Fast chaos in the outer planetary region

G. B. Valsecchi[1,2]

[1]IAPS-INAF, Roma, Italy

[2]IFAC-CNR, Sesto Fiorentino, Italy

Dynamics in the outer planetary region

The orbits of comets and asteroids in the outer planetary region are strongly affected by close encounters with the giant planets.

- An important parameter is the planetocentric velocity:
  - fast encounters – with hyperbolic planetocentric orbits – are effective only if deep;
  - slow encounters – with temporary satellite captures – can greatly modify cometary orbits even if rather shallow.
- The outcomes can be extremely sensitive to initial conditions.
- Here we deal only with fast encounters, in which the planetocentric velocity of the small body is hyperbolic.

Planetocentric velocity

In the the analytic theory of close encounters (Öpik 1976, Carusi et al. 1990, Valsecchi et al. 2003, Valsecchi 2006), let us consider the orbit of given a, e, i of a small body that can encounter a planet on a circular orbit of radius a_p.

From a, e, i we compute U (the modulus of the planetocentric velocity of the small body) and θ (the angle between $\vec{U}$ and the heliocentric velocity of the planet):

$$\begin{aligned} U &= \sqrt{3 - T} = \sqrt{3 - \frac{a_p}{a} - 2\sqrt{\frac{a(1-e^2)}{a_p}} \cos i} \\ \cos\theta &= \frac{1 - U^2 - \frac{a_p}{a}}{2U}; \end{aligned}$$

U is in units of the heliocentric velocity of the planet.

Planetocentric velocity and Tisserand parameter

- In the expression for U, T is the Tisserand parameter (Tisserand 1889a, 1889b).
- Kresák (1972) set the dividing line between asteroids and Jupiter family comets (JFCs, historically defined as with $P < 20$ yr) at $T_J = 3$.
- Carusi et al. (1987) proposed the dividing line between JFCs and Halley type comets (HTCs, historically defined as with $20 < P < 200$ yr) at $T_J = 2$.
- Thus, JFCs have $3 > T_J > 2$, while HTCs have $T_J < 2$; note that many HTCs are on retrograde orbits.

Asteroids in retrograde orbits

Not only HTCs, but also an increasing number of asteroids are found to be on retrograde orbits. Among them, not surprisingly, many Damocloids, defined as having $T_J < 2$.

What processes have transformed the orbits of these bodies from direct to retrograde?

Pathways to retrograde orbits

Possibilities include:

- residence in the Oort cloud, followed by planetary capture; in this case, the asteroid may be an extinct comet;
- chaotic evolution to mean-motion or degenerate secular resonance (like the ν_6), with large amplitude changes to inclination (Greenstreet et al. 2012), helped by the eccentric Lidov-Kozai mechanism (Lithwick and Naoz 2011, Naoz et al. 2017);
- close encounter with a planet (Rickman et al. 2017).

Here, we concentrate on the last mechanism, in the framework of the analytical theory of close encounters.

Transition prograde $\rightarrow$ retrograde

For $i = 90^\circ$, U becomes:

$$U = \sqrt{3 - \frac{a_p}{a}},$$

that implies:

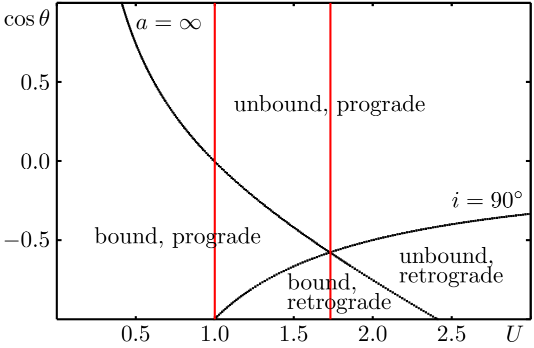
$$\frac{a_p}{a} = 3 - U^2.$$

Substituting back in the expression for θ:

$$\cos\theta_{i=90^\circ} \quad = \quad -\frac{1}{U}.$$

This implies that transitions to retrograde orbits can take place only if $U \geq 1$, no matter what the mass of the planet is.

