INRiM
ISTITUTO NAZIONALE
DI RICERCA METROLOGICA

Pseudodensity Matrix of a quatum optical system as a tool for visualizing open timelike curves

M.G.

BeCOMe

Light-matter interplay for optical metrology beyond the classical spatial resolution limits

SIQUST

Single-photon sources as new quantum standards

European Commission

FET Open project - Pathos

cost
EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

COST Action MP1405

FQXi
FOUNDATIONAL QUESTIONS INSTITUTE

FQXi Project

Nato grant G5263

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Quid est ergo tempus? si nemo ex me quaerat, scio; si quaerenti explicare velim, nescio.

Aurelius Augustinus Hipponensis, Confessiones, XI, 14

πάντα ῥεῖ ὡς ποταμός [Eraclitos]

“It is what passes (*shi* 逝) like that, indeed, not ceasing day or night.” [Confucius, The Analects, 2491]

Tempus item per se non est, sed rebus ab ipsis consequitur sensus, transactum quid sit in aevo, tum quae res instet, quid porro deinde sequatur

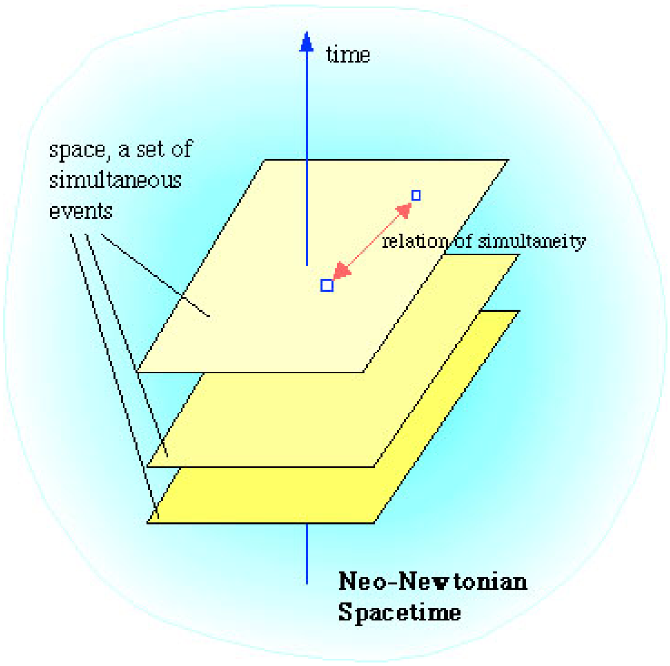
Titus Lucretius Carus, De Rerum Natura

[Time also exists not by itself, but simply from the things which happen the sense apprehends what has been done in time past, as well as what is present and what is to follow after.]

- Earlier ideas of **Parmenides of Elea** (Παρμενίδης ὁ Ἐλεάτης): the phenomena of movement and change are simply appearances of a static, eternal reality.

Newtonian Absolute Time (QM):
a fixed background parameter

Absolute, true, and mathematical **time**, from its own nature, passes equably without relation to anything external, and thus without reference to any change or way of measuring of time

