Quantum time,
and quantum time measurements

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FQXi Foundation,
"The physics of what happens"

What I'm going to talk about

Time in quantum mechanics

a consistent formalization

based on conditional probability amplitudes

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... and an application: how to define a time observable in QM

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BUT... **classical systems don't exist**

in a consistent theory of quantum mechanics
(they're just a limiting situation)

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“what is shown on a clock”

then use a **quantum** system as a clock

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e.g. a quantum particle on a line
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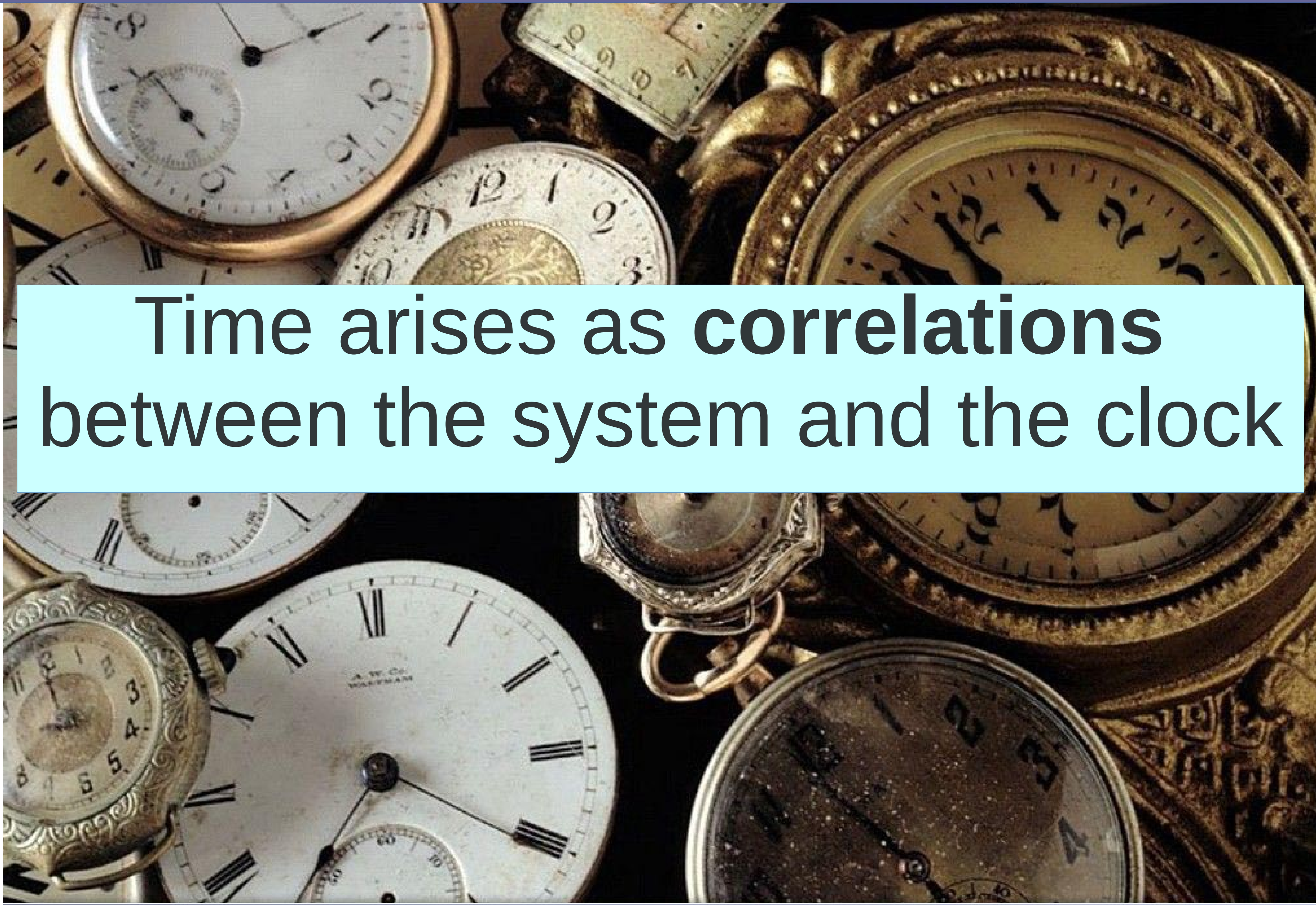
$$\mathcal{H} \equiv \mathcal{L}^2(\mathbb{R})$$

eigenbasis $\{|x\rangle\}$

$|t\rangle$

Time and entanglement



Time arises as **correlations** between the system and the clock

The PWAK mechanism

Page and Wootters [PRD **27**,2885 (1983)]
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time Hilbert space

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clock "momentum"

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time Hilbert space → $\mathcal{H}_T$

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This means that for physical states the *system* Hamiltonian is the generator of *clock* time translations

