



Project SoIMAG

The Real Winding Number of Coronal Flux Ropes

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Flux Rope Twist

- Flux rope commonly defined as having a coherent bundle of field lines that completes one full turn about its axis – a winding number of 1.
- Difficult to compute unless cylindrical symmetry is assumed.
- In practice, approximated by geometry-independent T_w (Berger & Prior, 2006):

$$T_w = \int_L \frac{\mu_0 J_{\parallel}}{4\pi B} dl = \int_L \frac{\nabla \times \mathbf{B} \cdot \mathbf{B}}{4\pi B^2} dl$$

- This measures how two infinitesimally close field lines wind about each other.
- Able to be computed quickly and easily.



Flux Rope Twist

- However, if the axis is known the winding number can be computed (Berger & Prior, 2006; Liu et al., 2016)

rotation angle made by
field line about axis x

unit tangent
vector to axis x

unit vector
normal to axis x

$$T_g = \frac{1}{2\pi} \int_x \frac{d\varphi}{ds} ds = \frac{1}{2\pi} \int_x \hat{\mathbf{T}}(s) \cdot \hat{\mathbf{V}}(s) \times \frac{d\hat{\mathbf{V}}(s)}{ds} ds$$



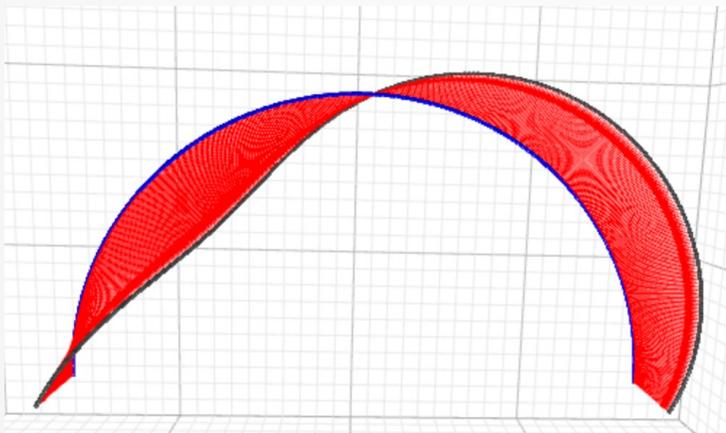
Single Field Line Testing

- Defined an axis with length $s = [0, \pi]$.
- Axis coordinates taken as: $(0, \cos(s), \sin(s))$.
- 'Field line' described by helical equation: $\cos(2Ns) + \sin(2Ns)$.
- Where N is the winding number.