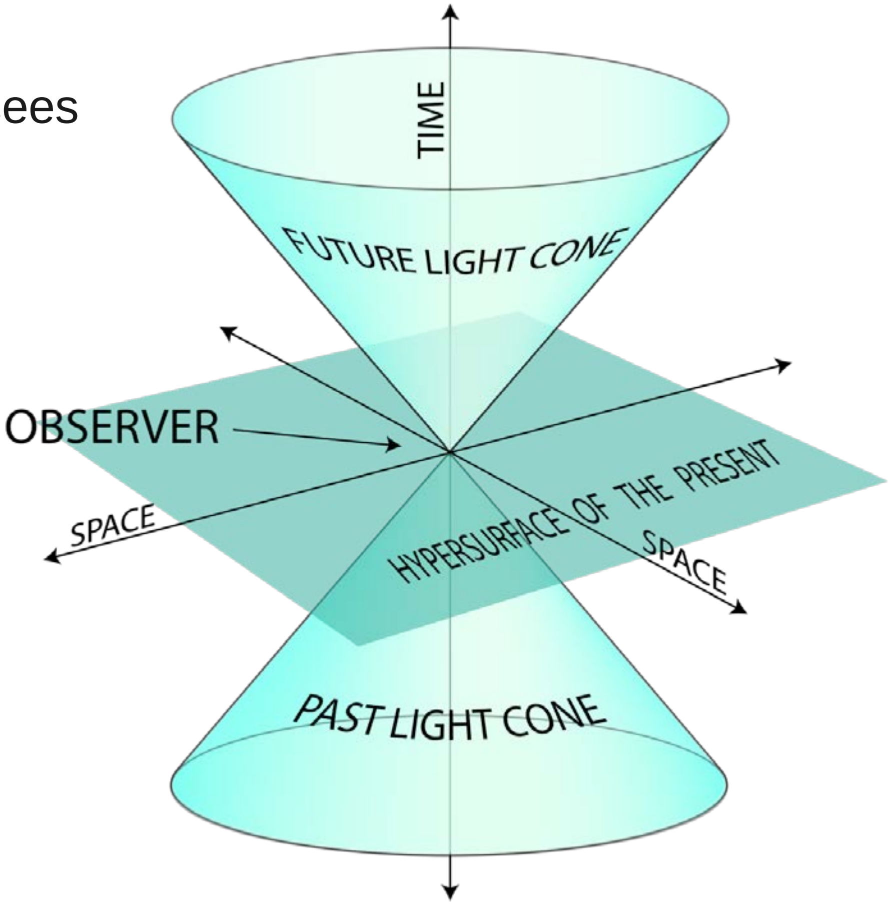


Non relativistic QM

SR Time: a proper time for each observer, time as 4th coordinate (set of privileged inertial frames)

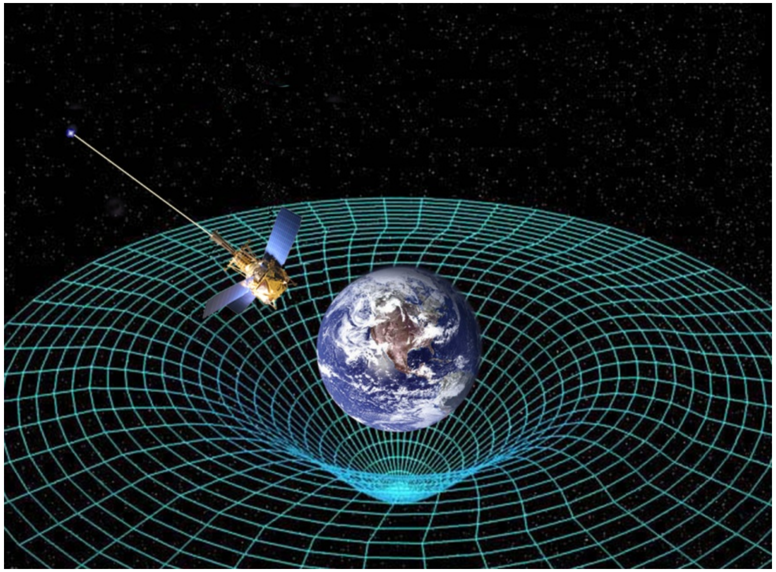
proper time : time that an observer sees on a clock at rest

$$\Delta\tau = \int_P \frac{1}{c}\sqrt{\eta_{\mu\nu}\,dx^\mu\,dx^\nu}$$
$$= \int_P \sqrt{dt^2 - \frac{dx^2}{c^2} - \frac{dy^2}{c^2} - \frac{dz^2}{c^2}}$$
$$= \int \sqrt{1 - \frac{1}{c^2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right]}\,dt$$
$$= \int \sqrt{1 - \frac{v(t)^2}{c^2}}\,dt = \int \frac{dt}{\gamma(t)},$$



GR Time: not at all an absolute time. Time is a general spacetime coordinate (time non orientability, closed timelike curves…)

$$\Delta\tau = \int_P d\tau = \int_P \frac{1}{c}\sqrt{g_{\mu\nu}\, dx^\mu\, dx^\nu}$$



