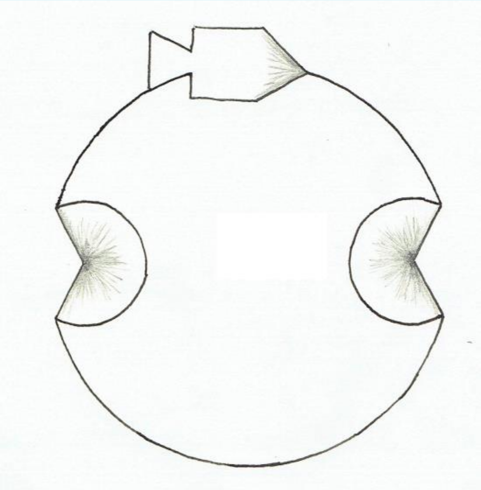
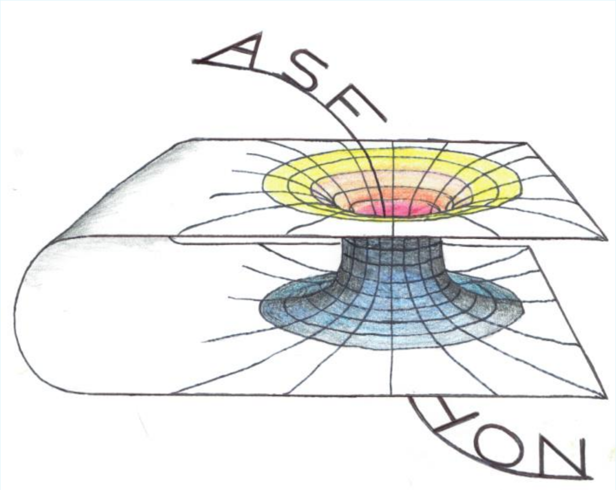
On closed timelike curves, cosmic strings and conformal invariance

Reinoud Jan Slagter

Univ of A'dam The Netherlands

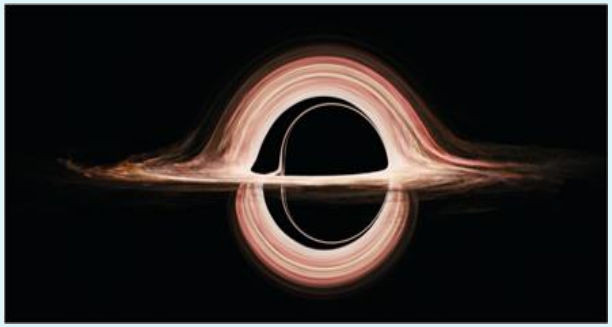
Asfyon, **Astronomisch fysisch onderzoek Nederland**





Two most famous compact objects in GRT
i.e. stationary axially symmetric

The Kerr solution



The spinning cosmic string

exact sol. with correct phys properties:

- ► No-rotation: Schwarzschild solution
- ► Asymptotical flat
- ► No CTC's [at least hidden]

<u>Precursor:</u>

van Stockum solution: unusual behavior [rotating dust cylinder]

- ► Problems matching interior to exterior
- ► Asymptotic conical!
- ► periodic time coordinate
- ► CTC's ??

what is the status of these objects concerning CTC's ?

Bonner (2008) on CTC's:

I believe there is an urgent need to find a convincing physical interpretation of CTC. They can no longer be dismissed as curiosities occurring in non-physical solutions. We now know that there are simple physical situations, such as that of the charge and the magnet, within the terms of reference of general relativity, of which the theory as currently understood does not give a satisfactory account.

Problematic causality issues (possible not hidden behind horizon):

A. Extreme BTZ- black hole [(2+1)-dim.]
B. Spinning Cosmic strings ►►► U(1) scalar-gauge field
C. **In general**: approaching the Planck scale
complementarity by conformal invariance :
will causality and locality survive?? [**'t Hooft 2017**]

Historical attempts: **Gödel, van Stockum cylinder, rotating dust-cylinder, Gott-spacetime**,....
[**all unphysical**]

At the front: **There is no black hole interior ?**

t' Hooft-conference on blackhole complementarity and AdS [**july 2019**]

Interesting discussions:

't Hooft ; Maldacena ; Strominger Susskind ; Verlinde ; Duff ; Rovelli .....

ER =EPR ?

Rebirth of the wormhole solution?

Why scalar-gauge field?

The abelian scalar(Higgs) field with gauge group U(1) has **lived up to its reputation**!

1. As **order parameter** in super conductivity: Ginzburg-Landau model

2. The U(1)-scalar-gauge field in **standard model** of particle physics (**Higgs mech**.)

3. The special ϕ^4 self interacting **Nielsen-Olesen vortex** solution. Gives insight in class. gauge theories Yang-Mills-Higgs equations for monopoles.

4. Needed in **inflationairy** model [horizon-flatness problems solved?]

5. **General Relativistic-cosmic** string solution

6. **Super-massive cosmic strings**: can build-up huge mass in the extra-dimension of the bulk spacetime (warped spacetimes: hierarchy problem solved)

7. ►►► **quasar alignment? Quasar-confinement for large red-shift must be of primordial origin** [**Slagter, IJMP D, 2018**]

The axially symmetric spacetimes

$$ds^2 = -e^{-2f}(dt - J\,d\varphi)^2 + e^{2f}\left[L\,d\varphi^2 + e^{2\gamma}\left(dr^2 + dz^2\right)\right]$$

f, J, L and γ functions of (r,z) **Note:** **radiating spacetimes:** $t \to iz,\ z \to it,\ J \to iJ$

Asymptotically:

$$ds^2 = -\left(1 - \frac{2M}{r} + A_0)dt^2\right) + \left(4\epsilon_{ijk}\frac{S^j x^k}{r^3} + A_i\right)dt dx^i \quad + \left[\left(1 + \frac{2M}{r}\right)\delta_{ij} + A_{jk}\right]dx^j dx^k$$

Weyl sol:

$$ds^2 = -e^{2\psi}dt^2 + e^{-2\psi}\left[r^2\,d\varphi^2 + e^{2\gamma}\left(dr^2 + dz^2\right)\right]$$

$$\nabla^2\psi = 0, \qquad \gamma_z = 2r\sigma_r\sigma_z, \qquad \gamma_r = r\left(\psi_r^2 - \psi_z^2\right)$$

$$\psi = \frac{m}{2z_0} ln\left[\frac{R^+ + R^- - 2z_0}{R^+ + R^- + 2z_0}\right], \quad \gamma = -\frac{m^2}{2z_0^2} ln\left[\frac{4R^+R^-}{(R^+ + R^-)^2 - 4z_0^2}\right], R^\pm = \sqrt{r^2 + (z \pm z_0)^2}$$

Exterior of a thin uniform rod, density ~ δ, $-z_0 < z < z_0$. Correct asym form: $\psi \sim 1 - \frac{2m}{r} + \frac{2m^2}{r^2} + \cdots$.

