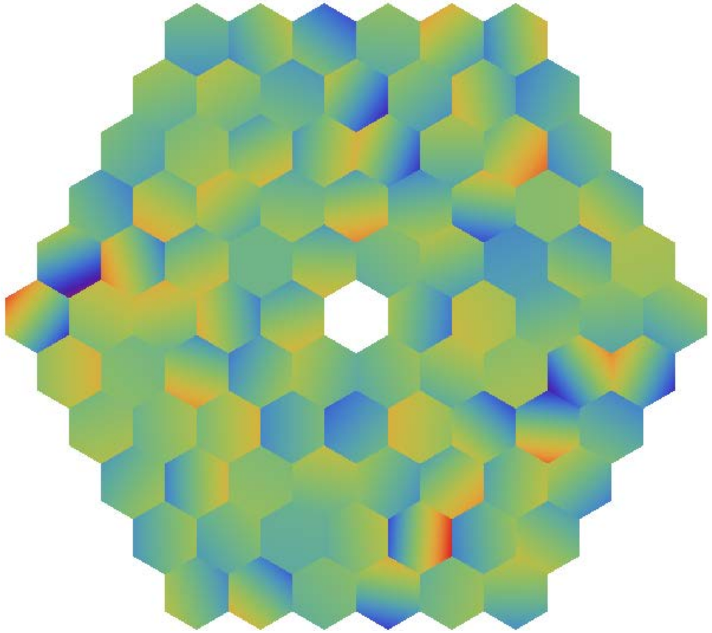
Maximum Likelihood Co-phasing of Large Segmented Mirrors under Turbulence with a Pyramid



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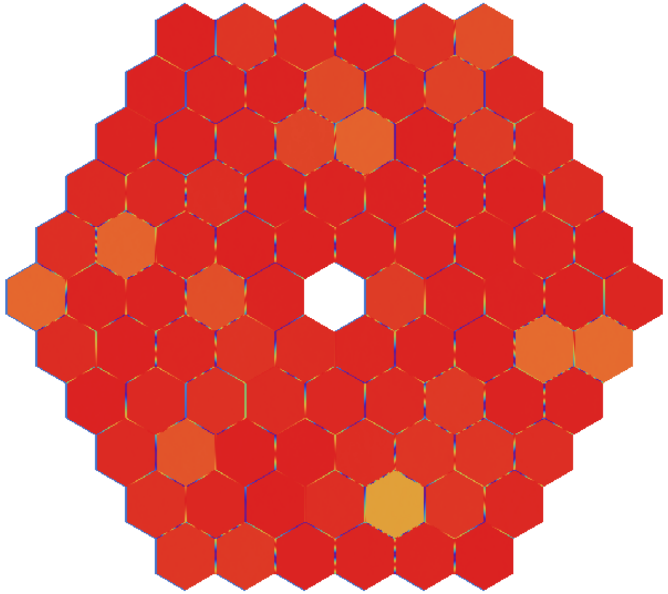
Wavefront Sensing Workshop

Oct 2019



Physical Optics Simulation Environment

→ ***Talk tomorrow at 17:20*** ←

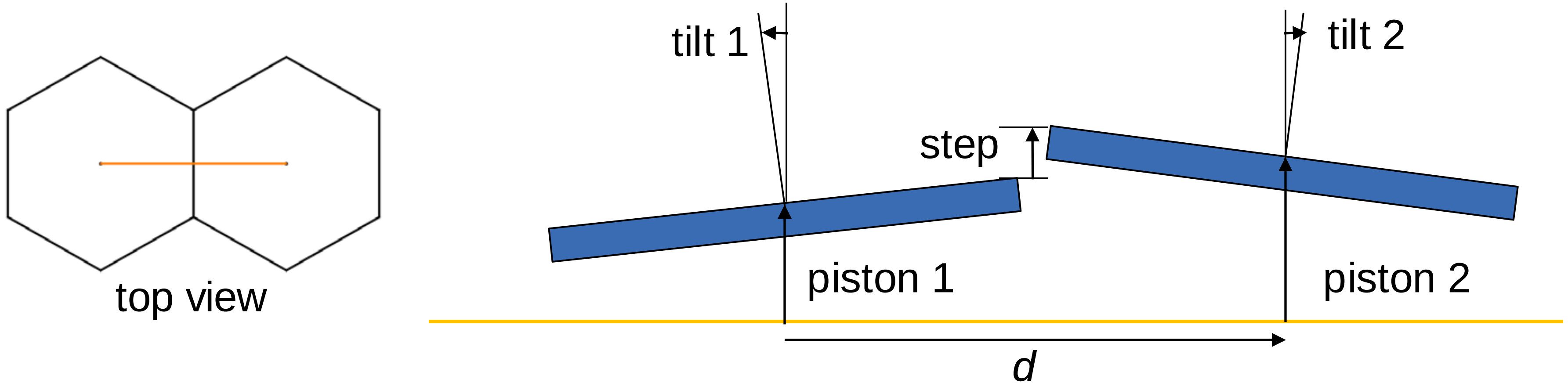
- Continuous need in ESO's Optics Department to simulate optical subsystems, often involving diffraction effects and turbulence
- "Numerical Test Bench": Deepen understanding of optical performance, compare technical options, ability to validate technical requirements, develop algorithms
- Fourier optics code, implemented in *Mathematica*
- Take advantage of *Mathematica's* strong analytical math capabilities, rapid code development, integrated graphical output, multiprocessing
- We welcome comparison of algorithms and results with OOMAO, YAO

Test Case: Study of Segment Phasing Scenarios Objective and Assumptions

- Phase large segmented mirrors for diffraction-limited observation (hundreds of segments, full pupil at the same time)
- Assume that segments are already coarsely stacked and piston errors are down to a few tens of waves
- Goal: Phase the mirror with a single WFS in as few exposures and position corrections as possible (one?), without need for (strong) AO
- Must be robust and registration/measurement error tolerant
- Neglecting detector noise, sky background, segment-to-pupil distortion for the moment; assuming bright star in long exposure (30–50 s)
- **This is a technical study**: Evaluating trades between different wavefront sensors and options for phase reconstruction methods

Geometric Equations

- ELT M1 segment state variables: {piston,tip,tilt} $\times$ 798 = 2394
- Each edge between two segments yields one equation:



- Link two pistons and tilts via the segment step:

$$step_{1\rightarrow 2} = piston_2 - piston_1 - (tilt_1 + tilt_2)\, d/2$$

➔ Entire mirror M1: Highly overdetermined linear equation system

Wavefront Sensor Wishlist

- Sense steps *and* segment tip/tilt in parallel; robust to petaling
- Simple, linear response function (weak saturation and low cross-talk)
- Keep structures resolved (no smearing) even in good seeing
- Can be imaged with a reasonable number of detector pixels
- Easy registration: Does not require accurate optical pupil alignment

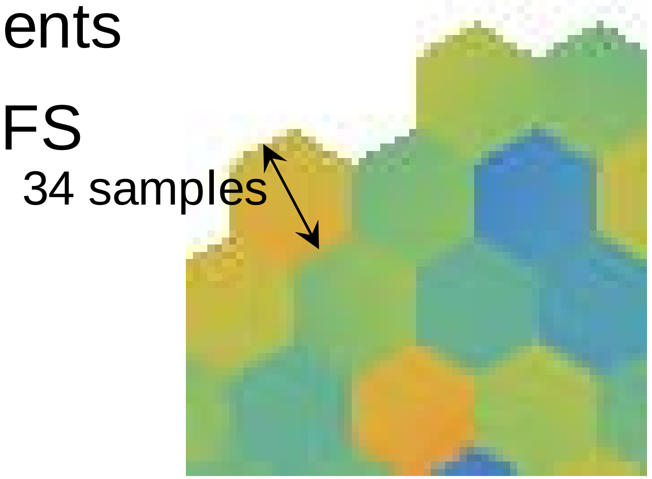
Some well-known candidates:

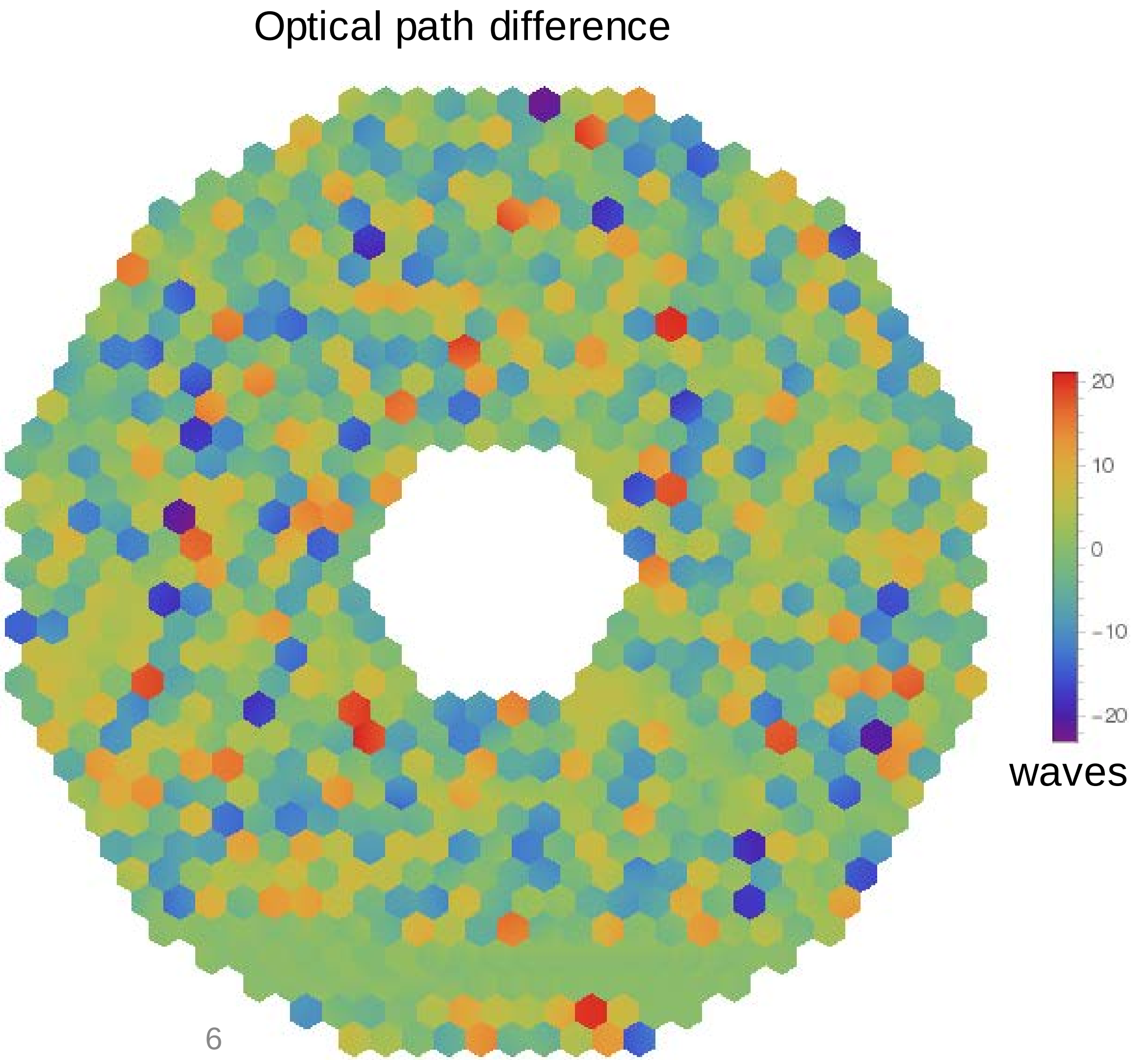
- Shack-Hartmann (with custom lenslet geometry?, in APE: *SHAPS*)
- Phase contrast WFS (phase contrast mask in the focal plane, *ZEUS*)
- Ext. Hartmann phase mask WFS (Ronchi grating/shearing like *Phasics SID-4*)
- Pyramid WFS with modulation (order 500–1000 pixels across pupil, *PYPS*)