Transition prograde $\rightarrow$ retrograde



The plane U-$\cos\theta$; close encounters displace the orbit vertically in this plane.

For $1 \leq U$, orbits bound to the Sun can only be prograde.

For $U \geq \sqrt{3}$, prograde orbits can only be unbound from the Sun.

The b-plane

In the analytic theory of close encounters:

- the b-plane of an encounter (Kizner 1961)
  - is the plane containing the planet and
  - is perpendicular to the planetocentric unperturbed velocity $\vec{U}$;
- the vector from the planet to the point in which $\vec{U}$ crosses the plane is $\vec{b}$, and the coordinates of the crossing point are ξ, ζ;
- the coordinate $\xi = \xi(a, e, i, \omega, f_b)$ is the local MOID;
- the coordinate $\zeta = \zeta(a, e, i, \Omega, \omega, f_b, \lambda_p)$ is related to the timing of the encounter.

In the above expressions, a, e, i, Ω, ω are the elements of the pre-encounter small body orbit, while f_b is the small body true anomaly and λ_p the longitude of the planet, both evaluated at the crossing of the b-plane.

The b-plane circles

The locus of b-plane points for which the post-encounter orbit has a given value of a', i.e. of θ', is a circle (Valsecchi et al. 2000) centred on the ζ-axis at $\zeta = D$, of radius $|R|$, with

$$D = \frac{c \sin\theta}{\cos\theta' - \cos\theta} \qquad R = \frac{c \sin\theta'}{\cos\theta' - \cos\theta},$$

where the scale factor $c = m/U^2$ is the value of the impact parameter corresponding to a velocity deflection of 90°.

These b-plane circles

- are a building block of the algorithm allowing to understand the geometry of impact keyholes (Valsecchi et al. 2003);
- can be used to explain the asymmetric tails of energy perturbation distributions (Valsecchi et al. 2000).

Transition prograde → retrograde

To obtain a transition from prograde to retrograde, we need a close encounter that changes θ into $\theta' > \theta_{i=90^\circ}$.

This is something that we know how to obtain: the *b*-plane coordinates must be within the circle of radius $|R_{i'=90^\circ}|$ centred in:

$$\begin{aligned} \xi &= 0 \\ \zeta &= D_{i'=90^\circ}, \end{aligned}$$

with $D_{i'=90^\circ}, R_{i'=90^\circ}$ given by:

$$\begin{aligned} D_{i'=90^\circ} &= \frac{c \sin\theta}{\cos\theta'_{i'=90^\circ} - \cos\theta} \\ R_{i'=90^\circ} &= \frac{c \sin\theta'_{i'=90^\circ}}{\cos\theta'_{i'=90^\circ} - \cos\theta}. \end{aligned}$$

Mapping the b-plane on the $\delta\omega$-$\delta\lambda_p$ plane

Valsecchi et al. (2018) show that small displacements from the origin in the b-plane, keeping constant a, e, i, Ω, can be mapped linearly onto the $\delta\omega$-$\delta\lambda_p$ plane.

Thus, the b-plane circles become ellipses when the corresponding initial conditions are considered in the ω-λ_p plane.

(5335) Damocles

Damocloids take their name from (5335) Damocles, discovered in 1991.

Damocles' orbit: $a = 11.83$ au, $e = 0.867$, $i = 61.^\circ8$, $T_J = 1.15$.

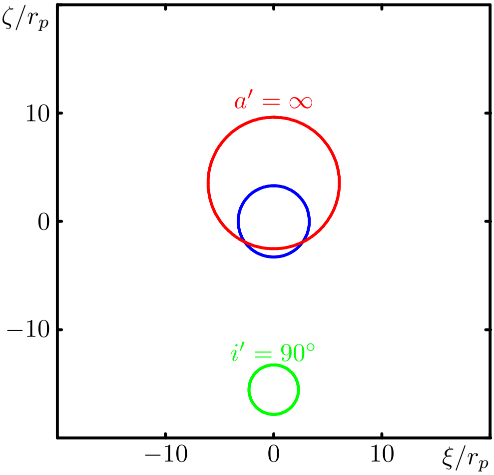
Can encounter Uranus: $MOID_U = 0.31$ au, $T_U = 1.99$.