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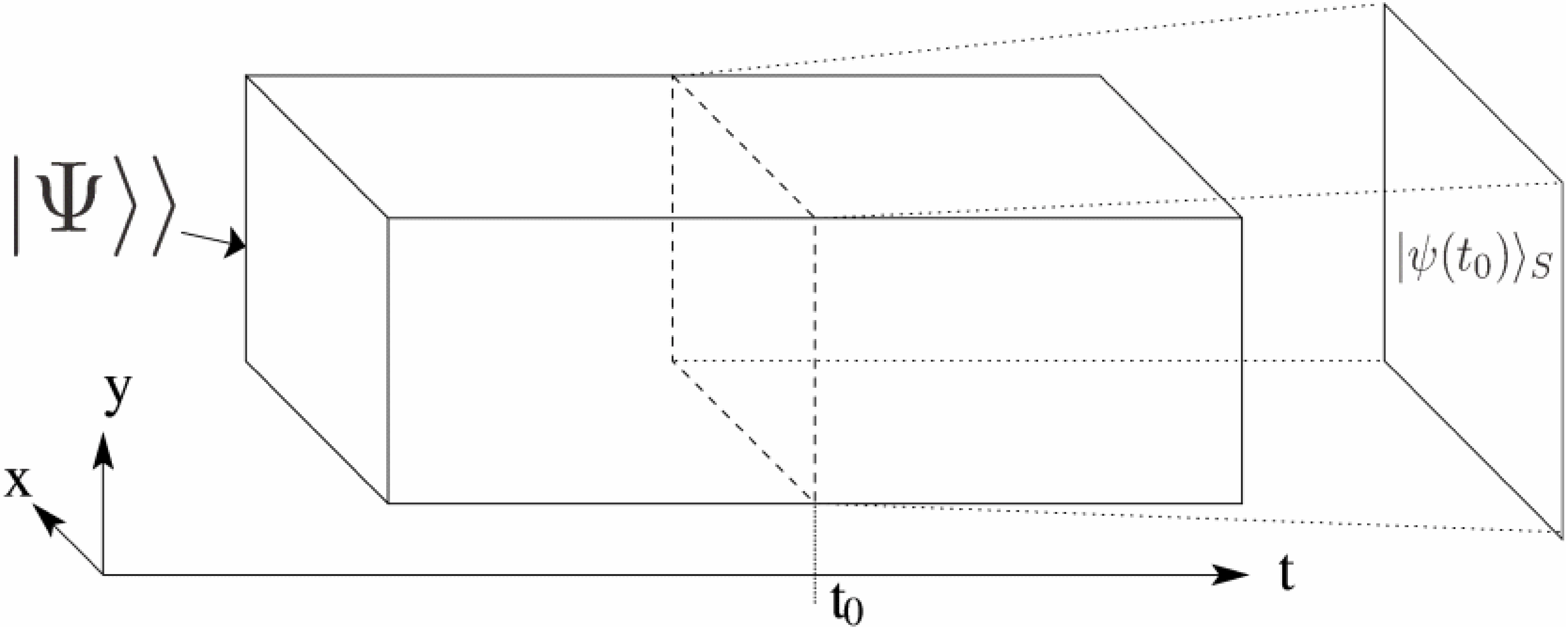
"momentum" representation=time indep. Schr eq.

what I've been saying is that

conventional qm arises in this framework through conditioning.



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is a **conditioned state**: the state *given that* the time was *t*

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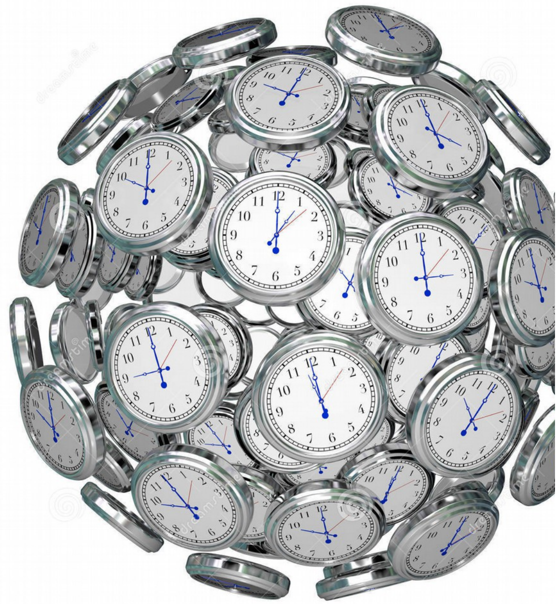
Other choices are possible!!

Physical interpretation

The time Hilbert space is the Hilbert space of the clock that **defines** time

remember: “time is what is measured by a clock”!



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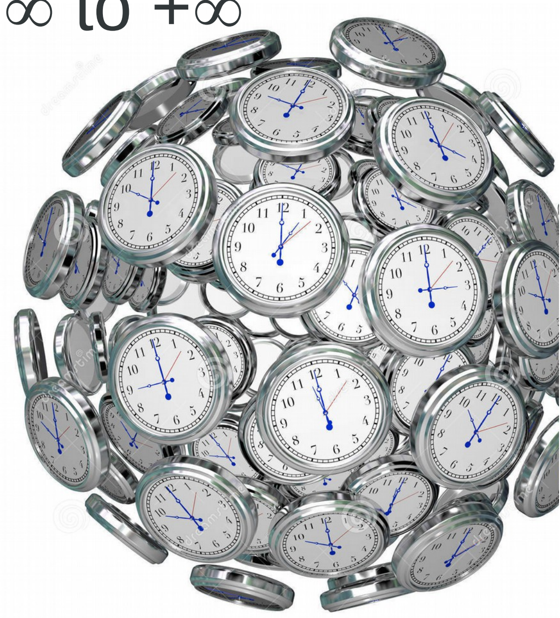

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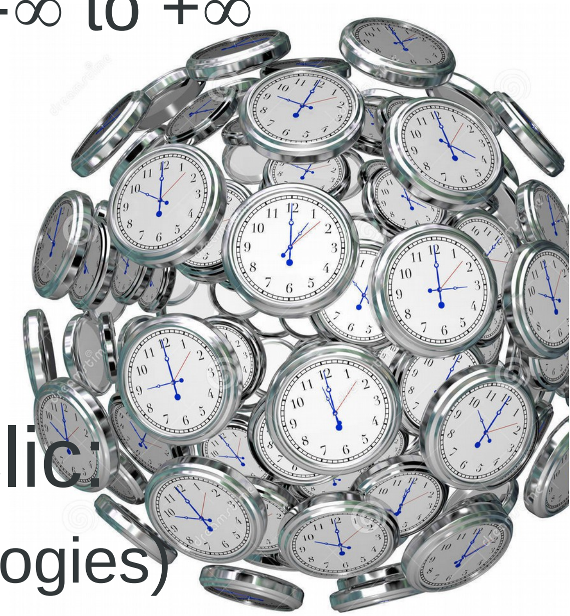

other choices are possible..

if the clock has finite energy, time is cyclic:
e.g. a spin (appropriate for certain closed cosmologies)

Physical interpretation 2

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alternative:

It can be seen as an **abstract purification space**

Entanglement?