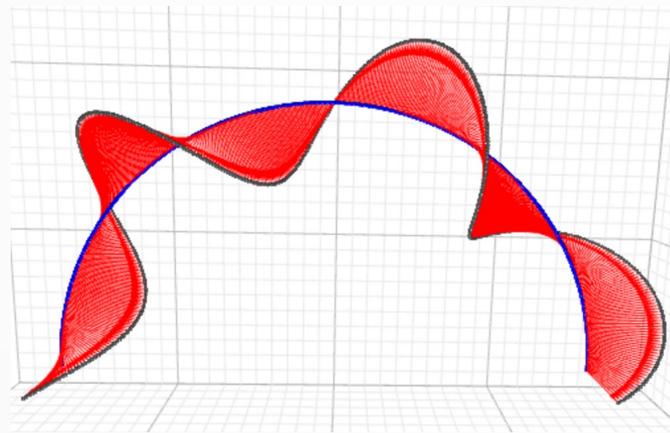


Single Field Line Testing

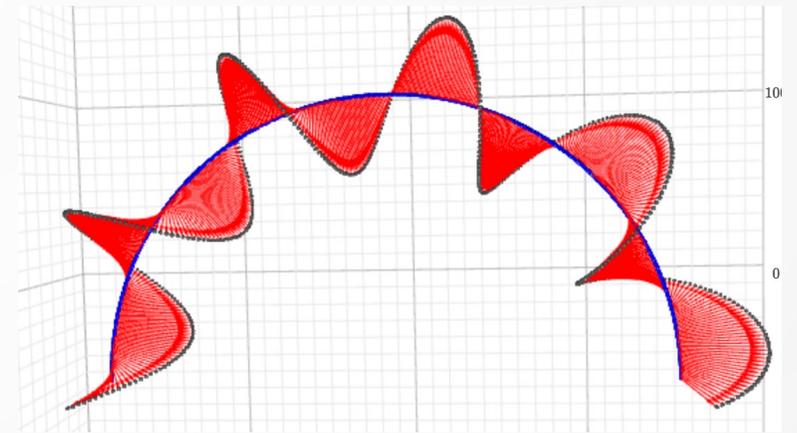
$N = 1$
 $T_g = 1.00$



$N = 3$
 $T_g = 3.01$



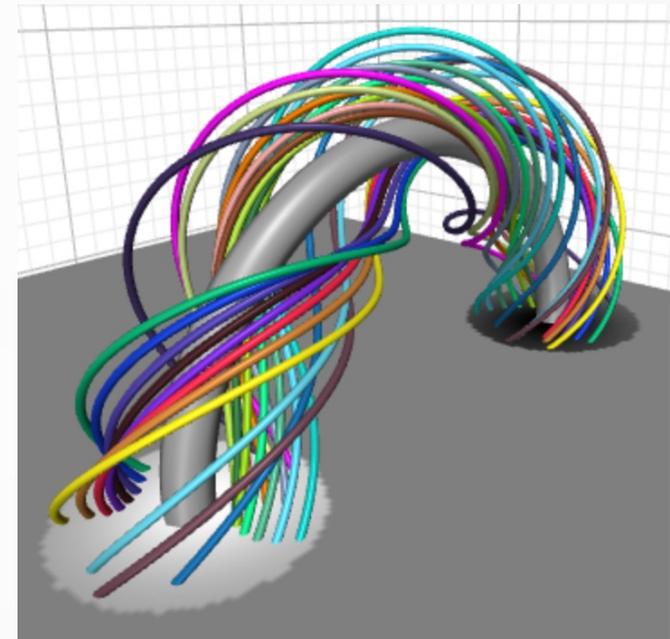
$N = 5$
 $T_g = 5.01$





Idealised Flux Rope Testing

- Approximately uniformly twisted flux rope from Vandas and Romashets (2017).



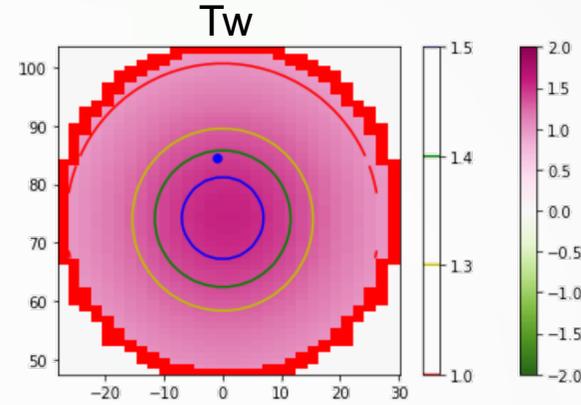
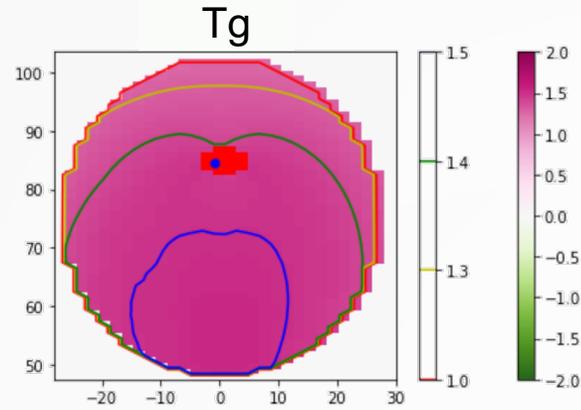