An encounter with Uranus can eject Damocles from the planetary system, or can flip its orbit into a retrograde one.

Circles... really?

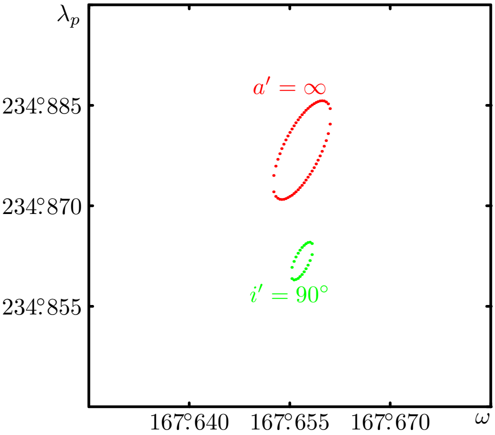
On the right: *b*-plane circles for $a' = \infty$ (red) and $i' = 90^\circ$ (green) for encounters of Damocles with Uranus, whose cross-section is the blue circle.

The dots come from a numerical integration of the restricted 3-dimensional 3-body problem, and confirm the accuracy of the analytical theory.



...and ellipses!

On the right: the points lying on the b-plane circles for $a' = \infty$ (red) and $i' = 90^\circ$ are arranged in ellipses in the initial conditions $\delta\omega$-$\delta\lambda_p$ plane.



Potentially Retrograde Asteroids (PRAs)

Let us consider asteroids on prograde orbits such that $1 \leq U \leq \sqrt{3}$ with respect to one or more of the outer planets.

These asteroids are "potentially retrograde" because an encounter with a planet with respect to which they have $1 \leq U \leq \sqrt{3}$ could make their orbit flip to retrograde, still remaining bound to the Sun.

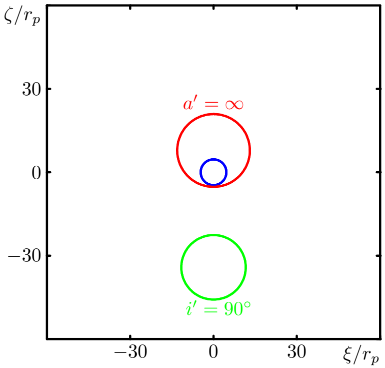
A Potentially Retrograde NEA: 2009 WN_{25}

The orbit of PRA 2009 WN_{25} has $a = 3.27$ au, $e = 0.66$, $i = 72^\circ$, $MOID_J = 0.03$ au.

With respect to Jupiter it has $U = 1.02$.

The cross-section for flipping to retrograde is 6.3 times larger than that for Jupiter collision.

The cross-section for ejection from the Solar System is 7 times larger than for Jupiter collision.



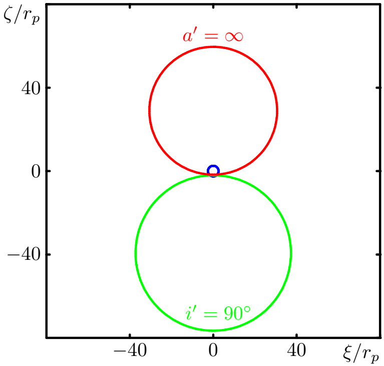
PRA 2014 TZ_{33}

The orbit of PRA 2014 TZ_{33} has $a = 38.32$ au, $e = 0.76$, $i = 86°$.

With respect to Saturn PRA 2014 TZ_{33} has $U = 1.60$.

The cross-section for flipping to retrograde is 215 times larger than that for Saturn collision.

The cross-section for ejection from the Solar System is 144 times larger than for Saturn collision.



Interstellar visitors

So far, two interstellar objects have been discovered, the first one on a retrograde orbit and the other on a prograde orbit.

	q (au)	e	a (au)	i (°)	V_{hyp} (km/s)	$b_{\odot}$ (au)
1I	0.256	1.201	−1.272	122.7	26.4	0.847
2I	2.006	3.354	−0.852	44.1	32.3	2.728

Can objects in orbits like these be captured by, say, an encounter with Jupiter?