Is entanglement important?
Could we do with classical correlations?

$$
\begin{aligned}
|\Psi\rangle\rangle &= \int dt\ |t\rangle_T \otimes |\psi(t)\rangle_S \\
&= \int d\mu(\omega)\ |\omega\rangle_T \otimes |\psi(\omega)\rangle_S\ ,
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$$

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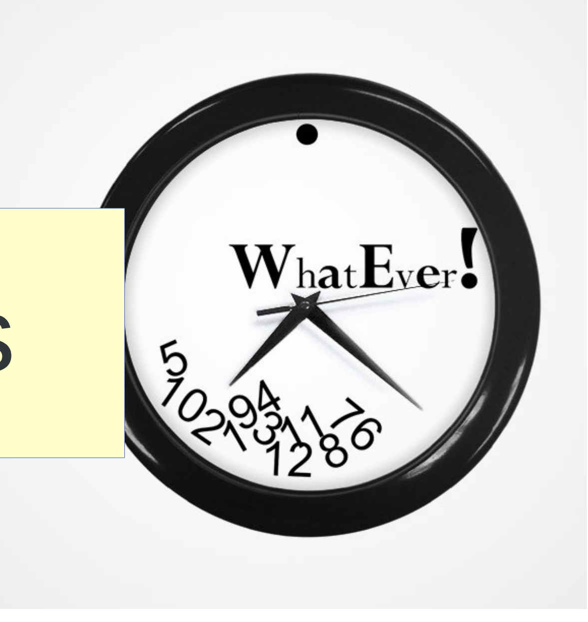
Our contribution

These ideas were basically abandoned in the 80s: because of objections (Kuchar, Unruh, etc.)

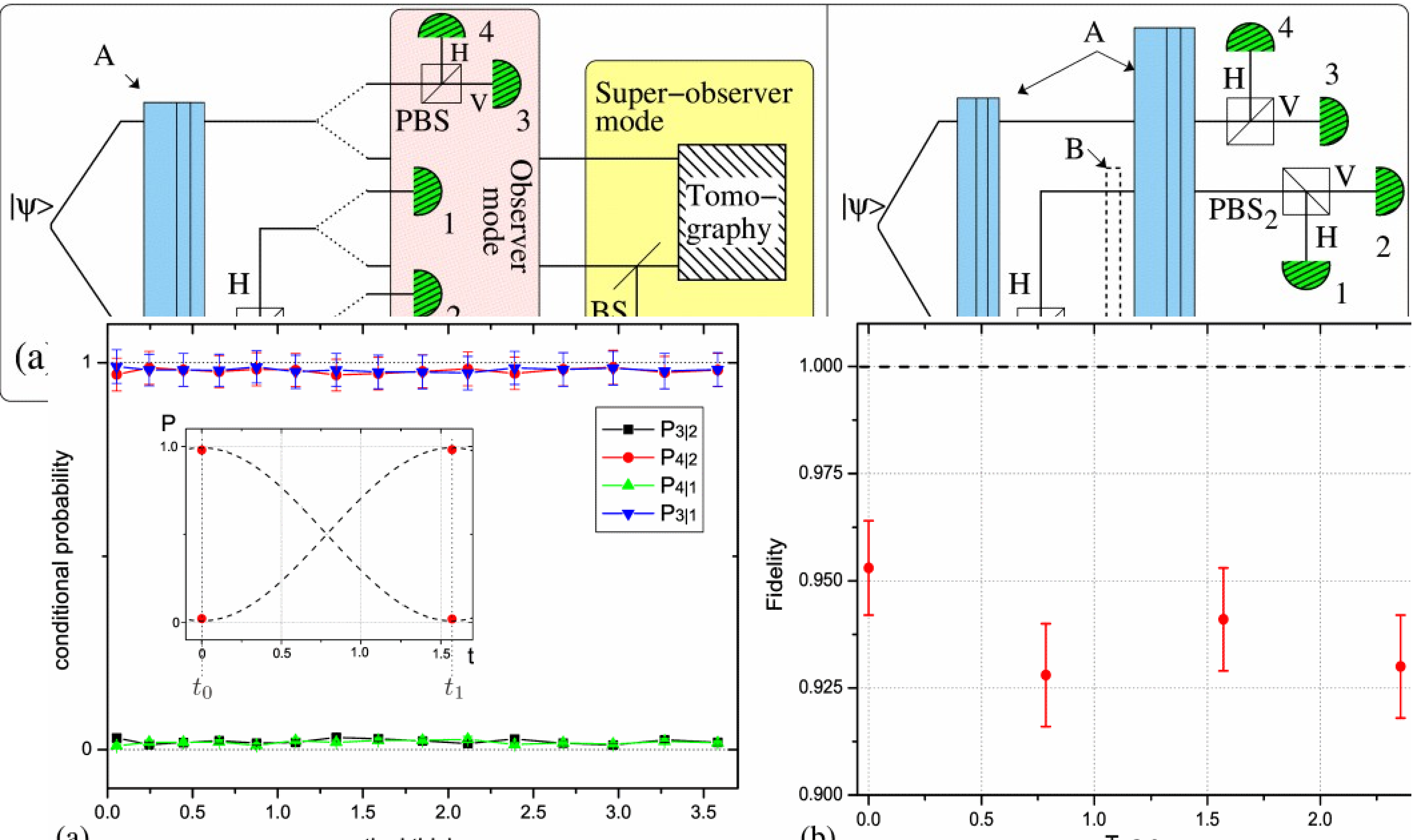


We removed these objections

... and also perfected the model

(e.g. role of entanglement, momentum representation)

Experimental realization
(collaboration with the INRIM group)



Different "times":

- Quantum time
- Time of arrival (quantum mechanics)
- Proper time (or clock time)
- Coordinate time
- Entropic time (arrow of time)
- WdW (no time)
- Conscience time
- "Time" (??)
- ...