Schwarzschild sol:

$$m = z_0, \quad transformation: \qquad r = \sqrt{\rho^2 - 2z_0\rho}\,sin\theta, \quad z = (\rho - z_0)cos\theta$$

$$\rho = z_0 + \frac{1}{2}(R^+ + R^-), \qquad cos\theta = \frac{1}{2z_0}\frac{1}{2}(R^+ - R^-)$$

Kerr solution: $J \neq 0$ [see any textbook]

The axially symmetric spacetimes

Papapetrou sol: $ds^2 = F(dt - W d\varphi)^2 - \frac{r^2}{F} d\varphi^2 - e^{\mu}(dr^2 + dz^2)$

$$F = \frac{1}{\alpha \cosh\left(\frac{z}{(z^2+r^2)^{3/2}}\right) - \beta \sinh\left(\frac{z}{(z^2+r^2)^{3/2}}\right)} \qquad W = -\frac{\sqrt{\alpha^2-\beta^2} r^2}{(z^2+r^2)^{3/2}}$$

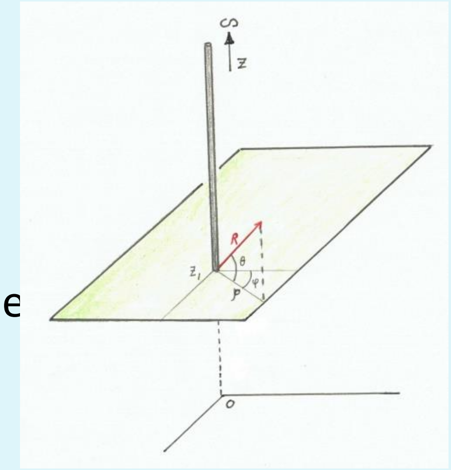
Has the correct asymp form $F \approx \frac{1}{\alpha} + \frac{\beta z}{\alpha^2 r^3} - \frac{3}{2}\frac{\beta z^3}{\alpha^2 r^5} + \cdots$. However no term $\sim \frac{1}{r}$ (no mass)

Lewis-Van Stockum sol (rotating dust cylinder).
- Not correct asymp. form
- Manifestly CTC's

Semi infinite line-mass [SILM]:

$$\psi = c_1 ln\left[\epsilon(z-z_1) + \sqrt{(z-z_1)^2 + r^2}\right] + \ln(C)$$

$$\gamma = 2c_1^2 ln\left[\frac{\varepsilon(z-z_1}{2\sqrt{((z-z_1)^2+r^2}} + \frac{1}{2}\right] + \ln(EC)$$



Ricci-flat; if $E = \frac{1}{C}$ then $\lim_{r\to 0}\gamma = 0$ then C can be transformed away e

Flat for $c_1 = 0$ and $c_1 = \frac{1}{2}$

E,C constant and $\varepsilon = \pm 1$; c_1 *related to the mass*

Infinite line-mass [ILM]:

Why are axially symmetric spacetime [with U(1) –Higgs] so interesting?

A. **"Kerr "-spacetime:** ► $ds^2 = -(dt - Jd\varphi)^2 + b^2 d\varphi^2 + dr^2 \ [+dz^2]$

There is no structure in z-direction: so suppress

B. **(2+1)- dim** spacetime: **local flat**, but **CTC's** for **b < J**

- Einstein: (wire-approx) **angle deficit** and J=const.

- planar "gravity" $ds^2 = -(dt - J_0 d\varphi)^2 + (1-4G\mu)^2 r^2 d\varphi^2 + dr^2$

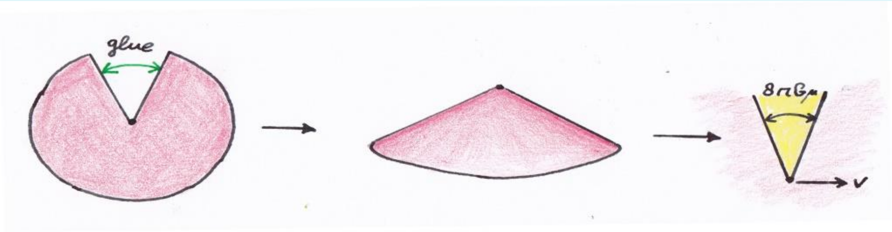
  transf to Minkowski: .... $+ r^2 d\bar{\varphi}^2 \quad 0 < \bar{\varphi} < (1-8G\mu)2\pi$

- **massive spinning point-source ("cosmon")** **mass density** μ **and** **intrinsic spin** J_0
- CTC' for $r_s < \frac{J_0}{(1-4G\mu)}$ **[can one confine the source within a small enough region?]**
- distributional enery momentum $T^{tt} \sim 4G\mu\delta(r)^2 \quad T^{ti} \sim J\varepsilon^{ij}\partial_j\delta(r)^2$
- if one tries to **hide** the string by $t \to t - J\varphi$: helical time coordinate: CTC's **everywhere**!

C. **where do we need these properties?** Quantum gravity!! **Planar gravity fits in very well!**

$R_{\mu\nu} = 0$ but we have point-masses! [quantized version: **'t Hooft 2002**,..]

mass is a local topological defect proportional to the wedge cut out the 2-plane.



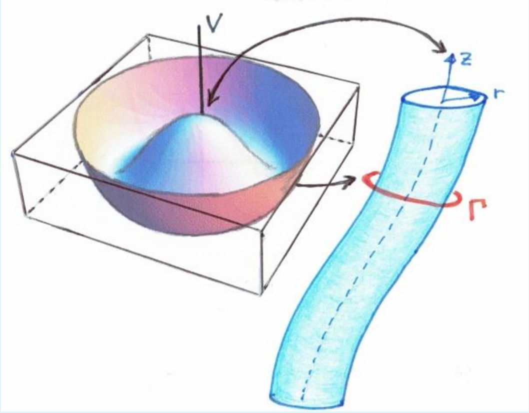
"Cosmon": z suppressed

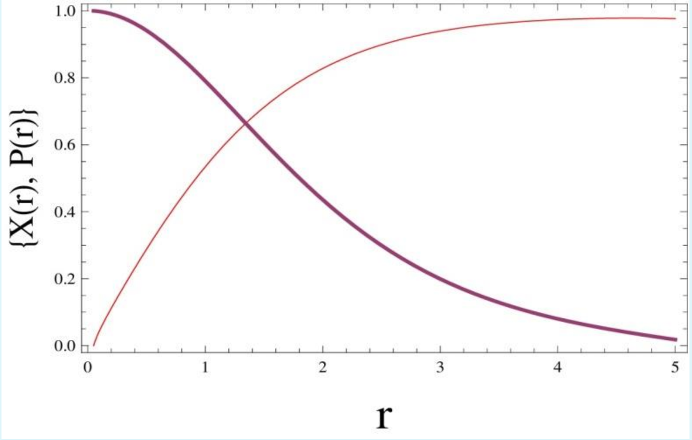
When "lifting-up" to (3+1): ► infinite line-mass no longer tenable [**Geroch et al.1987**]

► the **self-gravitating cosmic string** necessary with 2 parameters $\alpha = \frac{e^2}{\beta}$ and η

$$G_{\mu\nu} = \kappa^2{}_4 T_{\mu\nu} \qquad D_\mu D^\mu \Phi - 2\frac{\partial U}{\partial \Phi^*} = 0 \qquad \nabla^\mu F_{\mu\nu} - \frac{1}{2} ie[\Phi(D_\nu\Phi)^* - \Phi^*(D_\nu\Phi)] = 0$$

► The only physical acceptable mass distribution will be the U(1) scalar-gauge field.





So compactifying one of the space coordinates runs into problems.

