Principal Component Analysis of the Primordial Tensor Power Spectrum

arXiv:1905.08200

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Inflation and CMB Polarization

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Inflationary Paradigm

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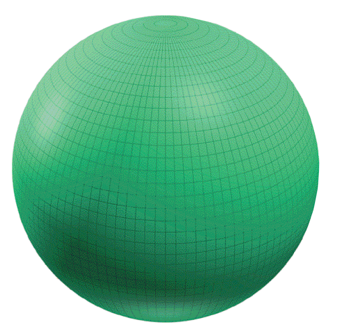

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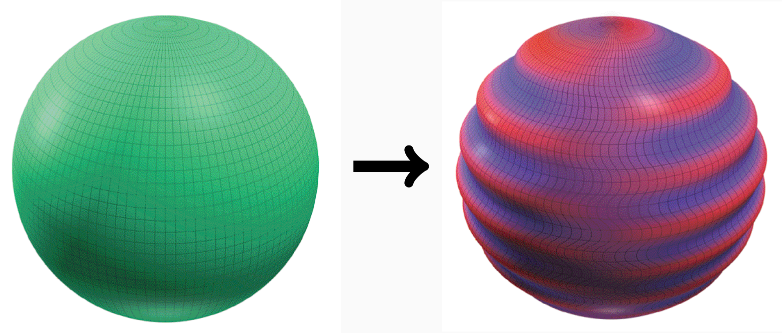

Inflation and CMB Polarization

- **CMB is polarized!** $\Rightarrow$ Thomson scattering at recombination
- Polarization state $\rightarrow$ **Stokes parameters** Q **and** U form **Polarization Tensor**
- Helmholtz decompose it in:
  - **Curl** component $\rightarrow$ **B-modes** $\rightarrow$ divergence-free
  - **Gradient** component $\rightarrow$ **E-modes** $\rightarrow$ curl-free

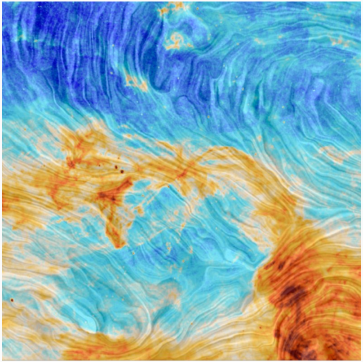

Figure 1: *From Planck website*

The Quest for Primordial Gravitational Waves

- **E-modes** produced by *scalar* ***and*** *tensor* perturbations
- **Primordial B-modes produced ONLY by tensor perturbations!**
- **IF Detected Primordial Gravitational waves** will give:
    - "Smoking gun" for inflation
    - Identify energy scale of inflation **for the simplest models!** (single scalar field slow-roll)
- **What about more complex models? Beyond the standard model of Early Universe? Lots of Physics to be understood in this primordial signal!**

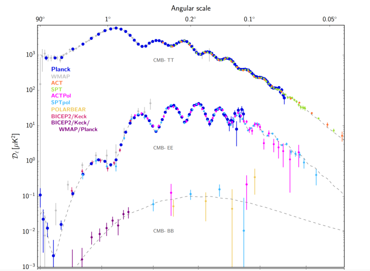

Figure 2: *From Planck results 2018*

Goals and Motivations

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1. Examples of **non-standard B-mode emission in the literature**:
    - ***massive gravity inflation*** (Domenech et al. 2017)
    - ***open inflation***(Yamauchi et al. 2011)
    - ***topological defects/cosmic strings*** (Lizarraga et al. 2014)
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2. Establish the constraining power of **future B-mode probes** on the **shape** of primordial tensor power spectrum
3. Sensitivity to **features, deviations from power-law behaviour**
4. We use **Principal Component Analysis** on Tensor Power Spectrum for a **model independent** approach

Power spectra, Parameters and Observations

Standard Parametrization

Primordial Tensor Power Spectrum (Standard Power-Law)

$$\mathcal{P}_T(k) = A_T \left(\frac{k}{k_0}\right)^{n_T}$$

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- **Tensor contribution**

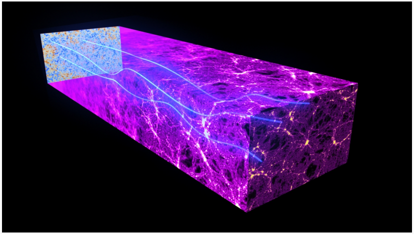
$$C_{\ell,t}^{XX'} \propto \mathcal{P}_T(k)$$

