New Classes of Warp Drive Solutions in General Relativity

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The Alcubierre Drive

$$ds^2 = -c^2 dt^2 + (dx - f(r_s) v_s dt)^2 + dy^2 + dz^2$$

(Alcubierre, 1994)

Shape function: $$f(r_s) = \frac{\tanh(\sigma(r_s + R)) - \tanh(\sigma(r_s - R))}{2\tanh(\sigma R)}$$

$$r_s = \sqrt{(x - v_s t)^2 + y^2 + z^2}$$

Properties:

- **Superluminal**
- **Event horizons**
- **Negative energy (a lot of)**

Energy density: $$T^{00} = -\frac{{f'_r}^2 \rho^2}{4 r_s^2} v_s^2$$

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Warp Drive Research

Most studied directions:

- **Negative energy, e.g. Phenning & Ford (1997), Olum (1998)**
- **Quantum instabilities, e.g. Vollick (2000)**
- **Causality/existence, e.g. Coutant (2012)**
- **Wormholes, e.g. Rahaman (2007)**
- **Time machines, e.g. Amo (2005)**
- **Physical properties, e.g. Natario (2006)**

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Existing Optimizations

- Expanding the space inside the warp (van den Broek, 1999)
- Warp drive without volume deformations (Natario, 2002)
- Time-related modifications (Loup et al. 2001), (Janka, 2007)
- Krasnikov tubes, wormholes, e.g. Krasnikov (1998)
- NASA Eagleworks Drive, e.g. White (2011)

What more can be done?

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Optimising the Shape Function

Original shape function:

$$f(r_s) = \frac{\tanh(\sigma(r_s + R)) - \tanh(\sigma(r_s - R))}{2\tanh(\sigma R)}$$

Optimising the total energy:

$$E = \int \mathrm{d}^3x \sqrt{-g}\, T^{00} \rightarrow \min$$

Optimised shape function:

$$\bar{f}(r_s) = \frac{A}{r_s} + B$$

Relation to point-mass geometry?

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Deforming the Alcubierre Drive

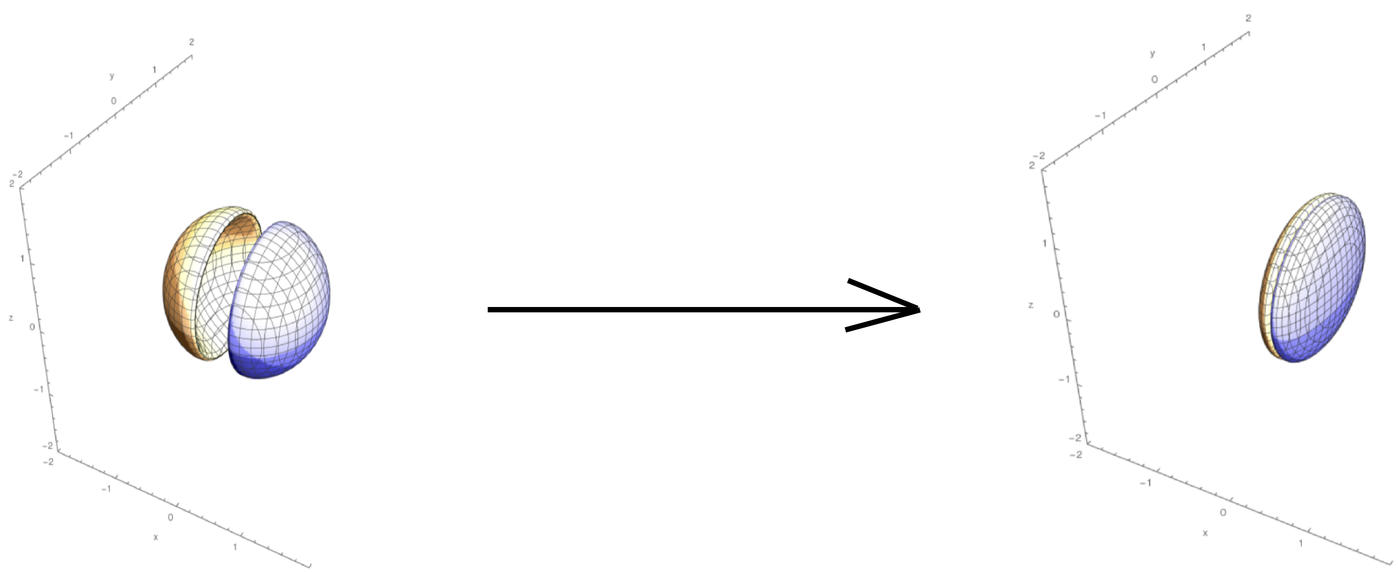
$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + (\mathrm{d}x - f(r_s)v_s\mathrm{d}t)^2 + \mathrm{d}y^2 + \mathrm{d}z^2$$

