Tensor Network Simulations of QFT in Curved Spacetime

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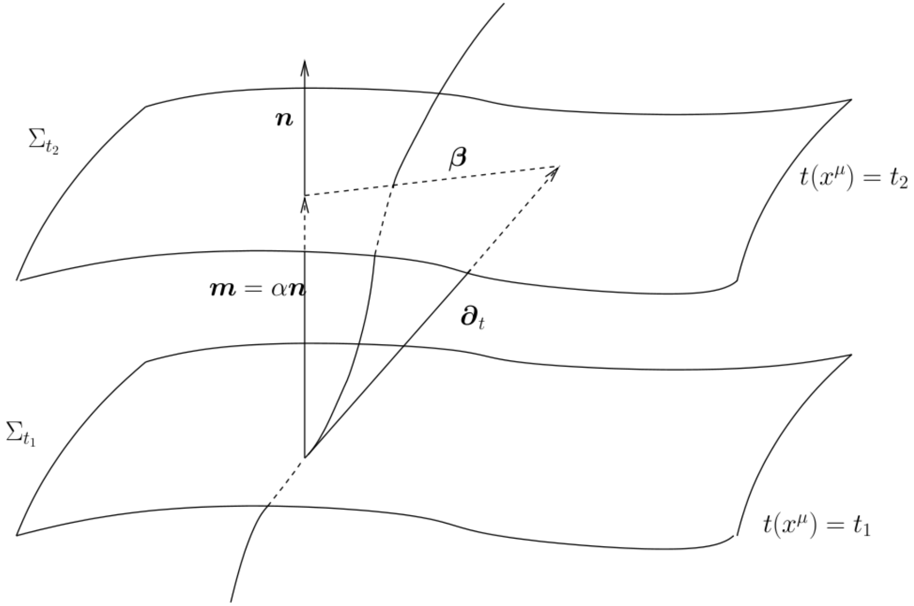
Motivation: Simulate Semiclassical Gravity

-If e.g. CTCs occur, they are a quantum effect.

-Detailed *quantitative* understanding of semiclassical gravity remains unavailable - need nonperturbative methods.

-Inflation? Black hole evaporation? Black hole formation? With interactions?

Numerical Relativity



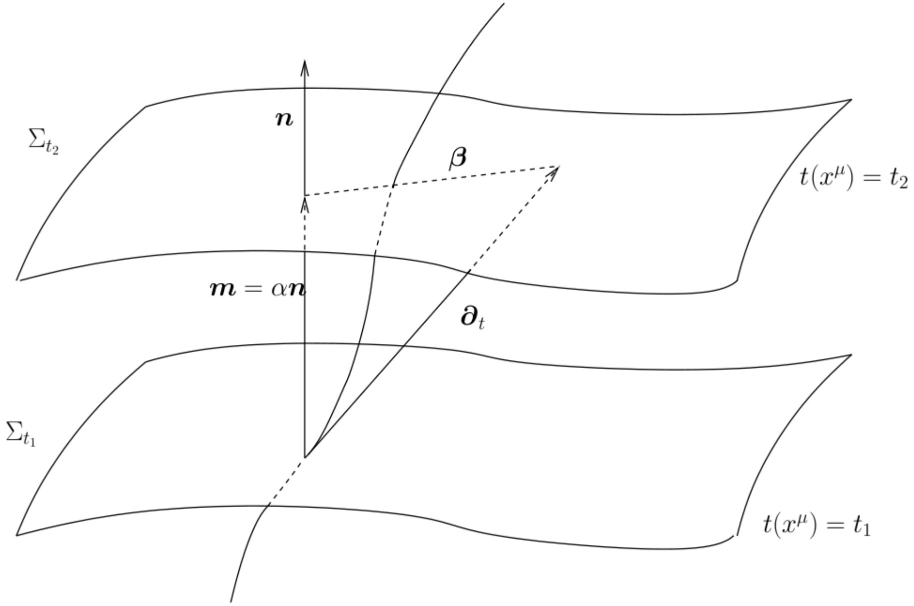
"Exploring New Physics Frontiers Through Numerical Relativity"
Vitor Cardoso and Leonardo Gualtieri and Carlos Herdeiro and Ulrich Sperhake

Foliate spacetime to get initial value problem.

EFEs -> PDE system evolving hypersurface-local quantities.

We can project other equations onto the hypersurfaces just as well.

Numerical QFTCS?



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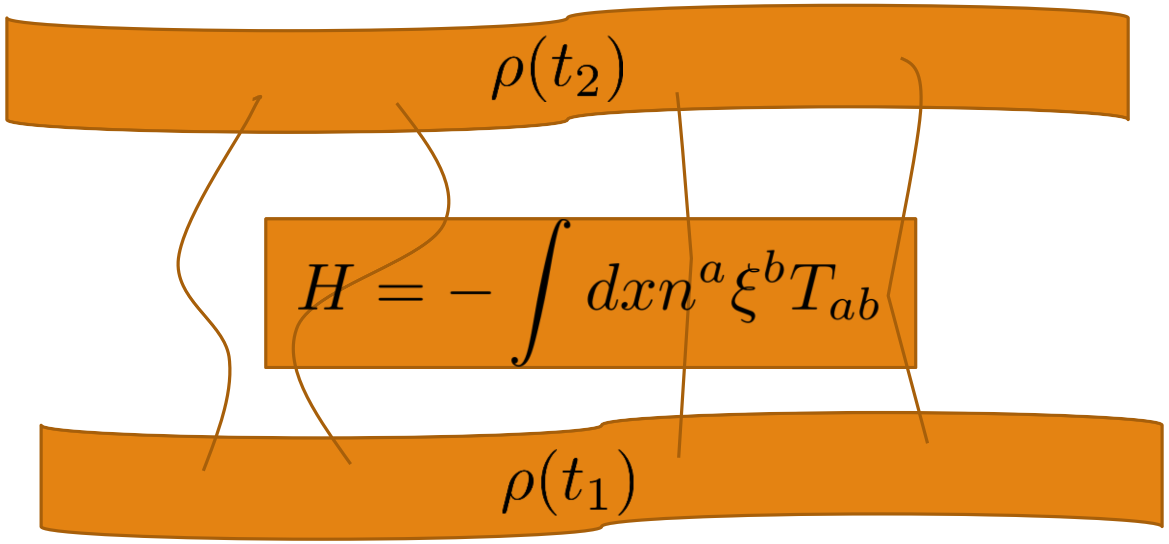
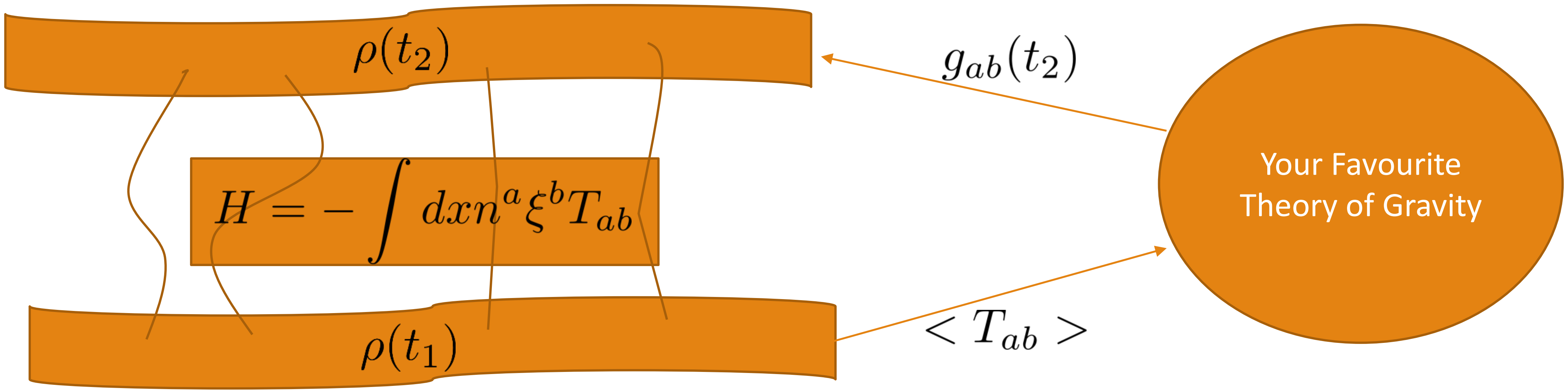