Max(T_g) = 1.54

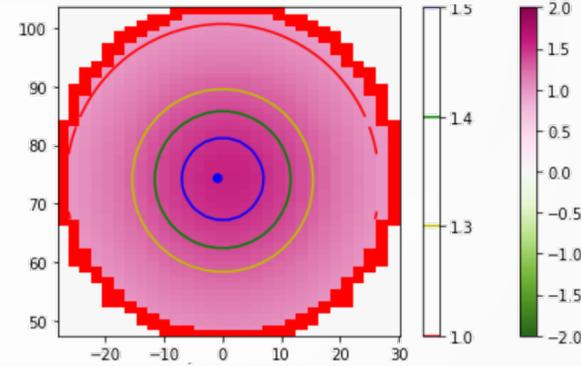
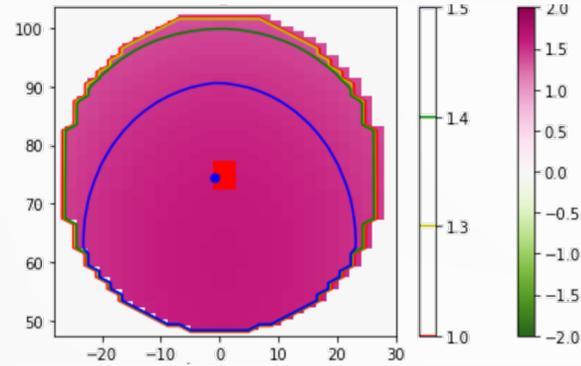
Max(T_g) = 1.64

Max(T_g) = 1.47

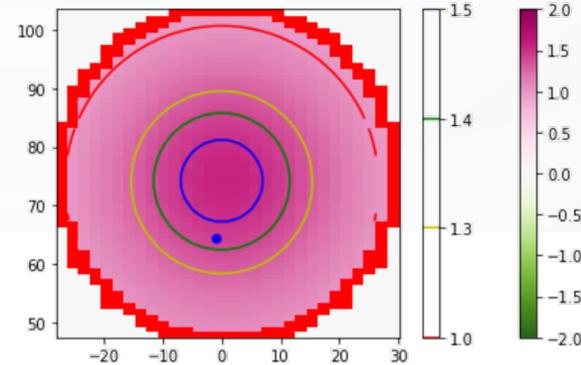
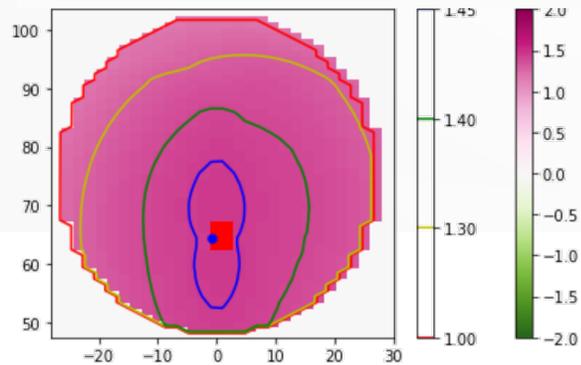
Dependency on Axis Location



