World Quantum Gravity

QFT with Quantum Causal Structure

arXiv: 1909.05322

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Abstract

A new approach of quantum gravity based on the world function (invariant distance) is presented. The approach takes a relational scalar quantity as a basic variable, conveniently incorporates matter, and facilitates the study of quantum causal structure of spacetime. The core of the approach is an application of Parker's observation that under a Feynman sum, a gravitational phase

arXiv Moderation über uwaterloo.ca 11.09.2019, 18:45 (vor 13 Tagen)

an Ding

Englisch > Chinesisch (Traditionell) Nachricht übersetzen Deaktivieren für: Englis

Dear arXiv user,

Our moderators suggest that you consider rewriting the title for this submission. They suggest:

"Synge's world function and quantum gravity" or possibly "Quantum gravity formulated in terms of Synge's world function".

Regards,
arXiv moderation

World quantum gravity: An approach based on Synge's world function

Ding Jia (贾丁)[1,2,*]

[1]*Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada*

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Abstract

A new approach of quantum gravity based on the world function (invariant distance) is presented. The approach takes a relational scalar quantity as a basic variable, conveniently incorporates matter, and facilitates the study of quantum causal structure of spacetime. The core of the approach is an application of Parker's observation that under a Feynman sum, a gravitational phase can be traded into the Van Vleck-Morette determinant – a functional of the world function. A formula for quantum amplitudes of processes on quantum spacetime is obtained. Quantum gravity

A simple idea

$$g_{ab} \to \sigma(x, y)$$

world function

Path integral approach

$$A = \sum_{g} A_{QG}[g] \sum_{m} A_M[g, m]$$

QFT

$$A = \int D g_{ab}\ e^{iS_{EH}[g_{ab}]} \int D\phi\ e^{iS_M[g_{ab},\phi]}$$

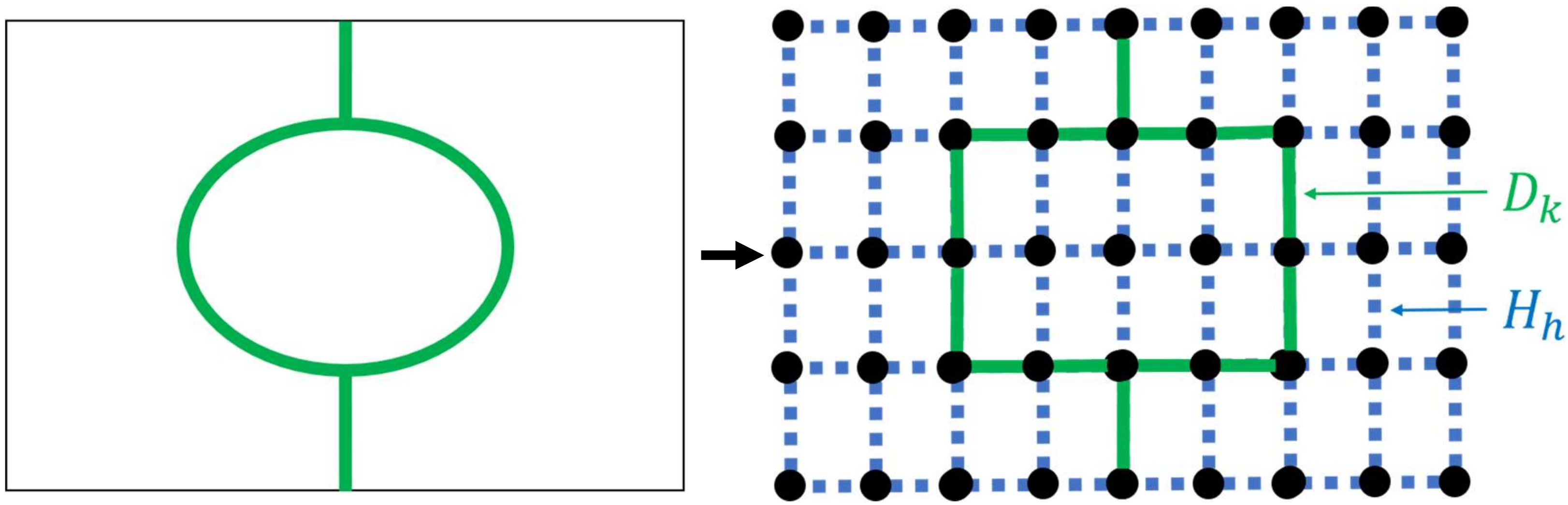
World Quantum Gravity (WQG)

$$A = \sum_{\sigma} A_{QG}[\sigma] \sum_{\gamma} A_M[\sigma, \gamma]$$

VVM determinant for gravity

Worldline formalism for matter

Main result



$\sum_\Gamma \prod_i \int dx_i \frac{V[\Gamma]}{N[\Gamma]} \prod_{k\in\Gamma} G_k$

Non-perturbative, background independent

$$\sum_\Gamma \sum_\sigma \frac{V[\gamma_\Gamma]}{N[\gamma_\Gamma,\sigma]} \prod_{h\notin\gamma_\Gamma} \underbrace{\Delta_h^{3\alpha_h \Delta_h^{-1}}}_{H_h} \prod_{k\in\gamma_\Gamma} \underbrace{\int \frac{dl_k}{(4\pi i l_k)^2} \Delta_k^{3C_k} \exp\left\{ i\frac{\sigma_k}{2l_k} - i m^2 l_k \right\}}_{D_k}$$

What is σ?

$$ds^2 = g_{ab} dx^a dx^b$$

$\sigma(x,y) = \frac{1}{2}\int_x^y ds^2$ - Synge world function

$$g_{ab}(x) = -\lim_{y\to x} \frac{\partial}{\partial x^a}\frac{\partial}{\partial y^b}\sigma(x,y)$$

Why σ?