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Can we use our quantum time
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YES!　(joint work with Krzysztof Sacha)

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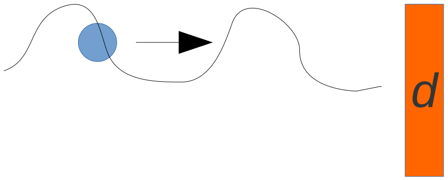

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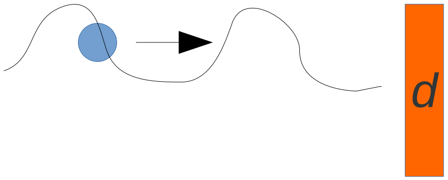

particle's spatial degrees of freedom

clock

$$|\Psi\rangle = \frac{1}{\sqrt{T}} \int_T dt \, |t\rangle |\psi(t)\rangle$$

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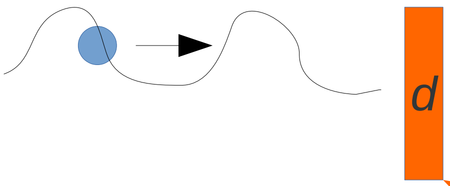

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$$\forall t : \Pi_t \equiv |t\rangle\langle t| \otimes P_d \,; \qquad \Pi_{na} = \mathbb{1} - \int dt\, \Pi_t$$

Outcome *t*="particle is at position *d* at time *t*", outcome *na*="not arrived"

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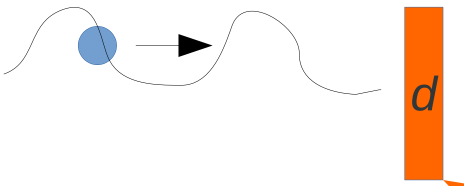

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Time of arrival quantum observable:

$$A = \int dt\, t\, |t\rangle\langle t| \otimes P_d + \mathbb{1}_c \otimes \lambda \int_{x \notin D} dx\, |x\rangle\langle x|$$

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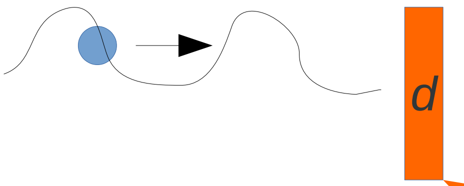

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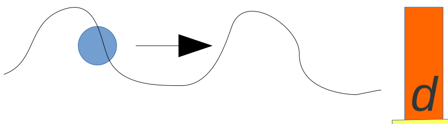

particle's spatial degrees of freedom

clock

$|\psi(t)\rangle$

Projec...

$\forall t$: ... 1_t

Outcome t= ... not arrived"

A property ***of the CLOCK***!!
(not *of the particle*, as in
~~most~~ competing proposals)
all?

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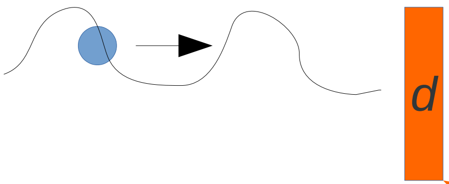

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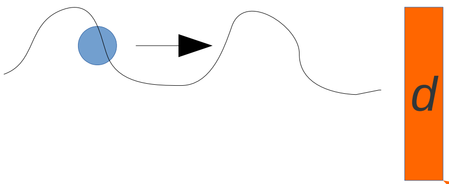
Born's rule

Time of arrival



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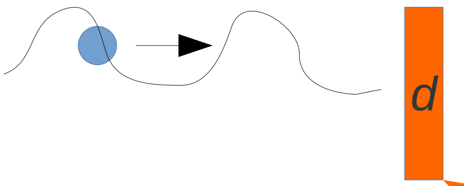
$$p(t, x = d) = \mathrm{Tr}[|\Psi\rangle\langle\Psi|\, \Pi_t] = \frac{1}{T} |\psi(d|t)|^2 ,$$

Born's rule

with $\psi(x|t) \equiv \langle x|\psi(t)\rangle$

Time of arrival



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$$|\Psi\rangle = \frac{1}{\sqrt{T}} \int_T dt\, |t\rangle |\psi(t)\rangle$$

Projective POVM:

$$\forall t : \Pi_t \equiv |t\rangle\langle t| \otimes P_d \,; \qquad \Pi_{na} = \mathbb{1} - \int dt\, \Pi_t$$

Outcome *t*="particle is at position *d* at time *t*", outcome *na*="not arrived"

$$\Downarrow$$

$$p(t, x = d) = \mathrm{Tr}\big[|\Psi\rangle\langle\Psi|\, \Pi_t\big] = \frac{1}{T} |\psi(d|t)|^2 \,,$$

Born's rule

with $\psi(x|t) \equiv \langle x|\psi(t)\rangle$

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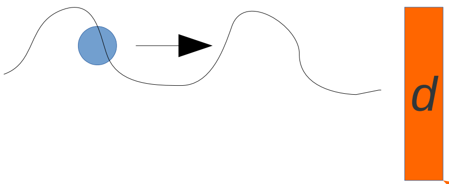
$$p(t|x = d) = \frac{p(t, x = d)}{p(x)}$$