Axis set 10 Mm
above the true axis



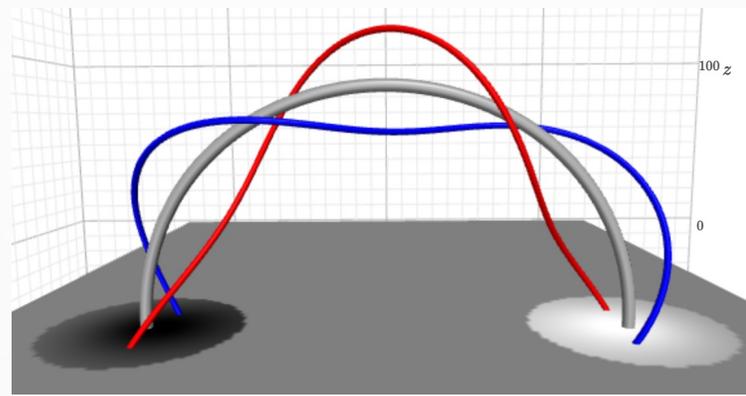
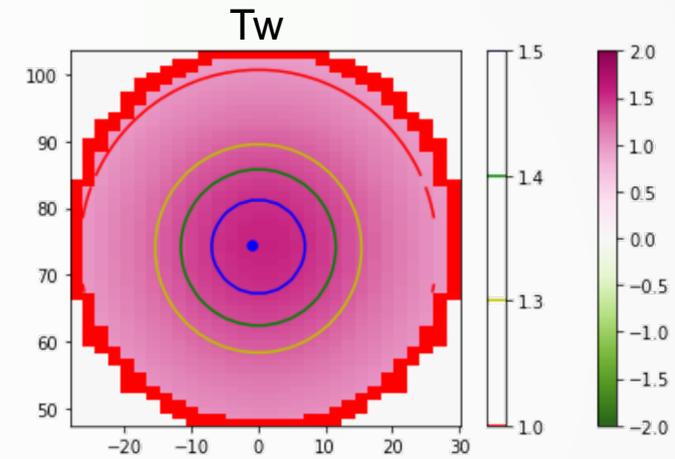
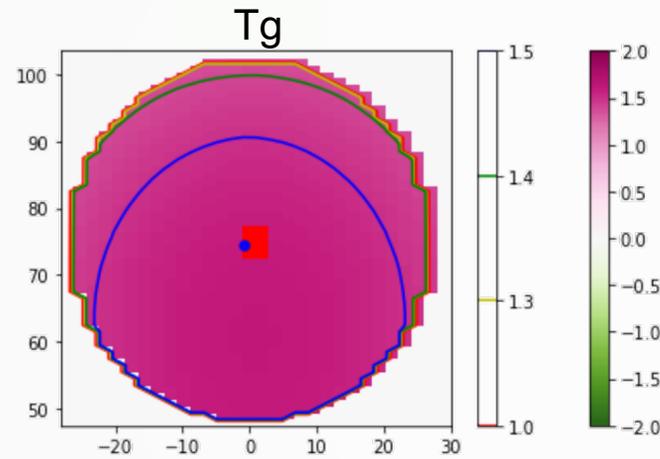
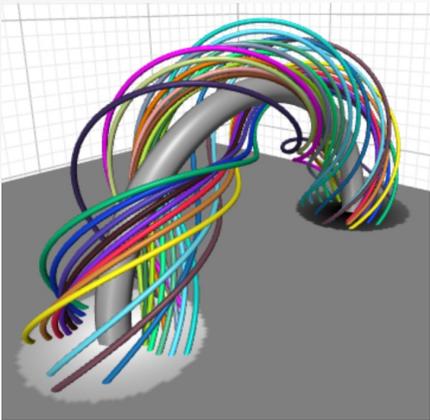
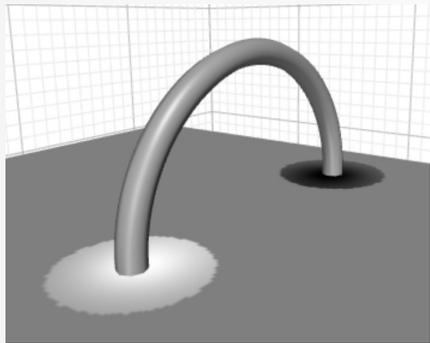
True axis