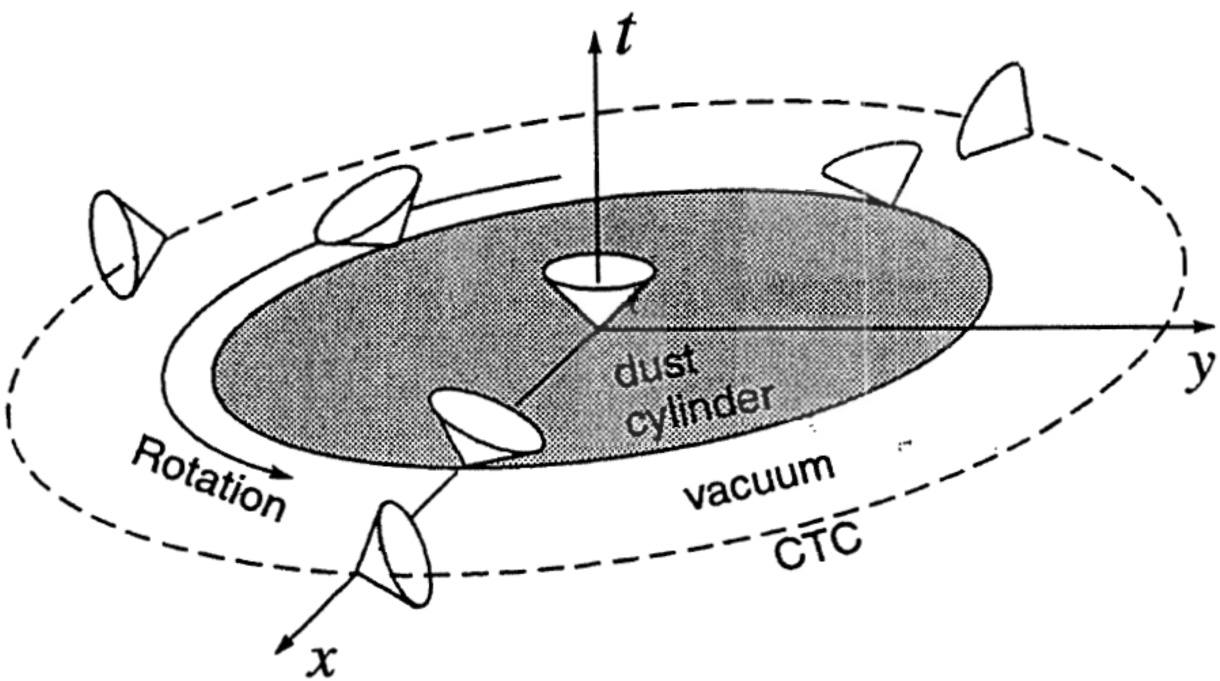
Problems in quantum cosmology

In canonical quantum gravity general relativistic evolution and constraints → Wheeler – De Witt equation for "the universe wave function" **ψ**

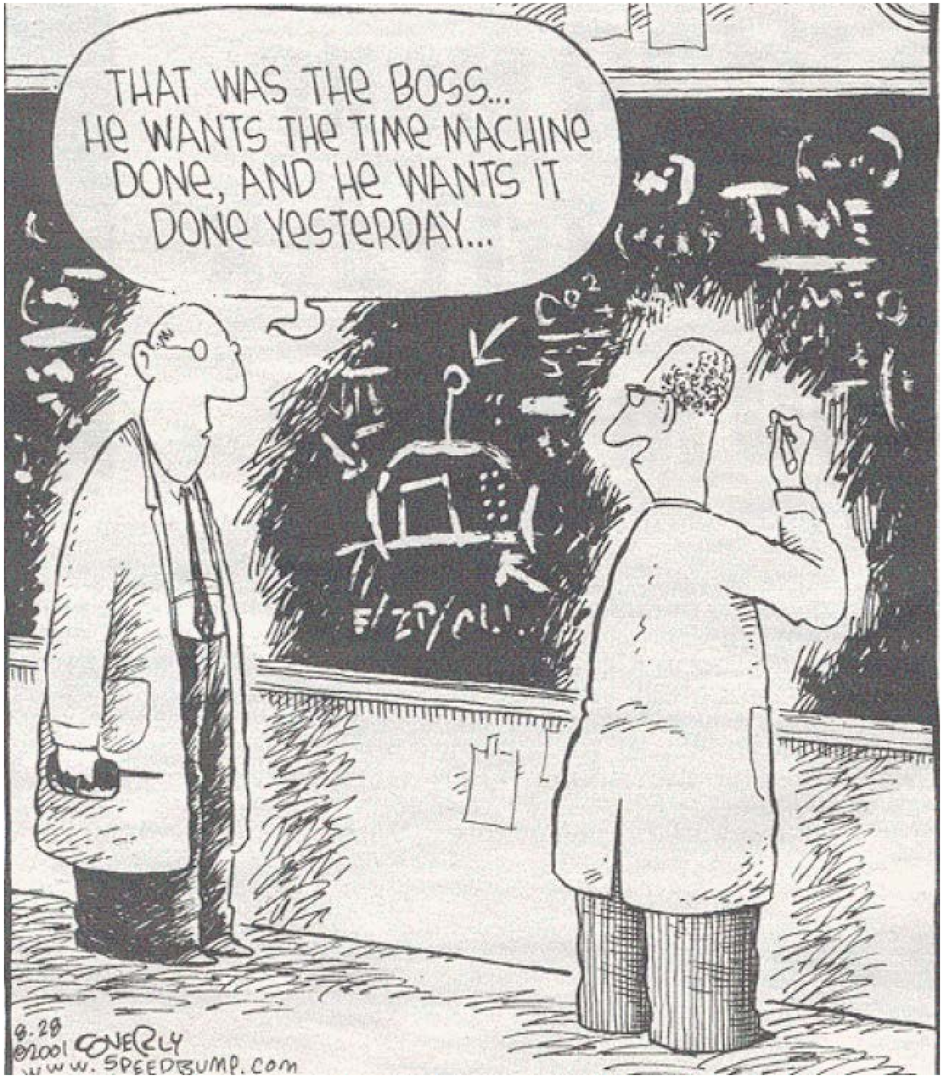
$H_{tot}\,\psi = 0$ Stationary equation!! **→ see Katia's talk**