- Practical

$$\sigma(x,y) = \sigma(x',y')$$

- Matter-friendly

$$A_M \to e^{i\sigma/2l - im^2 l}$$

- Causal structure manifest

$$\sigma \quad =,<,> \quad 0$$

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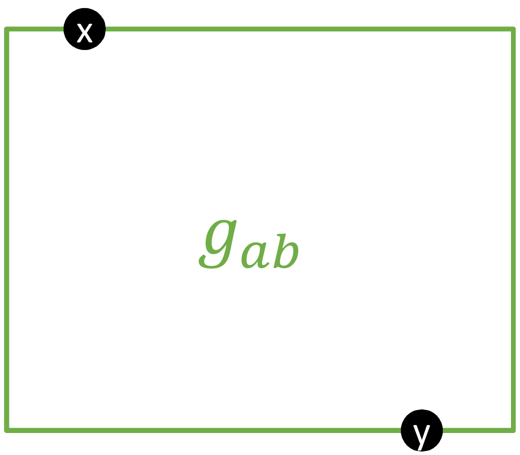
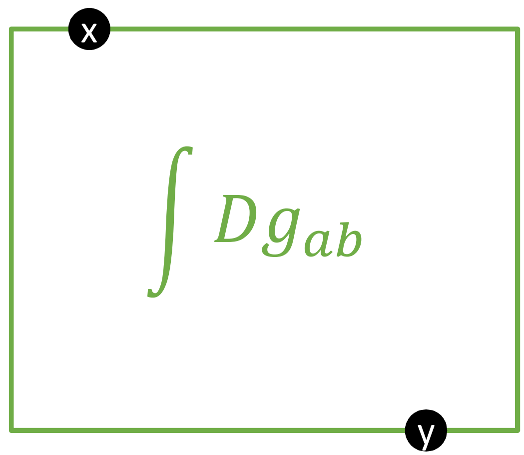
Quantum causal structure - Gravity

Classical



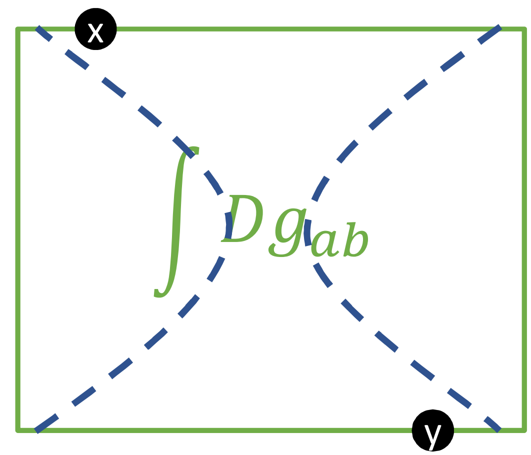
$x \to y$ or
$x \leftarrow y$ or
$x - y$

Quantum



$|x \to y>$
$|x \leftarrow y>$
$|x - y>$

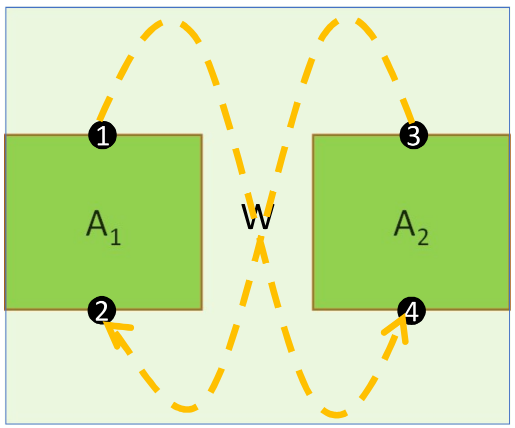
Quantum Time Machines



E.g., Black hole transition

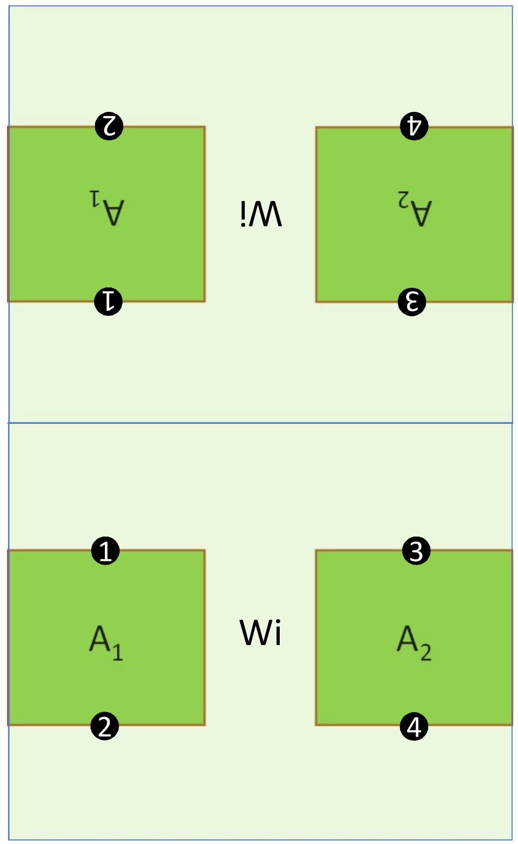
$|x \to y>$
$|x \leftarrow y>$
$|x - y>$

Quantum causal structure - Information



Hardy 2005;
Chiribella, D'Ariano, Perinotti, Valiron 2009;
Oreshkov, Costa, Brukner 2011
...

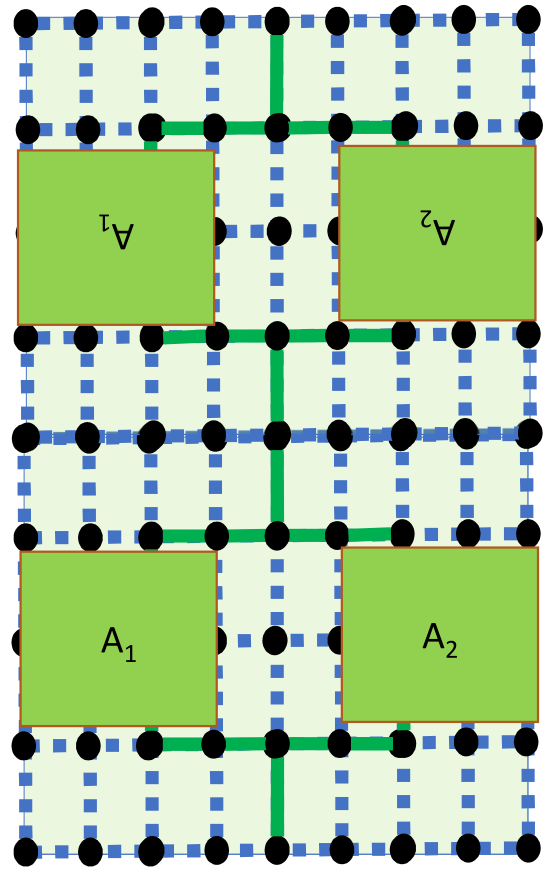
Σ_i



$N : \rho \sum_i \mapsto K_i \rho K_i^\dagger$

Quantum information circuits

Σ_γ



Theory

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$$\sigma \quad =, <, > \quad 0$$