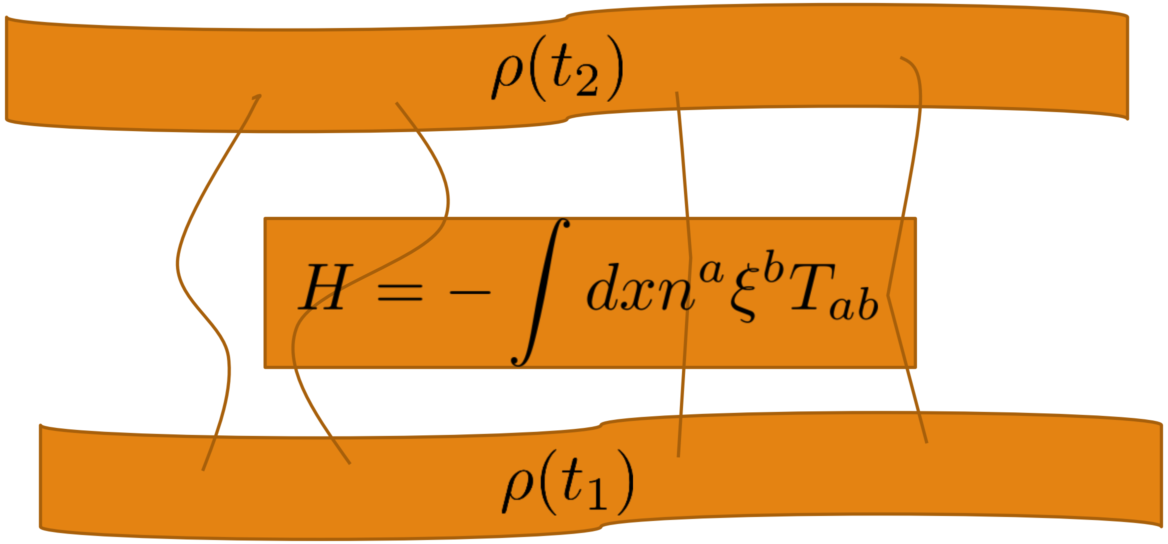
Numerical Semiclassical Gravity?



Hypersurface-local Hamiltonian

$$H = -\int dx T_{00} = \frac{1}{2}\left(\psi^\dagger\gamma^5\psi_{,1} - \psi^\dagger_{,1}\gamma^5\psi\right) + \frac{\Omega_{,1}}{2\Omega}\psi^\dagger\gamma^5\psi - m\Omega\psi^\dagger\gamma^0\psi + (g/4)(\bar{\psi}\psi)^2$$

$$g_{ab} = \Omega^2(x)\eta_{ab}$$



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$$g_{ab} = \Omega^2(x)\eta_{ab}$$

"Staggered Fermions" –
my laptop is finite

$$H \to \frac{i}{2}\left(\frac{1}{l} + \frac{\Omega_{,1}}{2\Omega}\right)\left(\phi^\dagger_n\phi_{n+1} - \phi^\dagger_{n+1}\phi_n\right) + (-1)^n m\Omega_n\phi^\dagger_n\phi_n - (g/4l)\phi^\dagger_n\phi_n\phi^\dagger_n\phi_n$$

"Jordan-Wigner transform" then maps to a spin chain Hamiltonian.

Strategy

$$H \rightarrow \frac{i}{2}\left(\frac{1}{l} + \frac{\Omega_{,1}}{2\Omega}\right)\left(\phi_n^\dagger \phi_{n+1} - \phi_{n+1}^\dagger \phi_n\right) + (-1)^n m \Omega_n \phi_n^\dagger \phi_n - (g/4l)\phi_n^\dagger \phi_n \phi_n^\dagger \phi_n$$

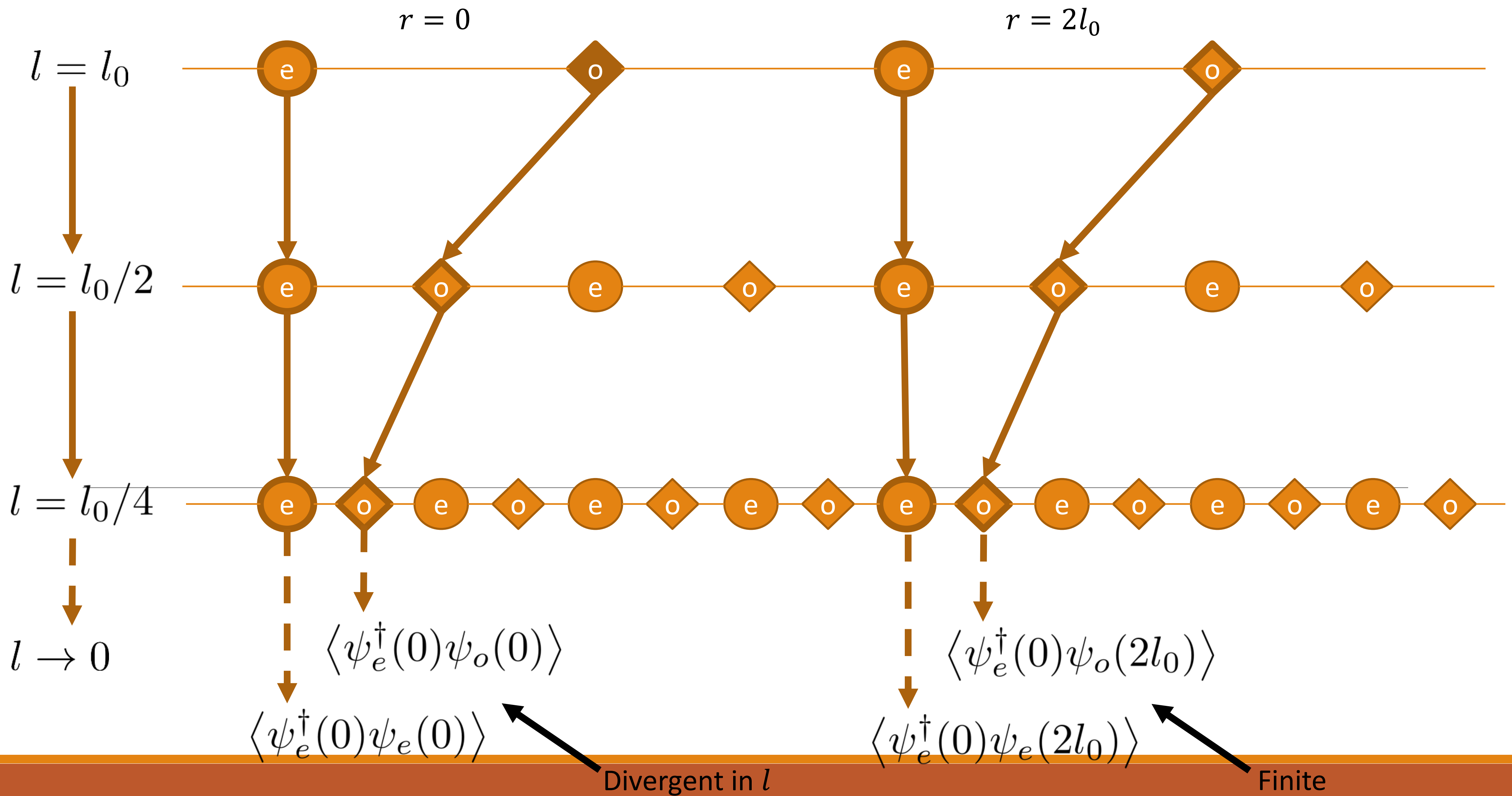
-Prepare state using H.