Bayes rule

Time of arrival



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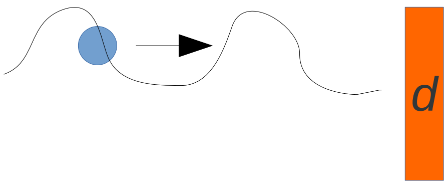
Time of arrival prob. distribution

Time of arrival



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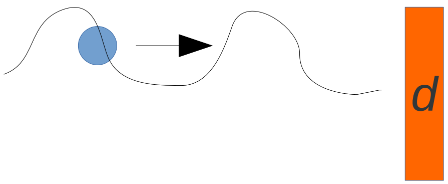

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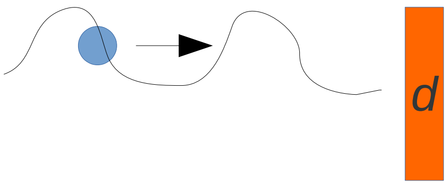

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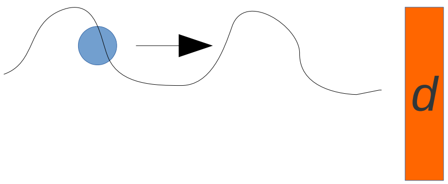

Summary:

- take the projector for the particle at *d* and for the clock at *t.*
- build a joint observable from this
- from the joint probability of clock+particle, get the clock probability through the Bayes rule.

Only “time of arrival”?

Only “time of arrival”? → NO!

Extensions to other time measurements:

a **generic** time measurement is

“At what time did the event *E* happen?”

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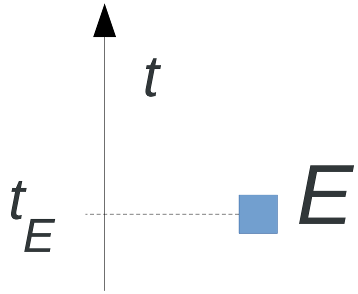

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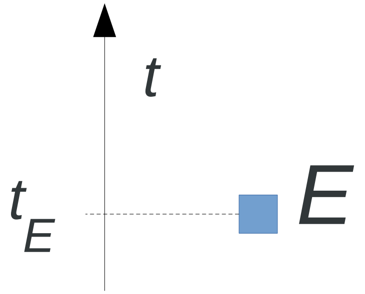

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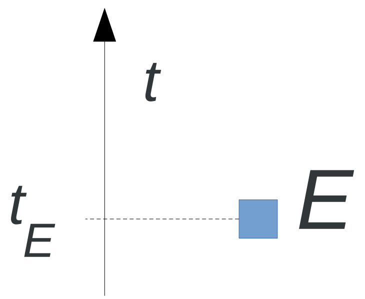

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e.g. “at what time is the spin up?” The projector is $|\uparrow\rangle\langle\uparrow|$

All usual manipulations →

for observables can be done:

- Expectation values
- Probability distributions
- Eigenstates, eigenvalues, etc.

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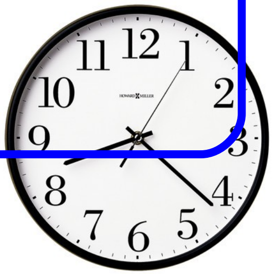
Criticisms to time quantizations

The Pauli argument

Pauli: “time **cannot be quantized**, because a time operator that is a generator of energy translations implies that the energy is unbounded (also from below)”



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- in conventional qm, time is not a dynamical variable $\Rightarrow$ no problem.

- in our case, time *is* a dynamical variable, but its translations are NOT generated by $\hat{H}_S$ (but by $\hat{\Omega}$)

The Kuchar argument against PaW

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$$|\Psi\rangle\rangle = \int dt\ |t\rangle_T \otimes |\psi(t)\rangle_S$$

time t

$$|\psi(t)\rangle$$

after a measurement of time, the state collapses to $|\psi(t)\rangle$: successive measurements give wrong statistics: **no more evolution**

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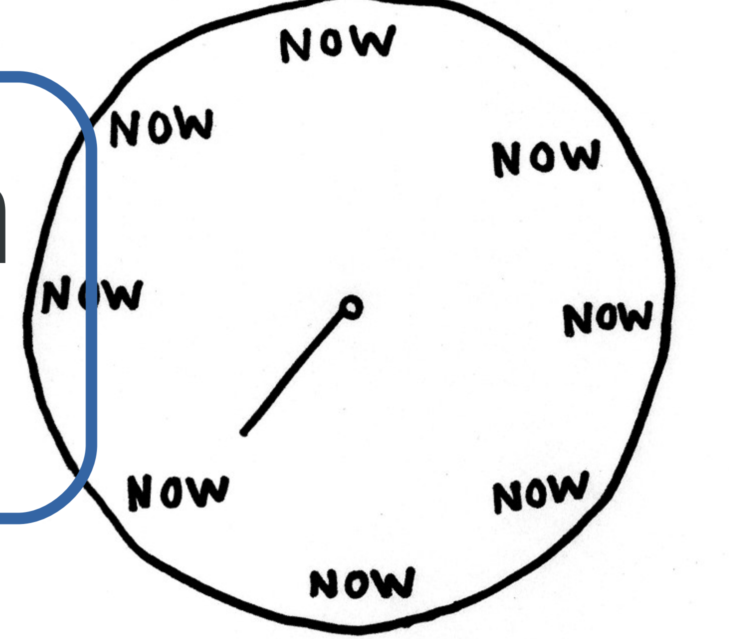
a careful formalization of **what a two-time measurement is** solves the problem!