Now: ► **Lifting procedure via holographic principle ?**

► **use conformal invariant Higgs gravity model !! [Slagter, Dustin, JHEP, 2019]**

The (2+1) dim black hole connection

In (2+1)-dimensional gravity: The **Banados-Teitelboim Zanelli** (BTZ) black hole:

Einstein: $G_{\mu\nu} = -\frac{1}{l^2} g_{\mu\nu}$ **l = length scale where curvature sets in**

$$ds^2_{BTZ} = \left(8GM - \frac{r^2}{l^2}\right) dt^2 + \frac{1}{\frac{16G^2J^2}{r^2} + \frac{r^2}{l^2} - 8GM} dr^2 - 8GJtd\varphi + r^2 d\varphi^2$$

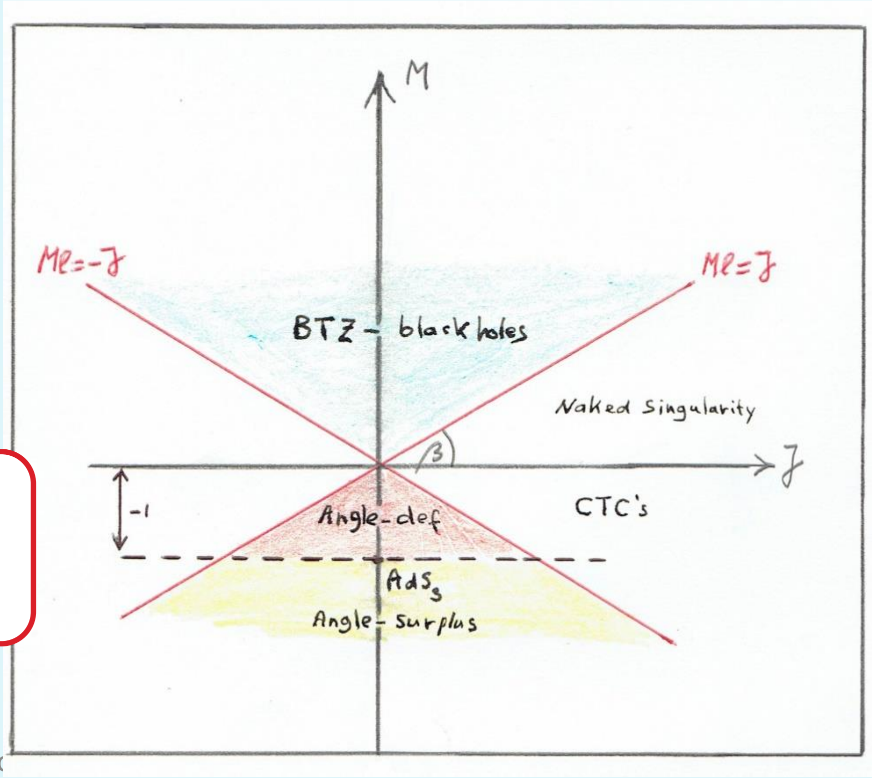
There is an inner and outer horizon and ergocircle

► M=-1/8G, J=0: global AdS_3

► $l \to \infty$: Killing horizon and surface gravity:
Hawking temperature

► **Coord transf**: $r^2 = \frac{2GJ^2}{M} - 8GM\bar{r}^2 \quad [\bar{r} \neq \frac{J}{2M}]$

$$ds^2 = -\left(\sqrt{8GM}dt - \sqrt{\frac{2G}{M}}Jd\varphi\right)^2 + d\bar{r}^2 + 8GM\bar{r}^2 d\varphi^2$$

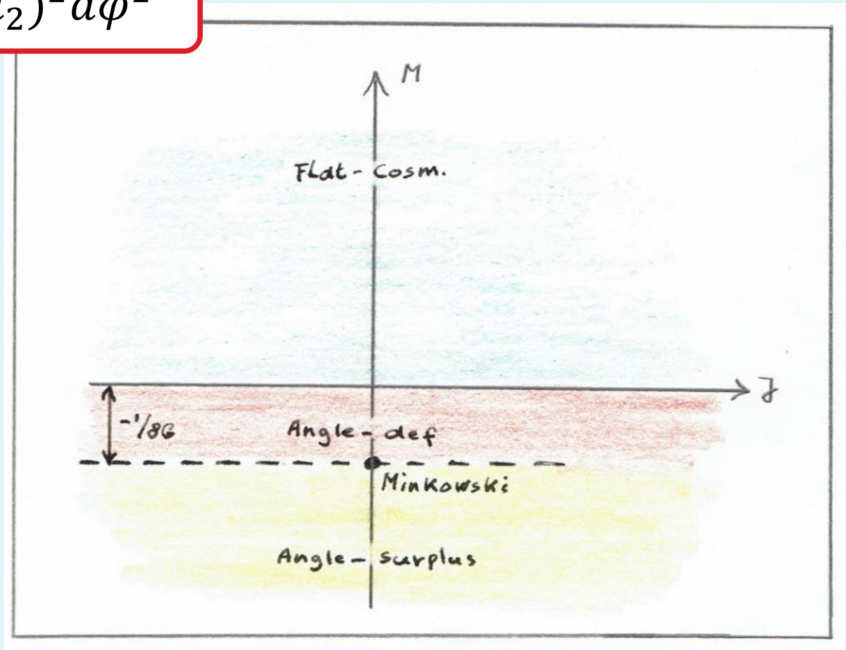


Just the **spinning particle** solution!

Remember: cosmic string solution asymptotically:

$$\boxed{ds^2 = -e^{a_1}(dt^2 - dz^2) + dr^2 + e^{-2a_1}(kr + a_2)^2 d\varphi^2}$$

a_i integration const, but k [mass] determined by field eq., so by the string variables



We already noticed:
there is an obscurity by defining mass M by surface charges associated to the 2 killing vectors
And:
One cannot ignore the interior of the spinning "object" and it will not consists of "ordinary matter"

Motivations for Conformal Invariant [CI] Gravity

1. **Mainly quantum-theoretical: opportunity for a renormalizable theory with preservation of causality and locality [alternative for stringtheory?]**

note: *"formulating GR as a gauge group was not fruitful", so "add" CI to gauge*

2. **Formalism for disclosing the small-distance structure in GR**

Note: *"there seems to be no limit on the smallness of fundamental units in one particular domain of physics, while in others there are very large scales and time scale"*

consider: **local exact CI, spontaneously broken just as the Higgs mechanism**

3. **CI can be used for** *"black hole complementarity"* **and information paradox [related to holography ['t Hooft** 1993, 2009**]**

4. **Alternative to dark energy/matter issue** [**Mannheim**, 2017];
   Construct traceless $T_{\mu\nu}$ [needed for CI: particles massless] and use spontaneous symmetry breaking!

5. **Explore issues such as "trans-Planckian" modes in Hawking radiation calculation and the nature of "entanglement entropy" [ER=EPR?]**

Example: warped 5D model: dilaton from 5D Einstein eq [**Slagter**, 2016]

Some results of Conformal Invariance

- ► CI in GR should be a ***spontaneously broken exact symmetry***, just as the Higgs mechanism
- ► **One splits the metric:** $\boxed{g_{\mu\nu}(x) = \omega(x)^2 \tilde{g}_{\mu\nu}(x)}$ $\tilde{g}_{\mu\nu}$ **the "unphys. metric"**

treat ω and scalar fields on equal footing!

- ► CI is well define on Minkowski: null-cone structure is preserved.
- ► If $\tilde{g}_{\mu\nu}$ *is (Ricci?) flat*: ω is unique (QFT is done on flat background!)
- ► If $\tilde{g}_{\mu\nu}$ *is non-flat*: additional gauge freedom: $\tilde{g} \rightarrow \Omega^2 \tilde{g},\ \omega \rightarrow \frac{1}{\Omega}\omega,\ \ \Phi \rightarrow \frac{1}{\Omega}\Phi, \ldots \ldots$

  [no further dependency on Ω, ω]

  SO: can we generate $\tilde{g}_{\mu\nu} = \Omega^2 \eta_{\mu\nu}$? ***I will present 2 examples*** (see next)

- ► **conjecture**: avoiding *anomalies* we generate constraints which will determine the physical constants such as the *cosmological constant*
- ►Consider conformal component of metric as a ***dilaton*** (ω)with only renormalizable interactions.
- ► **Small distance behavior** (ω→0) regular behavior by imposing *constraints* on model
- ► **Spontaneously breaking:** *fixes all parameters* (mass, cosm const,…) **['t Hooft, 2015]**

" In quantum field theory we work on a flat background. Then ω is unique.
On non-flat background: sizes and time stretches and become ambiguous"

Some results of Conformal Invariance

► Dilaton field ω need to be ***shifted to complex*** contour (Wick rotation) to ensure that ω has the same *unitary* and positivity properties as the scalar field. [**for our 5D model**: ω has ***complex solutions***!]