-Compute things with state.

-Take continuum limit.

-Already successfully applied to Schwinger model (1+1 QED)
-See e.g. works by Karl Jansen, MC Banuls, PhD thesis of Kai Zapp…

Extraction of Finitely Separated Correlators



Tensor Networks (Matrix Product States)

-Classically tractable Ansatz for ground states of (gapped) local Hamiltonians.

-As DMRG, a major tool of e.g. computational chemistry.

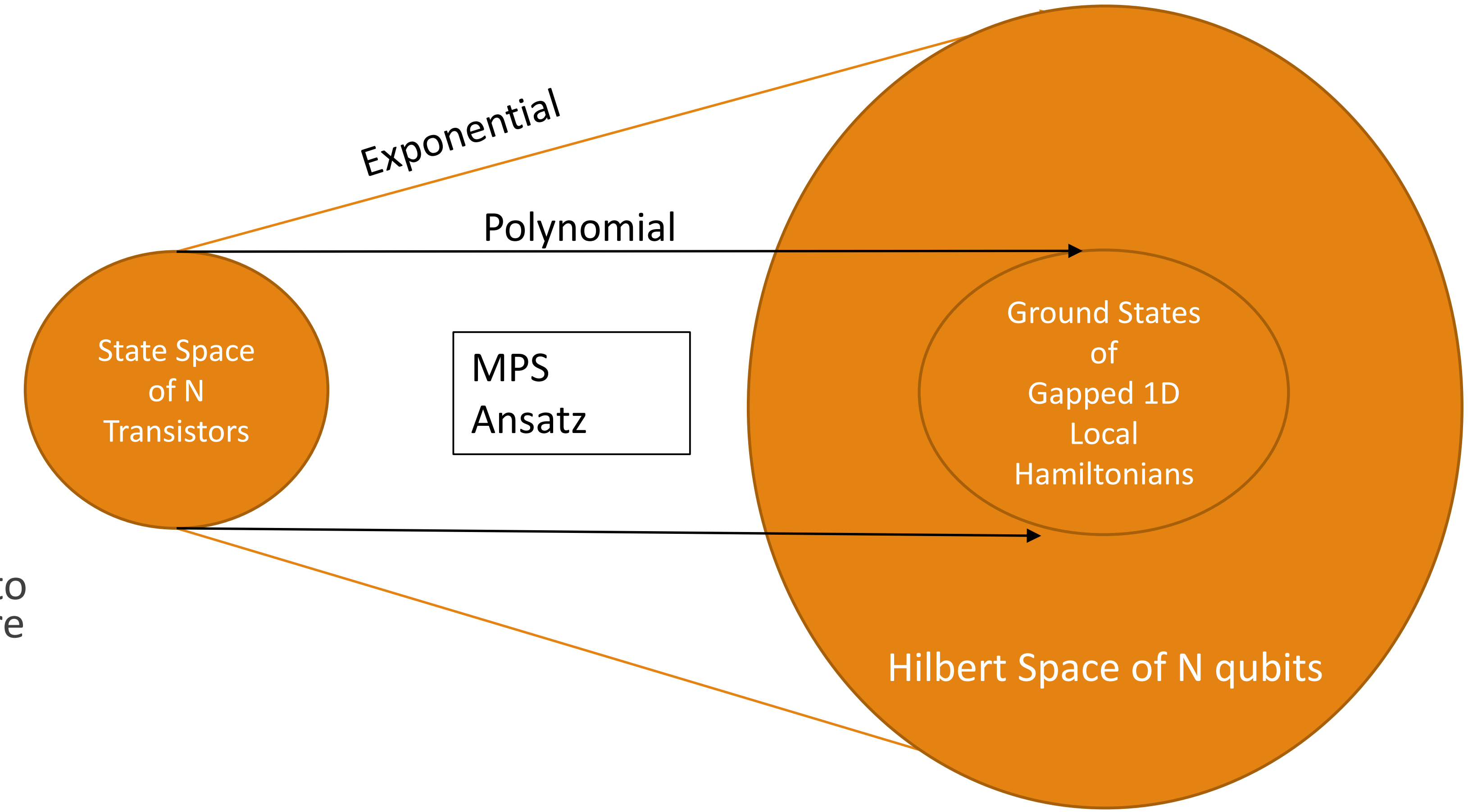
-Increasing use in QFT.

-Direct access to state.

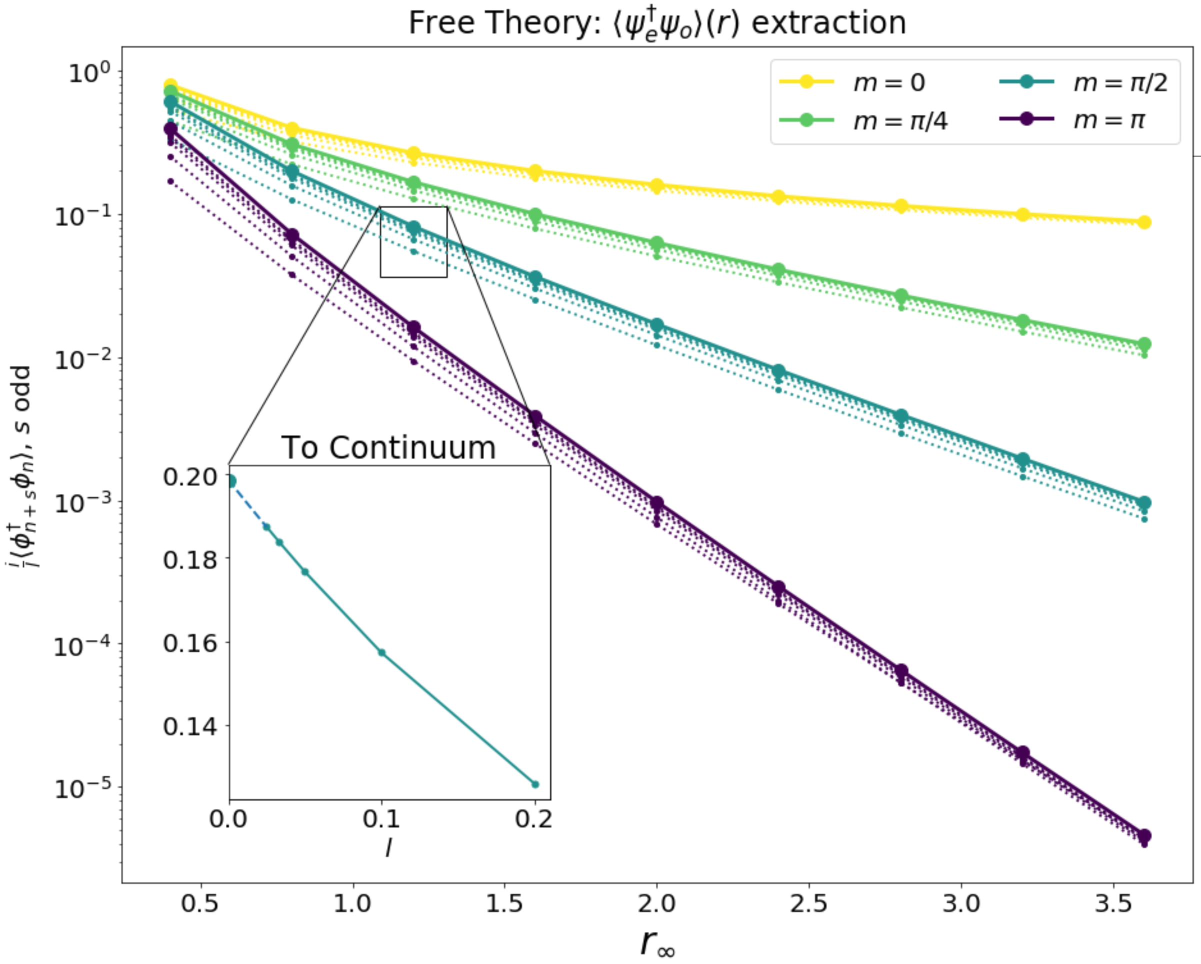
-Can handle time-dependence!

