Phenomenology of non-singular black holes beyond general relativity

R. Carballo-Rubio, F.D.F, S. Liberati, M. Visser, Phys.Rev. D98 (2018) no.12, 124009.

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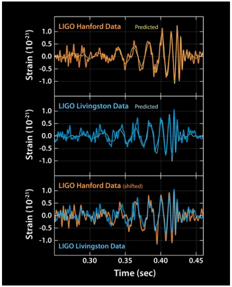
Introduction

- General relativity is a well tested and motivated theory.
- An ubiquitous prediction of general relativity is the presence of singularities: general relativity predicts its own failure!
- It is reasonable to assume that quantum gravity will somehow prevent the formation of singularities.



Outline

1. Possible alternatives
2. Parametrization
3. Example of an observational channel

Outline

Regular black holes

- A very conservative possibility involves departures from general relativity only close to the would be singularity

$$ds^2 = -e^{2\phi(t,r)}F(t,r)dt^2 + \frac{1}{F(t,r)}dr^2 + r^2 d\Omega^2$$

- Asymptotically

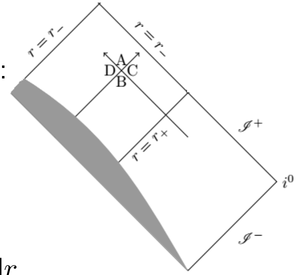
$$F(t,r) \longrightarrow 1 - \frac{2M}{r}, \qquad \phi \longrightarrow 0.$$

- To avoid the singularity, as $r \to 0$;

$$F(t,r) = 1 + \mathcal{O}(r^3).$$

- The horizon condition is $F(t,r) = 0$.
  - There is an even number of horizon.

Regular black holes



- Price law for the dumping of the perturbations:

  [Price 1972; Gundlach, Price, Pullin 1994; Dafermos, Rodnianski 2005]

$$m_{\rm in} \propto v^{-\gamma}$$

- Behavior of $F_B(r_0)$:

$$\frac{\mathrm{d}r}{\mathrm{d}v} = \frac{e^{-\phi(r)}F(r)}{2} \quad \Longrightarrow \quad \mathrm{d}v = \frac{2\mathrm{d}r}{e^{-\phi(r_-)}F'(r_-)(r-r_-)} + \mathfrak{o}(r-r_H).$$

- Integrating this equation

$$|F_B(r_0(v)|_{u=u_0})| \propto e^{-|\kappa_-|v}.$$

Putting these together,

$$m_{\rm A} \propto v^{-\gamma} e^{|\kappa_-|v}.$$

See R. Carballo Rubio, F.D.F., S.Liberati, C. Pacilio, M. Visser, *10.1007/JHEP07(2018)023*

Wormholes

- For simplicity let us focus on Morris–Thorne traversable wormholes [Morris, Thorne (1988)]

$$\mathrm{d}s^2 = -e^{-2\phi(x)}\mathrm{d}t^2 + \mathrm{d}x^2 + r^2(x)\mathrm{d}\Omega^2 \qquad r_{\min} \neq 0,$$

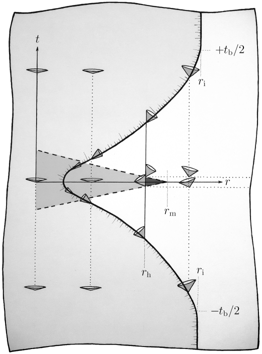
- where $x \in (-\infty, +\infty)$. Asymptotic flatness requires

$$\lim_{x\to\pm\infty} \frac{r(x)}{|x|} = 1, \qquad\qquad \lim_{x\to\pm\infty} \phi(x) = \Phi_\pm \in \mathbb{R}.$$

- This geometry correspond to flat spacetime far form the throat. It can be generalized to be asymptotically Schwarzschild [Visser(1997)], or to include rotation [Teo (1998)].
- We are not going to deal with the effect of the matter maintaining the object.

Bouncing geometries

- A more radical possibility involves a transition between a black hole to a white hole state.



- It is easy to write down a metric that describe such object. Using Painlevé-Gullstrand coordinates:

$$ds^2 = -dt^2 + [dr - f(r,t)v(r) = \sqrt{r_s/r}dt]^2 + r^2 d\Omega^2,$$

- where $f(t,r)$ interpolates between the values $f = \mp 1$ corresponding to a black hole or a white hole in these coordinates.

- An important issue regards the timescale of this process.

Quasi black holes

- Let us define a static and spherically symmetric quasi-black hole in a rough way as a spacetime satisfying the following conditions:
  - the geometry is Schwarzschild above a given radius R;
  - the geometry for $r \leq R$ is not Schwarzschild;
  - there are no event or trapping horizons.
- There are many geometries satisfying such criteria but few is known about:
  - Dynamical processes for their formation;
  - Stability of such objects.