Plan of talk

I. Main formula
(QFT on quantum spacetime)

II. Numerical analysis
(if time permits)

Why σ?

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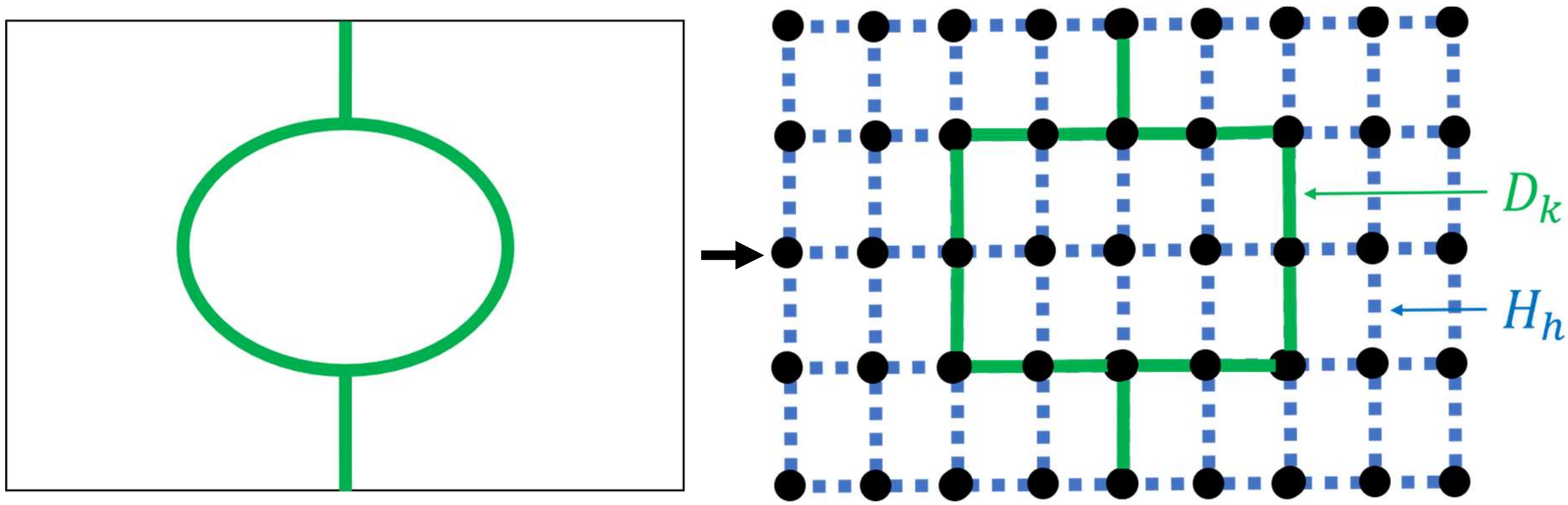
I. Main formula: QFT on quantum spacetime



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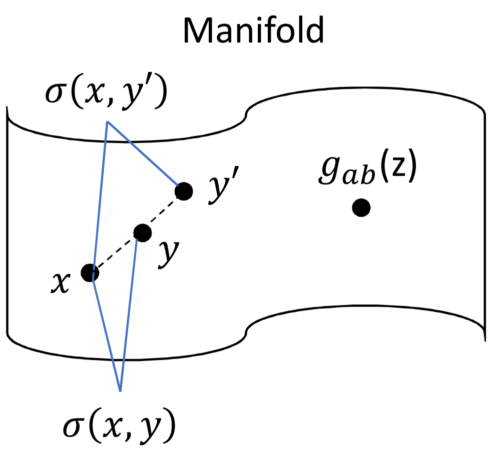
Topological setup

g_{ab}(x) – pointwise defined



$$\sigma(x,y) = \frac{1}{2}(l_y - l_x) \int_{l_x}^{l_y} g_{ab}(z)\frac{dz^a}{dl}\frac{dz^b}{dl}dl$$