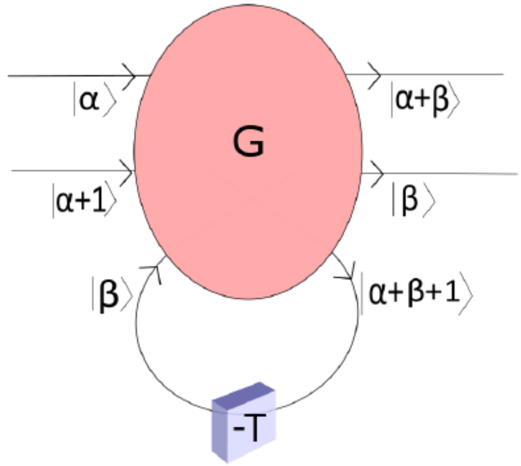
[E.Anderson, Ann.Phys. 524 (2012) 757;R.Sorkin, Int J.Th.Phys 85 (1994) 523; W.Unruh and R.Wald,PRD 40 (1989) 2598; D.Page and W.Wootters,PRD 27 (1983) 2885; W.Wootters, IJTP 23 (1984) 701; V.Giovannetti, S.Lloyd, L.Maccone, PRD 92 (2015) 045033; *V. Giovannetti, S.Lloyd, L.Maccone Phys. Rev. D 92, 045033;* E.Moreva,M.Gramegna,G.Brida,V.Giovannetti,L.Maccone,M.Genovese *Phys. Rev. A 89 (2014) 052122* …]

Closed (Open) timelike curves



Grand father paradox





α (β) = 0, 1 (presence, lack in the path)

Problem of consistency in classical case

Interaction

$$|\alpha\rangle\,|\alpha+1\rangle\,|\beta\rangle \;\rightarrow |\alpha+\beta\rangle\,|\alpha+\beta+1\rangle\,|\beta\rangle$$