-Also prepares for quantum simulators.

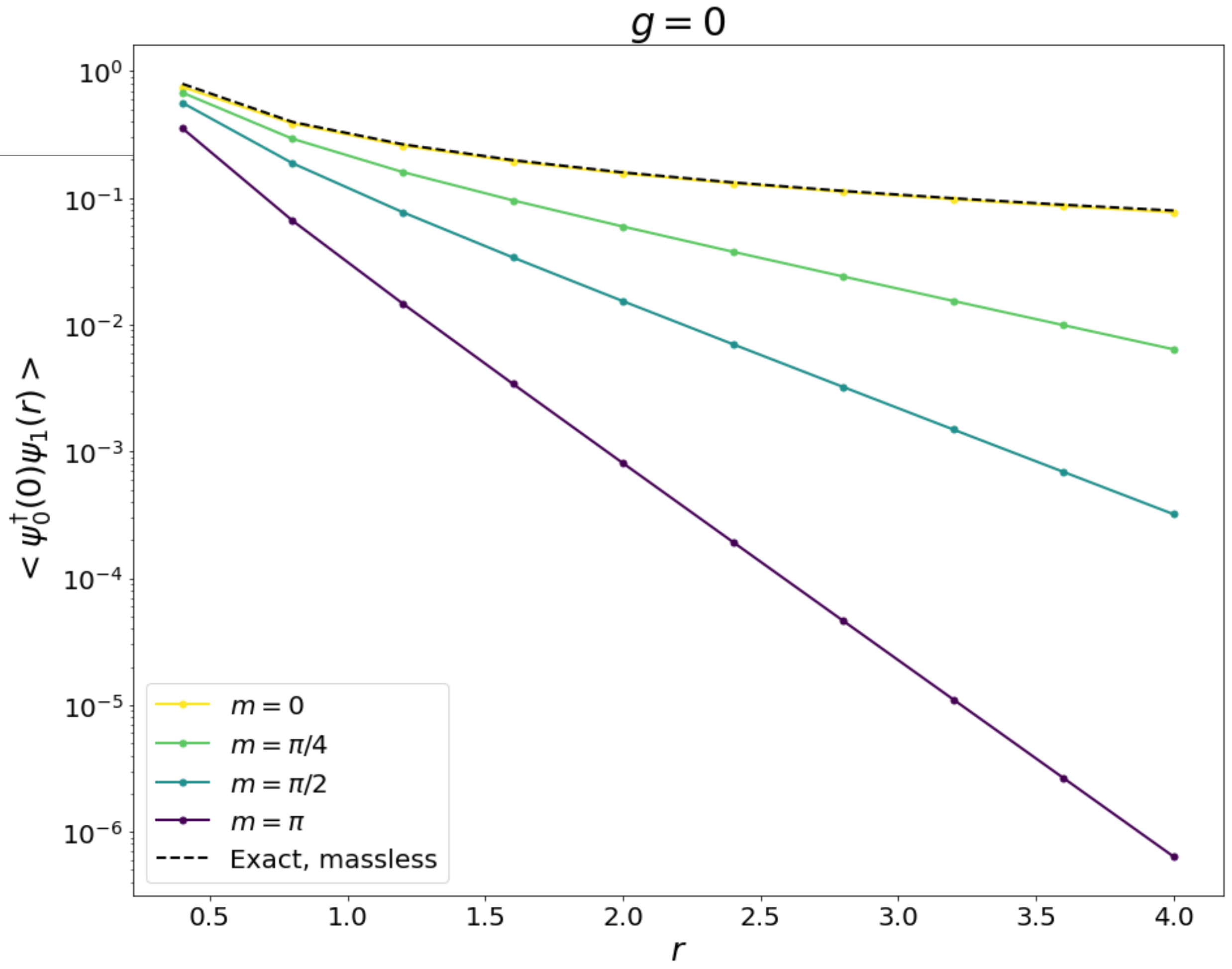
-We use the “VUMPS” algorithm to find ground states; ask me if you’re interested.



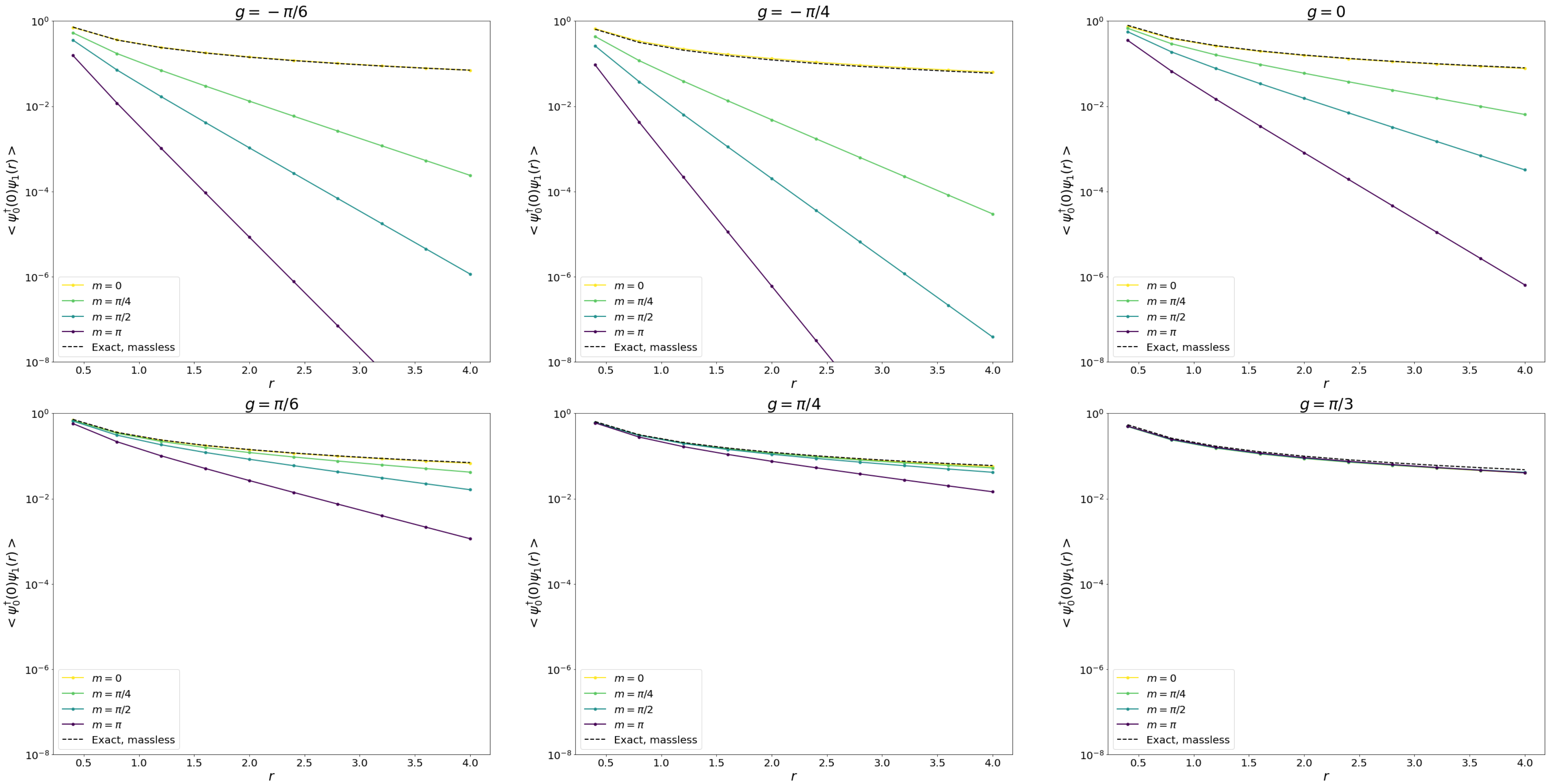
Correlation Functions - Minkowski



Free Fermions



MPS - VUMPS



$g = -\pi/6$
$g = -\pi/4$
$g = 0$
$g = \pi/6$
$g = \pi/4$
$g = \pi/3$
$< \psi_0^\dagger(0)\psi_1(r) >$
r
$m = 0$
$m = \pi/4$
$m = \pi/2$
$m = \pi$
Exact, massless