pairwise defined

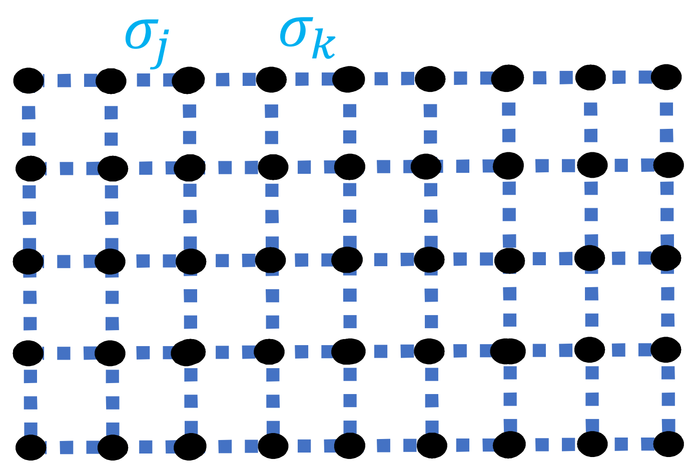
relational

Integral expression

local

Skeleton graph

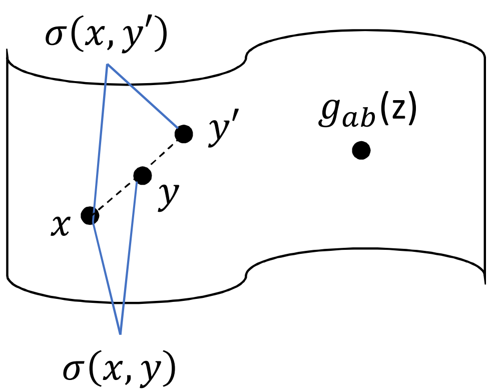
Hypercube+limit



Topological setup

g_{ab}(x) – pointwise defined

Manifold



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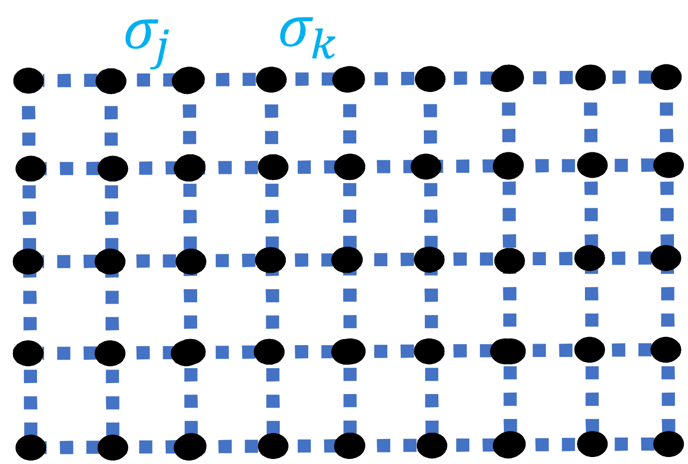
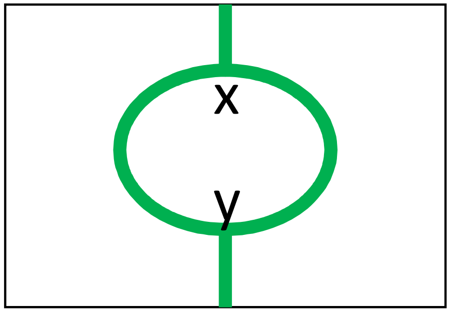


Algorithmic discreteness,
not necessarily fundamental!

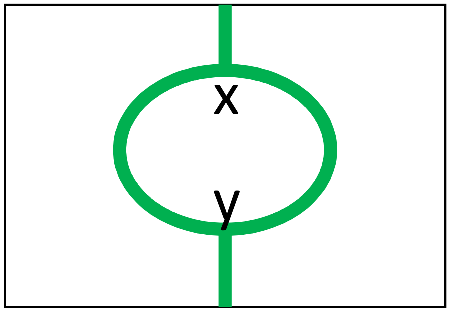
Matter amplitude: A_M



Γ Feynman diagrams

$$(\Box + m^2 + \xi R)\phi(x) = 0$$

Matter amplitude: A_M



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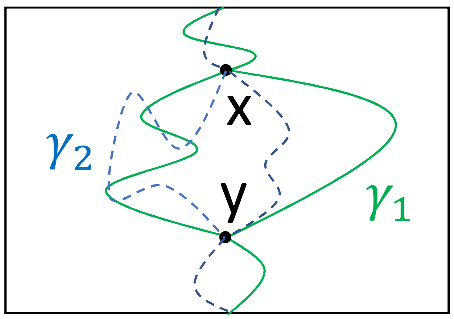

γ correlation diagrams

Feynman 1950;
"Worldline formalism"

$$G(x,y) = i\int_0^\infty \langle x,l|y,0\rangle\, e^{-im^2 l} dl$$

$$\langle x,l|y,0\rangle = \int d[x(l')]\exp\left\{i\int_0^l dl'[\tfrac{1}{4}g_{ab}\tfrac{dx^a}{dl'}\tfrac{dx^b}{dl'} - (\xi - \tfrac{1}{3})R(l')]\right\}$$

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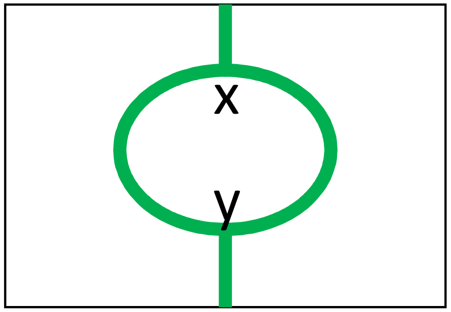

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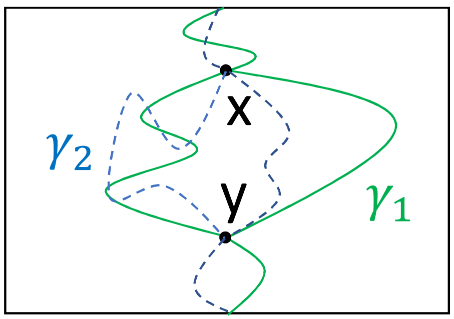

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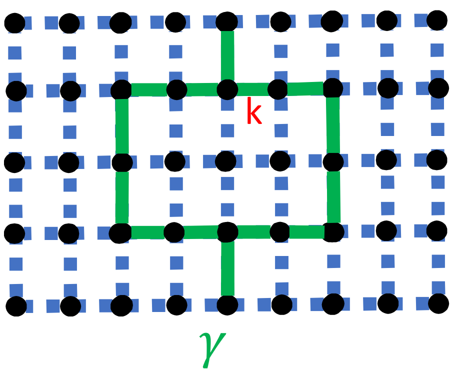

$$A_M[k \in \gamma, g] = \int \frac{dl_k}{(4\pi i l_k)^2} \exp\Big\{ i\frac{\sigma_k}{2l_k} - i(\xi - \frac{1}{3}) R_k l_k - im^2 l_k \Big\}$$

$$A_M[\gamma, g] = \prod_{k\in\gamma} A_M[k \in \gamma, g] V[\gamma]$$

Gravity amplitude : A_{QG}

$$\exp\{i(\frac{\sigma}{2l} + clR)\} \overset{\sum_{\text{path}}}{\longleftrightarrow} (\Delta[\sigma])^{3c} \exp\{i\frac{\sigma}{2l}\}$$

(Parker 1979, Bekenstein & Parker 1981)