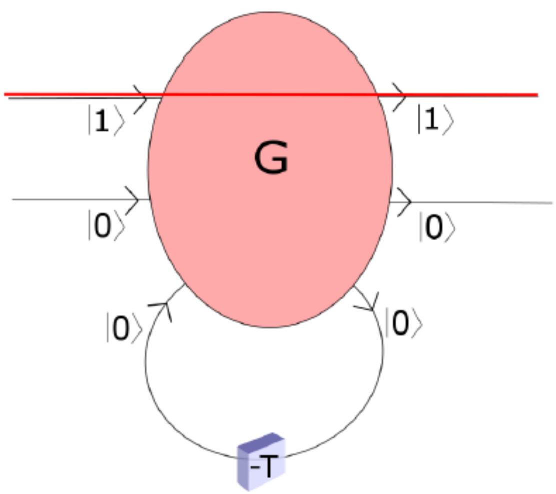
Consistency condition

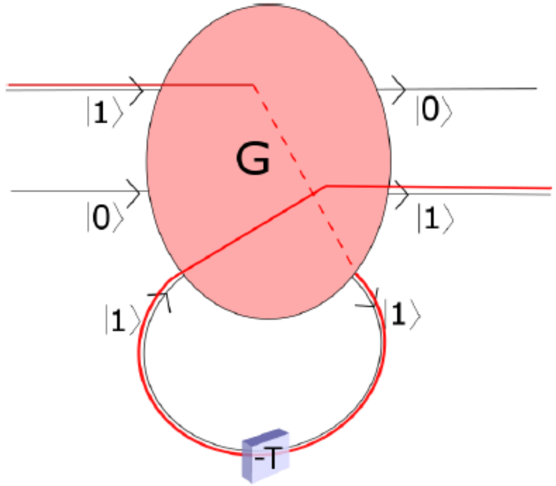
Two paths, lower one going to CTC

$$|\beta\rangle = |\alpha+\beta+1\rangle$$

α = 0 no consitent solution

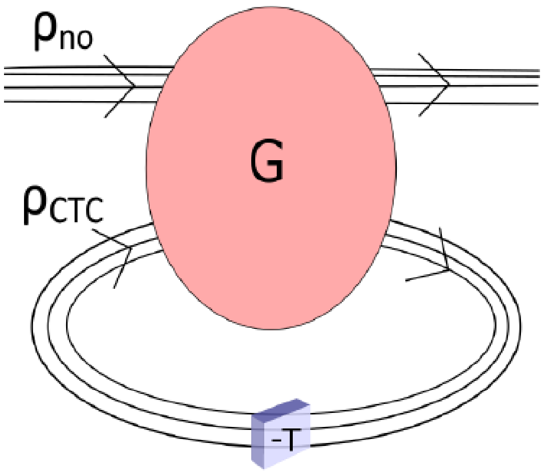
α =1 possible consistency





Solution thanks to quantum superpositions

[Deutsch Phys. Rev. D 44, 10 (1991)]



$$Tr_{no}\left(U(\rho_{no} \otimes \rho_{CTC})U^{\dagger}\right) = \rho_{CTC}.$$

Tracing out the degrees of freedom corresponding to the system not crossing CTC one must find ρ(CTC)

But: non-linearity due to CTC

- perfect discrimination of non-orthogonal states
- violation of the uncertainty principle
- violation of no-cloning theorem
- ….

Optical simulation: S.Lloyd et al. PRL **106** 040403 (2011) (P-CTC);
Ringbauer et al.; Nat. Comm **5** 4145 (2014)

Problems even in an open time-like curve (OTC), when the qubit does not interact with its past copy, but it is initially entangled with another, chronology-respecting, qubit.

To avoid **violating entanglement monogamy**, one has to postulate a non-linear evolution.

We propose an alternative approach to OTCs, to preserve linearity and avoid all other drastic consequences →

qubit state in the OTC described by pseudo-density operator

Pseudo density operator [JF. Fitzsimons, JA. Jones & V.Vedral Sc. Rep. 5:18281]
Includes also temporal correlations

The Pseudo Density Operator

SCIENTIFIC REPORTS

OPEN

Quantum correlations which imply causation

Joseph F. Fitzsimons[1,2], Jonathan A. Jones[3] & Vlatko Vedral[2,3,4]

Attempt to generalize the concept of density operator for multiple spatial and temporal measurements

A generic density operator

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

In terms of n-qubit Pauli operators

$$\rho = \frac{\langle \mathbb{I} \rangle}{2^n}\mathbb{I} + \sum_{i=1}^{4^n-1} \frac{\langle P_i \rangle}{2^n} P_i$$

When interested in correlations one can express n-qubit Pauli operators in terms of single qubit Pauli operators

$$\rho = \frac{1}{2^n}\sum_{i_1=0}^{3}\cdots\sum_{i_n=0}^{3}\left\langle \bigotimes_{j=1}^{n}\sigma_{i_j}\right\rangle\left(\bigotimes_{j=1}^{n}\sigma_{i_j}\right) \tag{3}$$

where $\sigma_0 = \mathbb{I}, \sigma_1 = X, \sigma_2 = Y$ and $\sigma_3 = Z$.

This above equation can be taken as defining a generalization of the density matrix. We consider a set of events $\{E_1 \dots E_N\}$, where at each event E_j a von Neumann measurement of a single qubit Pauli operator $\sigma_{i_j} \in \{\sigma_0, \dots, \sigma_3\}$ can be made. For a particular choice of Pauli operators $\{\sigma_{i_j}\}_{j=1}^{n}$, we take $\langle\{\sigma_{i_j}\}_{j=1}^{n}\rangle$ to be the expectation value of the product of the result of these measurements. Then we can define a pseudo-density matrix

$$R = \frac{1}{2^n}\sum_{i_1=0}^{3}\cdots\sum_{i_n=0}^{3}\langle\{\sigma_{i_j}\}_{j=1}^{n}\rangle \bigotimes_{j=1}^{n} \sigma_{i_j}. \tag{4}$$

Properties [JF. Fitzsimons, JA. Jones & V.Vedral Sc. Rep. 5:18281] :

Hermiticity. All pseudo-density matrices are necessarily Hermitian.

Tr[R] = 1

Partial trace. Given a pseudo-density matrix R_{AB} defined over two sets of events A and B, the pseudo-density matrix obtained from the set of events A can be obtained from R_{AB} by tracing over the subsystem corresponding to B

Measurements. The pseudo-density matrix contains information not only about Pauli measurements, but also about the expectation value of the product of any set of local measurements with eigenvalues restricted to ± 1.