Simulations

- Monochromatic in NIR, physical optics with FFT size 1176^2, non-elongated point source, average over 4000 independent phase screens
- 798 ELT-size hexagonal segments (1.22 m edge-to-edge), 2 edges aligned with pixel grid
- Gaussian random distribution of tip/tilt and piston misalignments
- Study the pyramid WFS
- Resolution:

3.6 cm, angle: 8.6 mas

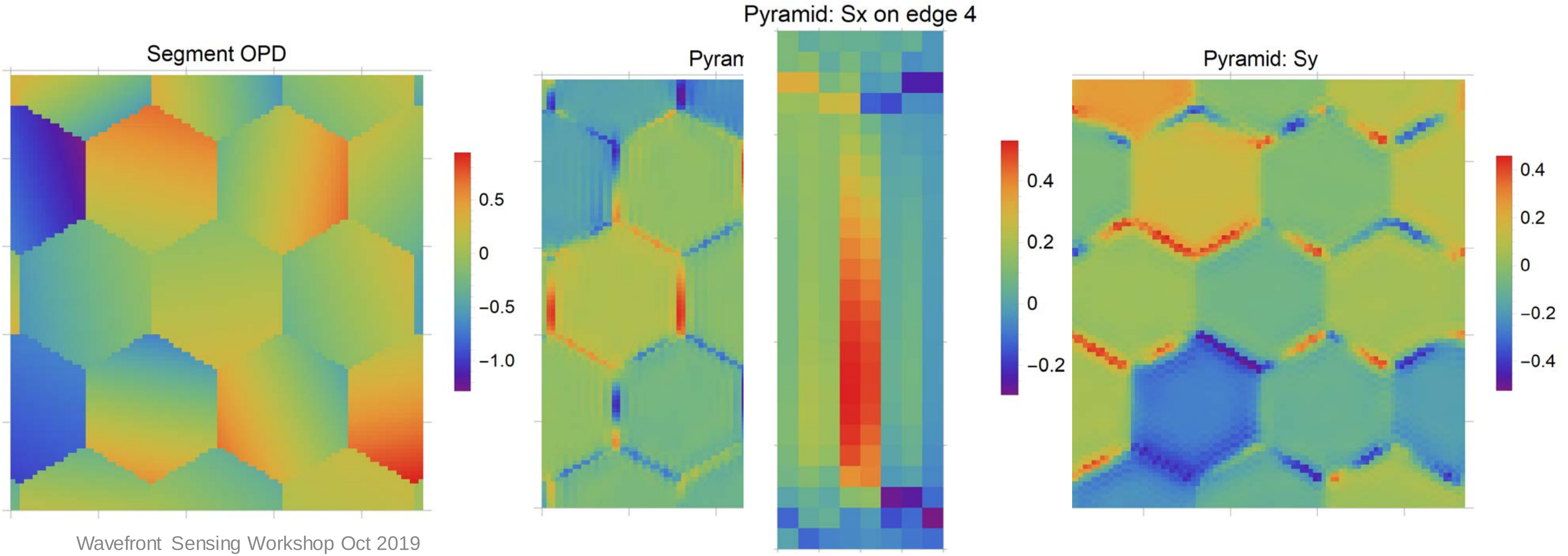




Modulated Pyramid WFS

- 1780 nm (H-band), averaged over 4000 phase screens, X- and Y-slopes (S_x, S_y)
- Seeing: 0.67” at 500 nm, IQ: 0.37”, r_0 : 602 mm (14.6 samples)

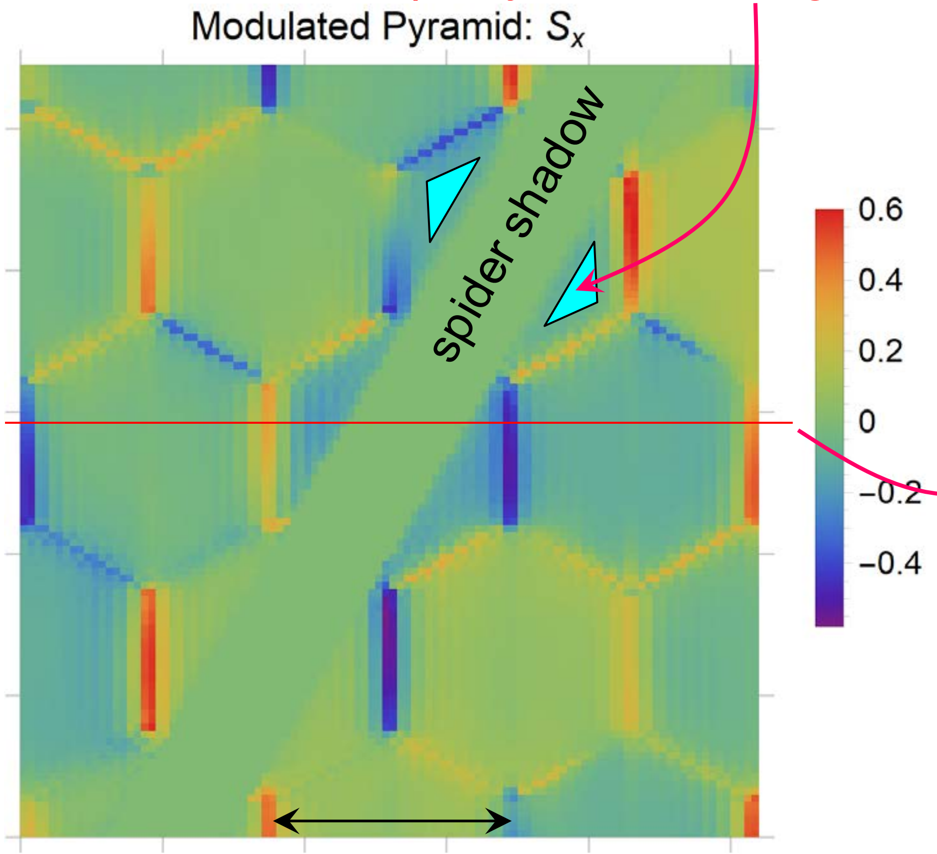
PWFS modulation radius:
0.47 " = 1.6 × λ/edge



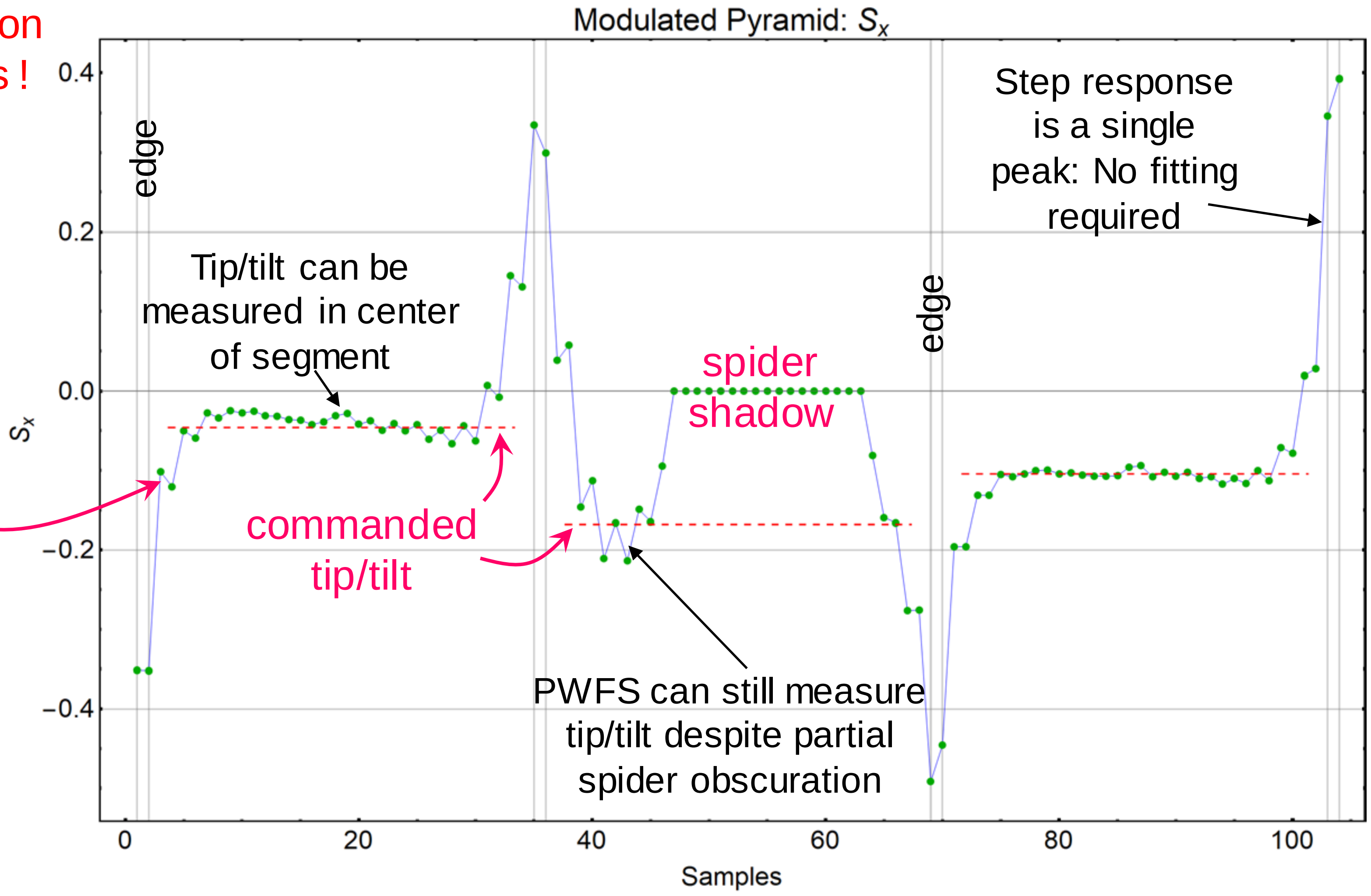
PWFS Slice Across Spider Shadow

1780 nm, r_0 : 602 mm (14.6 samples)

➔ Can sense tip/tilt even on partly obscured segments !