$X, X' \in \{T, E, B\}$

Noise, Lensing and Foregrounds Contribution

$$C_\ell^{XX'} = C_\ell^{XX',\,prim} + C_\ell^{XX',\,noise} + \lambda C_\ell^{XX',\,lens} + C_\ell^{XX,\,fgs}$$

- Total $C_\ell^{XX'}$ contains:
  1. **Primordial spectrum** $C_\ell^{XX',\,prim}$
  2. **Instrumental noise after Component Separation** $C_\ell^{XX',\,noise}$
  3. **CMB lensing contribution** $\lambda C_\ell^{XX',\,lens}$ (λ **delensing factor**)
  4. **Foregrounds contribution** $C_\ell^{XX,\,fgs}$



Foregrounds and Lensing

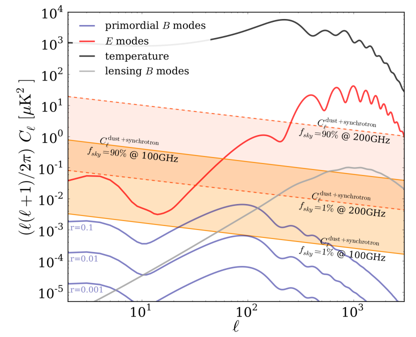


Figure 3: *from Errard et al. 2016*

- **Lensing** → dominant at **intermediate – small scales**
- **Foregrounds** → Dominant at **large scales**
- **Dust** + **Synchrotron**
- **Parametric maximum-likelihood component separation**
- **Residual foregrounds** in maps → **residuals power spectrum C_ℓ^{fgs}**
- **FGBuster** Code (Poletti & Errard)

Introduction to Principal Component Analysis

Principal Component Analysis (PCA)

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- Diagonalize Fisher matrix

$$\boldsymbol{F} = \boldsymbol{S}^T \boldsymbol{E} \boldsymbol{S}$$

- **PCA modes** $\rightarrow$ **eigenvectors** of **F** (Rows of **S**)
- e_i **eigenvalues** of **F** ordered from largest to smallest

$$\boldsymbol{E} = diag(e_i)$$

- **PCA amplitudes** $\rightarrow$ New **uncorrelated** parameters set **m** linear combination of original parameters
- **Compression of information** in the first best measured modes

Principal Components of Tensor Power Spectrum

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$$F_{ij} = f_{sky} \sum_{\ell=2}^{\ell_{\max}} \frac{2\ell+1}{2} \mathrm{Tr}\left[\mathbf{D}_{\ell i}\mathbf{C}_{\ell}^{-1}\mathbf{D}_{j\ell}\mathbf{C}_{\ell}^{-1}\right]$$

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- Obtain $\mathcal{S}_a(k) \to$ **basis** for tensor spectrum

PCA modes for Model Testing: 2 Approaches

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Fisher approach

- Projecting power spectrum model over PCA modes
- **Uncertainties** $\boldsymbol{\sigma_{Fisher}}$ on m_a from **Fisher matrix**
  - **Advantages** → fast & easy
  - **Caveats** → **insensitive to physicality prior** $\boldsymbol{P_T > 0}$

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MCMC approach

- Constrain m_a using simulated C_ℓ spectra (or data)
  - **Cosmological + Power spectrum parameters**

$$\{m_1, ..., m_n, A_s, n_s, \tau, \Omega_b h^2, \Omega_D h^2, \theta\}$$

  - **Impose** $\boldsymbol{P_T > 0}$ **in MCMC**
  - **Advantages** $\rightarrow$ **impact of physicality priors** ($\boldsymbol{\sigma_{Fisher}}$ vs σ_{MCMC}), **correlations**
  - **Disadvantages** $\rightarrow$ slow convergence

Future Probes Specifications

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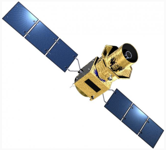