► In canonical gravity: ***quantum amplitudes*** are obtained by integration of the action over all components of $\boldsymbol{g_{\mu\nu}}$.

Now: first over $\boldsymbol{\omega}$; and then over $\tilde{g}_{\mu\nu}$; **then**: constraints on $\tilde{g}_{\mu\nu}$ and matter fields

$$\int d^5W \int d^4\omega \int d\tilde{g}_{\mu\nu} \ldots\ldots e^{iS}$$

[$\tilde{g}_{\mu\nu}$still inv. under local conv. trans.]

S gauge fixing constraints.

► Vacuum state would have normally **R=0**; **now:** $\boldsymbol{R \to \frac{R}{\Omega^2} - \frac{6}{\Omega^3}\nabla^\mu\nabla_\mu\Omega}$

so the vacuum breaks local CI spontaneously

Nature is not scale invariant, so the vacuum transforms into another unknown state.

► **Conjecture: conformal anomalies must be demanded to cancel out**

$\longrightarrow$ **all renormalization group β-coeff must vanish**

$\longrightarrow$ **constraints to adjust all physical constants!**

► **Ultimate goal**: all parameters of the model computable (including masses and Λ)

black hole complementarity

►It was believed that information would disappear in the central singulatity
► In-falling particle entangled. Firewall?
CI can do better: local breaking of exact CI

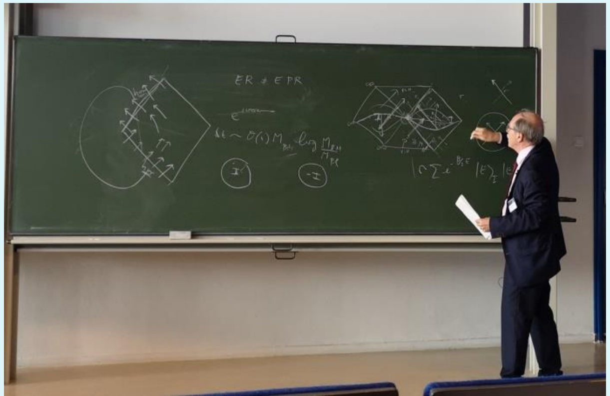
Distinction between **infalling** and **outside** observer: they experience a **different ω**

"the infalling observer passing the horizon experience $g_{\mu\nu}$ and mass M, The outside observer experience Hawing radiation and shrinking mass of the hole: the disagree about dilaton field (in a dynamical setting)"

►One could say that ω is "unobservable"
►Adjust ω, when a singularity is encountered (hide behind horizon) .
►When a local observer encounters a singularity, it,s clock will slow down an infinite time.
►the two observers disagree about the vacuum state of ω

So CI offers a handle for quantum gravity

Further reading: **anti-podal identification** crossing the firewall
[restores time reversal symmetry!!]



Connection with 5D Warped Spacetime

Consider on a 5D warped spacetime [NOT yet CI] [Slagter, Found of Phys, 2016]

$$ds^2 = \mathcal{W}(t,r,y)^2\left[e^{2(\gamma(t,r)-\psi(t,r))}\left(-dt^2+dr^2\right)+e^{2\psi(t,r)}dz^2+r^2e^{-2\psi(t,r)}d\varphi^2\right]+\Gamma dy^2$$

U(1) scalar-gauge field on the brane + empty bulk. Gravity can propagate into the bulk.

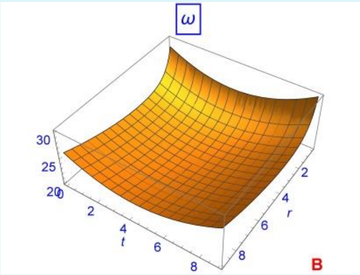
5D:

$${}^5G_{\mu\nu} = -\Lambda_5\,{}^5g_{\mu\nu} + \kappa_5^2\delta(y)[-{}^4g_{\mu\nu}\Lambda_4 + {}^4T_{\mu\nu}]$$

On the brane:

$${}^4G_{\mu\nu} = -\Lambda_{eff}\,{}^4g_{\mu\nu} + \kappa_4^2\,{}^4T_{\mu\nu} + \kappa_5^4\,S_{\mu\nu} - \mathcal{E}_{\mu\nu}$$

From 5D:



$$\mathcal{W} = \frac{e^{\sqrt{-\frac{1}{6}\Lambda_5}(y-y_0)}}{\alpha\sqrt{r}}\sqrt{(d_1e^{\alpha t}-d_2e^{-\alpha t})(d_3e^{\alpha r}-d_4e^{-\alpha r})}$$

$$\Phi = \eta X(t,r)e^{in\varphi}, \qquad A_\mu = \frac{1}{\epsilon}[P(t,r)-n]\nabla_\mu\varphi$$

Scalar-gauge field eq.:

$$D^\mu D_\mu\Phi = 2\frac{\partial V}{\partial\Phi^*} \qquad {}^4\nabla^\mu F_{\mu\nu} = \frac{1}{2}i\epsilon[\Phi(D_\nu\Phi)^* - \Phi^*D_\nu\Phi]$$

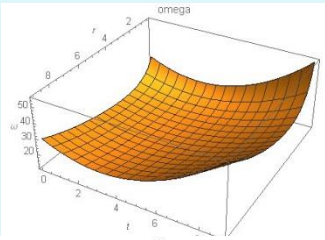
One could say that the "**information about the extra dimension**" translates itself as a curvature effect on spacetime of one fewer dimension!!

Warped 5D spacetime conformally revisited

We rewrite our metric

$$ds^2 = \omega(t,r)^2 W(y)^2 \tilde{g}_{\mu\nu} + n_\mu n_\nu \Gamma(y)^2$$

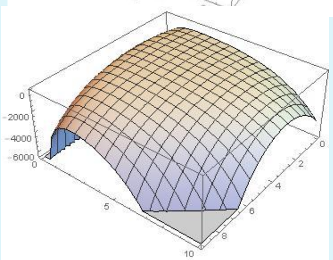
$\downarrow \qquad \downarrow$



$\leftarrow$ real solution.

dilaton — **"unphysical metric"** [Bondi-Marden.]

$$(\partial_{tt} - \partial_{rr} - \frac{2}{r}\partial_r)\omega + \frac{\partial_r\omega^2 - \partial_t\omega^2}{\omega} = 0$$



$\leftarrow$ solution: $\omega^2 < 0$ needed : integration over complex contour['tHooft..]
and ω has same unitary and positivity prop as Φ

write the action **conformal invariant** **[i.e. :** $\tilde{g}_{\mu\nu} \to \Omega^2 \tilde{g}_{\mu\nu} \quad \bar{\omega} \to \frac{1}{\Omega}\bar{\omega} \quad \tilde{\Phi} \to \frac{1}{\Omega}\tilde{\Phi}$]

$$A = \int d^4x \sqrt{-\tilde{g}} \Big[-\frac{1}{12}(\Phi\Phi^* + \bar{\omega}^2)\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}(\tilde{\partial}_\mu\bar{\omega}\tilde{\partial}_\nu\bar{\omega} + D_\mu\tilde{\Phi}D_\nu\tilde{\Phi}^*)] - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}$$
$$- V(\tilde{\Phi}, \bar{\omega}) - \frac{1}{36}\kappa_4^2 \Lambda \bar{\omega}^4 \Big]$$

$$\omega^2 = -\frac{1}{6}\kappa_4^2 \bar{\omega}^2$$

$$V(\tilde{\Phi}, \bar{\omega}) = \frac{1}{8}\beta\eta^2\kappa_4^2 \tilde{\Phi}\tilde{\Phi}^* \bar{\omega}^2 + \lambda\tilde{\Phi}^4$$

<u>**Note:**</u>

* CI broken by mass term via $V(\tilde{\Phi}, \bar{\omega})$
* we take $\Lambda=0$
* Newton's const hidden in $V(\tilde{\Phi}, \bar{\omega})$, so **re-appears** when CI is **broken**