Capture from initial hyperbolic orbits

Using Gauss' units, consider a planet of mass m on a circular orbit of radius a_p, and a small body on an initial hyperbolic orbit, with heliocentric velocity at infinity V_{hyp}, impact parameter with respect to the Sun $b_{\odot}$, and inclination with respect to the orbit of the planet i.

The problem is then similar to that of studying impacts of near-Earth asteroids on the Moon, treated analytically in Valsecchi et al. (2014).

Capture from initial hyperbolic orbits

Semimajor axis, eccentricity and perihelion distance of the orbit are (k is Gauss'constant):

$$
\begin{aligned}
a &= -\frac{k^2}{V_{hyp}^2} \\
e &= \frac{\sqrt{k^4 + b_{\odot}^2 V_{hyp}^4}}{k^2} \\
q &= \frac{\sqrt{k^4 + b_{\odot}^2 V_{hyp}^4} - k^2}{V_{hyp}^2}.
\end{aligned}
$$

Capture from initial hyperbolic orbits

The analytical theory can be applied only if $q \leq a_p$; the condition $q = a_p$ sets a maximum for $b_\odot$, equal to:

$$b_{\odot max} = \frac{\sqrt{a_p(a_p V_{hyp}^2 + 2k^2)}}{V_{hyp}};$$

moreover, in order to allow a close encounter with the planet, the heliocentric distance of at least one of the nodal points of the orbit must be equal to a_p, which implies:

$$\cos\omega = \pm\frac{q(1+e) - a_p}{a_p e}.$$

Capture from initial hyperbolic orbits

We can then compute the planetocentric velocity U (U is in units of the heliocentric velocity of the planet):

$$U = \sqrt{3 + \frac{a_p V_{hyp}^2}{k^2} - \frac{2 b_\odot V_{hyp} \cos i}{k \sqrt{a_p}}},$$

and θ (θ is the angle between the heliocentric velocity of the planet and the planetocentric velocity of the small body):

$$\cos\theta = \frac{1 - U^2 - \frac{a_p}{a}}{2U}.$$

Capture from initial hyperbolic orbits

For given values of V_{hyp} and of $\cos i$, in order to allow encounters with a planet on circular orbit of radius a_p, the impact parameter $b_\odot$ must obey:

$$0 \leq b_\odot \leq \frac{\sqrt{a_p(a_p V_{hyp}^2 + 2k^2)}}{V_{hyp}}.$$

Capture from initial hyperbolic orbits

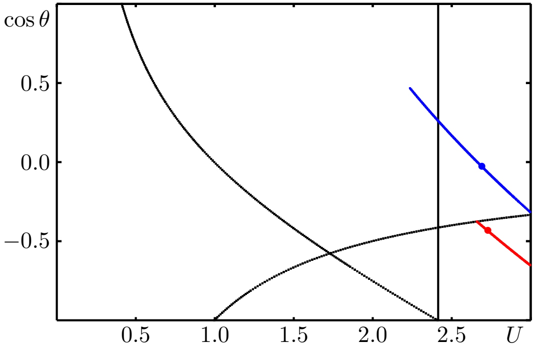
As a consequence, for positive $\cos i$ the planetocentric velocity U is bounded by:

$$
\begin{aligned}
U_{min} &= \sqrt{3 + \frac{a_p V_{hyp}^2}{k^2} - \frac{2\cos i \sqrt{a_p V_{hyp}^2 + 2k^2}}{k}} \\
U_{max} &= \sqrt{3 + \frac{a_p V_{hyp}^2}{k^2}},
\end{aligned}
$$

while for negative $\cos i$ the planetocentric velocity U is bounded by:

$$
\begin{aligned}
U_{min} &= \sqrt{3 + \frac{a_p V_{hyp}^2}{k^2}} \\
U_{max} &= \sqrt{3 + \frac{a_p V_{hyp}^2}{k^2} - \frac{2\cos i \sqrt{a_p V_{hyp}^2 + 2k^2}}{k}}.
\end{aligned}
$$

Capture from initial hyperbolic orbits



The hyperbolic objects 1I (red dot) and 2I (blue dot) in the plane U-$\cos\theta$ relative to Jupiter.