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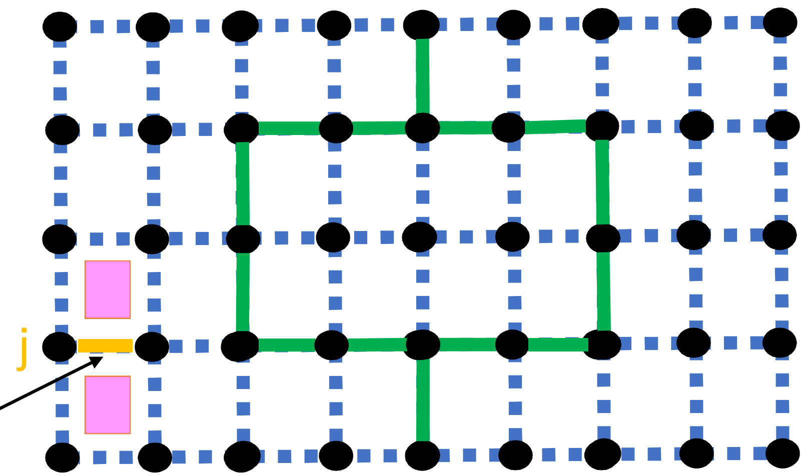
$$c_j \to \alpha_j \Delta_j^{-1}$$
$$l_j \to s_j$$



$$\exp\{i\alpha_j s_j \Delta_j^{-1} R_j\}$$

$$\alpha_j = \alpha \sum_{\{k,m,n\}} s_k s_m s_n$$

$$\exp\{i\bar{\alpha} d^4 x_j \sqrt{-g_j} R_j\}$$

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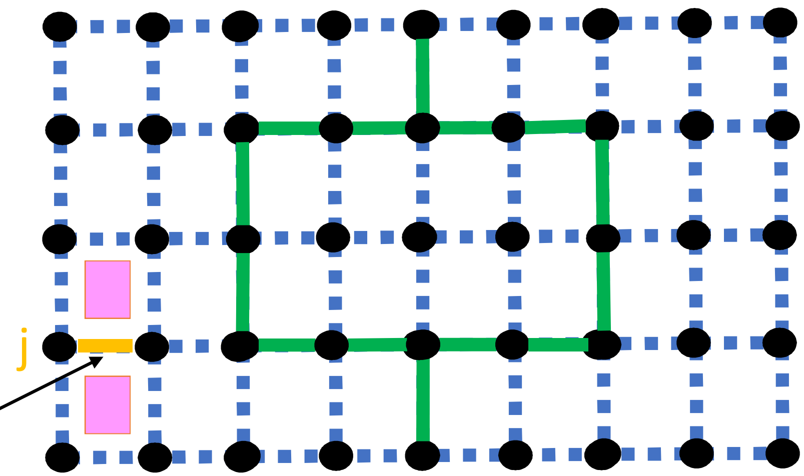

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$$\alpha_j = \alpha \sum_{\{k,m,n\}} s_k s_m s_n$$

$$\exp\{i\bar{\alpha} d^4 x_j \sqrt{-g_j} R_j\}$$

Van Vleck-Morette determinant:

$$\Delta = C\exp\{-\int \theta ds'\}/s^{-d}$$

1. Meaning: Measures curvature by geodesic (de)focusing

(Visser 1993)

curved spacetime geodesic density / Flat spacetime geodesic density

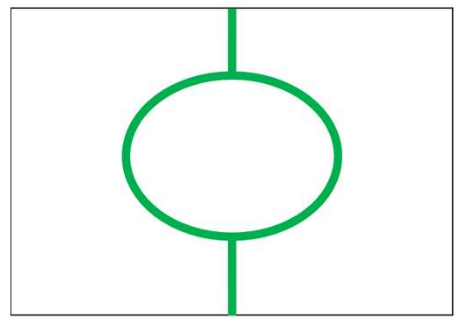
2. World function $\Delta = \Delta[\sigma]$

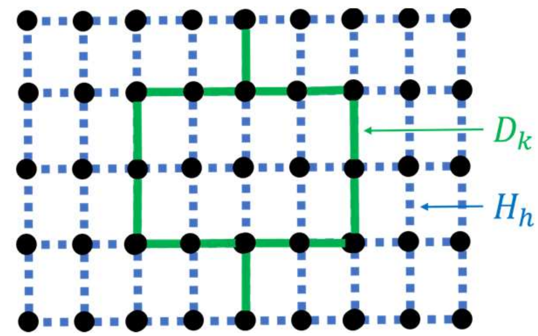
$$s = |2\sigma|^{1/2} \qquad \theta(x) = \frac{\sigma^a{}_a(x,y) - 1}{s}$$

3. Wilson line $\mathcal{P}\exp\{\int A_a dx^a\}$

QFT on quantum spacetime

$$\sum_\Gamma \prod_i \int dx_i \frac{V[\Gamma]}{N[\Gamma]} \prod_{k\in\Gamma} G_k$$





①Difference: Sum over geometry

$$\sum_{[\sigma]} \sum_{\sigma\in[\sigma]}$$

$$\|$$

③ Matter: Propagators broken into pieces

$$\sum_\Gamma \sum_\sigma \frac{V[\gamma_\Gamma]}{N[\gamma_\Gamma,\sigma]} \prod_{h\notin\gamma_\Gamma} \underbrace{\Delta_h^{3\alpha_h \Delta_h^{-1}}}_{H_h} \prod_{k\in\gamma_\Gamma} \underbrace{\int \frac{dl_k}{(4\pi i l_k)^2} \Delta_k^{3C_k} \exp\left\{i\frac{\sigma_k}{2l_k} - im^2 l_k\right\}}_{D_k}$$