Warped 5D spacetime conformally revisited

Field equations rewritten[**Slagter**,2019]

$$\tilde{G}_{\mu\nu}=\frac{1}{\left(\bar{\omega}^2+\tilde{\Phi}\tilde{\Phi}^*\right)}\left[\tilde{T}^{(\bar{\omega})}_{\mu\nu}+\tilde{T}^{(\tilde{\Phi},c)}_{\mu\nu}+\tilde{T}^{(A)}_{\mu\nu}+\frac{1}{6}\tilde{g}_{\mu\nu}\Lambda_{eff}\kappa_4^2\bar{\omega}^4+\kappa_5^4 S_{\mu\nu}+\tilde{g}_{\mu\nu}V(\tilde{\Phi},\bar{\omega})\right]-\mathcal{E}_{\mu\nu}$$

$$\tilde{\nabla}^\alpha\tilde{\partial}_\alpha\bar{\omega}-\frac{1}{6}\bar{\omega}\tilde{R}-\frac{\partial V}{\partial\bar{\omega}}-\frac{1}{9}\Lambda_4\kappa_4^2\bar{\omega}^3=0$$

$$D^\alpha D_\alpha\tilde{\Phi}-\frac{1}{6}\tilde{\Phi}\tilde{R}-\frac{\partial V}{\partial\tilde{\Phi}^*}=0$$

$$\tilde{\nabla}^\nu F_{\mu\nu}=\frac{i}{2}e\left(\tilde{\Phi}\left(D_\mu\tilde{\Phi}\right)^*-\tilde{\Phi}^* D_\mu\Phi\right)$$

Calculate **Trace**: rest term as expected:

$$\frac{1}{\bar{\omega}^2+X^2}\left[16\kappa_4^2\beta\eta^2X^2\bar{\omega}^2\;-\;\kappa_5^4\left(\frac{\partial_r P^2-\partial_t P^2}{r^2e^2}\right)^2 e^{8\tilde{\psi}-4\tilde{\gamma}}\right]$$

Bianchi: $\nabla^\mu\,\mathcal{E}_{\mu\nu}=\kappa_5^4\,\nabla^\mu\,S_{\mu\nu}$ so (3+1) spacetime variation in matter-radiation on brane can source KK modes

$$\tilde{T}^{(\bar{\omega})}_{\mu\nu}=\tilde{\nabla}_\mu\partial_\nu\bar{\omega}^2-\tilde{g}_{\mu\nu}\tilde{\nabla}_\alpha\partial^\alpha\bar{\omega}^2-6\partial_\mu\bar{\omega}\partial_\nu\bar{\omega}+3\tilde{g}_{\mu\nu}\partial_\alpha\bar{\omega}\partial^\alpha\bar{\omega}$$

$$\tilde{T}^{(\tilde{\Phi},c)}_{\mu\nu}=\tilde{\nabla}_\mu\partial_\nu\tilde{\Phi}\tilde{\Phi}^*-\tilde{g}_{\mu\nu}\tilde{\nabla}_\alpha\partial^\alpha\tilde{\Phi}\tilde{\Phi}^*\;-3\left(\mathcal{D}_\mu\tilde{\Phi}(D_\nu\tilde{\Phi})^*+(D_\mu\tilde{\Phi})^*D_\nu\tilde{\Phi}+3\tilde{g}_{\mu\nu}D_\alpha\tilde{\Phi}(D^\alpha\tilde{\Phi})^*\right)$$

$$\tilde{T}^{(A)}_{\mu\nu}=F_{\mu\alpha}F^\alpha_\nu-\frac{1}{4}\tilde{g}_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$$

New: Some applications

We will consider now two examples of the "**un-physical**" metric $\tilde{g}_{\mu\nu}$

A. Bondi-Marder spacetime [suitable for our scalar-gauge model]

I. With the contribution from projected Weyl tensor [**Slagter** ,**ArXiv:gr-qc**/171108193]
II. Without [Slagter, **Phys Dark Universe,2019**]

B. Spinning Cosmic String [**Slagter, Dustin**, **JHEP**, **2019**]

Stationary axially symmetric solutions: Kerr solution. CTC's hidden behind the horizon
Where are the others?

Weyl, Parapetrou, van Stockum, ... All are physically unacceptable: not the correct asymptotic behavior
CTC's are possible
matching problems at the boundary

However: cosmic string solution in GR : could be physically acceptable .

Now: spinning cosmic strings: Some additional fields are necessary to compensate the energy failure close to the core.
THEN: How do we solve the CTC problem and matching problem??

By Conformal invariant model?

Bondi-Marder spacetime as "unphysical" metric

Remember: Bondi-Marder spacetime [needed because $T_{tt} + T_{rr} \neq 0$ for CS]

$$ds^2 = e^{-2\psi}[e^{2\gamma}(dr^2 - dt^2) + r^2 d\varphi^2] + e^{2\psi+2\mu}dz^2$$
$$= \hat{\omega}^2[-dt^2 + dr^2 + e^{2\tau}dz^2 + r^2 e^{-2\gamma}d\varphi^2]$$

So $\tilde{g}_{\mu\nu} = \hat{\omega}^2 \bar{g}_{\mu\nu}$

↑ ↑ **Ricci-flat**

un-physical metric from 5D

$\hat{\omega}$ is a conformal factor. We consider first the exterior vacuum situation:

Einstein equation: $\hat{\omega}^2 \bar{G}_{\mu\nu} = T_{\mu\nu}^{(\hat{\omega})}$

$\hat{\omega}$ - equation: $\bar{\nabla}^\mu \partial_\mu \hat{\omega} - \frac{1}{6}\hat{\omega}\bar{R} = 0$

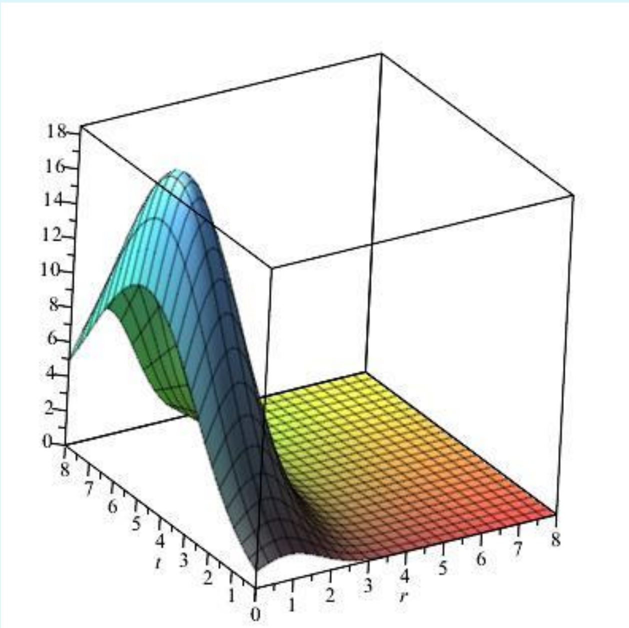
Check: $Tr\left[\bar{G}_{\mu\nu} - \frac{1}{\hat{\omega}^2} T_{\mu\nu}^{(\hat{\omega})}\right] = 0$