Outline

Phenomenological parameter

- Relaxation time τ_-: Time necessary to form the object;
- Lifetime τ_+: Timescale in which the object disappears completely;
- Size $R = r_S\,(1+\Delta)$: Value of the radius of the object. It can be useful to introduce $\mu = 1 - \frac{r_S}{R}$. For very compact objects $\mu \approx \Delta$.
- Absorption coefficient κ: Fraction of incoming energy that is (semi)permanently lost inside the region $r \leq R$;
- Elastic reflection coefficient Γ: Portion that is reflected at $r \geq R$ due to elastic interactions;
- Inelastic reflection coefficient $\tilde{\Gamma}$: Portion of energy that is temporarily absorbed by the object and then re-emitted.
- Tails $\epsilon(t, r)$: Modifications of the geometry that decay with radial distance.

Model	τ_-	τ_+	Δ	κ	Γ	$\epsilon(t,r)$
Classical black hole	$\sim 10M$	∞	0	1	0	0
Regular black hole	$\sim 10M$	U	0	1	0	MD
Wormhole	U	∞	> 0	MD	$1-\kappa$	MD
Bouncing geometries	MD	MD	0?	1?	0?	$r_\star = \mathscr{O}(r_s)$
Quasi-black hole	MD/U	∞	> 0	MD/U	MD/U	MD

MD: Model Dependent U:Unknown.

Outline

Accretion disk around Sgr A^*

- Accretion rate $\dot{M}$ from the accretion disk to the object;
- If the compact object is not a black hole you would expect an outgoing flux $\dot{E}$;
- Negative observation of such flux can bu used to cast a constraint on the phenomenological parameters;
- It was claimed that even sub-Planckian value of Δ were not compatible with the observation [Broderick, Narayan (2006-2007); Narayan, McClintock(2008)].

Accretion disk around Sgr A^*

The analysis was based on two hypotheses

- **Thermality**: The emitted radiation follow a thermal distribution;
- **Steady state**: A steady state between the compact object and the accreation disk has been reached ($\dot{E} = \dot{M}$).

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- When do we expect such hypotheses to hold?

Steady State

- First of all, a steady state is not possible until the first ingoing radial null geodesics can bounce back at the surface $r = R$,

$$T_{\text{bounce}} \approx 4M\,|\ln \mu| \qquad\qquad \mu = 1 - \frac{r_S}{R}.$$

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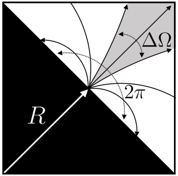
- This is not in contradiction with the steady state assumption.
- Strong lensing constitute a more important time delay for the steady state [Cardoso, Pani (2017)].
- Only the rays emitted inside a solid angle $\Delta\Omega$ reaches infinity.

$$\Delta\Omega = 2\pi\left[1 + \left(1 - \frac{3M}{R}\sqrt{1+\frac{6M}{R}}\right)\right] \sim \frac{27}{8}\mu$$



Steady state

- Assuming $\kappa = \Gamma = 0$,

$$\frac{\dot{E}}{\dot{M}} = 1 - \left(1 - \frac{\Delta\Omega}{2\pi}\right)^{T/T_{\rm bounce}}$$

- where T is the timescale over which the accretion rate is constant. For Sgr A^*, $\frac{T}{T_{\rm Bounce}} \approx 10^{15}$. So,

$$\frac{\dot{E}}{\dot{M}} \lesssim 10^{-2} \iff \mu \approx \Delta \lesssim 10^{-17}$$

Steady state

- For $\Gamma = 0$ but $\kappa \neq 0$

$$\frac{\dot{E}}{\dot{M}} = \frac{(1-\kappa)\Delta\Omega/2\pi}{\kappa + (1-\kappa)\Delta\Omega/2\pi}\left[1 - (1-\kappa)^{(T/T_{\rm bounce})}\left(1 - \frac{\Delta\Omega}{2\pi}\right)^{(T/T_{\rm bounce})}\right].$$

- For $\kappa \gg (T_{\rm bounce}/T)$,

$$\frac{\dot{E}}{\dot{M}} = \frac{(1-\kappa)\Delta\Omega/2\pi}{\kappa + (1-\kappa)\Delta\Omega/2\pi} < 10^{-2}$$

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- For completeness, if $\Gamma \neq 0$

$$\lim_{t\to\infty}\frac{\dot{E}}{\dot{M}} = \frac{(1-\kappa-\Gamma)(1-\Gamma)\Delta\Omega/2\pi}{\kappa + (1-\kappa-\Gamma)\Delta\Omega/2\pi}.$$

Conclusions

Model	Stars (EM)	Accretion (EM)	Shadows (EM)	Bursts (EM)	Coalescence (GW)	Echoes (GW)
Regular black hole	$\epsilon(t,r)$	✗	$\epsilon(t,r)$	✗	✗	✗
Wormhole	$\epsilon(t,r)$	✗ $(\Gamma+\kappa=1)$	$\epsilon(t,r)$	✗	τ_-, Γ	Γ, $[\mu]$
Bouncing geometries	$\epsilon(t,r)$	✗	$\epsilon(t,r)$	✓	τ_- (short-lived)	
Quasi-black hole	$\epsilon(t,r)$	μ, Γ, κ	$\epsilon(t,r)$	✗	τ_-, μ, Γ	Γ, $[\mu]$

- All the current observations are compatible with general relativity black holes;
- Alternatives are far from being excluded;
- A combined effort in different observational channel is needed.

Thank you for your attention