LiteBIRD

- Satellite
- Timescale 2027
- 15 frequency bands [40-402 GHz]
- Noise [36.1 - 4.7 μK-arcmin]
- Beams FWHM [69.2-9.7 arcmin]
- Sky fraction $f_{sky} = 60\%$
- 20% Delensing
- Multipole range $\ell \sim 2 - 1350$

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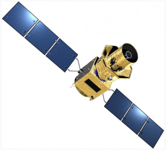

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- Ground-based
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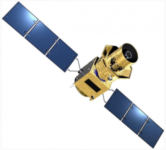

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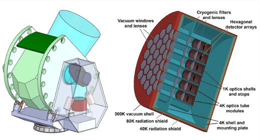
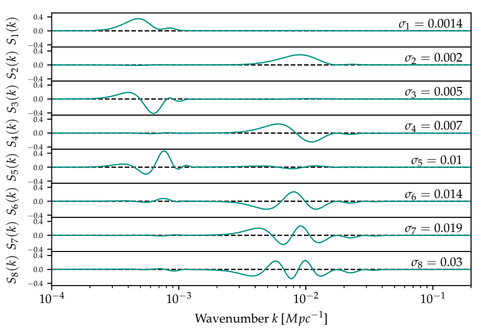


CMB-S4

- Ground-based
- Timescale 2027
- 9 frequency bands [20-270 GHz]
- Noise [14 - 1.3 μK-arcmin]
- Beams FWHM [76.6-8.5 arcmin]
- Sky fraction $f_{sky} = 3\%$
- 90% Delensing
- Multipole range $\ell \sim 30 - 4000$

PCA Basis

PCA Basis: Problems



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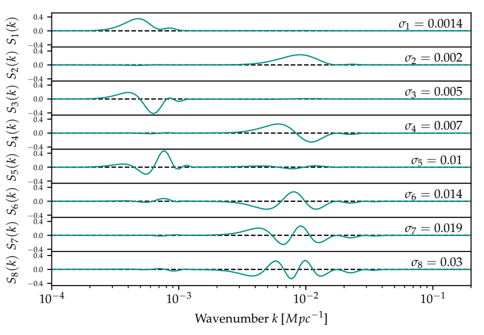

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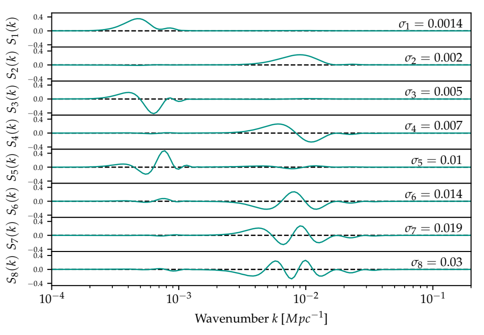

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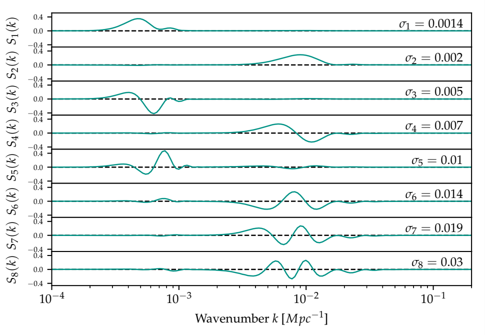

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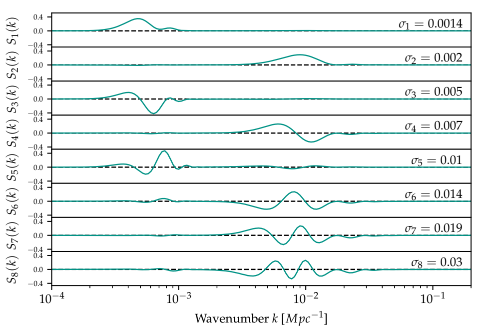

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- **BUT Information in C_ℓs with Tensors ($r > 0$) can be very different from Information matrix that defined PCA basis!**