Unruh Effect

$\Omega(x) = 1$: Minkowski

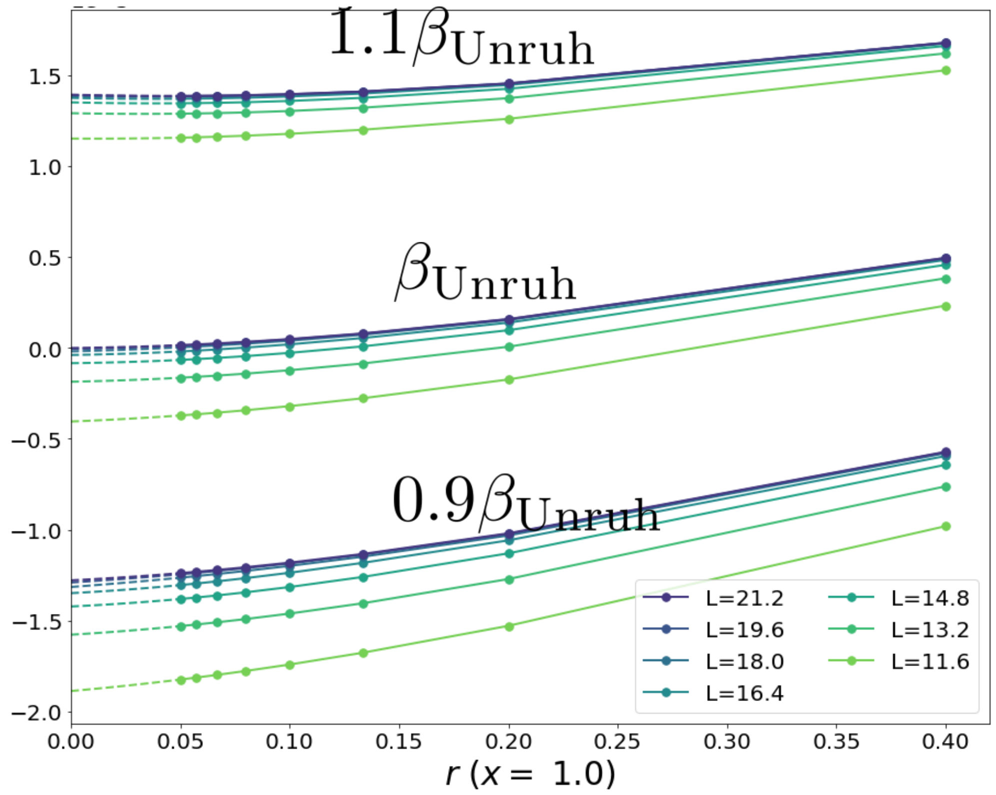
$\Omega(x) = e^{2x}$: Rindler

Unruh Effect: Thermal state of Rindler at Unruh temperature = Ground state of Minkowski.

$$H \to \frac{i}{2}\left(\frac{1}{l} + \frac{\Omega_{,1}}{2\Omega}\right)\left(\phi_n^\dagger \phi_{n+1} - \phi_{n+1}^\dagger \phi_n\right) + (-1)^n m \Omega_n \phi_n^\dagger \phi_n$$

Unruh Effect

$$\langle \psi_0^\dagger(1.0)\psi_1(r)\rangle_{\text{Rindler}} - \langle \psi_0^\dagger(1.0)\psi_1(r)\rangle_{\text{Minkowski}}$$



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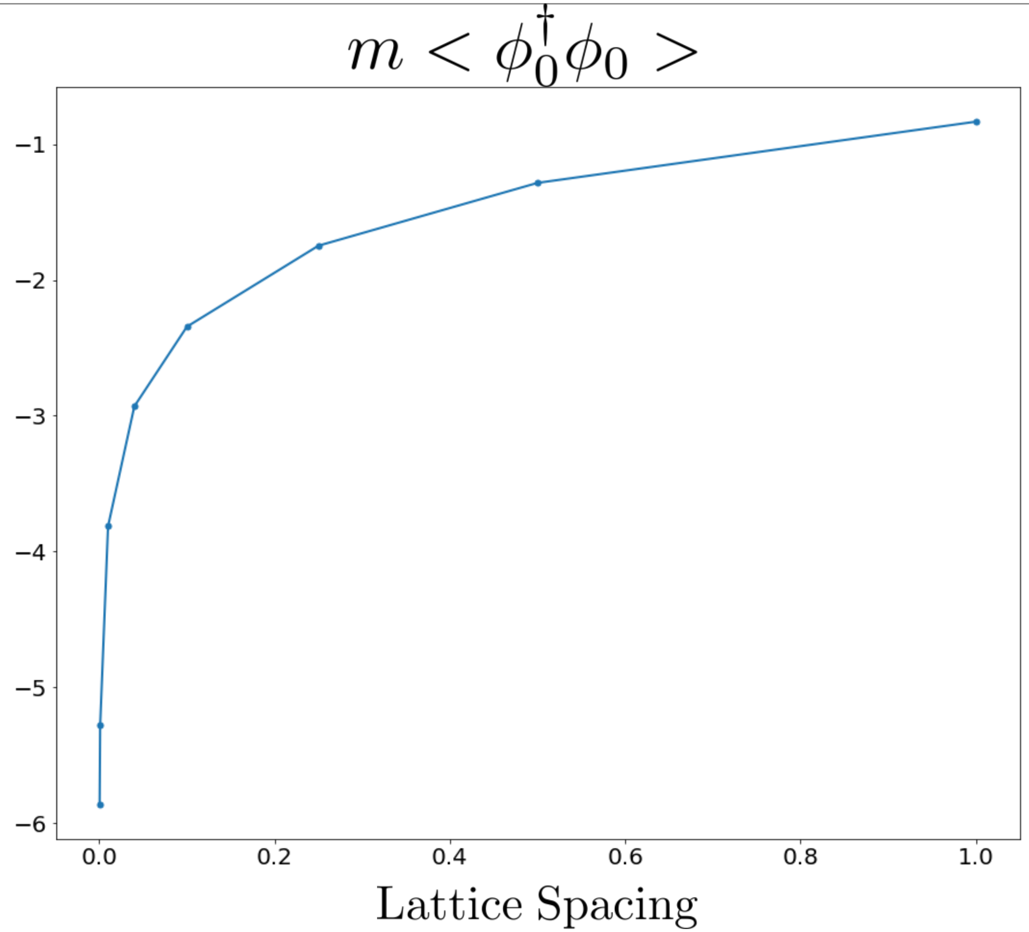
Quadratic Expectation Values



$$< T_{ab} >$$

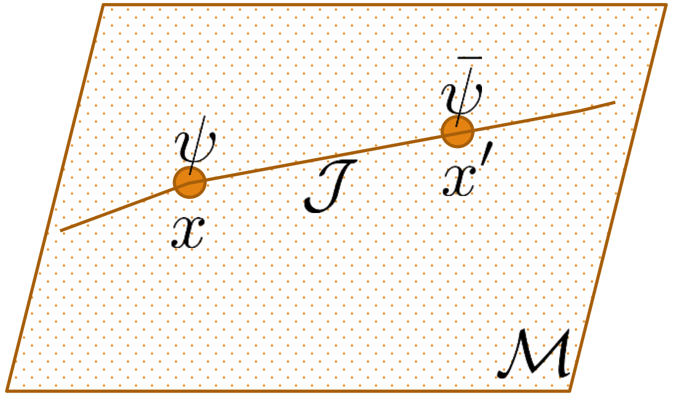
-The stress tensor is quadratic in the fields – such expectation values diverge.