1228 mm ≈ 2 r_0



Maximum Likelihood Phasing Approach

- ELT M1 has 798 segments with 2262 inner edges (TMT: 492 / 1386)
- Each segment has 3 DoF: {piston,tip,tilt}. But we sense steps and tip,tilt
- Multi-color measurement to overcome phase ambiguity (synthetic wavelength) *
- Must be based on WFS error model (function of phase)
- Adopt a multilevel approach, using multicolor PWFS measurements:
  1. Sense $\{S_x, S_y\}$ near the segment centers (tip/tilt) and across edges (steps) for each color
  2. Compile list of possible {step,likelihood} pairs, using accurately calibrated WFS model
  3. Evaluate likelihood of triples of steps around each inner vertex (geometric consistency)
     → two ranked lists of possible steps per edge; pick matching step solution
  4. Reconstruction: Find state vector x that minimizes $(r\, \boldsymbol{V}^{-1}\, r)$ with $r = \boldsymbol{A} \cdot x - b$ (GLS)
- Steps 2 and 3 are key (algorithm not limited to a specific WFS)

* Esposito & Devaney, Beyond Conv. AO (2001)

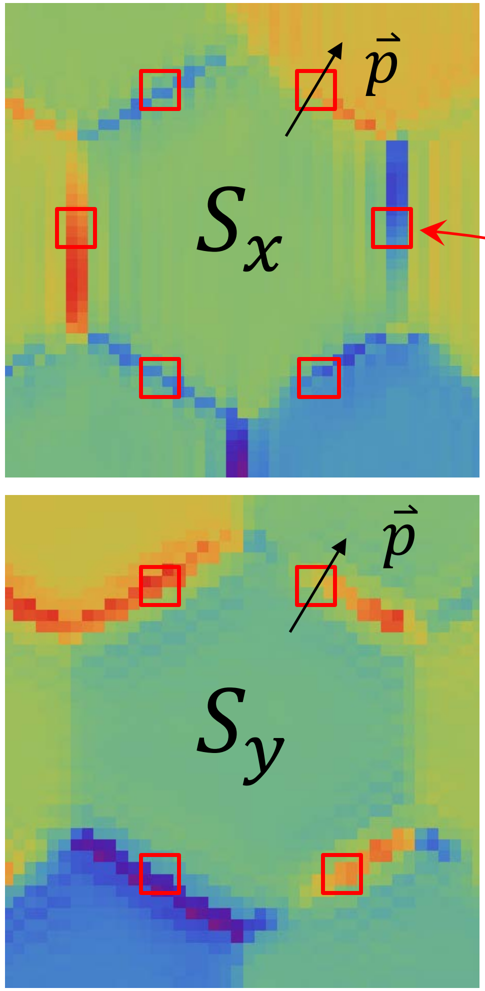
Step Reconstruction

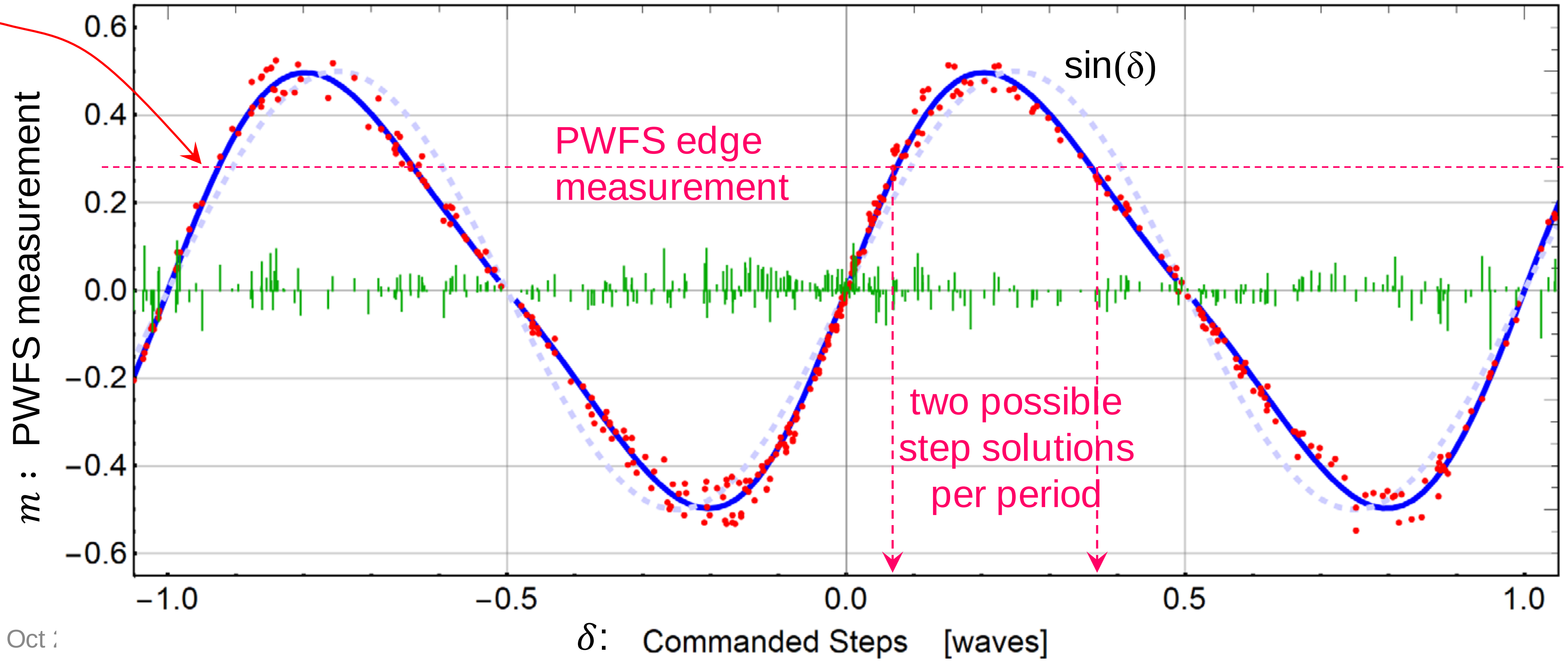
Fit function: skewed sine $m \coloneqq \left(\{S_x, S_y\} - \varepsilon\{R_x, R_y\}\right) \cdot \vec{p} = \dfrac{a\ \sin(2\pi\delta)}{1 - b\cos(2\pi\delta)}$

$\delta' \coloneqq \delta - [\delta]$: step in waves, mapped back to the basic period [−0.5, 0.5]

Small correction in m for tip/tilt $\{R_x, R_y\}$ of the two adjacent segments

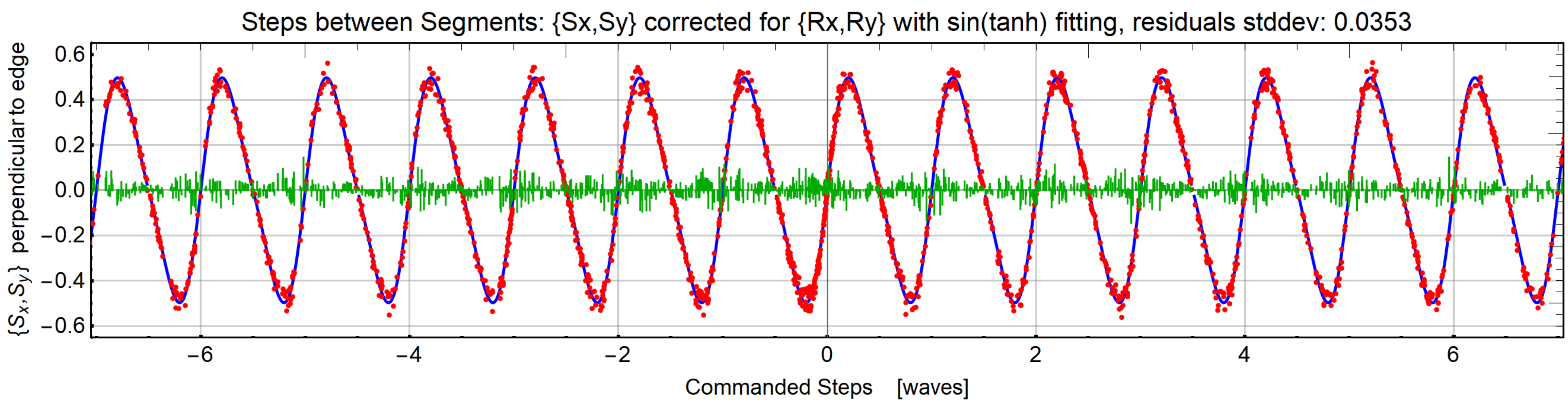
Parameter b to model residual saturation; smaller with larger modulation



