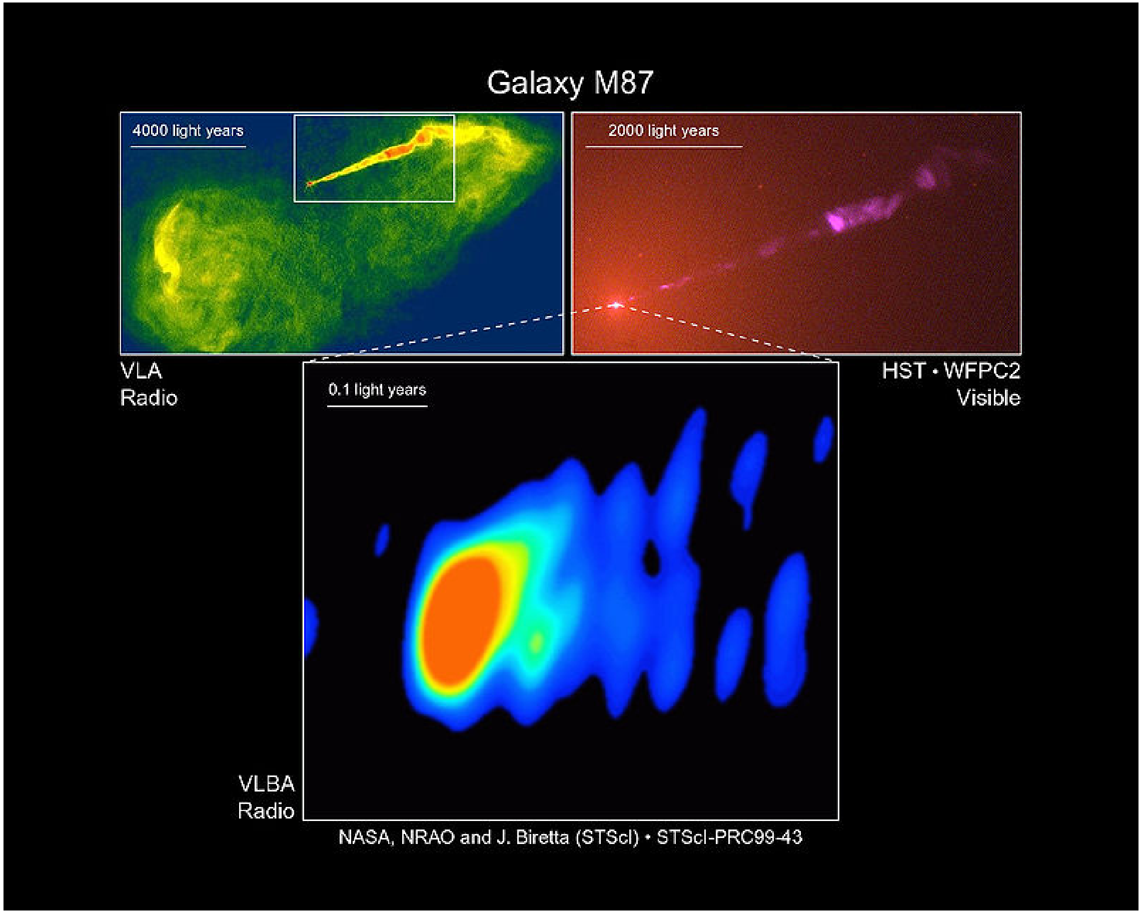


Location of M87 in the sky

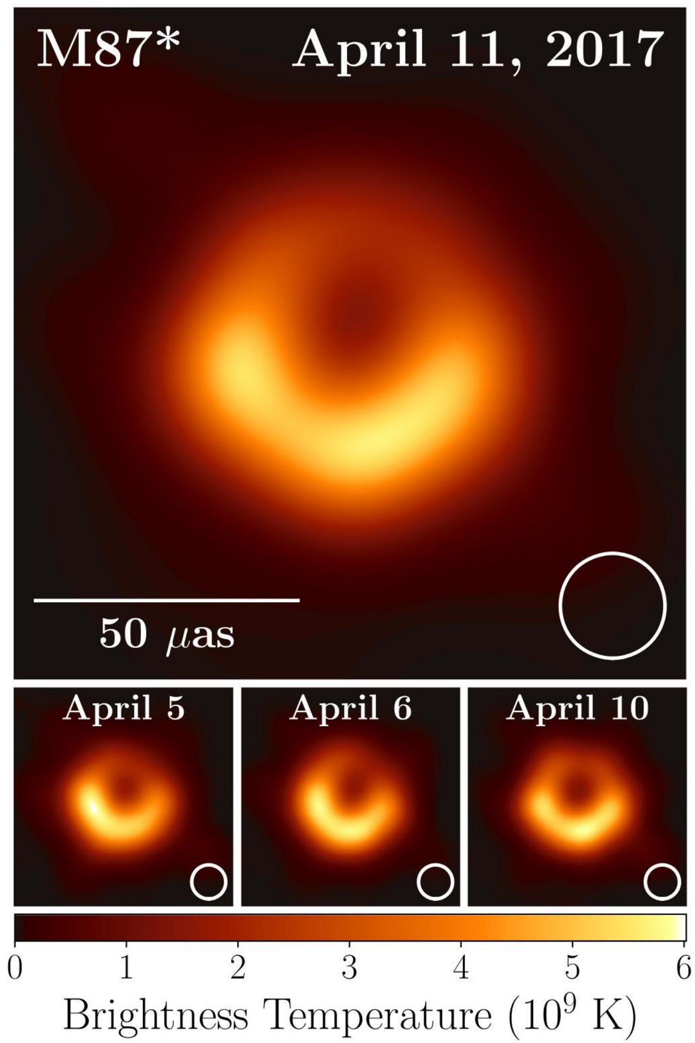


M87 in the optical

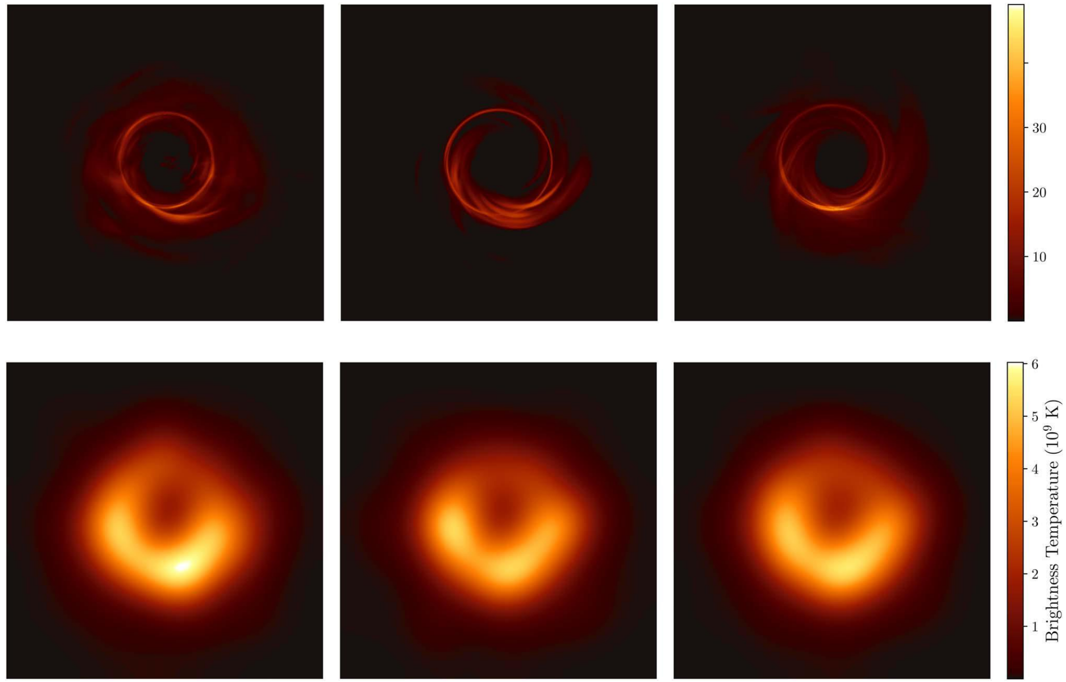
Picture: HST



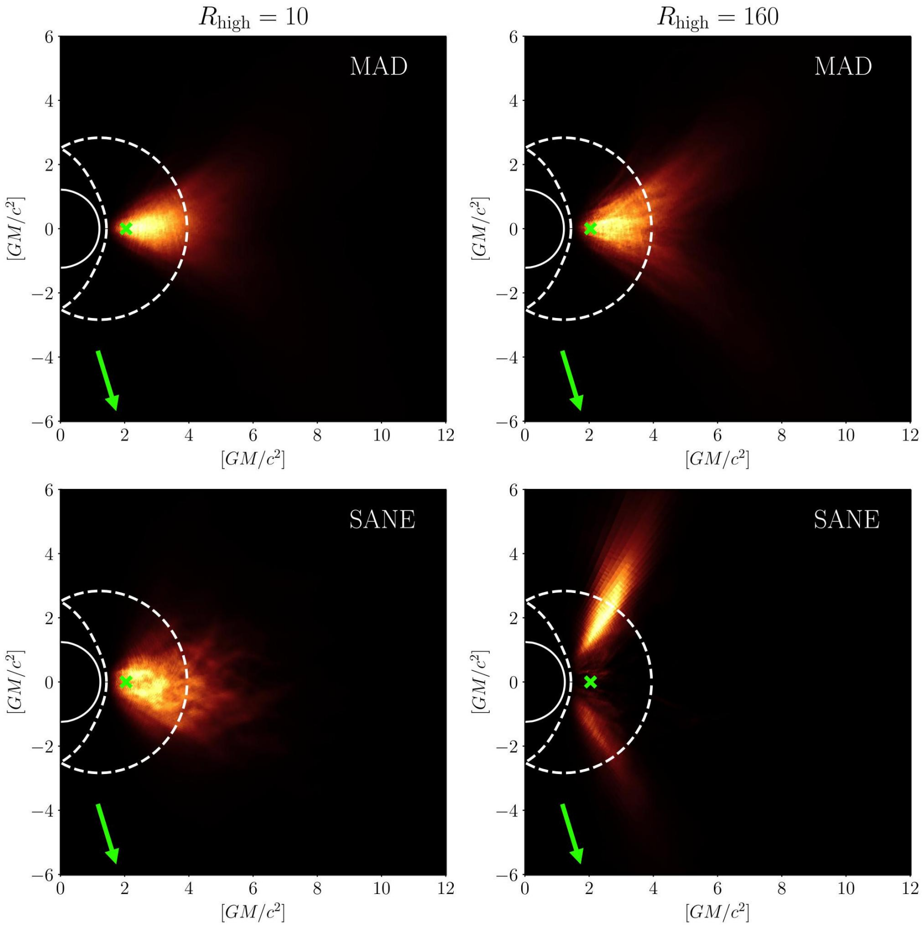
M87 in the radio (Virgo A)



EHT Collaboration, Astrophys. J. Lett. 875, L1 (2019)



EHT Collaboration, Astrophys. J. Lett. 875, L1 (2019)



EHT Collaboration, Astrophys. J. Lett. 875, L5 (2019)