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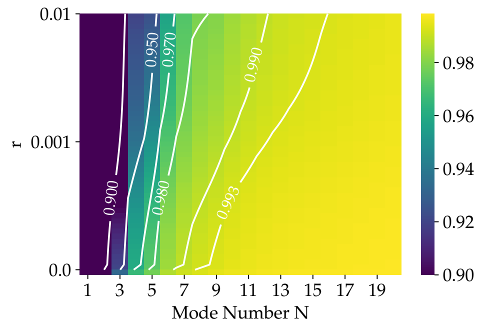

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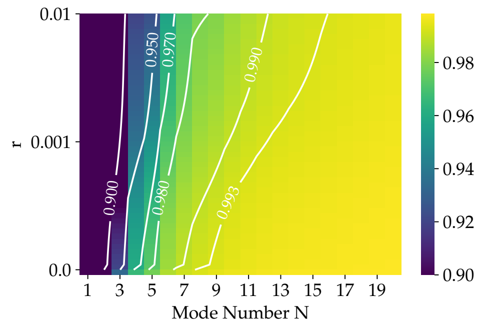

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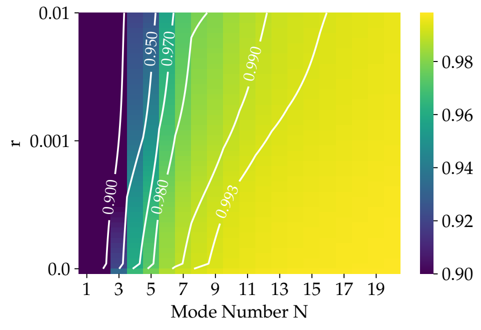

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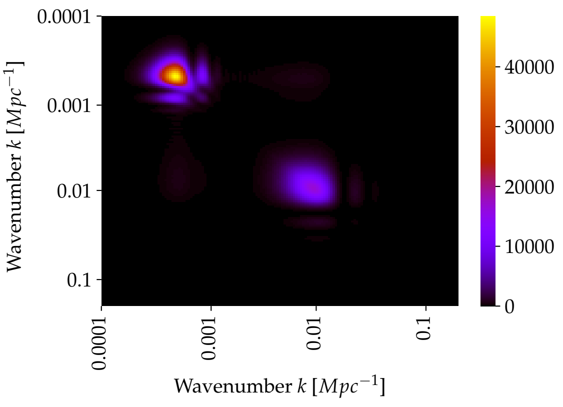
- e.g. LiteBIRD $\rightarrow$ set $N = 8$

Application to LiteBIRD

Application to LiteBIRD: Fisher Matrix



- **Fisher Matrix for LiteBIRD for** $r = 0$

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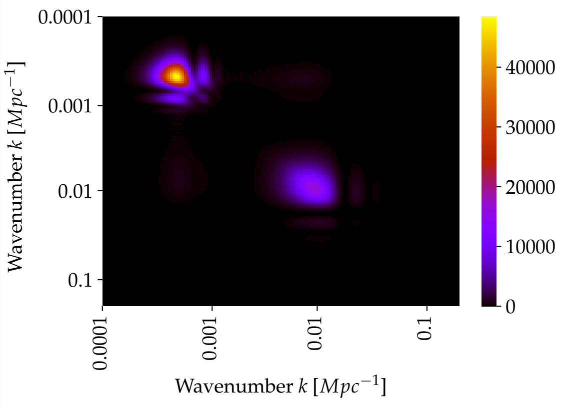

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- Main features: **recombination bump** ($k \approx 6 \times 10^{-3}\mathrm{Mpc}^{-1}$) and **reionization bump** ($k \approx 6 \times 10^{-4}\mathrm{Mpc}^{-1}$)

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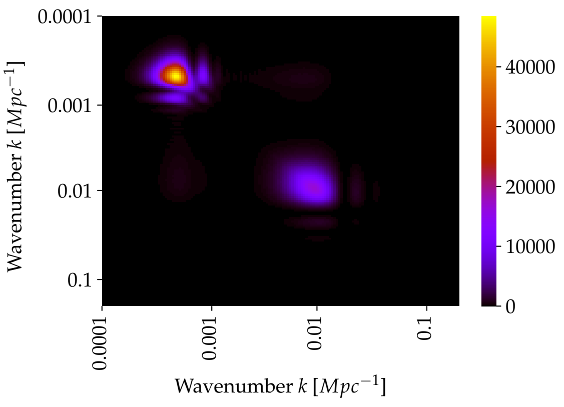