Cylindrical coordinates: $f(r_s) \longrightarrow f(x - x_s, \rho)$

Generally-shaped Alcubierre drive

New energy density: $T^{00} = -\frac{f_r'^{\,2}\rho^2}{4r_s^2}v_s^2 \longrightarrow T^{00} = -\frac{1}{4}f_\rho'^{\,2}v_s^2$

Optimized shape function: $\bar{f}(\rho) = A\ln\rho + B$



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What is a Warp Drive?

Alcubierre: $$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + (\mathrm{d}x - f(r_s)v_s\mathrm{d}t)^2 + \mathrm{d}y^2 + \mathrm{d}z^2$$

Inside: $$f = 1 \quad \mathrm{d}s^2 = -c^2\mathrm{d}t_{\mathrm{loc}}^2 + \mathrm{d}x_{\mathrm{loc}}^2 + \mathrm{d}y_{\mathrm{loc}}^2 + \mathrm{d}z_{\mathrm{loc}}^2$$

Outside: $$f = 0 \quad \mathrm{d}s^2 = -c^2\mathrm{d}t_{\infty}^2 + \mathrm{d}x_{\infty}^2 + \mathrm{d}y_{\infty}^2 + \mathrm{d}z_{\infty}^2$$

Coordinates:

$$\mathrm{d}t_{\mathrm{loc}} = \mathrm{d}t = \mathrm{d}t_{\infty}$$

$$\mathrm{d}x_{\mathrm{loc}} = \mathrm{d}(x_{\infty} - x_s(t_{\infty}))$$

$$\mathrm{d}y_{\mathrm{loc}} = \mathrm{d}y = \mathrm{d}y_{\infty}$$

$$\mathrm{d}z_{\mathrm{loc}} = \mathrm{d}z = \mathrm{d}z_{\infty}$$

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Generating New Classes

When generalised:

$$\mathrm{d}s^2 = -c^2\mathrm{d}t_\infty^2 + (\mathrm{d}x_\infty(1-f) + f\mathrm{d}x_{\mathrm{loc}})^2 + \mathrm{d}y_\infty^2 + \mathrm{d}z_\infty^2$$

$$x_{\mathrm{loc}} = x_{\mathrm{loc}}(x_\infty, t_\infty)$$

- **Same possible for other coordinates**
- **With individual shape functions**

Can choose the internal spacetime!

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Lorenz Drive

Impose:

$$\mathrm{d}t_{\mathrm{loc}} = A(\mathrm{d}t_\infty - \frac{v_s \mathrm{d}x_\infty}{c^2})$$

$$\mathrm{d}x_{\mathrm{loc}} = A(\mathrm{d}x_\infty - v_s \mathrm{d}t_\infty)$$

Uniquely diagonal

New metric:

$$\mathrm{d}s^2 = -c^2 F^2 \mathrm{d}t^2 + F^2 \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$$

$$F^2 = 1 + 2f(1-f)(\gamma - 1)$$

Energy density:

$$T^{00} = -\frac{1}{\rho F^3}(\rho F'_\rho)'$$

Positive- and negative-energy regions

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Yet More Classes

Connecting metrics:

$$\mathrm{d}s^2 = (1-f)\mathrm{d}s_\infty^2 + f\mathrm{d}s_{\mathrm{loc}}^2$$

Alternative version of the Alcubierre drive:

$$T^{00} = -\frac{1}{4}{f'_\rho}^2 v_s^2 \longrightarrow T^{00} = -\frac{{f'_\rho}^2 v_s^2}{4\left(1-(1-f)f\frac{v_s^2}{c^2}\right)^2}$$