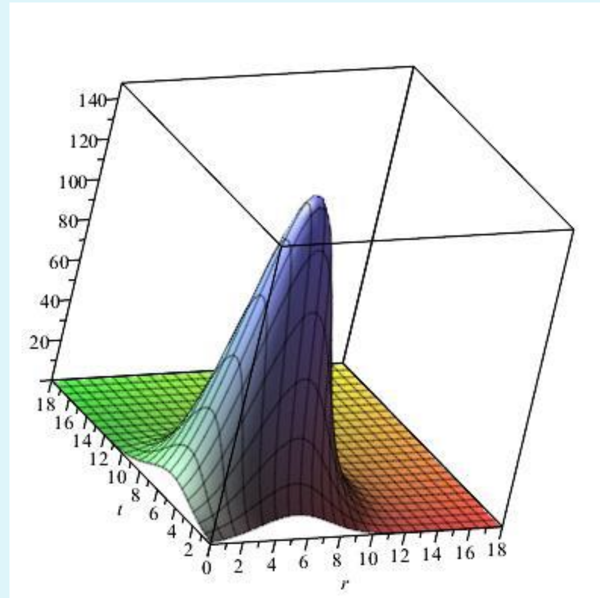
One can solve equation for $\hat{\omega}$:

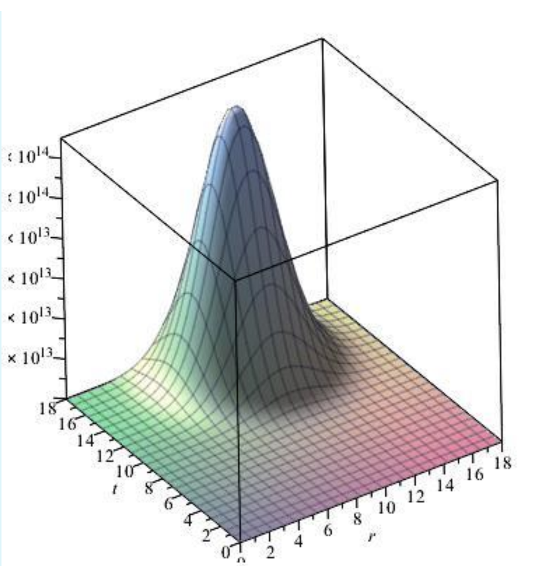
$$\hat{\omega} = \mathcal{B} e^{\frac{1}{2}\varsigma_1(r^2+t^2) - \frac{1}{2}\upsilon r^2 + \varsigma_2 t + r}$$

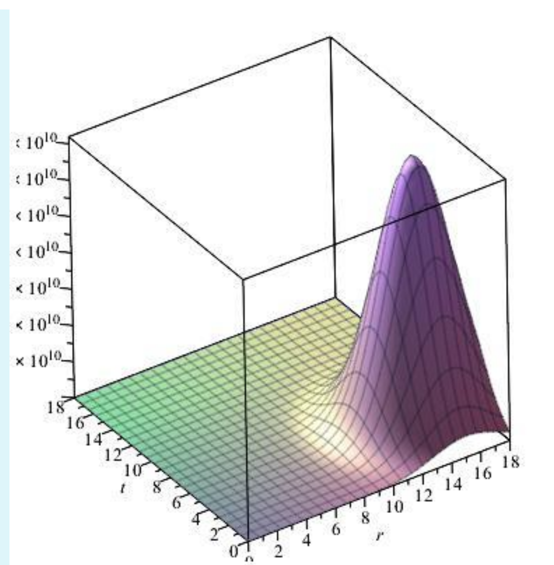
4 constants . Generation of curvature from Ricci flat spacetimes. [**Slagter**, Phys. Dark Univ.,2019]

Numerical solution ω









Quantum amplitudes are obtained by

$$\int D\omega(x) \ldots..$$

No problem here.

Spinning U(1) gauged cosmic strings

Let us consider now the 4D stationary axially symmetric spacetime with **rotation:**

[for the moment no t-dependency]

$$ds^2 = -e^{-2f(r)}(dt - J(r)d\varphi)^2 + e^{2f(r)}\left[l(r)^2 d\varphi^2 + e^{2\gamma(r)}(dr^2 + dz^2)\right]$$

rewritten as

$$\boldsymbol{ds^2 = \omega(r)^2\left[-(dt - J(r)d\varphi)^2 + b(r)^2 d\varphi^2 + e^{2\mu(r)}(dr^2 + dz^2)\right]}$$

dilaton $\downarrow$ **decoupled from $\tilde{g}_{\mu\nu}$**

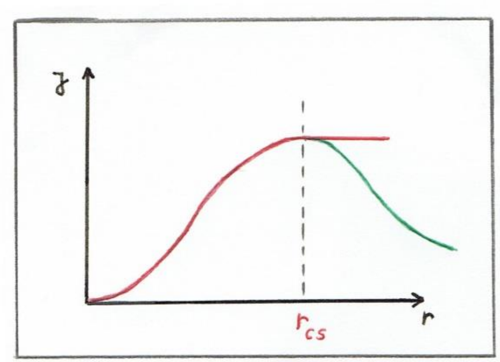
Some results:

1. obtainable from Weyl form by: $\boldsymbol{t \to iz}$, $\boldsymbol{z \to it}$, $\boldsymbol{J \to iJ}$
2. interesting relation with (2+1) dim gravity [cosmon's; **'tHooft ,**2000]
3. Gott-spacetime: no CTC's [parallel and opposite moving pair]
4. for constant J: ► conical exterior spacetime [**angle-deficit**]
   ► if one transform: $t \to t - J\varphi$: results in local Minkowski
   but then t jumps by $8\pi GJ$ [**helical time**]
   QM-solution? Quantized angular momentum→ also t !

5. What happed at the **boundary** r_c of the string?

r=0: J = 0 and b→r

r = r_c: J = constant and $b = B(r + r_c)$



Then:

problems at the boundary for $\boldsymbol{J_r}$ and **WEC** violated!!

Spinning U(1) gauged cosmic strings in CI gravity

No choice yet for $V(\omega, \Phi)$. From tracelessness and Bianchi:

$$\frac{2}{3}V = \widetilde{\Phi}^* \frac{dV}{d\widetilde{\Phi}^*} + \widehat{\omega}\frac{dV}{d\widehat{\omega}} \qquad \frac{1}{6}V' = \widetilde{\Phi}^{*\prime} \frac{dV}{d\widetilde{\Phi}^*} + \widehat{\omega}'\frac{dV}{d\widehat{\omega}}$$

For the **exterior** we obtain

$$J'' = J'\left(\frac{b'}{b} - 2\frac{\widehat{\omega}}{\widehat{\omega}}\right) \qquad b'' = \frac{1}{b}J'^2 - \frac{2}{\widehat{\omega}}b'\widehat{\omega}' \qquad \mu'' = \frac{1}{2b^2}J'^2 - \mu'\left(\frac{b'}{b} + 2\frac{\widehat{\omega}'}{\widehat{\omega}}\right)$$

$$\widehat{\omega}'' = -\frac{3\widehat{\omega}}{8b^2}J'^2 + \frac{\widehat{\omega}'^2}{2\widehat{\omega}} + \frac{1}{2}\mu'\left(\frac{\widehat{\omega}'b'}{b} + 2\widehat{\omega}'\right)$$