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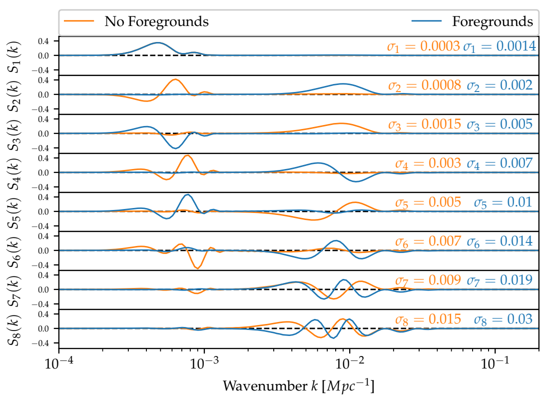
- Main features: **recombination bump** ($k \approx 6 \times 10^{-3}\mathrm{Mpc}^{-1}$) and **reionization bump** ($k \approx 6 \times 10^{-4}\mathrm{Mpc}^{-1}$)
- Most information for $r = 0$ comes from reionization peak

Are Foregrounds important?



- **Factor ~ 5 on σ_1 and ~ 3 on σ_2 due to foregrounds!**
- Foregrounds change relative importance of reionization and recombination peak

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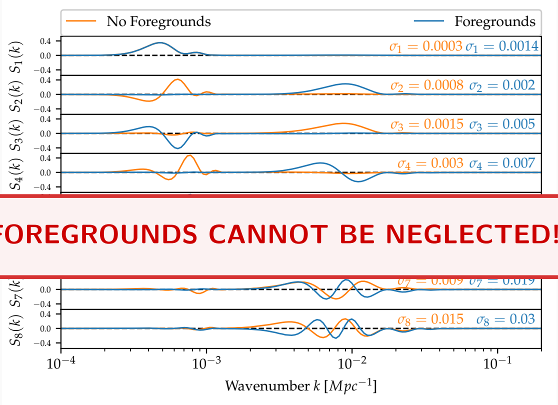

FOREGROUNDS CANNOT BE NEGLECTED!

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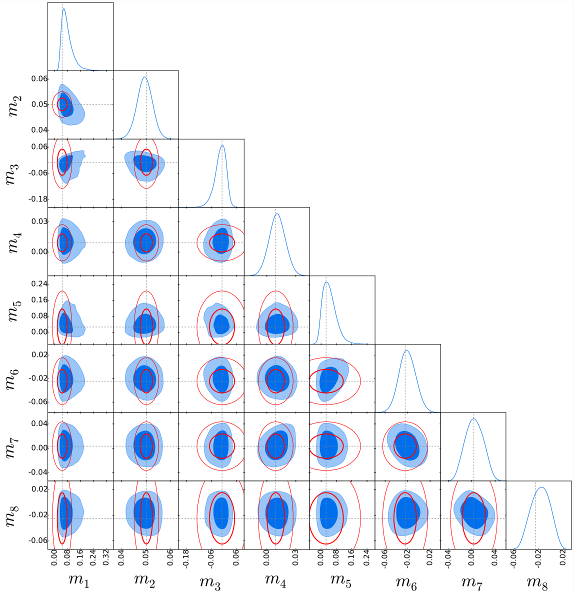
Limitations of the PCA Method and MCMC Exploration

MCMC Exploration: $r = 0.01$



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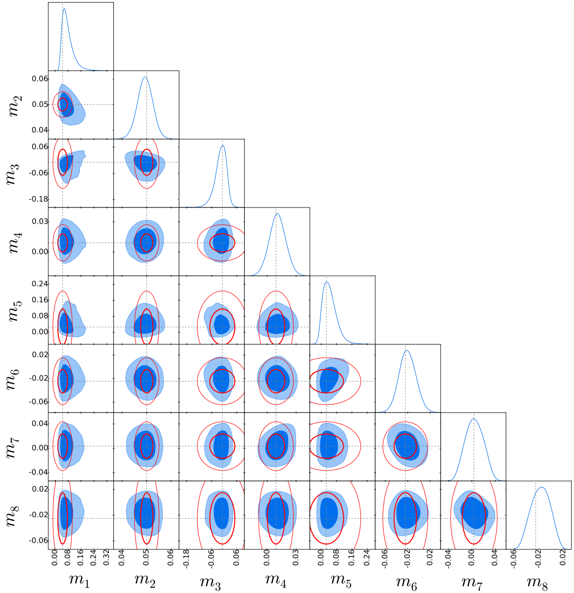