-Need a principled way to subtract divergent terms, formulated in coordinate space.

Covariant Point-Splitting

$$\mathrm{Tr}\left\langle \bar{\psi}\psi \right\rangle(x) \longrightarrow \lim_{x'\to x} \frac{1}{2}\mathrm{Tr}\mathcal{J}\left\langle \left[\bar{\psi}(x')\psi(x)\right]\right\rangle$$

$$\longrightarrow \lim_{x'\to x} \left(\frac{1}{2}\mathrm{Tr}\mathcal{J}\left\langle \left[\bar{\psi}(x')\psi(x)\right]\right\rangle - \mathrm{div.}\right)$$



"Hadamard renormalization"

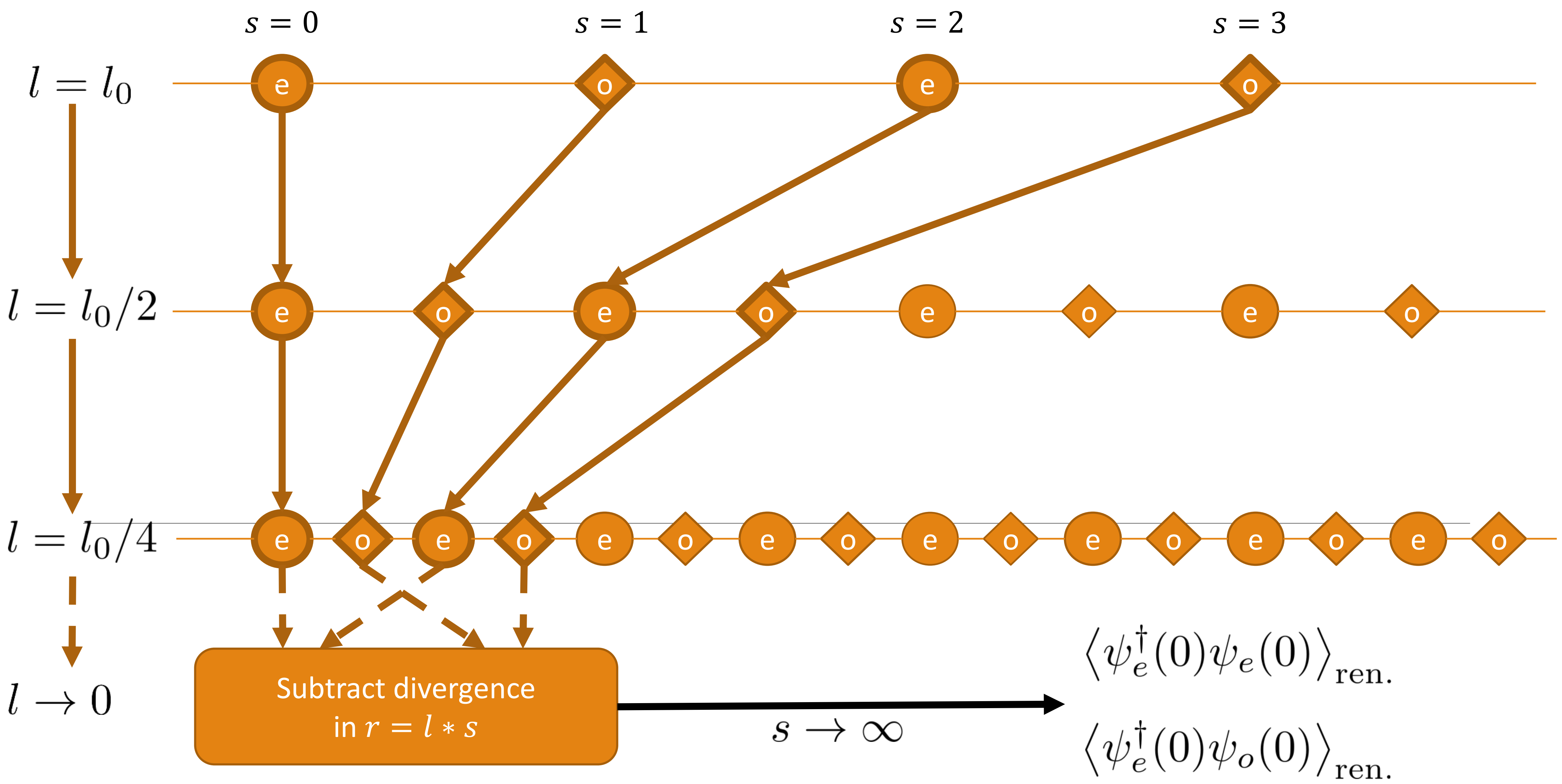
Hadamard Renormalized Stress Tensor

$$\mathcal{T}_{ab}^{\text{loc.}} \equiv \mathcal{T}_{ab}^{\text{div.}} + \mathcal{T}_{ab}^{\text{fin.}}$$

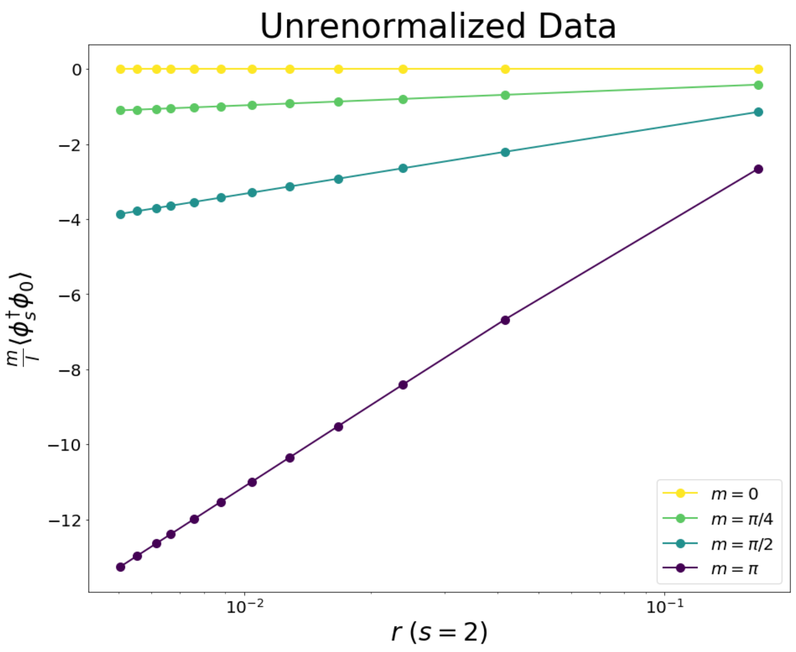
$$\mathcal{T}_{ab}^{\text{div.}} = \frac{1}{2\pi}\left[\frac{g_{ab}}{\sigma} - \frac{\sigma_{;a}\sigma_{;b}}{\sigma^2} + \frac{1}{2}m^2 g_{ab}\ln\mu\sigma\right]$$

$$\mathcal{T}_{ab}^{\text{fin.}} = \frac{1}{2\pi}\left[R\left(\frac{\sigma_{;a}\sigma_{;b}}{12\pi\sigma} - \frac{5}{48}g_{ab}\right) - \frac{1}{2}m^2\left(g_{ab} - \frac{\sigma_{;a}\sigma_{;b}}{\sigma}\right)\right]$$