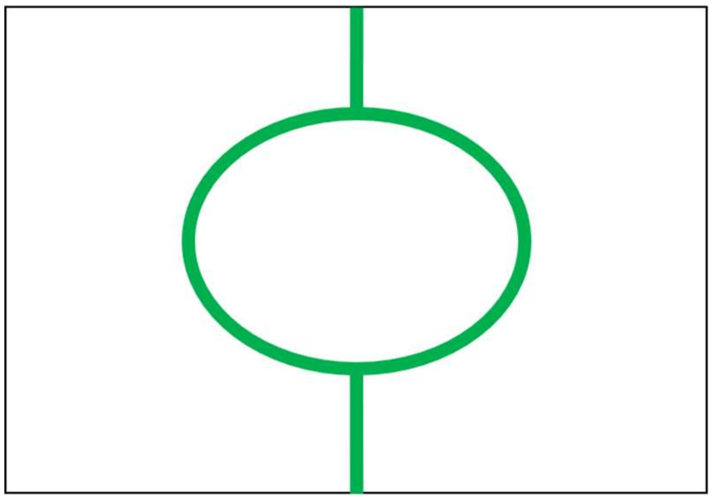
$$C_k = \left(\frac{1}{s_k} + \frac{1}{l_k}\right)\left[\alpha_k s_k \Delta_k^{-1} - \left(\xi - \frac{1}{3}\right) l_k\right]$$

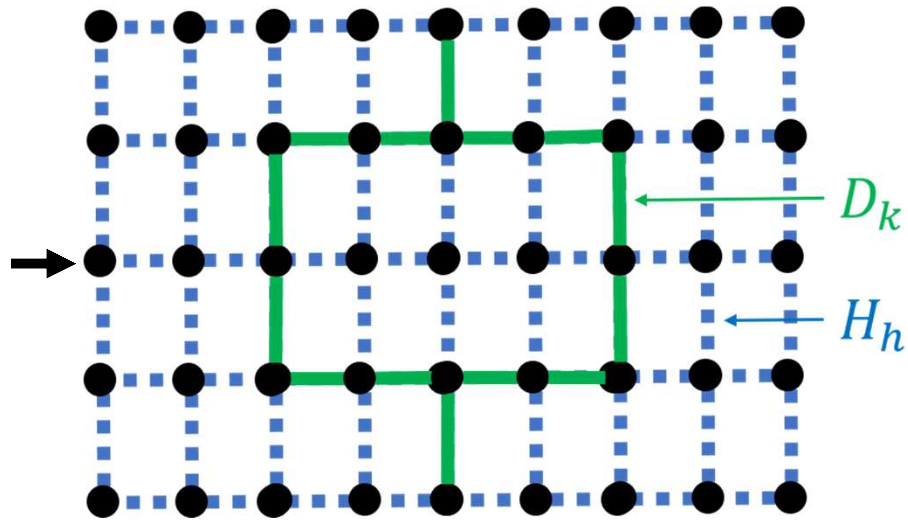
②Gravity: Non-perturbative treatment

II. Numerical analysis (Preliminary)

B \ α	0.	0.2	0.4	0.6	0.8	1.
0.25	4348.37	4814.91	5326.67	5932.13	6650.38	7510.03
0.5	4388.28	4858.77	5375.17	5986.14	6710.92	7578.33
0.75	4430.65	4905.33	5426.67	6043.47	6775.17	7650.82
1.	4481.73	4961.46	5488.75	6112.63	6852.69	7738.29
1.25	4545.96	5032.04	5566.83	6199.53	6950.14	7848.2
1.5	4627.45	5121.55	5665.88	6309.83	7073.77	7987.72
1.75	4730.38	5234.71	5791.02	6449.14	7229.86	8163.81
2.	4870.94	5376.13	5947.45	6623.36	7425.16	8384.1

Main formula





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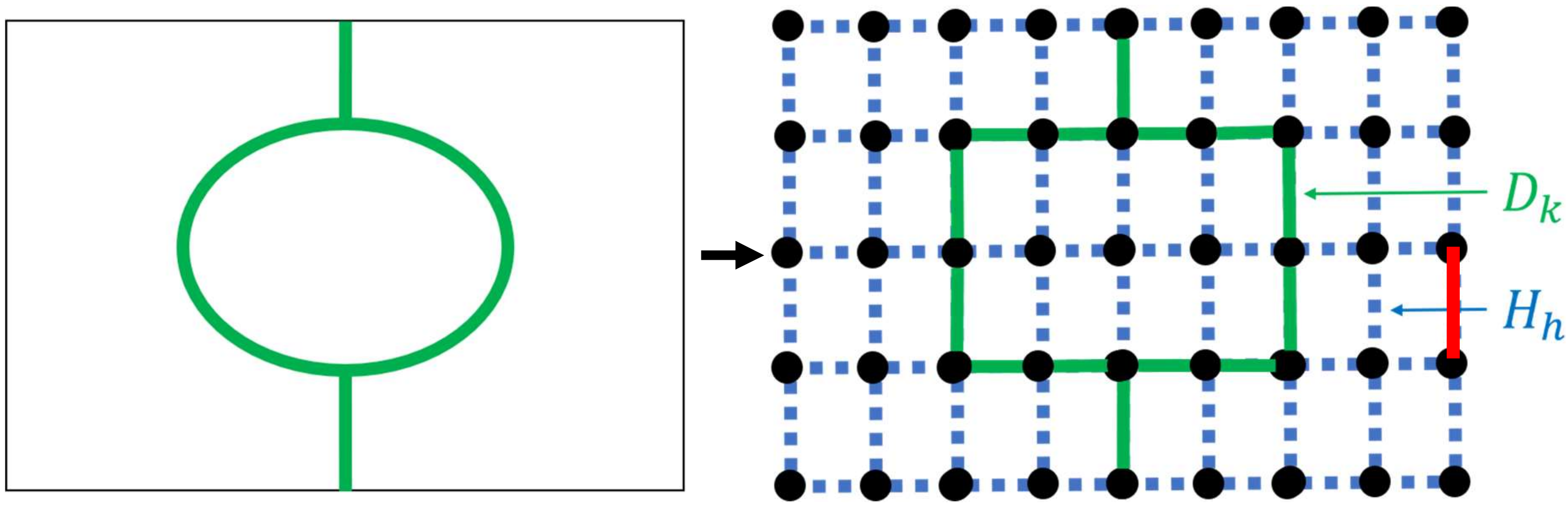