The second measurement is a joint measurement on the system and on the d.o.f. that stored the outcome of the first.

$\Rightarrow$ Kuchar's objection is defeated!



this argument can be extended to POVMs, propagators, etc...

Conclusions

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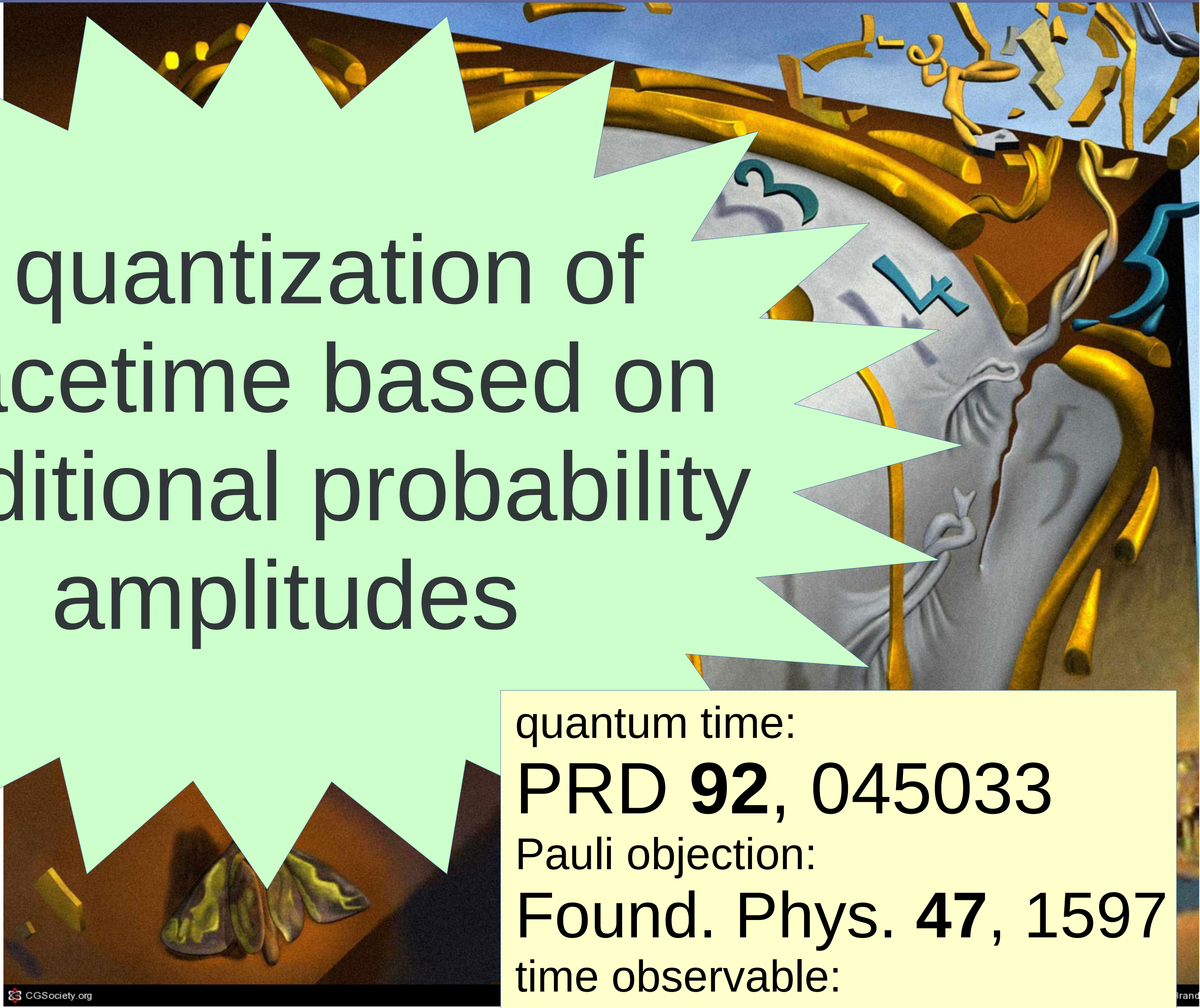
- Pauli objections and others..

Take home message



A quantization of spacetime based on conditional probability amplitudes

quantum time:

PRD **92**, 045033

Pauli objection:

Found. Phys. **47**, 1597

time observable:

arXiv:1810.12869

Lorenzo Maccone
maccone@unipv.it

- sostituire WdW con constraint for q reference frames
- aggiungere il caso di multiple clocks

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appropriate for a beginning of time?!?

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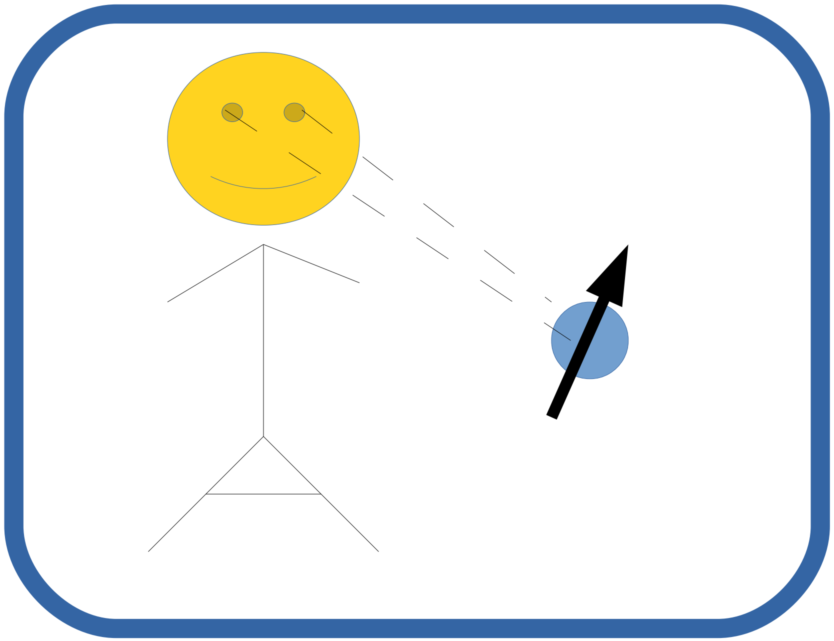