Extraction of Quadratic Expectation Values

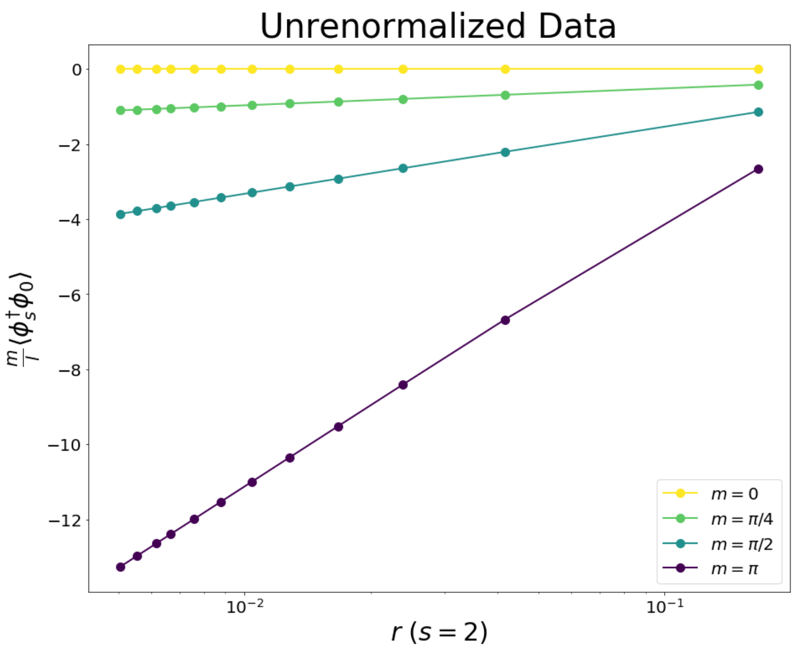


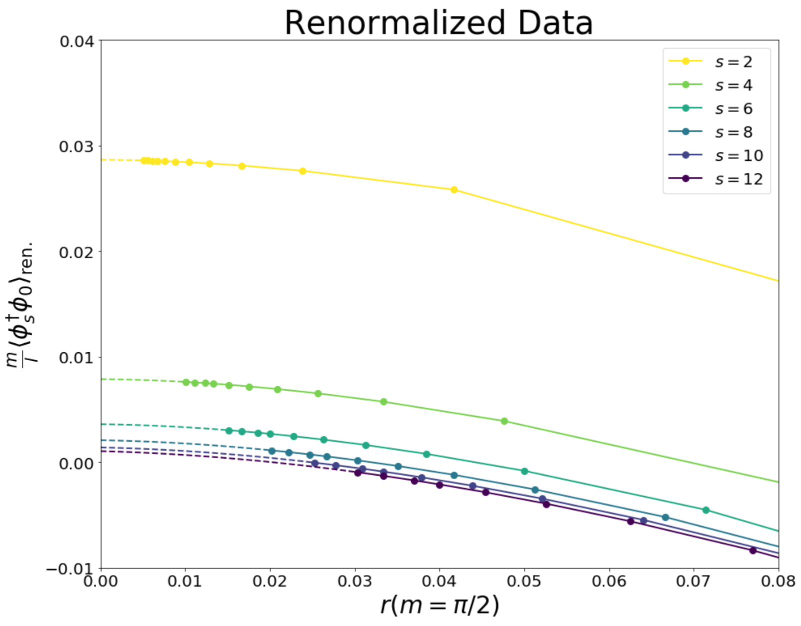
Extraction of Quadratic Expectation Values – Minkowski Spacetime



Get bare data from the lattice.

Extraction of Quadratic Expectation Values – Minkowski Spacetime





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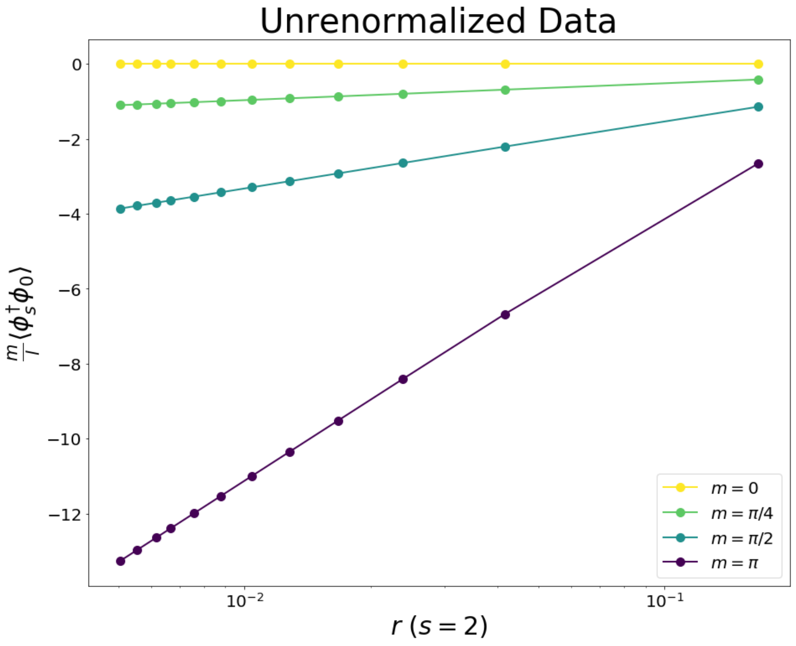
Subtract analytically computed divergences.

Extraction of Quadratic Expectation Values – Minkowski Spacetime



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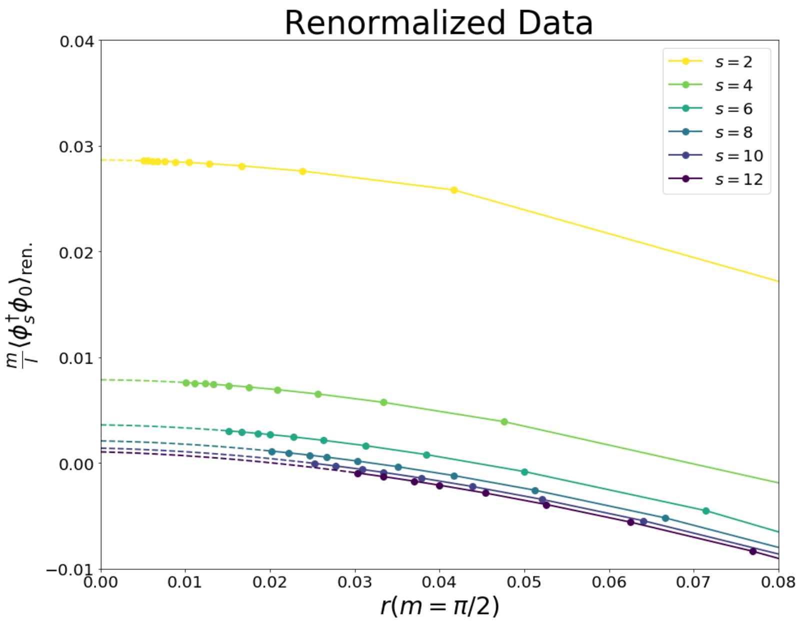

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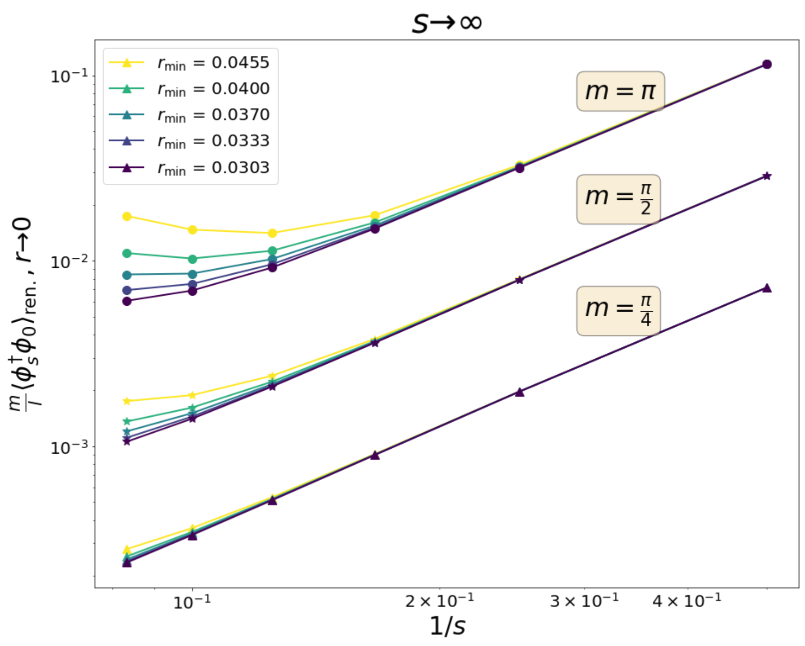


Extrapolate to zero coordinate and infinite lattice separation.