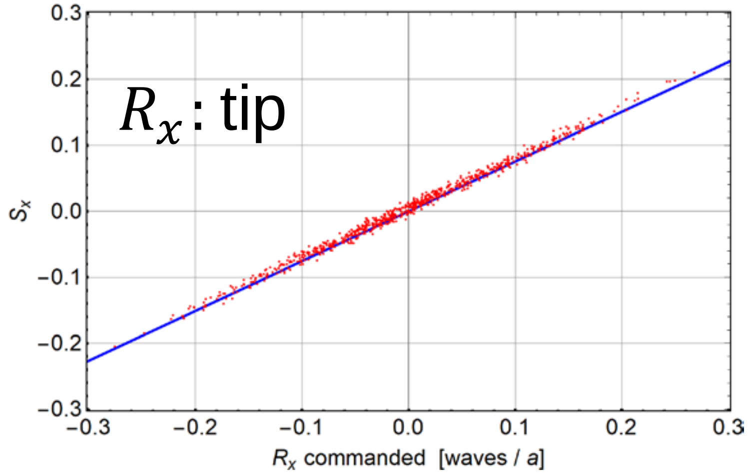


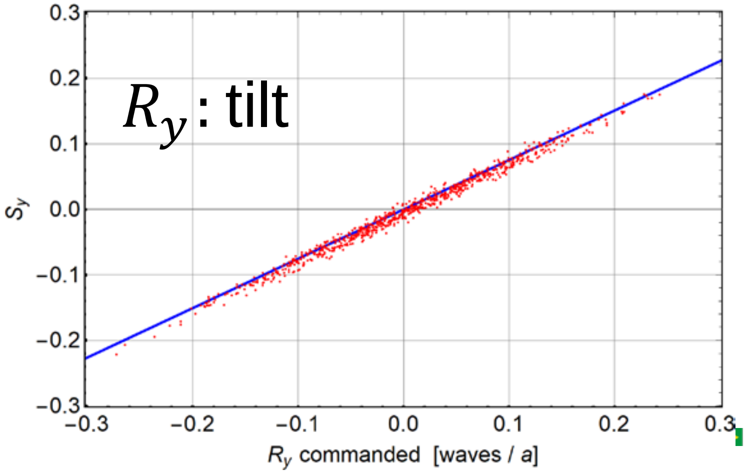
Fit Models for Steps and Tip/Tilt

Skewed sine is indeed a good fit function over many waves:

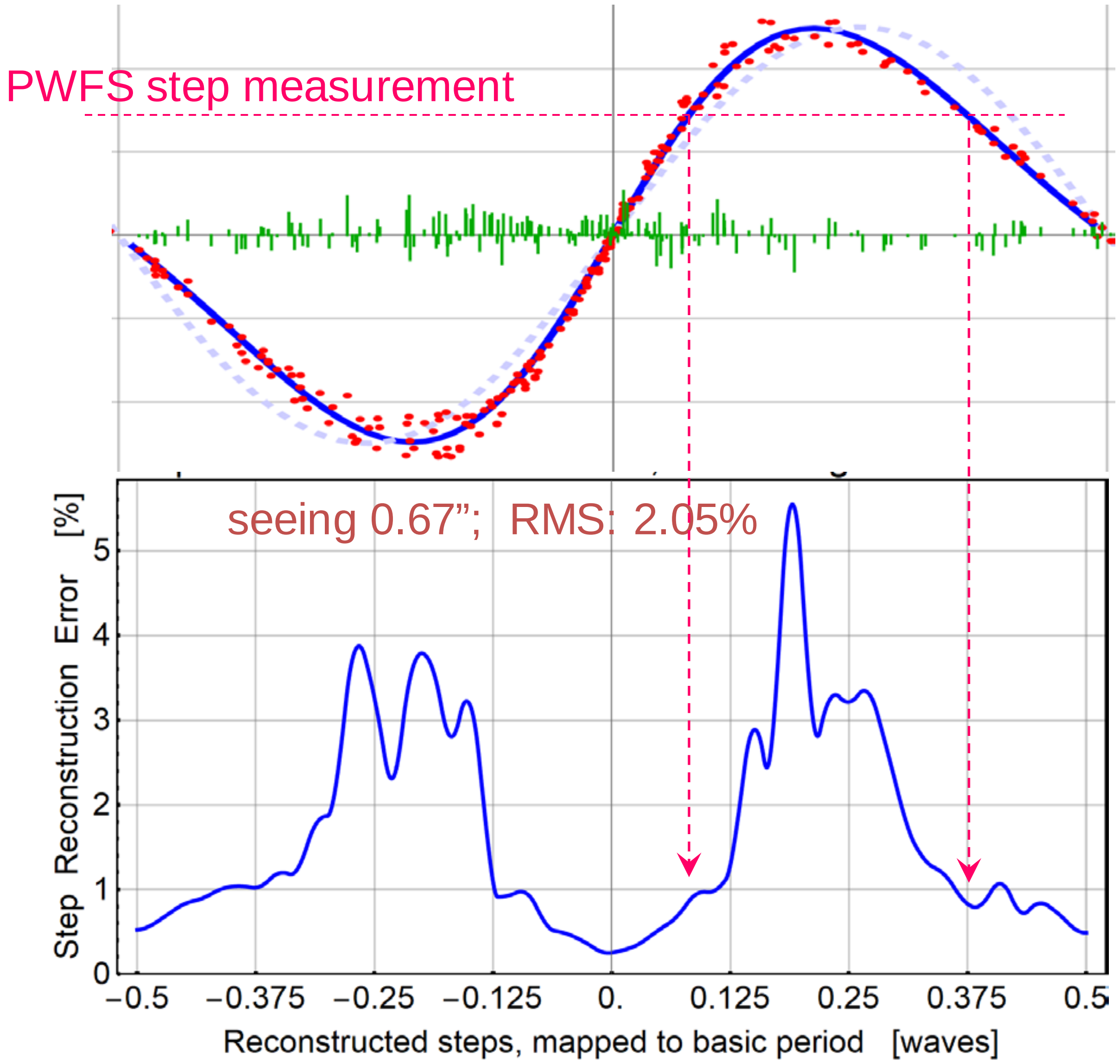


Random pistons picked from Gaussian PDF so that the steps have 6.3 waves OPD standard deviation; tip/tilt smaller by a factor 50

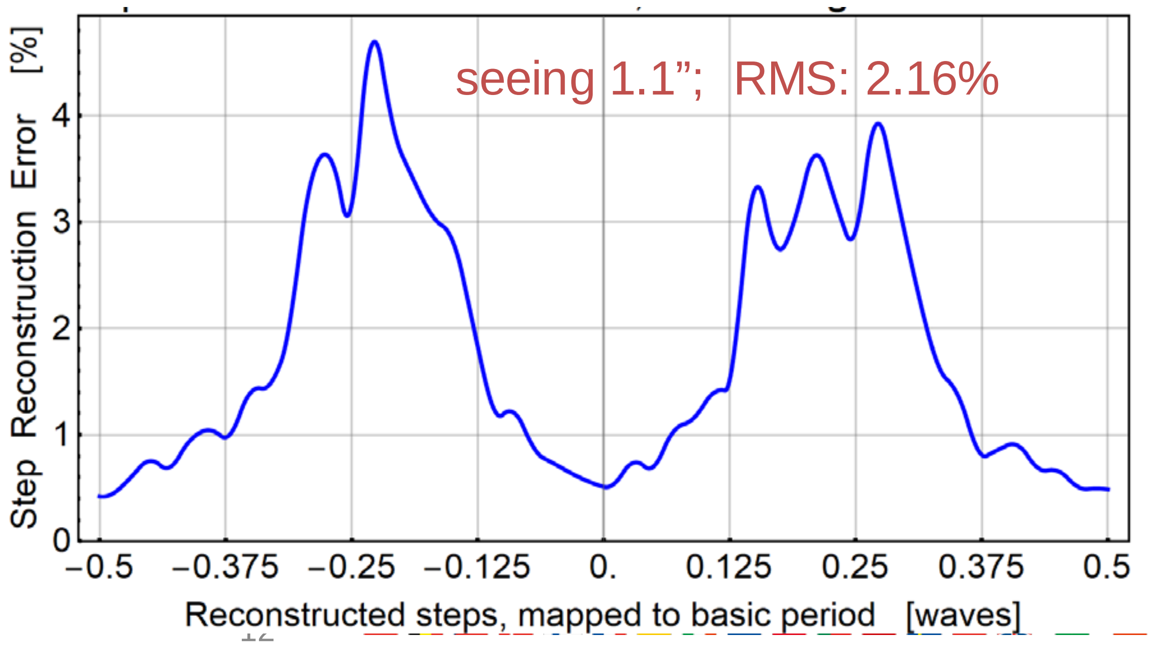




PWFS Step Inversion Error

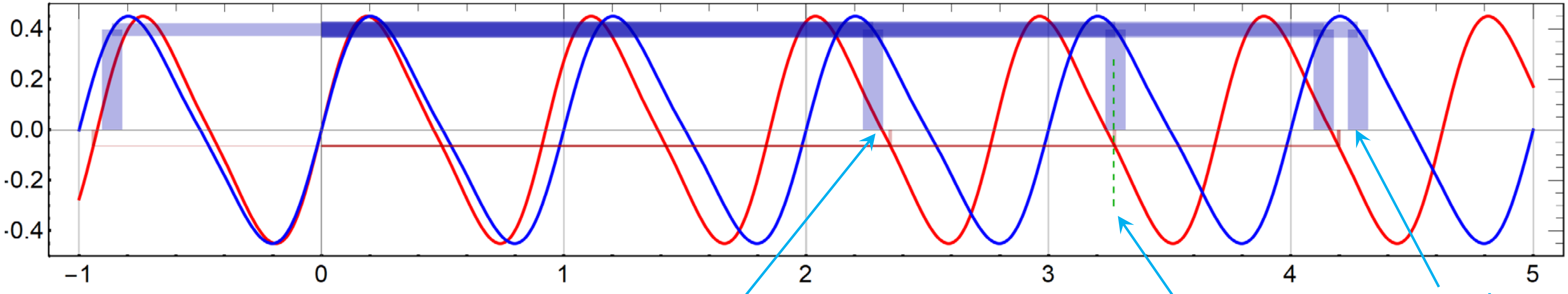


➔ Step inversion error is highest near the peaks of the skewed sine and lowest near the zeros. We use this information in the multicolor step reconstruction and later in the Generalized Least Squares method to find most likely sol’s



Multi-Color Step Solution Overlap

Likelihood of correct step solution is given by the solution overlap:



Assuming the individual color step measurements have Gaussian distributed errors (with different errors σ_i), integrate the product of the probability of several colors: This is the likelihood that the solutions pertain to the correct step

$$P_2 = \exp\left(-\frac{(x_1 - x_2)^2}{2(\sigma_1^2 + \sigma_2^2)}\right),$$

$$P_3 = \exp\left(-\frac{\sigma_1^2(x_2 - x_3)^2 + \sigma_2^2(x_1 - x_3)^2 + \sigma_3^2(x_1 - x_2)^2}{2(\sigma_1^2\sigma_2^2 + \sigma_3^2\sigma_2^2 + \sigma_1^2\sigma_3^2)}\right)$$