Alice's lab

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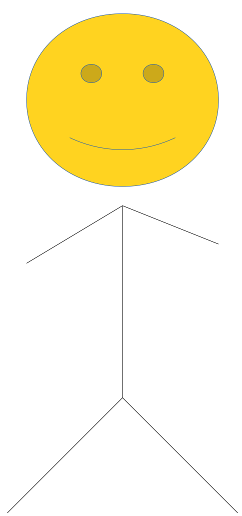
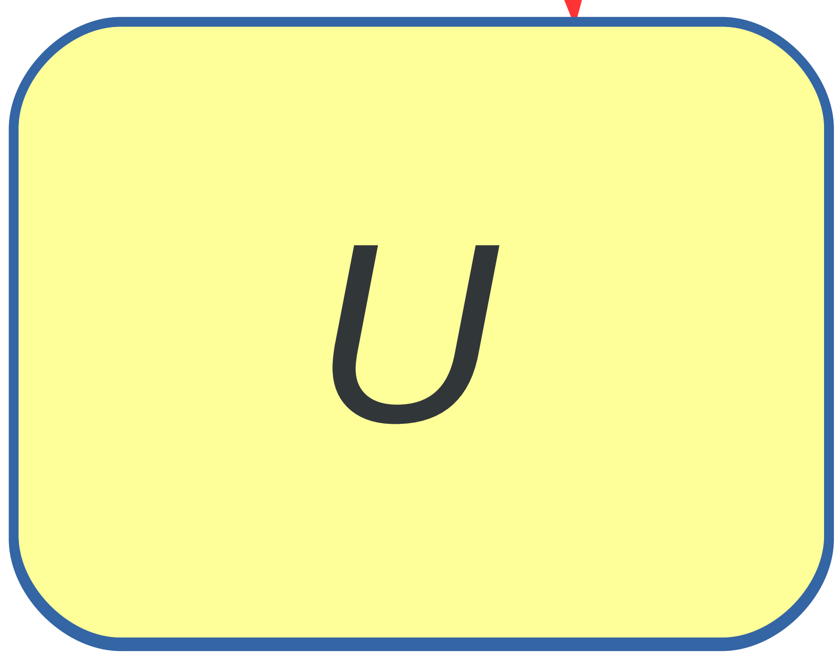


Bob's point of view



Alice's lab

Motivations

same treatment of time and space in qm

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Easy to quantize the position in space,
but **difficult** to quantize the position in time

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OUR FRAMEWORK permits the QUANTIZATION OF EVENTS

In what follows

↓

no relativity!

Just consider time (not spacetime)

Alternative way of defeating Kuchar's objection the Gambini et al. proposal [PRD 79,041501]

*Use Rovelli's **evolving constants of motion***

Average over the inaccessible coordinate time

$$p(d|t) = \frac{\int dT \, \mathrm{Tr}[P_{d,t}(T)\rho]}{\int dT \, \mathrm{Tr}[P_t(T)\rho]}$$

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Comparison to Stuekelberg's qm

$$|\Psi_{stu}\rangle = \int dt \int d^3x \, \Psi_{stu}(\vec{x}, t) |\vec{x}\rangle |t\rangle$$

prob. ampl. to find a particle in **spacetime position** *x,y,z,t*.

$$\int dt \, d^3x |\Psi_{stu}|^2 = 1$$

(good only for qft?)

$$|\Psi\rangle\rangle = \int_{-\infty}^{+\infty} dt \, |t\rangle |\psi(t)\rangle$$

prob. ampl. to find a particle at x,y,z **given that** the time is t

$$|\Psi_{screen}\rangle\rangle = \int_{-\infty}^{+\infty} dz \, |z\rangle |\chi(z)\rangle$$

prob. ampl. to find a particle at x,y and time t **given that** the screen is at z

Conditional prob. ampl.

need a framework where we can condition on all!

(qm for events?)

Question for you:

WHAT is an event?!??

a good definition? (“intersection of world lines” no good for qm)

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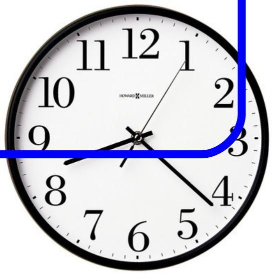

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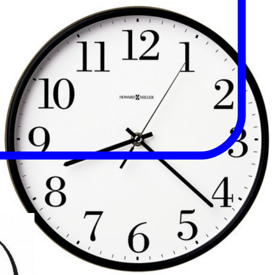

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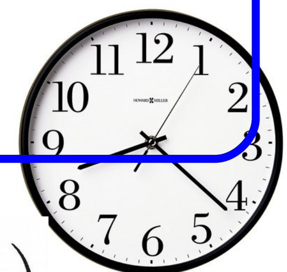

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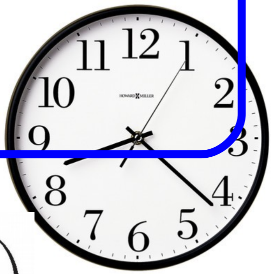

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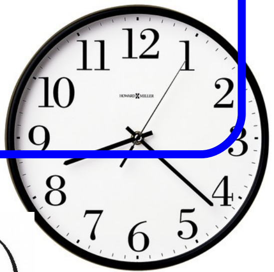