The lines show the range spanned by them keeping V_{hyp} and i fixed and varying $b_{\odot}$; 2I, for large $b_{\odot}$, falls on the left of the vertical line: in principle Jupiter could capture it to a bound retrograde helicentric orbit.

Capture from initial hyperbolic orbits

For the comet to be captured as a consequence of an encounter with the planet, we need to change θ into $\theta' \geq \theta'_{par}$, where θ'_{par} is the post-encounter value corresponding to parabolic orbits, given by:

$$\cos\theta'_{par} = \frac{1 - U^2}{2U}.$$

The b-plane circle corresponding to θ'_{par} is characterized by:

$$D_{par} = \frac{m \sin\theta}{U^2(\cos\theta'_{par} - \cos\theta)} \qquad R_{par} = \frac{mc \sin\theta'_{par}}{U^2(\cos\theta'_{par} - \cos\theta)}.$$

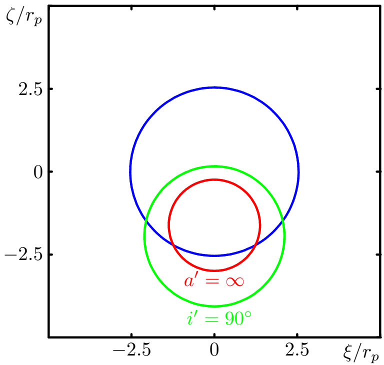
The probability of crossing the b-plane within this circle is:

$$p = \frac{m^2 a^2 (6U^2 - U^4 - 1)}{\pi U^3 a_p^2 \sin i \sqrt{2 - \frac{a_p}{a} - \frac{a(1-e^2)}{a_p}}}.$$

Capture from initial hyperbolic orbits

Assuming 2I to have the same V_{hyp} and i, but the maximum value of $b_{\odot}$ compatible with an encounter with Jupiter.

Then, it could be captured by Jupiter in a bound heliocentric retrograde orbit, but most of the initial conditions leading to capture would lead to a collision with the planet.



Conclusions

Close planetary encounters represent a viable mechanism by which a small Solar System body can be put in a retrograde orbit and vice-versa, and an interstellar object can be captured to a bound heliocentric orbit.

The analytical theory of close encounters allows to identify the regions of the b-plane and of the initial conditions space where these post-encounter states occur.

These region can be easily compared to the regions leading to other interesting orbital outcomes, like planetary collision or ejection from the Solar System.

References

- Carusi, Kresák, Perozzi, Valsecchi, A&A 187, 899 (1987)
- Carusi, Valsecchi, Greenberg, CeMDA 49,111 (1990)
- Greenstreet, Ngo, Gladman, Icarus 217, 355 (2012)
- Kizner, Planet. Space Sci. 7, 125 (1961)
- Kresák, BAICz 23, 1 (1972)
- Lithwick, Naoz, ApJ 742, 94 (2011)
- Naoz, Li, Zanardi, de Elía, Di Sisto, AJ 154, 18 (2017)
- Öpik, Interplanetary Encounters, Elsevier (1976)
- Rickman, Gabryszewski, Wajer, Wiśniowski, Wójcikowski, Szutowicz, Valsecchi, Morbidelli, A&A 598, A110 (2017)

References

- Tisserand, Bull. Astron. 6, 241 (1889a)
- Tisserand, Bull. Astron. 6, 289 (1889b)
- Valsecchi, Lect. Notes Phys. 682, 145 (2006)
- Valsecchi, Milani, Gronchi, Chesley, CeMDA 78, 83 (2000)
- Valsecchi, Milani, Gronchi, Chesley, A&A 408, 1179 (2003)
- Valsecchi, Alessi, Rossi, CeMDA 119, 257 (2014)
- Valsecchi, Alessi, Rossi, CeMDA 130, 8 (2018)