Not positive

Example: two times measurements on a maximally mixed single qubit

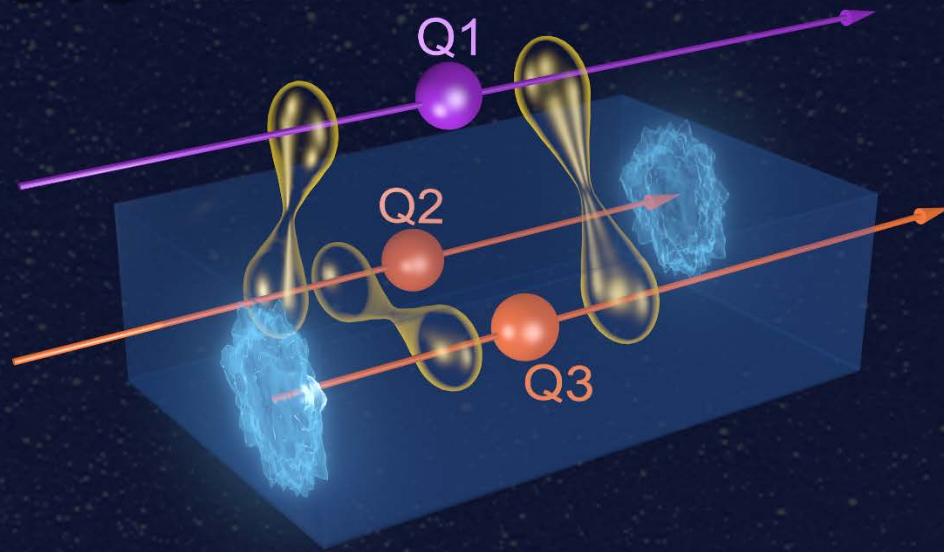
$$R_{12} = \frac{1}{4}\{I + X_1 X_2 + Y_1 Y_2 + Z_1 Z_2\}$$

Similarities with a singlet, but here:

- correlations all positive

- Non positive operator

PDO and OTC



[C.Marletto, V.Vedral, S.Virzì,E.Rebufello, A.Avella,F.Piacentini,M.Gramegna,I.P.Degiovanni,M.Genovese Nat. Comm. (2019) 10:182]

PDO for a qubit maximally entangled with a second, entering a CTC

$$R_{123} = \frac{1}{8}\{I_{123} - \Sigma_{12} + \Sigma_{23} - \Sigma_{13})\}$$

$$\Sigma_{ij} = X_i X_j I_k + Y_i Y_j I_k + Z_i Z_j I_k$$

Correlations described by a pseudodensity matrix do not need obeying monogamy relation