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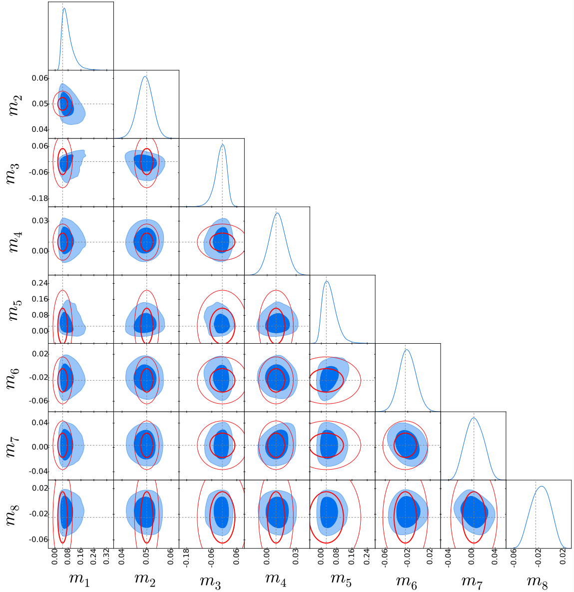

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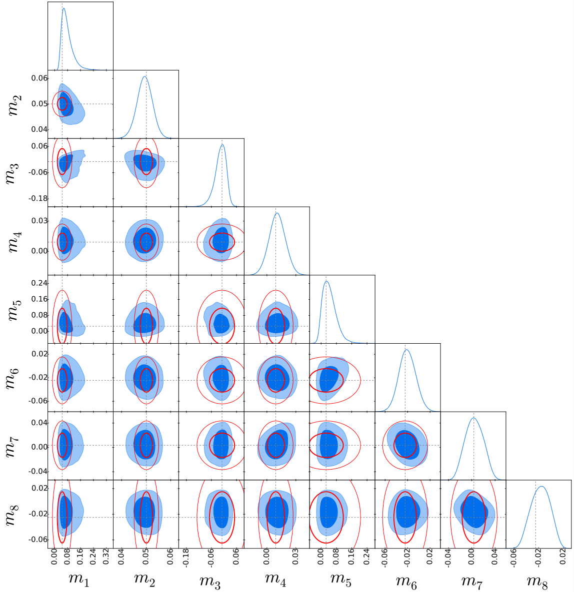
- Must have $\sigma_{\text{MCMC}} \geq \sigma_{\text{Fisher}}$ $\rightarrow$ **true without physicality prior**

MCMC Exploration: $r = 0.01$



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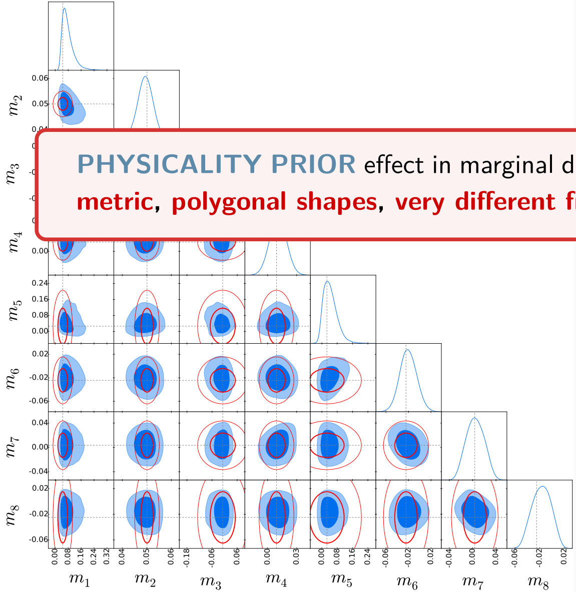
- Must have $\sigma_{\mathbf{MCMC}} \geq \sigma_{\mathbf{Fisher}}$ $\rightarrow$ **true without physicality prior**
- **With physicality prior** $\Rightarrow$ $\sigma_{\mathbf{MCMC}} < \sigma_{\mathbf{Fisher}}$ for most modes!