Curved Spacetime: Killing Vacua

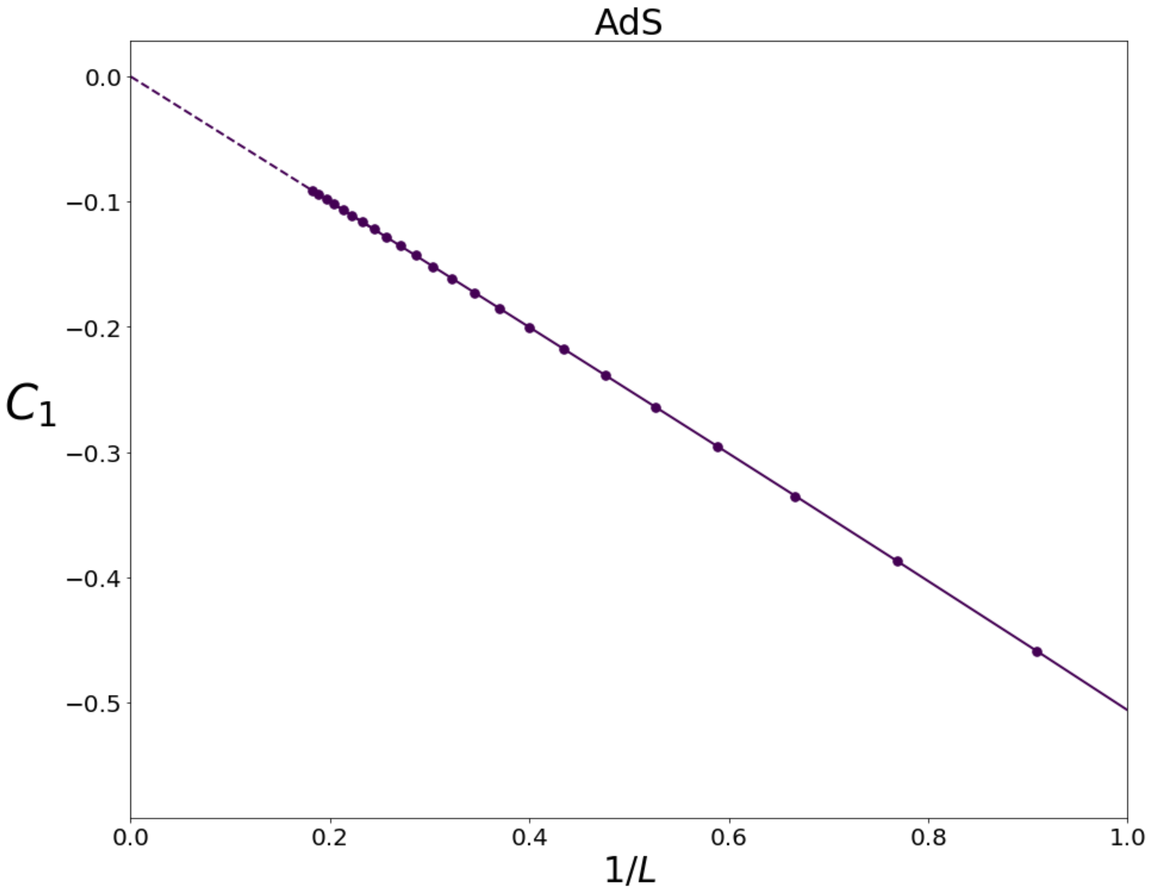
-QFTCS: the vacuum defined by a timelike Killing field.
-Lattice: The state minimizing a time-independent Hamiltonian.

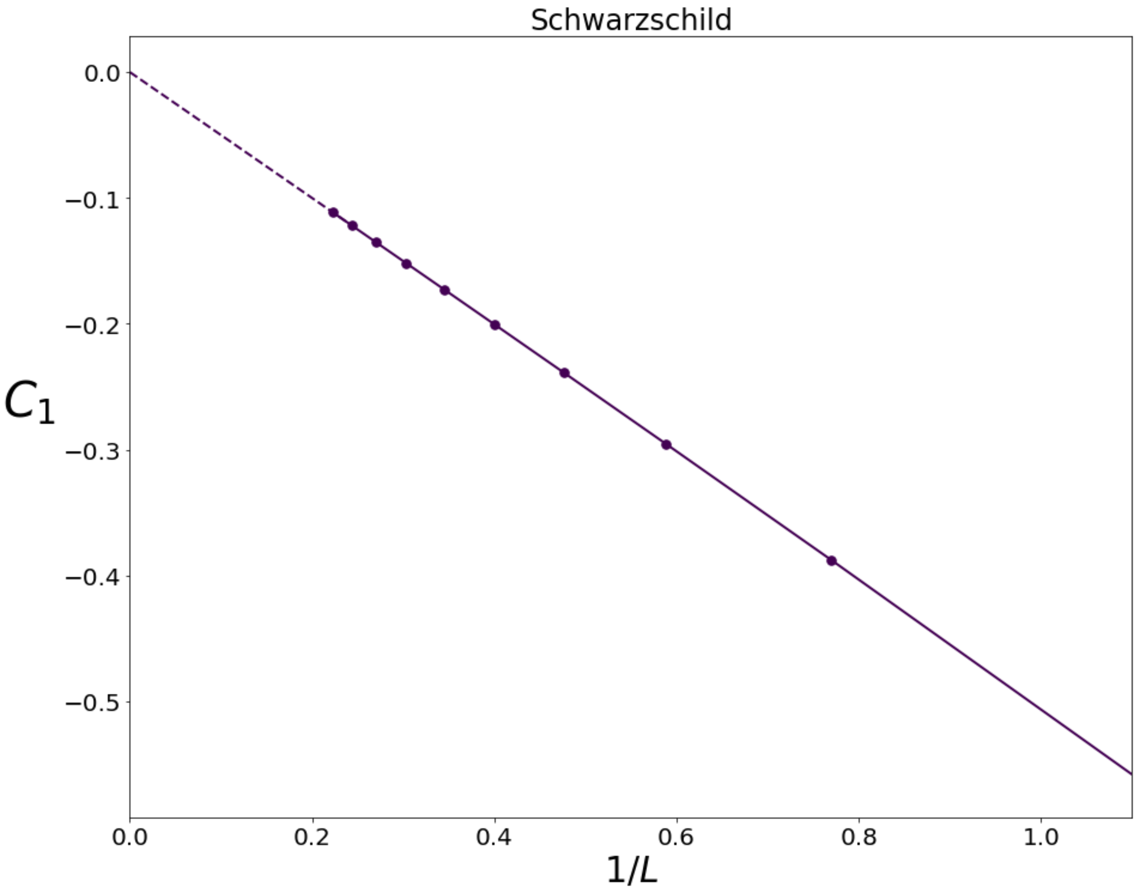
$$C_1 \equiv \lim_{x \to x'} \left(\langle \bar{\psi}(x') \gamma_1(x) \psi(x) \rangle - \text{div.} \right) = \frac{1}{2} \frac{\Omega'(x)}{\Omega(x)}$$

-We will recover this result numerically in (1+1) AdS and Schwarzschild.

$$H \to \frac{i}{2} \left(\frac{1}{l} + \frac{\Omega_{,1}}{2\Omega} \right) \left(\phi_n^\dagger \phi_{n+1} - \phi_{n+1}^\dagger \phi_n \right) + (-1)^n m \Omega_n \phi_n^\dagger \phi_n$$

cases.





$$\Omega(x) = 1/x$$

$$\Omega(x^*) = \sqrt{\left(1 - \frac{2M}{x}\right)}$$

Conclusion

-We can use MPS to simulate interacting QFT in curved backgrounds.

-I demonstrated the extraction of correlation functions at finite separation, and of Hadamard-renormalized quadratic expectation values.

-Please give me a job.

Thanks! (hire me)