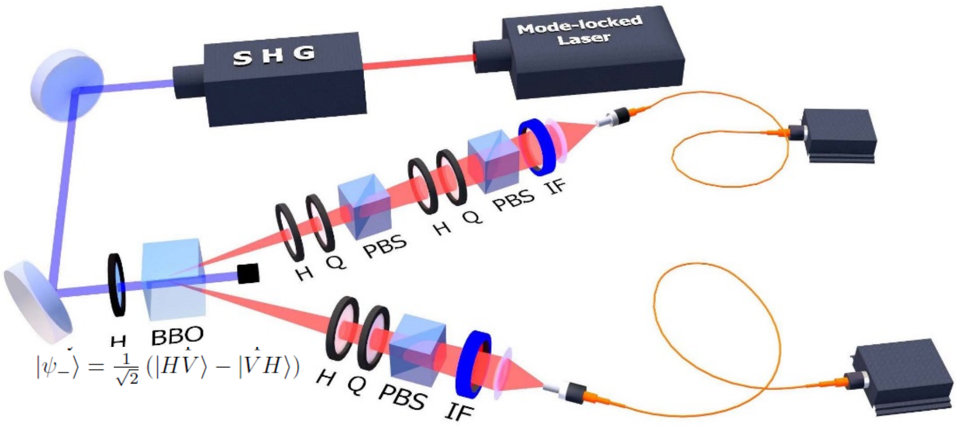
$$E_{12} + E_{23} \leq 1$$

Experimentally convenient using CHSH inequalities

$$C_{mk} + C_{nk} \leq 4$$

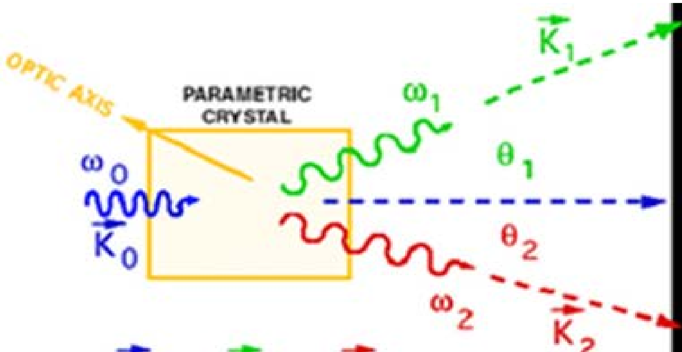
EXPERIMENTAL VISUALIZATION OF OCT

We intend simulating the pseudo-density matrix describing OCT



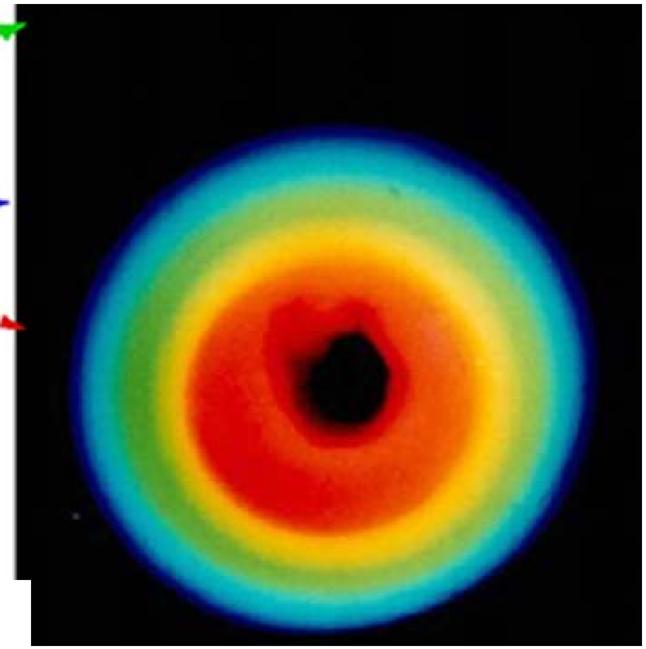
- Entangled pair of photons (A and B)
- A measured at two different times (t1 and t2)
  B is only measured once at time t1.
- The simulation consists in reconstructing all the statistics contained in R_{123}
- The photon measured at two different times represents the qubit entering the OTC and its copy emerging from the OTC.