$\downarrow$ "spin-mass rel"

$$J(r) = \text{const.} \int \frac{b}{\widehat{\omega}'^2}\, dr$$

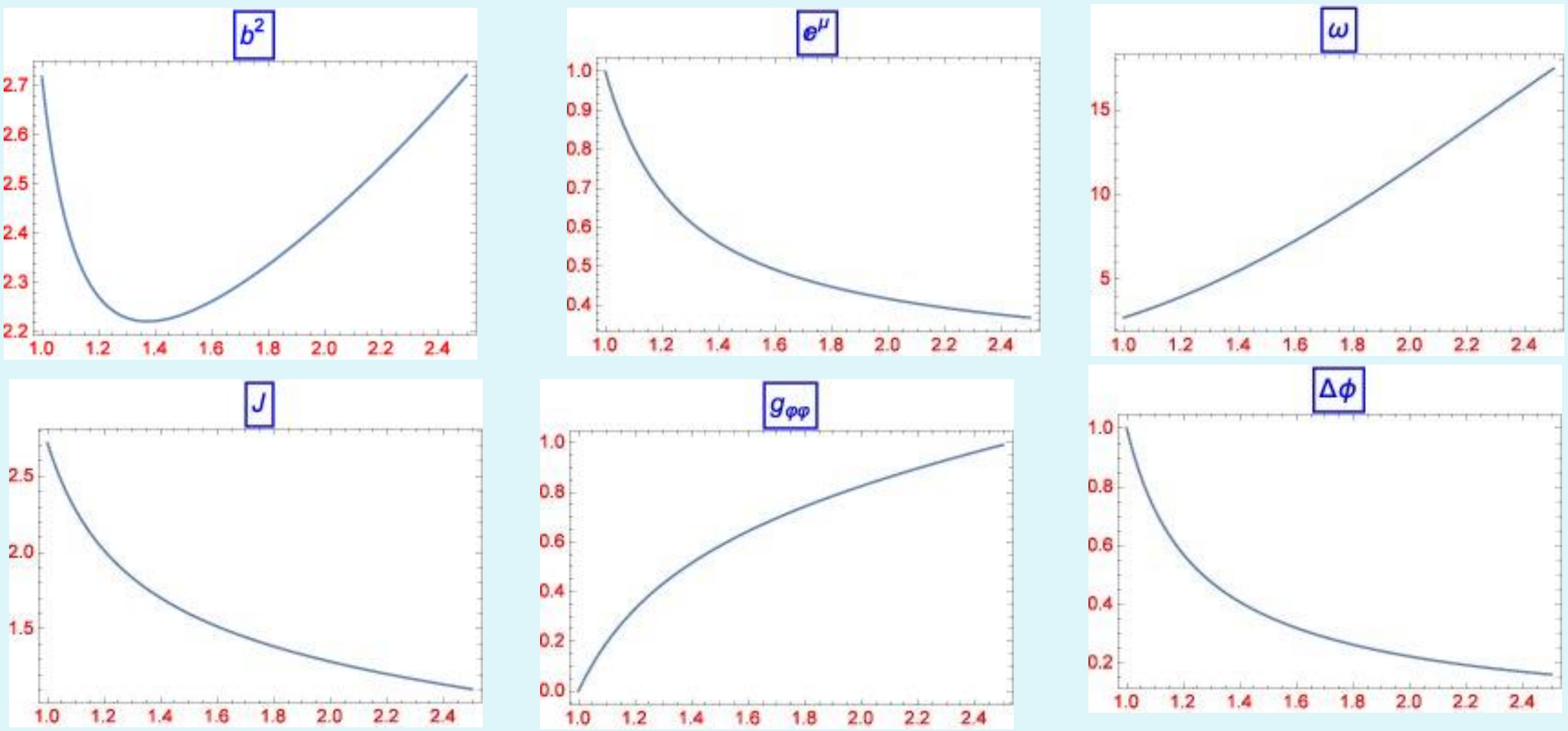
with **exact** solution:

$$\mu(r) = c_1 r + c_2 - \log\left(\sqrt{c_4 r + c_5}\right) \qquad b(r) = \frac{c_3}{2c_4 r + 2c_5} \qquad \omega(r) = \sqrt{2c_4 r + 2c_5}$$

$$J(r) = c_6 \pm \frac{c_3}{2c_4 r + 2c_5}$$

- ► J has correct asymptotic form!
- ► $g_{\mu\nu}$ **Ricci flat**! [while $\tilde{g}_{\mu\nu}$ *not*]
- ► CTC for $r = \frac{c_3 - c_5 c_6}{c_4 c_6}$ which can be **pushed to** $\pm\infty$ or 0.

Numerical verification



The interior solution

For the gauge field we can take: $\boldsymbol{A_\mu = \left[P_0(r), 0, 0, \frac{1}{e}(P(r) - n)\right]}$

The field equation contain now terms like

$$J'' = J'\,\partial_r\left[log\left(\frac{b}{\eta^2 X^2 + \widehat{\omega}^2}\right)\right] - 2\frac{P_0'(eJP_0' + P')}{e(\eta^2 X^2 + \widehat{\omega}^2)} + \cdots$$

The **"spin-mass"** relation becomes in case of **global strings** (P=P_0 =0)

$$J = const\int \frac{b}{\eta^2 X^2 + \widehat{\omega}^2}\,dr$$

Energy momentum:

$$T_{tt} = -\frac{3}{4b^2}{J'}^2 + \frac{\mu' b'}{b} + (\mu' + \frac{b'}{b})\partial_r(\log(\eta^2 X^2 + \widehat{\omega}^2))$$

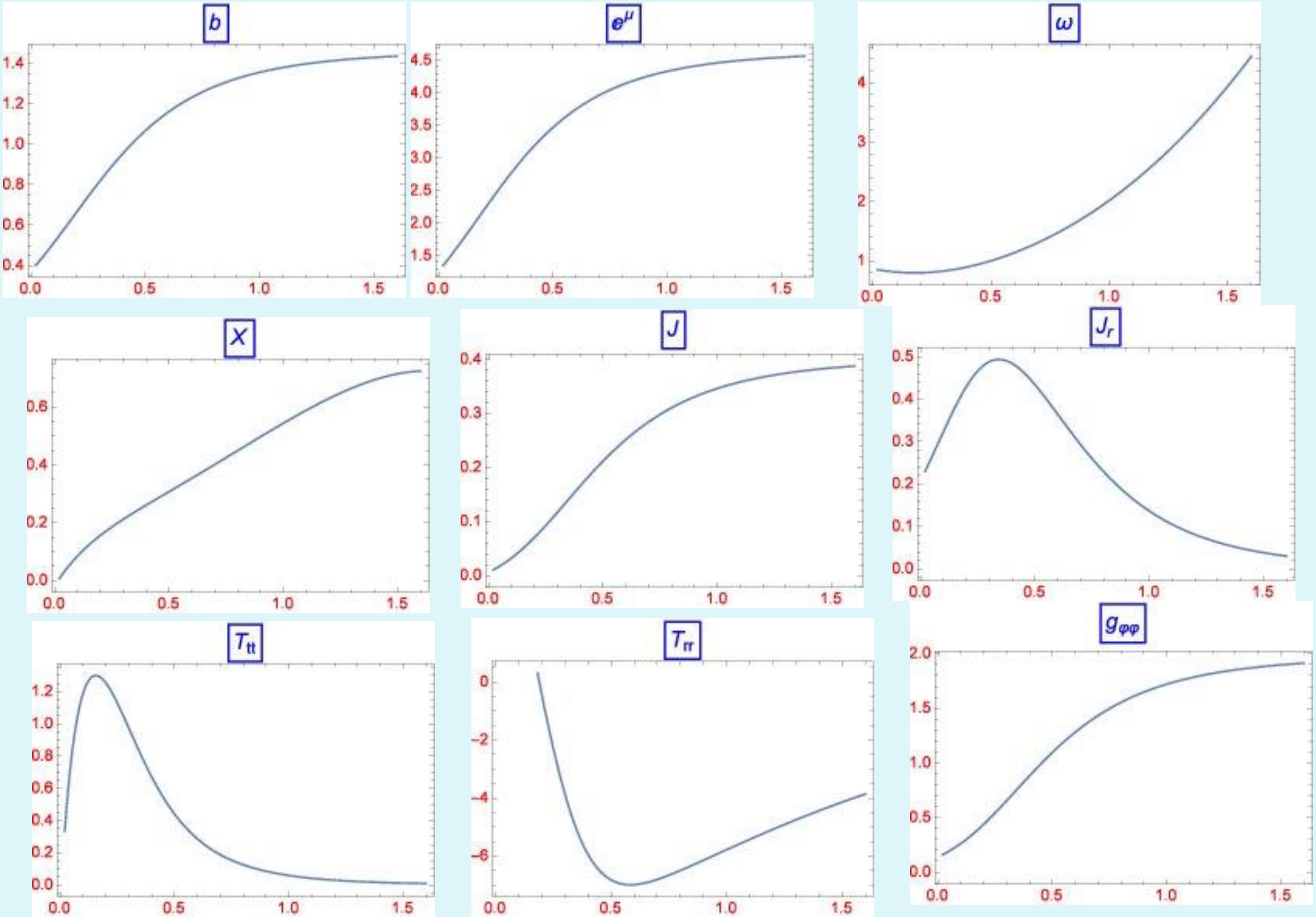
This can be made positive due to the additional matter!