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$$\sum_{\sigma_h} \Delta_h^{3\alpha_h \Delta_h^{-1}} B^{s_h}$$

$$\Delta = C\exp\{-\int \theta ds'\}/s^{-d} \qquad \theta(x) = \frac{\sigma^a{}_a(x,y)-1}{s} \qquad s = |2\sigma|^{1/2}$$

Raychaudhuri $\frac{d\theta}{ds} = -\frac{1}{3}\theta^2 - \bar{\sigma}^2 + \omega^2 - R_{ab}u^a u^b$

Approximations $\bar{\sigma} = \omega = 0;\ \rho := R_{ab}u^a u^b$

$$\theta(s) = \sqrt{3\rho}\cot\left(s\sqrt{\tfrac{\rho}{3}}\right) \qquad \Delta(s,\rho) = \left[\sqrt{\tfrac{\rho}{3}}\ s\ \csc\left(s\sqrt{\tfrac{\rho}{3}}\right)\right]^3$$

$$\int d\rho_h \int d\sigma_h \Delta_h^{3\alpha_h \Delta_h^{-1}} B^{s_h}$$

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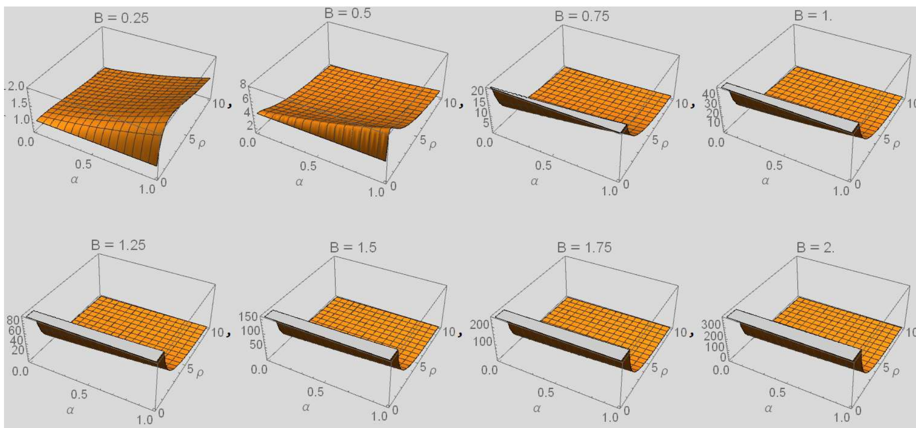
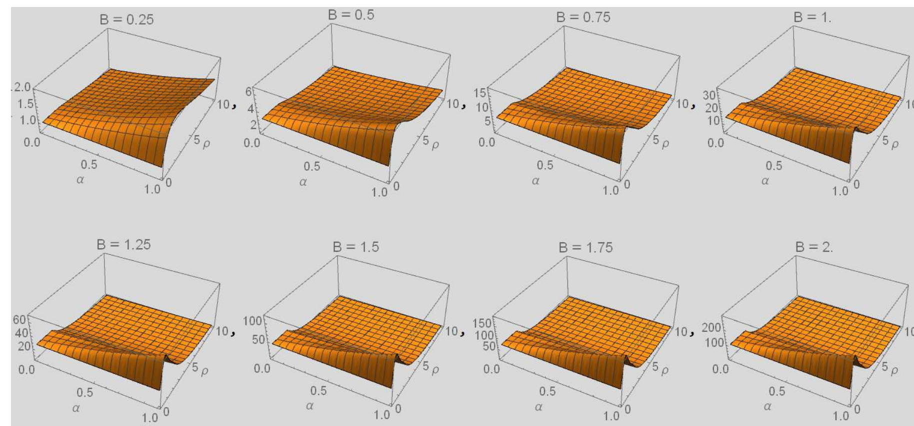
$$\Delta(s,\rho) = \left[\sqrt{\frac{\rho}{3}}\ s\ \csc\left(s\sqrt{\frac{\rho}{3}}\right)\right]^3$$

$$\int_{-a^2/2}^{a^2/2} d\sigma \Delta^{3\alpha\Delta^{-1}} B^s = 2\int_0^a ds\ s\Delta^{3\alpha\Delta^{-1}} B^s$$





$$\int_0^\infty d\rho \int_{-a^2/2}^{a^2/2} d\sigma \Delta^{3\alpha\Delta^{-1}} B^s$$