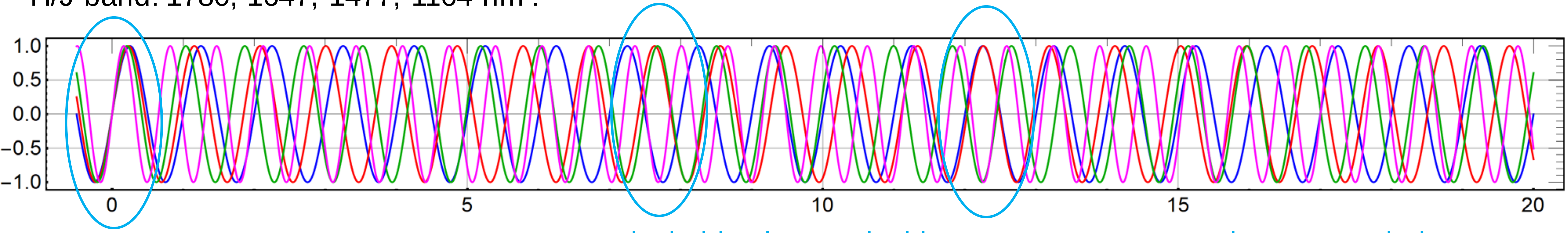
$$P_4 = \ \ldots$$

number of colors

Multicolor Sine Overlap

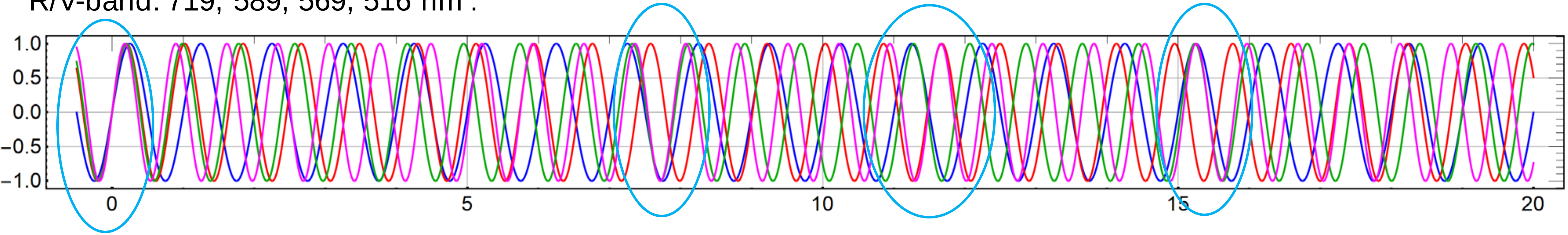
- Plot sine functions up to 20 waves and highlight areas where phases of 3 or even 4 colors agree (near peaks also phase $+\pi$ agreement)

H/J-band: 1780, 1647, 1477, 1164 nm :

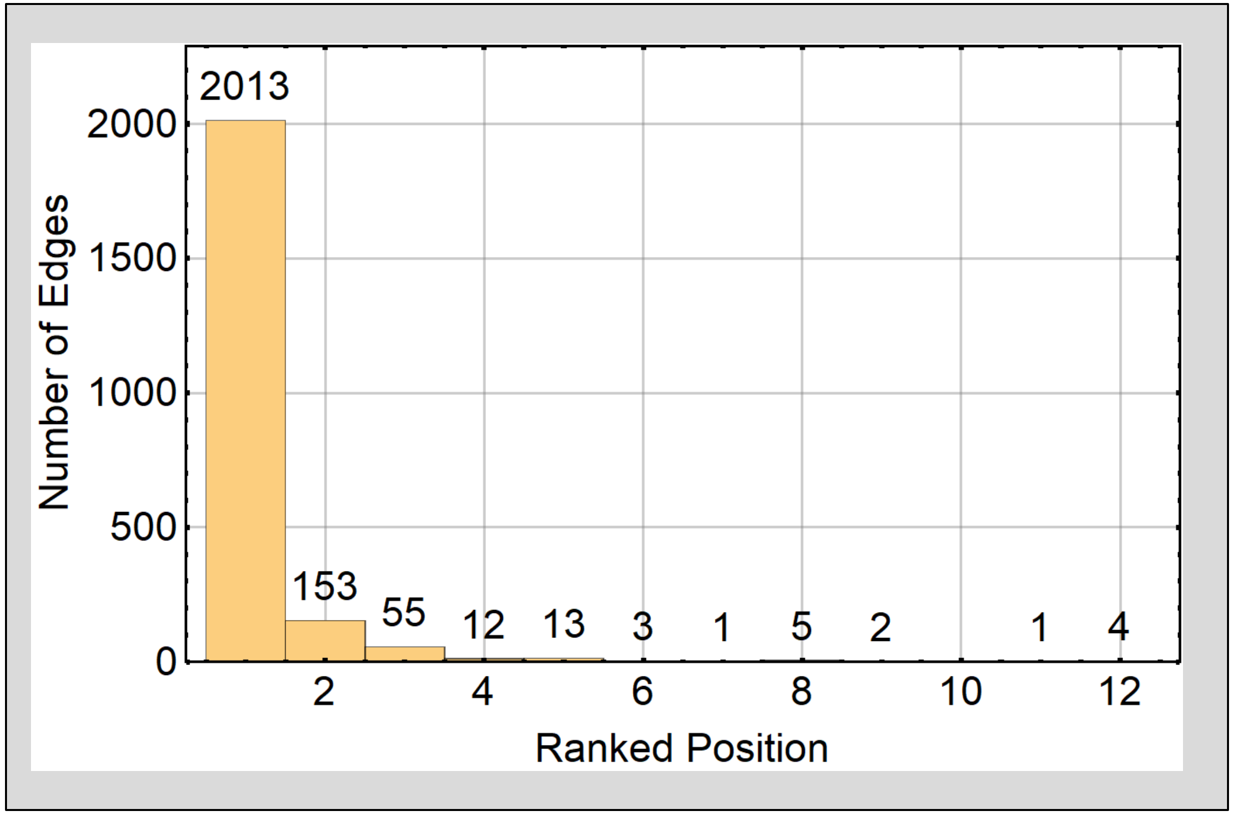


undesirable phase coincidences; may cause spurious step solutions

R/V-band: 719, 589, 569, 516 nm :



Step Solutions vs. Likelihood



Histogram showing the positions of the correct step solutions in a ranked list (2262 steps, 4 colors, 12 solutions per step in the list)

➔ Fraction of correct solutions in first position (90% in this example) is a strong function of the step inversion error

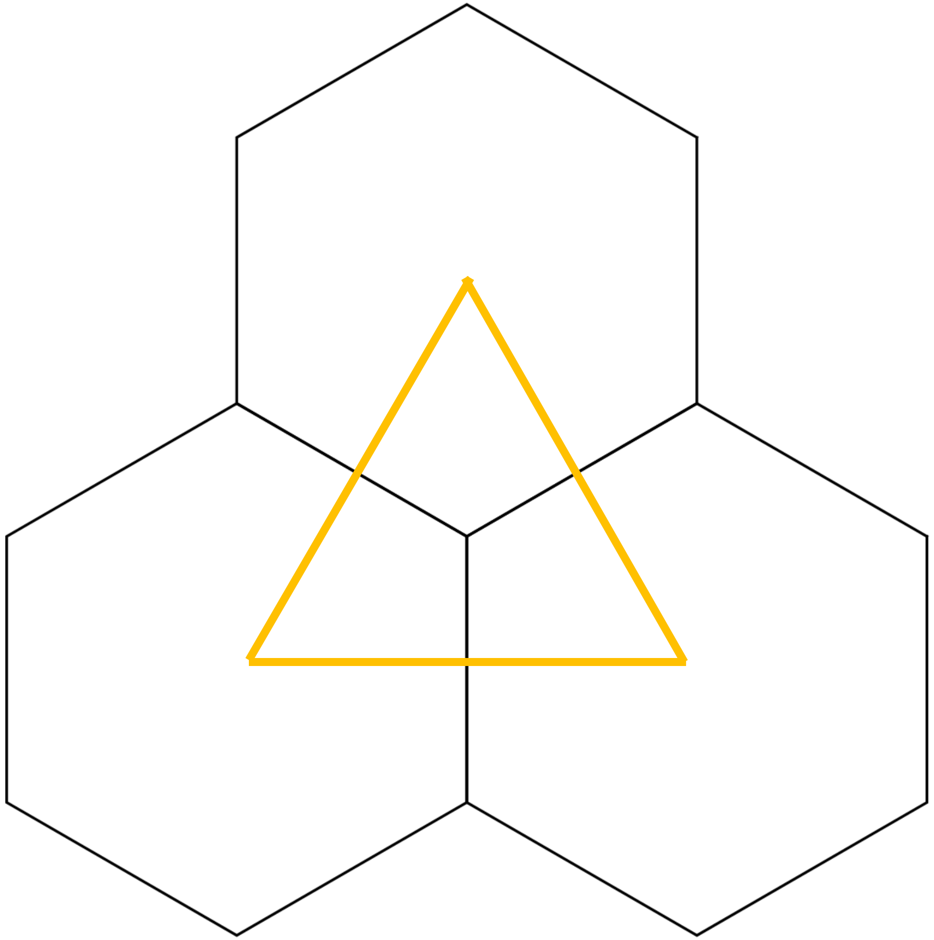
Exploit Geometric Redundancy on Clovers

- Clover: Set of three segments sharing a vertex
- Follow the yellow triangle: The sum of directed {tip,tilt} and steps must be zero ("phase closure")
- ***Step 3***: Set up list of step triples, compute the geometric error Δz in the directed loop sum
- Rank triples by combined likelihood:

$$P_{\text{clover}} = P_{\text{multicolor overlap}} \times P(\Delta z)$$

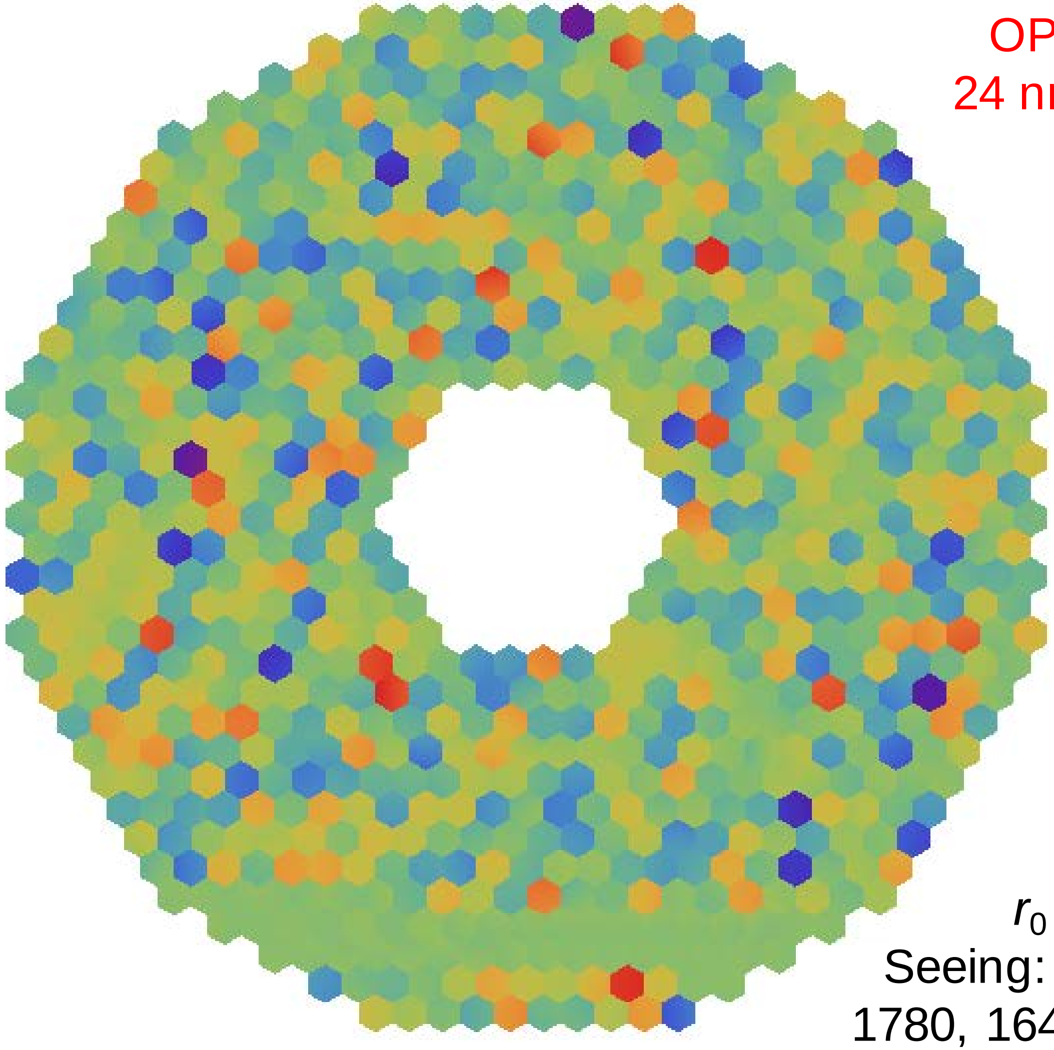
➔ Finally, compare the ranked triples for each edge between its two enclosing clovers and select most likely match(es)

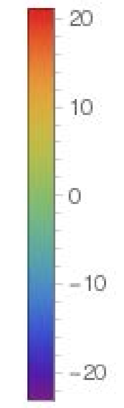
- Iterate with GLS; use large variances for high residual



H/J-Band Reconstruction (with spiders)

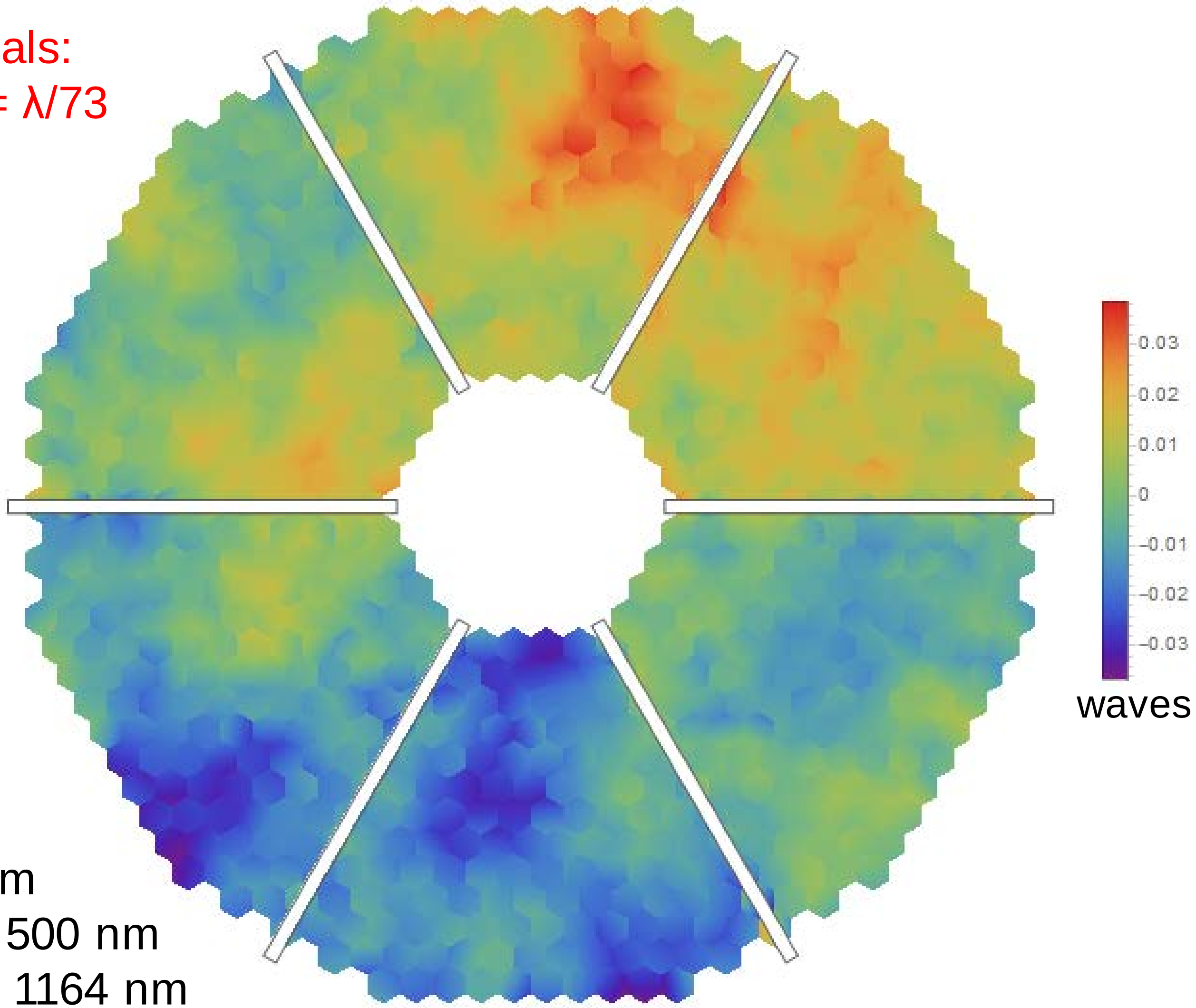
Before Phasing





Post-Phasing Residuals

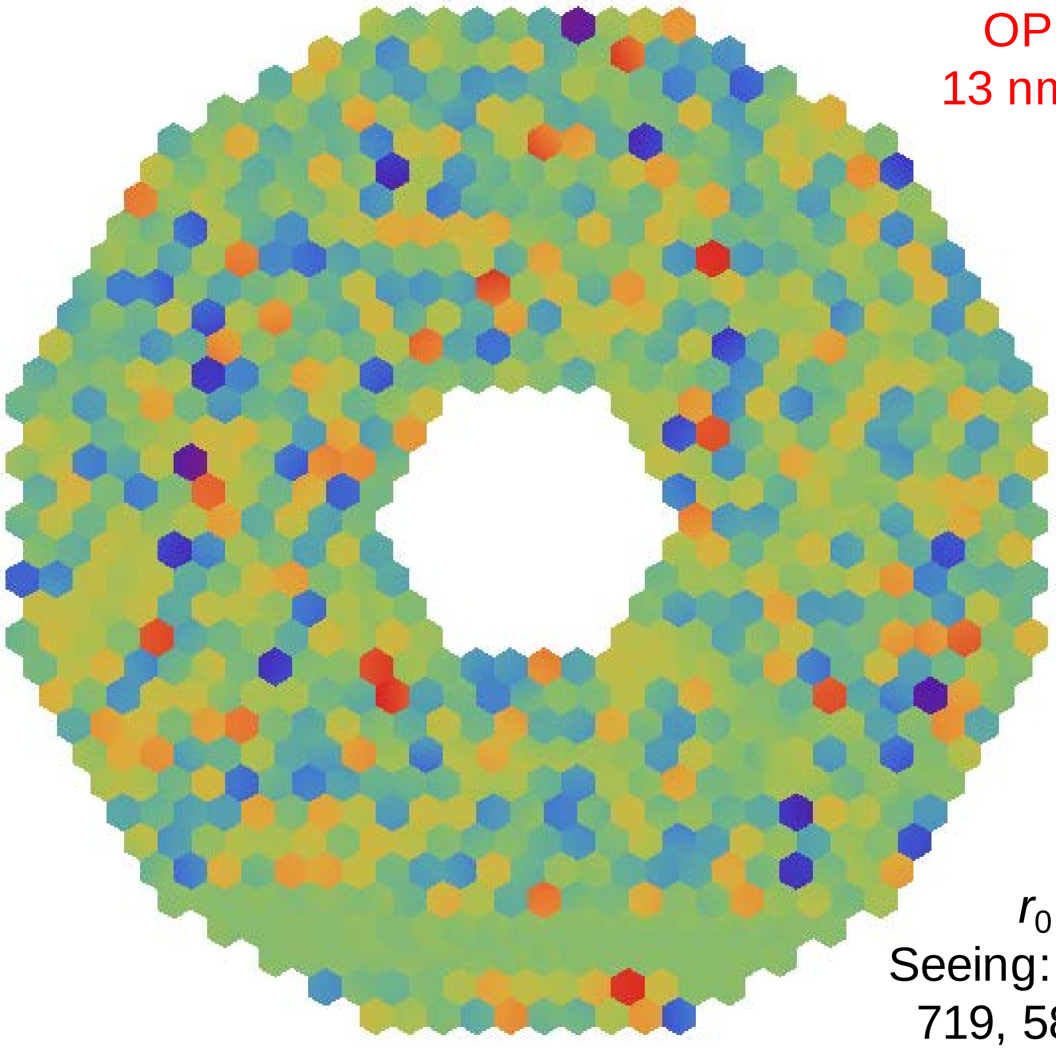
OPD residuals:
24 nm RMS = λ/73

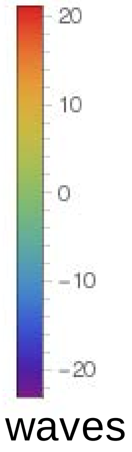


r_0 = 602 mm
Seeing: 0.67” @ 500 nm
1780, 1647, 1477, 1164 nm

R/V-Band Reconstruction (with spiders)