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Signature-Switch Drives

- **Arbitrary internal spacetimes**
- **But a fixed way of connecting**

Extreme example:

$$\mathrm{d}s^2_\infty = -c^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$$

$$\mathrm{d}s^2_{\mathrm{loc}} = c^2\mathrm{d}t^2 + (\mathrm{d}x - v_s\mathrm{d}t)^2 + \mathrm{d}y^2 + \mathrm{d}z^2$$

Energy density: $T^{00} \longrightarrow$ **Complexity**

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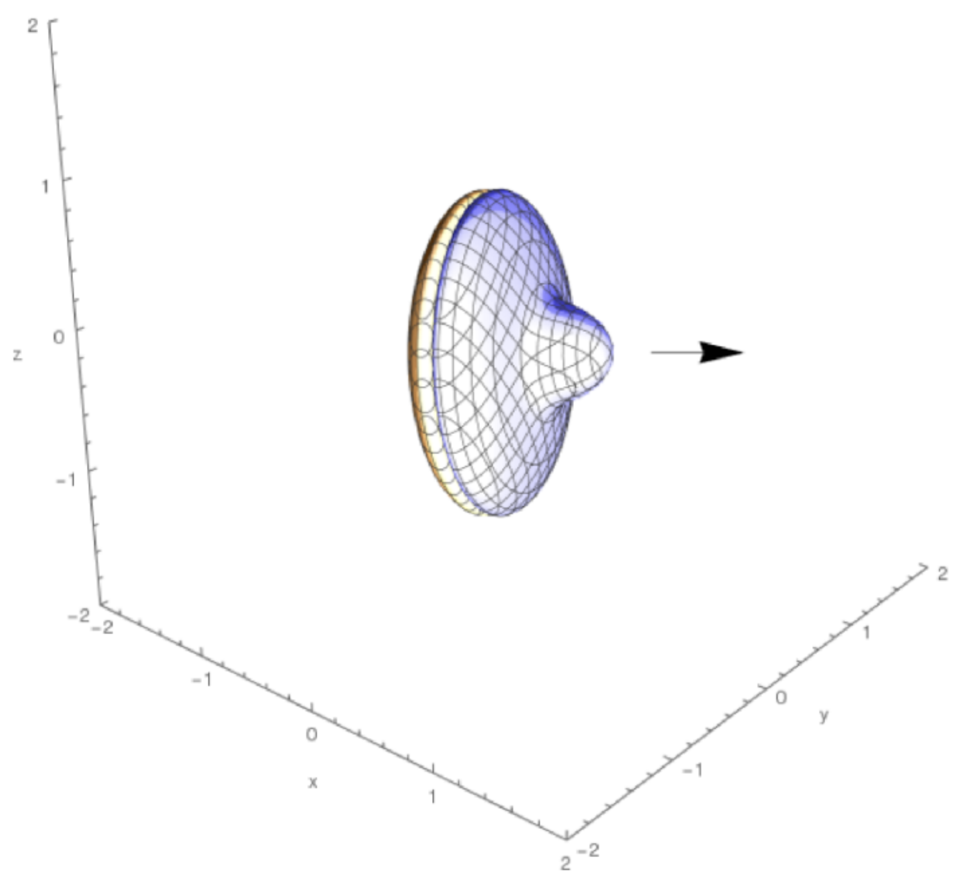
Realising the Penrose Process

Taking advantage of the metric:

$$\mathrm{d}s^2 = (1-f)\mathrm{d}s^2_{\infty} + f\mathrm{d}s^2_{\mathrm{loc}}$$

$$\mathrm{d}s^2_{\mathrm{loc}} = g\mathrm{d}s^2_{\mathrm{ergo}} + (1-g)\mathrm{d}s^2_{\mathrm{flat}}$$



Penrose process

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Summary

Warp drives:

- **Many optimizations possible**
- **Arbitrary internal spacetimes**
- **Energy optimizations**

Applied Physics Institute:

- **Comments welcome**
- **Collaborations welcome**
- **Positions open**

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