Scalar curvature:

$$R = \frac{k' b'}{b} + \left(k'\frac{b'}{b}\right)\partial_r(\eta^2 X^2 + \omega^2) + \frac{5}{2}\frac{\eta^2 {X'}^2 + {\omega'}^2}{\eta^2 X^2 + \omega^2}$$

regular everywhere

Numerical solution



Local observer

Local orthonormal frame: $\widehat{\Theta}^t = dt - Jd\varphi \quad \widehat{\Theta}^r = e^{\mu}dr \quad \widehat{\Theta}^z = e^{\mu}dz \quad \widehat{\Theta}^{\varphi} = bd\varphi$

Timelike 4-velocity: $U_{\widehat{\nu}} = \frac{1}{\varepsilon}[1,0,\alpha,\beta]$

Local energy density measured by the observer moving at constant $r = r_c$

$$\varepsilon^2 G^{\widehat{\mu}\widehat{\nu}} U_{\widehat{\mu}} U_{\widehat{\nu}} = \frac{(\beta^2 + \alpha^2)b' + \beta J'}{b} \partial_r[log(\eta^2 X^2 + \widehat{\omega}^2)] + \frac{2\alpha^2 - \varepsilon^2}{4b^2} J'^2$$

Can be made ***positive*** for suitable physically acceptable behavior of b', J', X', ω' and $\varepsilon^2 < 2\alpha^2$ (for sufficiently high velocity)

==

Main conclusion:

It seems that there are no obstructions for a physically acceptable solution for a spinning cosmic string in conformal gravity.

No CTC's   No violation of WEC   Interior: regular and easily matched on exterior

Thank you!

Extra background

Some history of QFT

Calculations in QFT:

- *In perturbation theory the effect of interactions is expressed in a powerserie of the coupling constant (<<1 !)*
- *Regularization scheme necessary in order to deal with divergent integrals over internal 4- momenta.*
- *Introduce cut-off energy/mass scale Λ and stop integration there.*
  [however, invisible in physical constants and partcle data tables]
  So renormalization comes in
- *Covariant theory of gravitation cannot be renormalized [in powercounting sense] Non-renormalizable interactions is suppressed at low energy, but grows with energy. At energies much smaller than this "catastrophe-scale", we have an effective field theory.*

Standard model is too an effective field theory.

- **In curved background: geometry of spacetime remaims in first instance non-dynamical!**
  However: in GRT it is.

String theory solution?

- **Nambu-Goto action (Polyakov)** $A = -T\int d^2\sigma\sqrt{-g}\,g^{\alpha\beta}h * \eta_{\alpha\beta}$

Some history of QFT

New gauge symmetry: $g_{\alpha\beta} \rightarrow \Omega(\sigma)^2 g_{\alpha\beta}$ [Ω smooth function on the worldsheet]

After quantization: $\langle T_\alpha^\alpha \rangle$ depends on Ω, unless a crucial number in 2d-CFT (central charge) is zero! [in conformal gravity $T_\alpha^\alpha = 0$]

The Fadeev-Popov ghost field (needed for quantisation) contribute a central charge of -26, which can be canceled by 26-dimensional background.

Can we do better? **New conformal field theory**

Suppose: QFT is correct and GRT holds at least to the Planck scale

■ **Advantages of CI:**

A. ***At high energy, the rest mass of partcles have negligible effects***
So no explicit mass scale. ***CI would solve this***

B. CI field theory ***renormalizable*** [coupling constants are dimensionless]

C. CI In curved spacetime: would solve the ***black hole complementarity*** through conformal transformations between infalling and stationary observers.

D. Could be ***singular-free***

E. Success in CFT/ADS correspondence

F. In standart model, symmetry methods also successful.

G. CI put constraints on GRT . Very welcome!

Related Issues

► **Asymptopia:** **How to handle:** **"far from an isolated source?"**

we have only locally: $\boxed{\nabla_\alpha T^{\alpha\beta} = 0}$

is there a Killing-vector k_μ: then

then integral conservation law.

gravitational energy and mass?

$$\boxed{\nabla_\alpha J^\alpha = \nabla_\alpha\left(T^{\alpha\beta}k_\beta\right) = 0}$$

► Isotropic scaling trick: $g_{\mu\nu} \rightarrow \hat{g}_{\mu\nu} = \omega^2 g_{\mu\nu}$ with $\omega \rightarrow 0$ far from the source.

[**note:** we shall see that Einstein equations yield: $G_{\mu\nu} = \frac{1}{\omega^2}(\dots)$, so small distance limit will cause problem, unless we add scalar field comparable with "dilaton "ω : $G_{\mu\nu} = \frac{1}{\omega^2+\Phi^2}(\dots)$]

Example: Minkowski: $\boldsymbol{ds^2 = -dvdu + \frac{1}{4}(v-u)^2\left[d\theta^2 + \sin^2\theta d\varphi^2\right]}$

one needs information about behavior of fields at $v \rightarrow \infty$

then: $\boldsymbol{ds^2 = \frac{1}{V^2}\left[dudV + \frac{1}{4}(1-uV)^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]}$ and infinity : $V \rightarrow 0$

so singular!

then: $\boldsymbol{g_{\mu\nu} \rightarrow \hat{g}_{\mu\nu} = \omega^2\eta_{\mu\nu} = V^2\eta_{\mu\nu}}$ *: smooth metric extended to V=0 and one can handle tensor analysis at infinity.*

Even better: $\boldsymbol{\hat{g}_{\mu\nu} = \frac{4}{(1+v^2)(1+u^2)}\eta_{\mu\nu}}$ with $T, R = \tan^{-1}v \pm \tan^{-1}u$

$$\boldsymbol{ds^2 = -dT^2 + dR^2 + \sin^2R\left(d\theta^2 + \sin^2\theta d\varphi^2\right)}$$

Static Einstein universe $S^3 \otimes \mathcal{R}$: conformal map $(\mathcal{R}^4, \eta_{\mu\nu}) \rightarrow (S^3 \otimes \mathcal{R}, \hat{g}_{\mu\nu})$

Related Issues CI

► If spacetime is fundamental discrete: then continuum symmetries (such as L.I.) are imperilled. To make it compatible: the price is locality. [**Dowker**, 2012; **'t Hooft**, 2016]

Can non-locality be tamed far enough to allow known local physics to emerge at large distances?

► The Causal Set approach to quantum gravity: atomic spacetime in which the fundamental degrees of freedom are discrete order relations. [**'tHooft, Myrheim, Bombelli, Lee, Myer and Sorkin**]

► The causal set approach claims that certain aspects of General Relativity and quantum theory will have direct counterparts in quantum gravity:

1. the spacetime causal order from General Relativity,
2. the path integral from quantum theory.

Then: Is it possible to obtain our familiar physical laws described by PDE's from discrete diff operators on causal sets? For example, discrete operators that approximate the scalar D'Alembertian in any spacetime dimension? Seems to be yes!

►ω is fixed when we specify our global spacetime and coordinate system, which is associated with the vacuum state.

[remember $\boldsymbol{R \to \frac{R}{\Omega^2} - \frac{6}{\Omega^3}\nabla^{\mu}\nabla_{\mu}\Omega}$ **]** If we not specify this state, then no specified ω.

't Hooft: *" In quantum field theory we work on a flat background. Then ω is unique On non-flat background: sizes and time stretches and become ambiguous"*