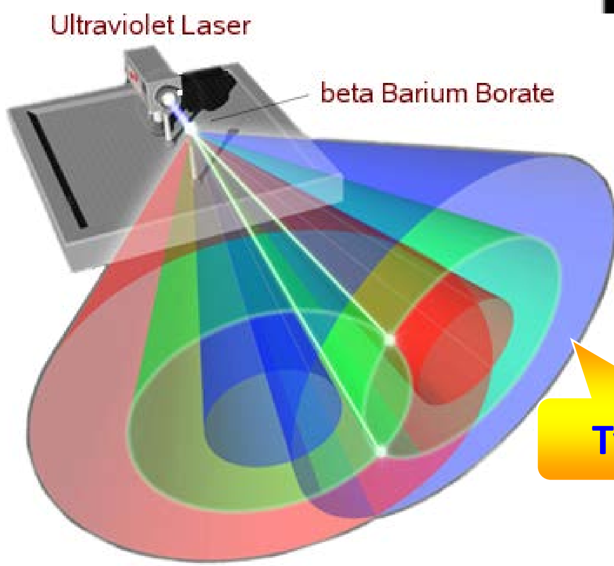
PDC: a brief summary



$$\vec{K}_0 = \vec{K}_1 + \vec{K}_2$$

$$\omega_0 = \omega_1 + \omega_2$$

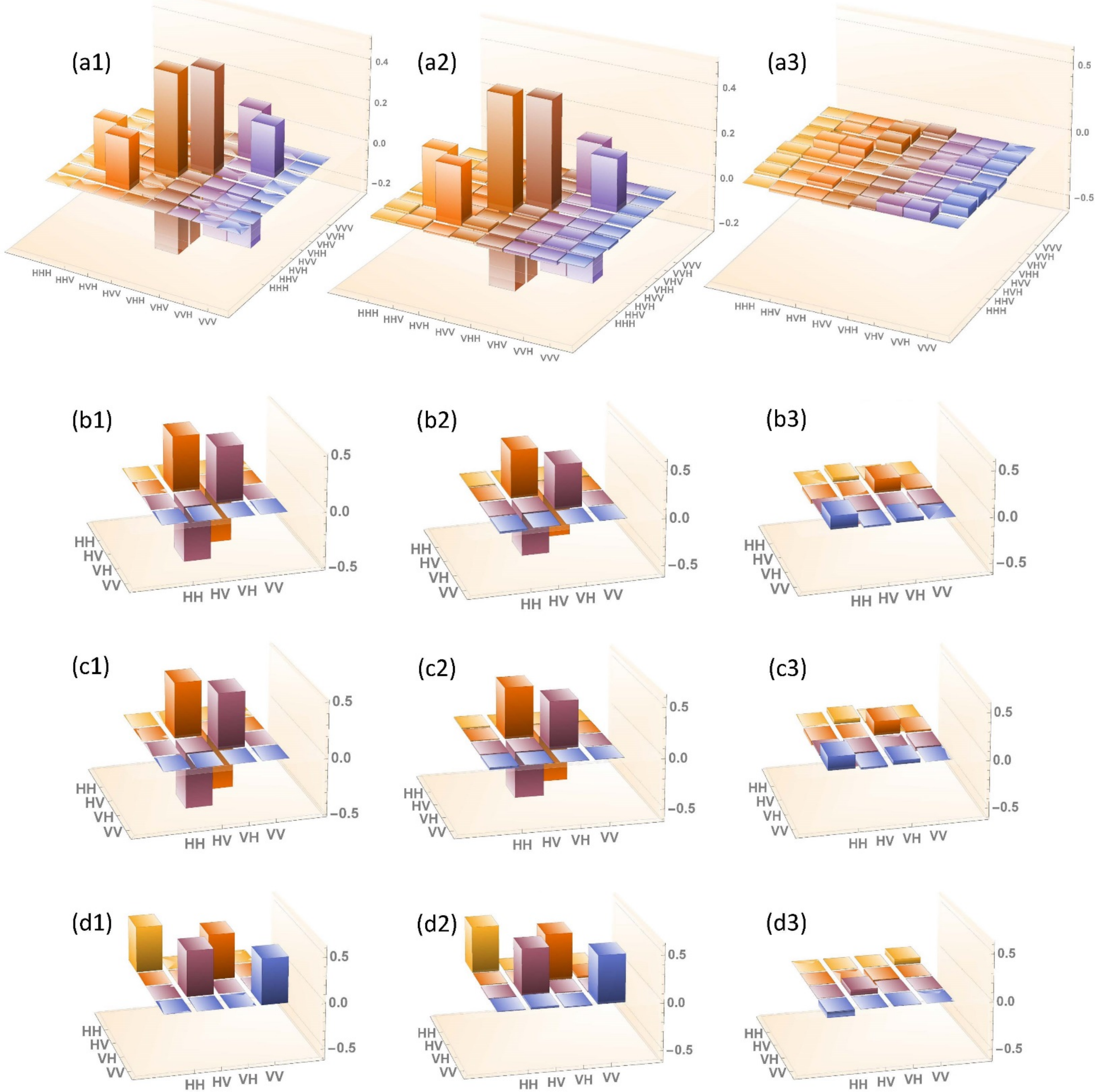




Type-II PDC

Type-I PDC

$$\frac{[(\,|H>|V> - |V>|H>)\,]}{\sqrt{(2)}}$$


(a1)
(a2)
(a3)
(b1)
(b2)
(b3)
(c1)
(c2)
(c3)
(d1)
(d2)
(d3)
HHH HHV HVH HVV VHH VHV VVH VVV
HH HV VH VV
0.5
0.0
−0.5

$F_{ab} > 96\%$

$$C_{12}^{(exp)} + C_{23}^{(exp)} = 5.52 \pm 0.03$$

$$C_{12}^{(exp)} + C_{13}^{(rec)} = 5.42 \pm 0.07$$

$$C_{23}^{(exp)} + C_{13}^{(rec)} = 5.55 \pm 0.07$$

This procedure highlights interesting properties of the PDO:

- the reduced PDO of some subsystems is obtained by taking the trace on the degrees of freedom of the rest of the systems, e.g. $R_{13} = Tr2(R_{123})$
- However, R_{13} cannot be reconstructed experimentally
  - acquiring measurements for the three-point correlations
  - averaging over the results of the measurements on the second qubit (i.e., photon A measured at time t_1).
- This is because the trace over a temporal degree of freedom is not equivalent to averaging with respect to all possible values of the observables measured at same time.
- $Tr(PR_{123})$ (P generic projector) could be negative

Extension to other situations where the same properties of quantum theory seem to be violated.

Entangled particle falling into an evaporating black hole→
PDO resolves the apparent violation of monogamy of entanglement.

When one of the two entangled particles falls into the black hole, time-like correlations between the two particles, out of what used to be spatial correlations, could be created.

Another particle entangled with the interior one
→ same structure as R_{123}.

In this state qubits 1 and 2 are temporally correlated while qubits 2 and 3 are in a spatially maximally entangled state.