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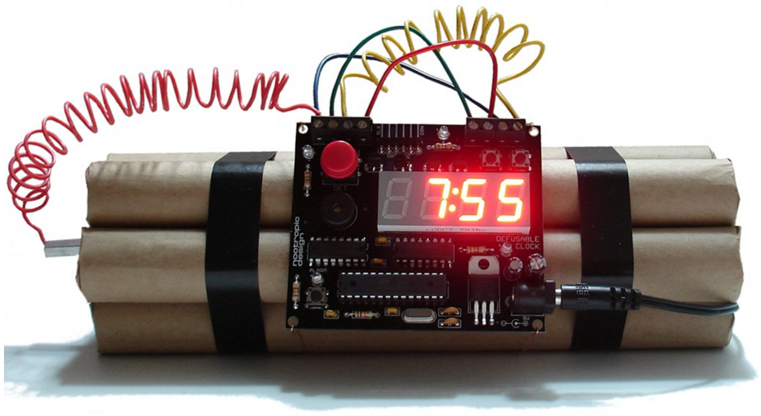

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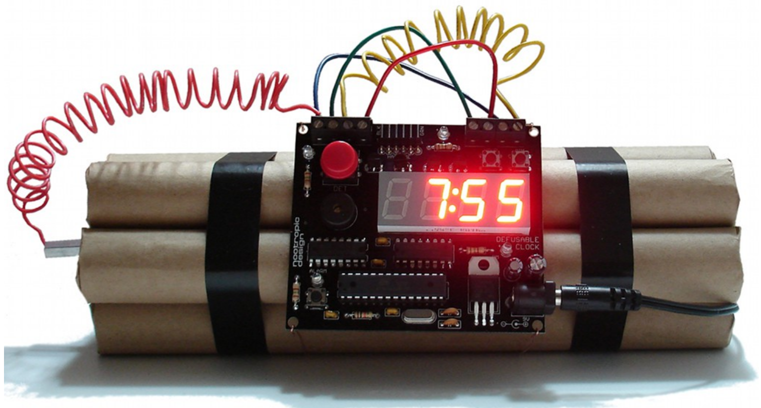

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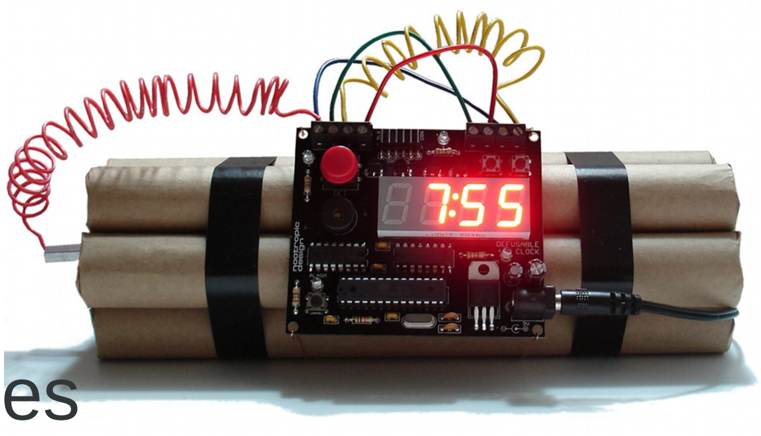

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Measurement of observable with eigenstates $|a\rangle$ at t_0:

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memory dof

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$$\Rightarrow \boxed{p(a|t_0) = |\langle a|\psi(t_0)\rangle|^2 = \|{}_m\langle a|_T\langle t_0|\Psi\rangle\rangle\|^2}$$

$$= |\psi_a(t_0)|^2$$

(Born's rule)

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$$+ \int_{t_1}^{\infty} dt \sum_{ab} \psi_a(t_0) U(t-t_1)|b\rangle_S \langle b|U(t-t_0)|a\rangle_S |a\rangle_m^N |b\rangle_{m'}^N |t\rangle_T$$

Bayes rule

$$p(b|a,t_1) = \frac{p(b,a,t_1)}{p(a,t_1)} = \frac{\|\langle a|\langle b|\langle t_1|\Psi\rangle\rangle\|^2}{\|\langle a|\langle t_1|\Psi\rangle\rangle\|^2}$$

two time measurements: same idea!!

$|a\rangle$ at t_0 and $|b\rangle$ at t_1 :

$$|\Psi\rangle\rangle = \int_{-\infty}^{t_0} dt \ldots + \int_{t_0}^{t_1} dt \sum_a \psi_a(t_0) U(t - t_0)|a\rangle_S |a\rangle_m^N |r\rangle_{m'}^N |t\rangle_T$$

$$+ \int_{t_1}^{\infty} dt \sum_{ab} \psi_a(t_0) U(t - t_1)|b\rangle_S \langle b|U(t - t_0)|a\rangle_S |a\rangle_m^N |b\rangle_{m'}^N |t\rangle_T$$

Bayes rule

$$p(b|a, t_1) = \frac{p(b, a, t_1)}{p(a, t_1)} = \frac{\| \langle a|\langle b| \langle t_1|\Psi\rangle\rangle \|^2}{\| \langle a|\langle t_1|\Psi\rangle\rangle \|^2}$$

$$= |\langle b|U(t_1 - t_0)|a\rangle|^2$$

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Bayes rule

$$p(b|a,t_1) = \frac{p(b,a,t_1)}{p(a,t_1)} = \frac{\|\langle a|\langle b|\langle t_1|\Psi\rangle\rangle\|^2}{\|\langle a|\langle t_1|\Psi\rangle\rangle\|^2}$$

$$= \left|\langle b|U(t_1-t_0)|a\rangle\right|^2$$

The expected outcome!! (Born's rule)

⇒ Kuchar's objection is defeated!