MCMC Exploration: $r = 0.01$

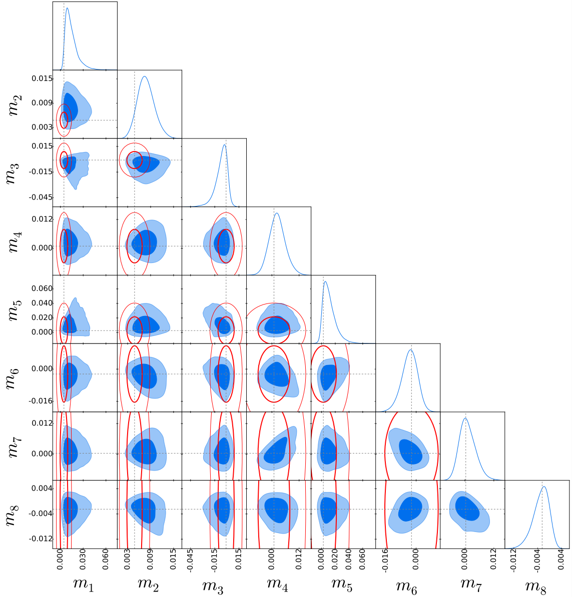


- **Fisher estimates** for PCA can be **inconsistent**! $\rightarrow$ insensitive to **physicality prior** $\mathcal{P}_T > 0$
- ... **true** ... **prior**
- **With physicality prior** $\Rightarrow$ $\sigma_{\text{MCMC}} < \sigma_{\text{Fisher}}$ for most modes!

PHYSICALITY PRIOR effect in marginal distributions $\rightarrow$ **asymmetric**, **polygonal shapes**, **very different from Fisher**

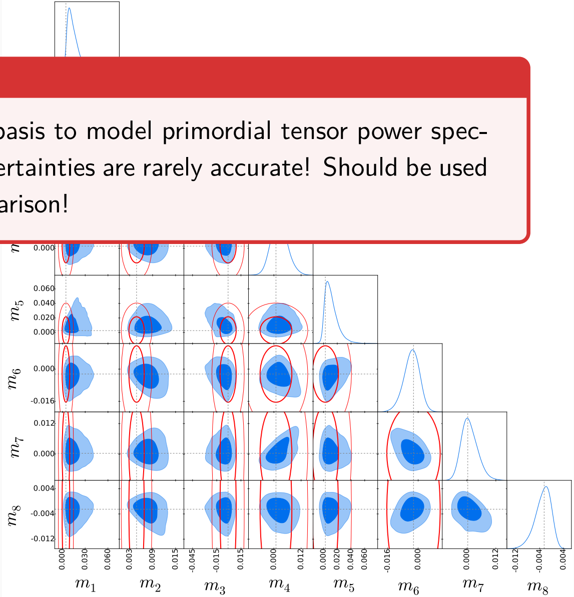
MCMC Exploration: $r = 0.001$

- Physicality prior effect **even more evident** for smaller r ($r = 0.001$)!



MCMC Exploration: $r = 0.001$

- Physicality prior effect **even more evident**



CONCLUSION

Can always use PCA basis to model primordial tensor power spectrum **BUT** Fisher uncertainties are rarely accurate! Should be used only for relative comparison!

Conclusions

- Applied **PCA** to **Tensor primordial power spectrum**
- Detect in **B-modes deviations from scale-invariance** in **model-independent** way
- Constraints for **LiteBIRD**, **SO** and **CMB-S4**
- **Foregrounds cannot be neglected!**
- **Our Basis** (**no tensors**) $\rightarrow$ preferable to the **Constant Mode Basis**
- **Fisher uncertainties** can be affected by **Physicality prior**!
- Can be applied to any **Early Universe scenario**

Thanks For Your Attention