Axis set 10 Mm
below the true axis



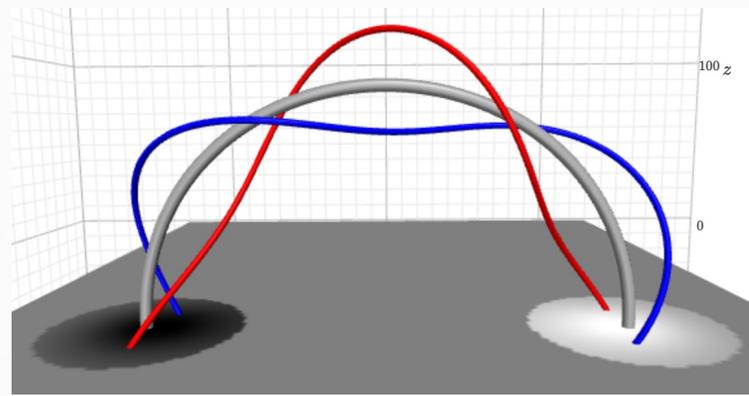
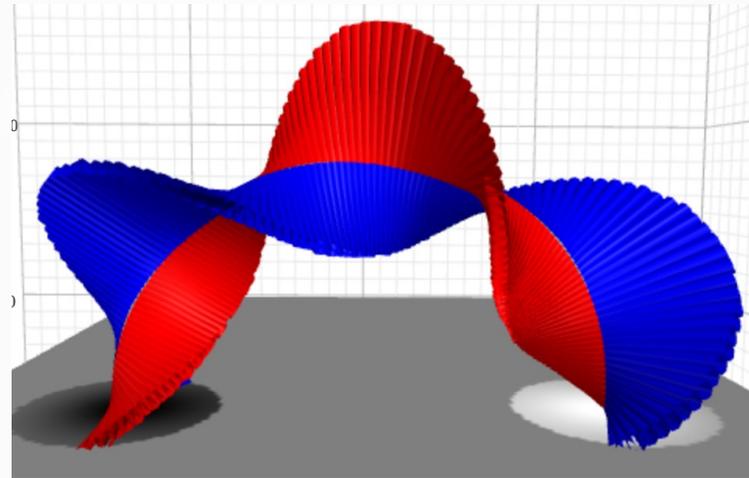
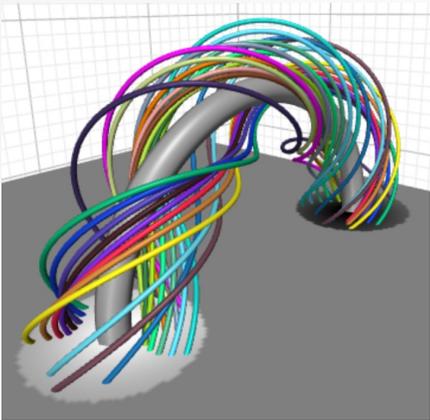
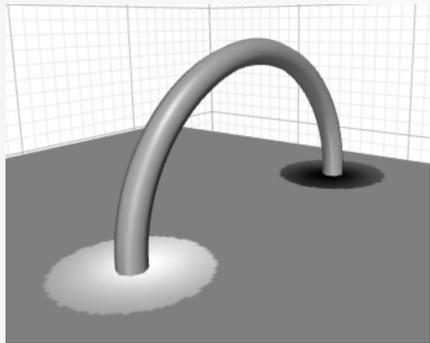
Distribution Investigation



Core T_w = 1.57
Red line T_g = 1.50
Blue line T_g = 1.63



Geometric Effects



Normal vector length
proportional to
contribution to T_g

Geometrical effects
visually evident

Core $T_w = 1.57$

Red line $T_g = 1.50$

Blue line $T_g = 1.63$



Summary and Future Work

- Implemented a method for computing the winding number of a flux rope.
- Tested against a single 'field line' and an idealised flux rope.
- Showed the importance of selecting the correct axis location.
- Future work
 - Test against more idealised and real data-driven flux ropes.
 - Further examine the effects of non-cylindrical symmetry as in Liu et al. (2016).



References

- Berger, M. A., Prior, C.: 2006, The writhe of open and closed curves, *Journal of Physics A: Mathematical and General*, **39**, 8321. <https://doi.org/10.1088/0305-4470/39/26/005>
- Liu, R., Kliem, B., Titov, V. S., Chen, J., Wang, Y., Wang, H., Liu, C., Xu, Y., Wiegelmann, T.: 2016, Structure, stability, and evolution of magnetic flux ropes from the perspective of magnetic twist, *The Astrophysical Journal*, **818**, 148. <https://doi.org/10.3847/0004-637X/818/2/148>
- Vandas, M., Romashets, E.: 2017, Magnetic cloud fit by uniform-twist toroidal flux ropes, *Astronomy & Astrophysics*, **608**, A118. <https://doi.org/10.1051/0004-6361/201731412>