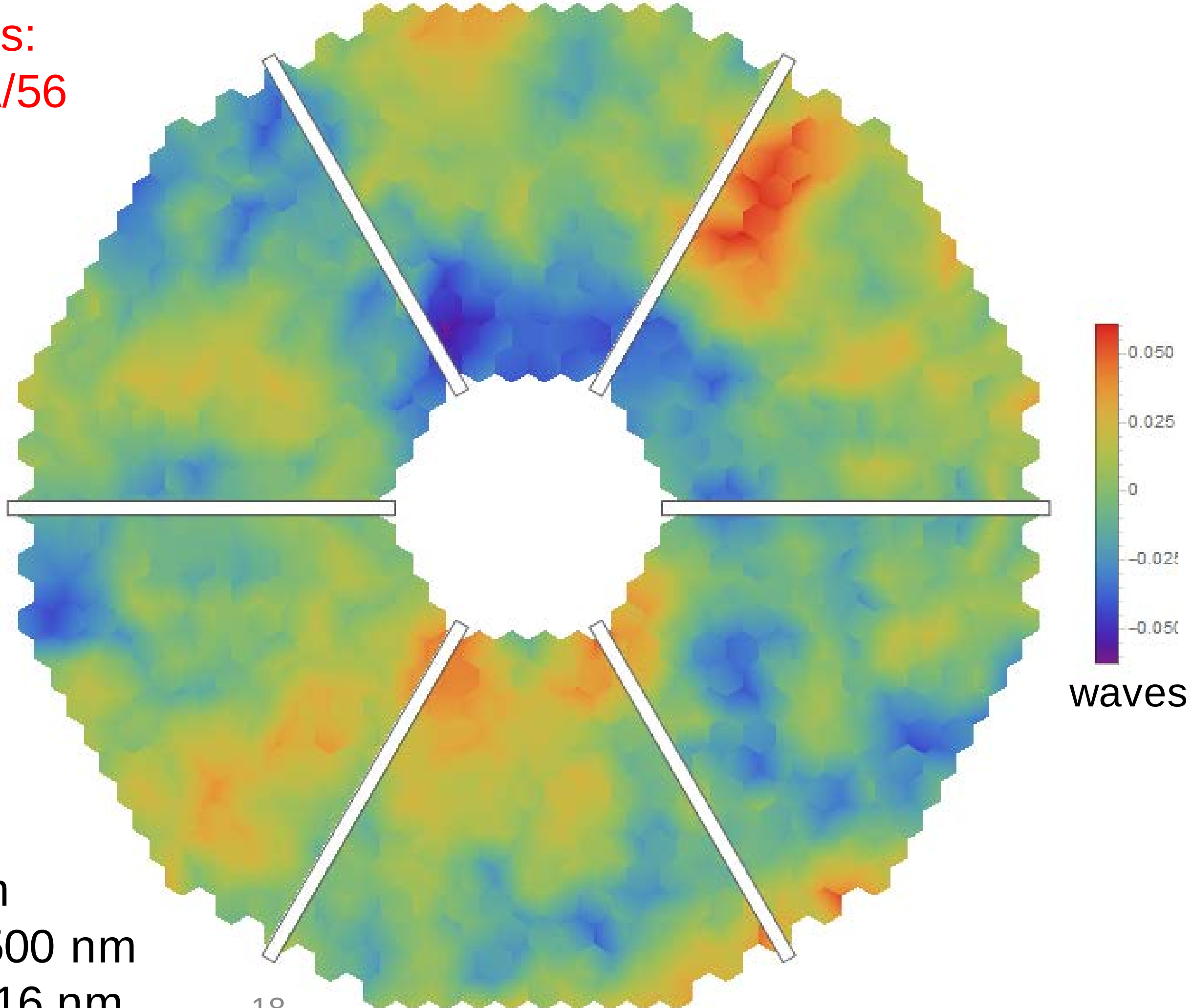
Before Phasing





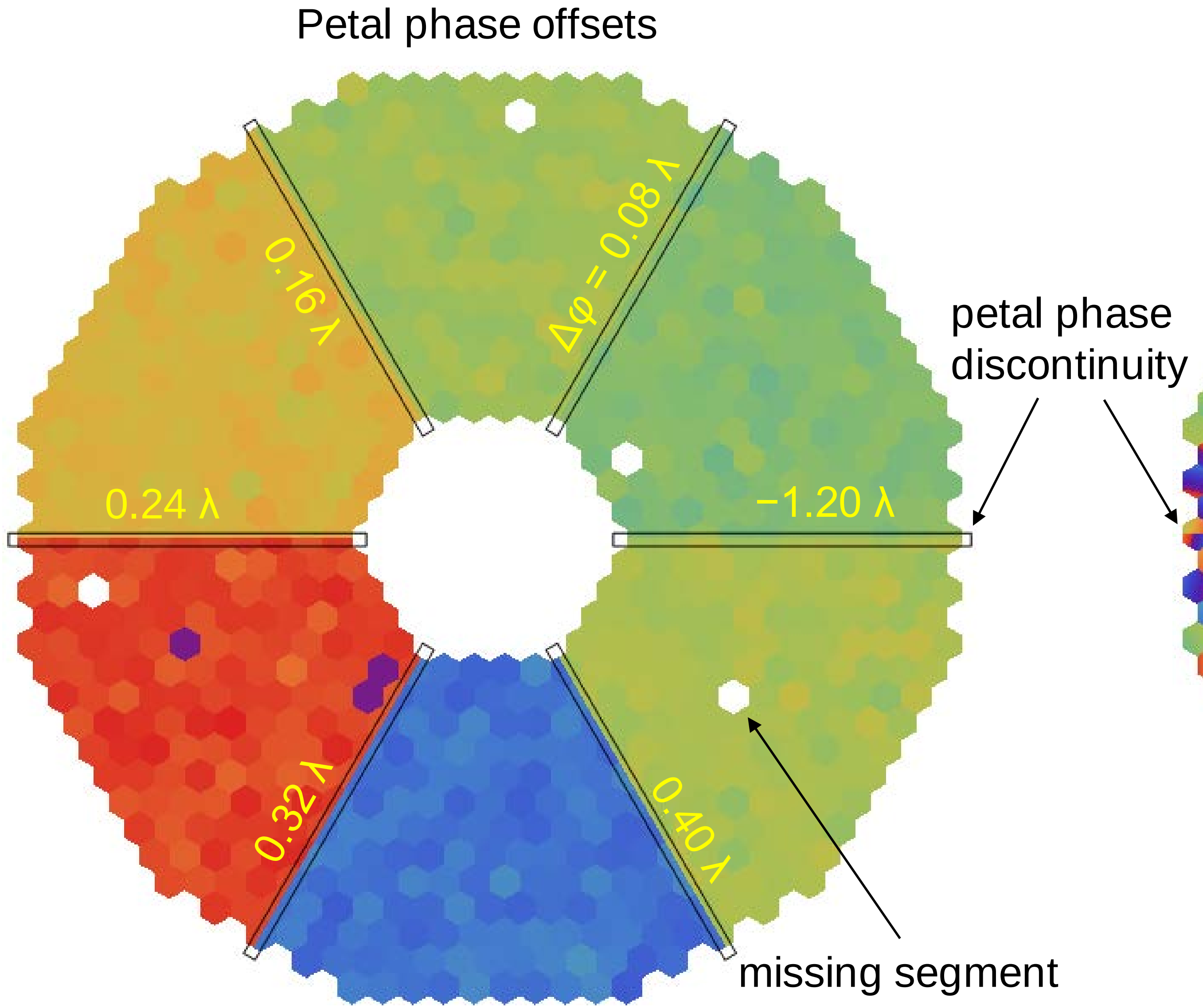
OPD residuals:
13 nm RMS = λ/56

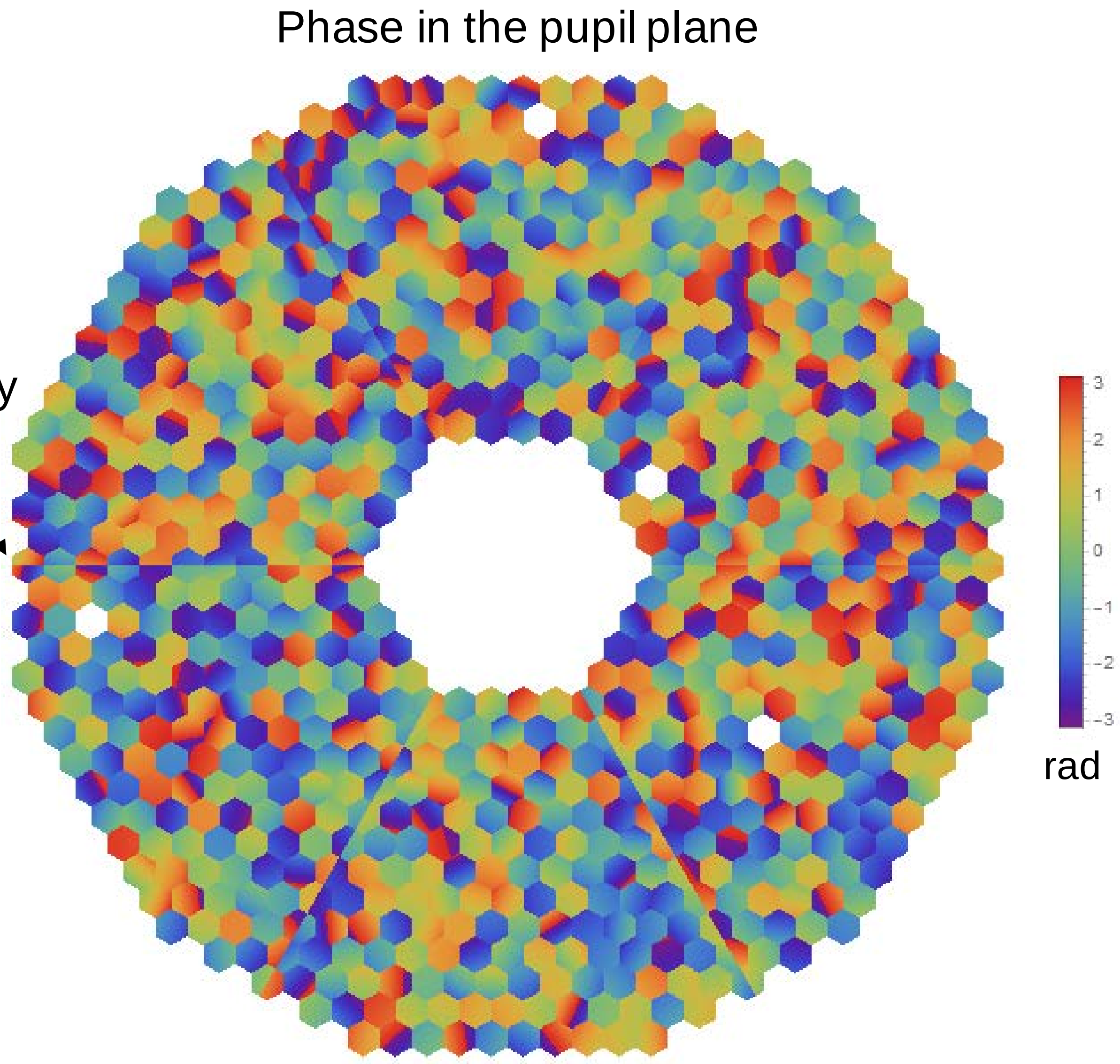
Post-Phasing Residuals



r_0 = 203 mm
Seeing: 0.67” @ 500 nm
719, 589, 569, 516 nm

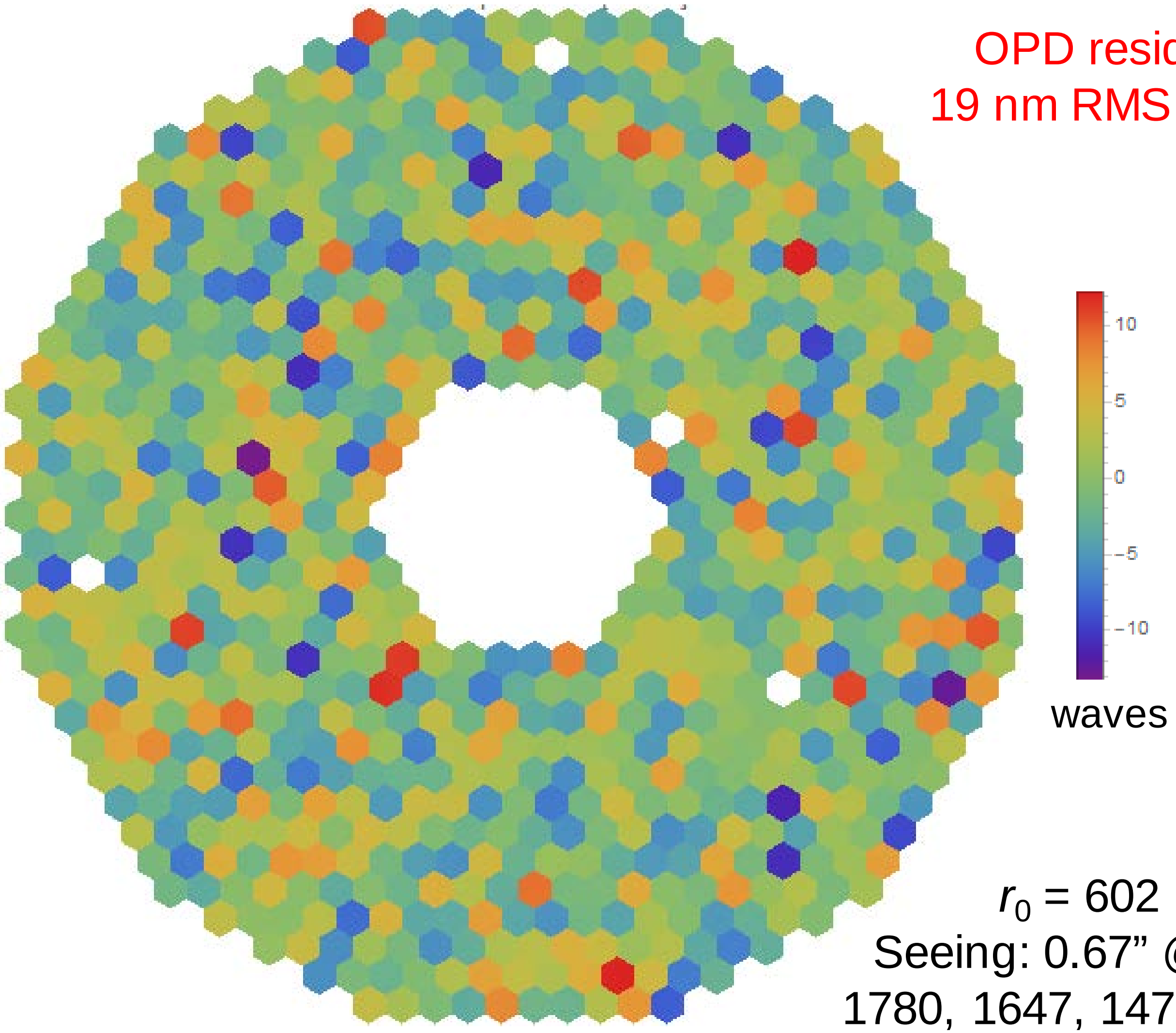
Add Petal Phase Offsets + Missing Segments





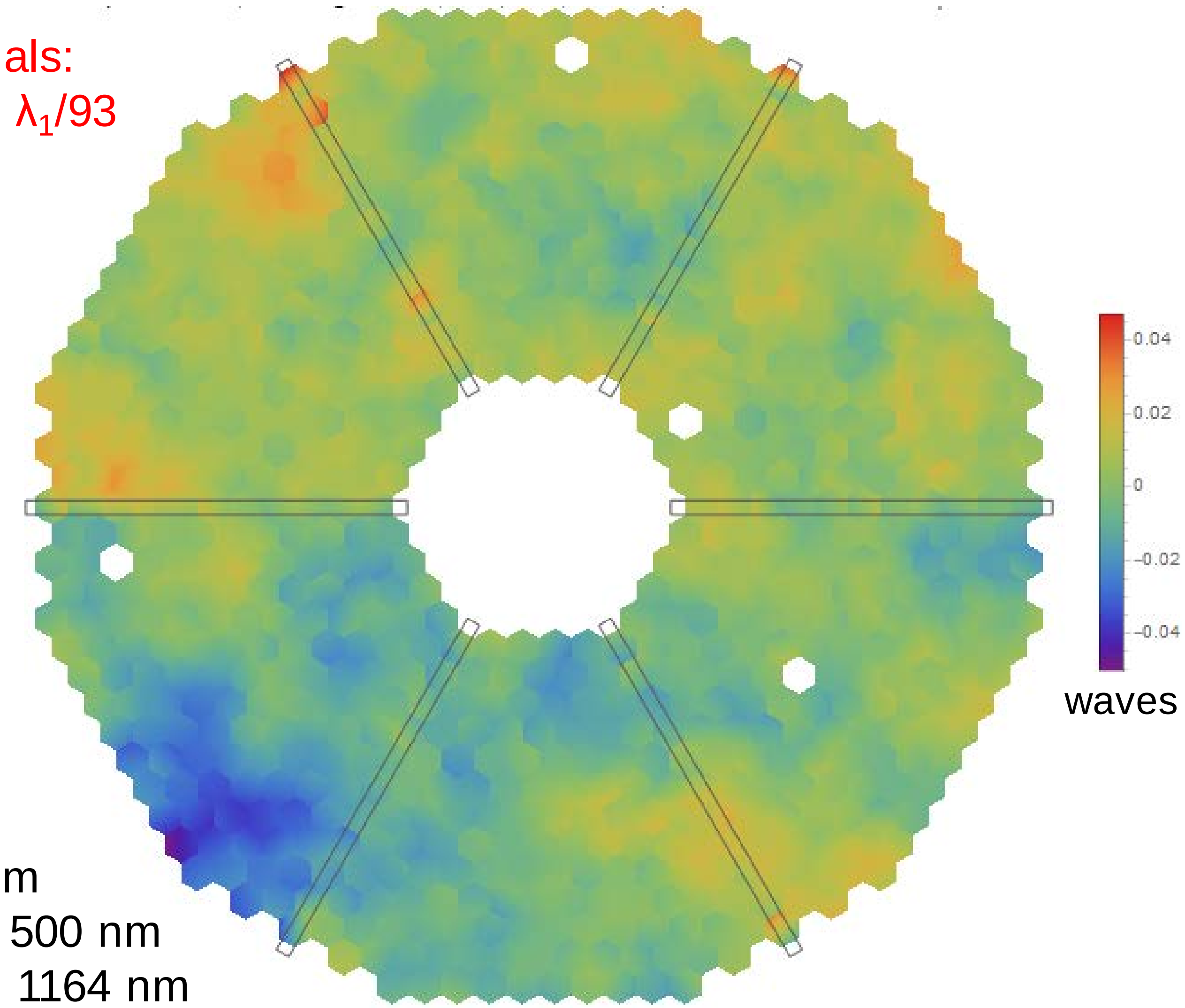
H/J-Band: Missing Segments + Petaling

Before Phasing



OPD residuals:
19 nm RMS = $\lambda_1/93$

Phasing Residuals



r_0 = 602 mm
Seeing: 0.67” @ 500 nm
1780, 1647, 1477, 1164 nm

Conclusions

- Building up / extending a physical optics simulation environment
- Primary mirror phasing becomes more demanding in ELTs
  - Ability to quickly/frequently phase M1 would be a valuable asset
  - Desirable to sense both, segment tip/tilt and steps, in parallel
  - Response function, cross-talk, linearity vary with WFS type
  - Segment registration and reconstruction algorithm must be optimized for WFS type to get best performance. Room for performance increase!?
- Numerical demonstration of “one shot” multicolor segment phasing with PWFS in the low-noise limit, both in R/V and H/J bands
  - Works with spider obscuration, missing segments and petaling (talk tomorrow 17:20)
  - To be done: model detector/sky noise, radial segment-to-pupil compression, segment registration on skewed pixel grid, lower pixel count, op. scenarios…

Ideas for Experiments!?