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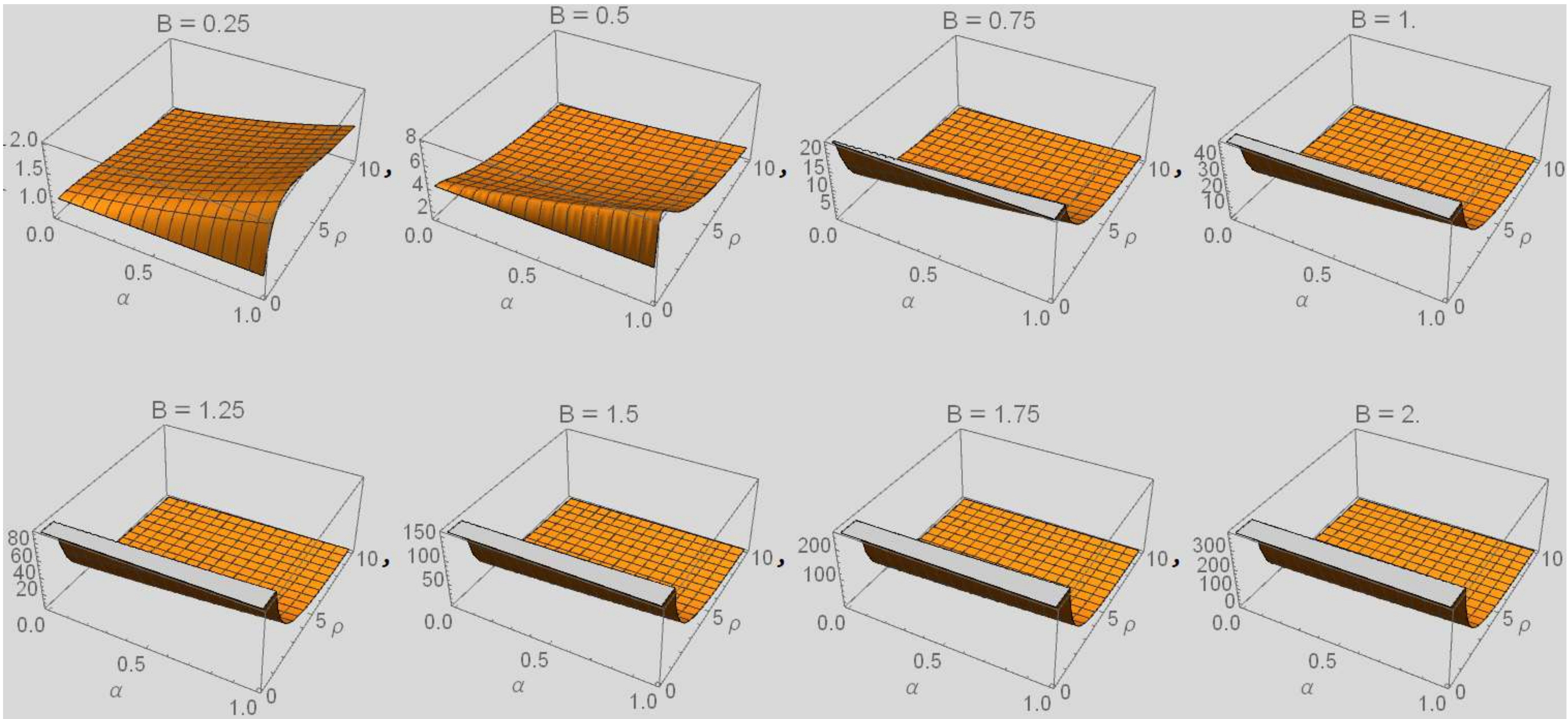
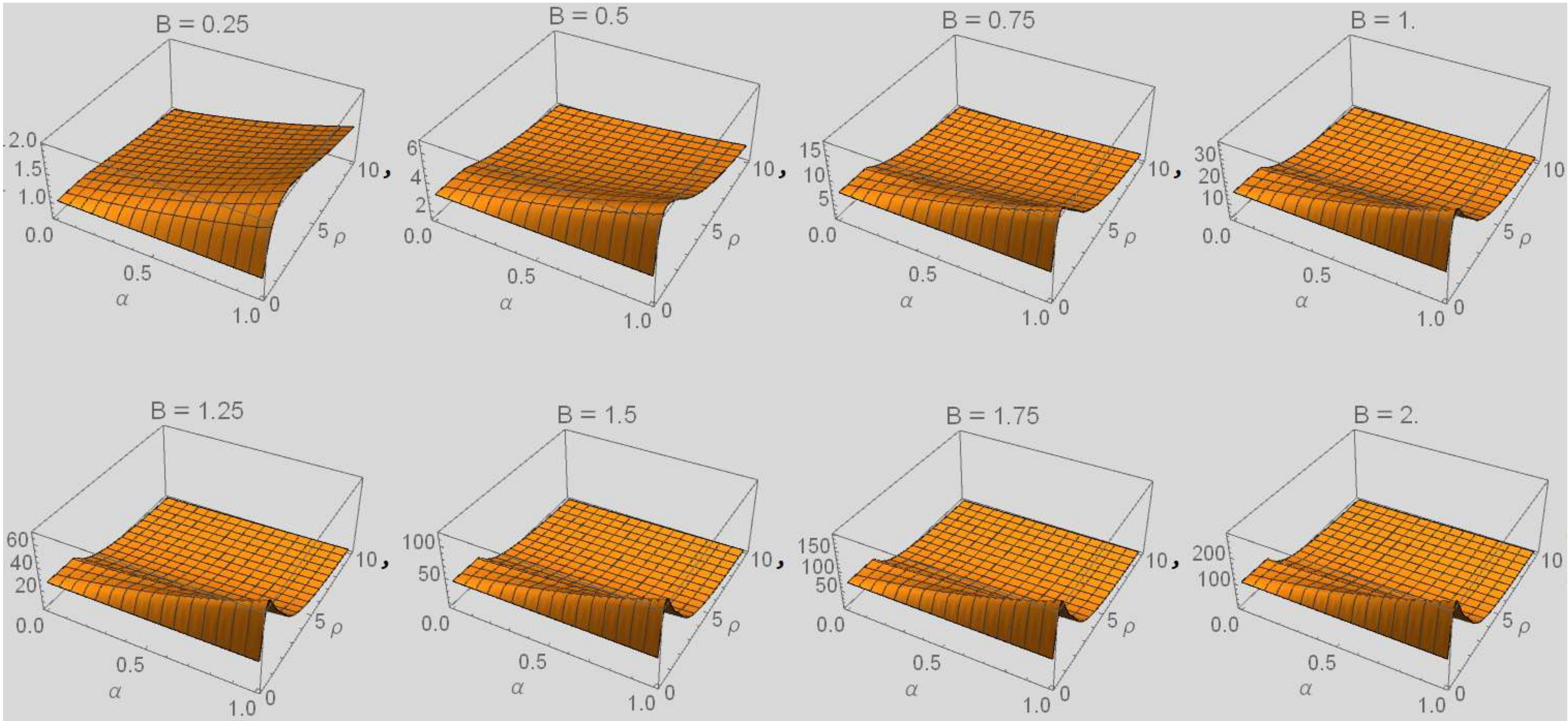


B = 0.25
B = 0.5
B = 0.75
B = 1.
B = 1.25
B = 1.5
B = 1.75
B = 2.
α
ρ


B = 0.25
B = 0.5
B = 0.75
B = 1.
B = 1.25
B = 1.5
B = 1.75
B = 2.
α
ρ

Summary

arXiv: 1909.05322

- A new approach to QG
- Relational variable $\sigma(x, y)$
- Suitable for studying quantum causal structure
- Incorporates matter with ease
- Hopefully practical for calculations. Test by applying to:
  -QG modification of matter QFT (UV regularization by smearing?)
  -Black hole transitions
  -Quantum cosmology

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Thank you!