

Energy flux of Alfvén waves in the stratified solar atmosphere: propagation and reflection

Yuriy Tsap (Crimean Astrophysical Observatory)

Petr Jelínek (University of South Bohemia, Faculty of Science, Department of Physics)

Victoria Smirnova (Crimean Astrophysical Observatory)

Types of harmonic linear modes:

- Propagating waves: $\sim \sin(\omega t - kx)$
- Standing waves: $\sim \cos(\omega t) \sin(kx)$
- Evanescent waves: $\sim \cos(\omega t) \exp(-kx)$
- Intermediate modes $\sim \exp(-i\omega t) f(x)$, where $f(x)$ is the oscillating complex function
- How to describe the intermediate modes on the basis of analytical methods?

Method to determine cutoff frequencies for acoustic waves propagating in nonisothermal media

Z. E. Musielak, D. E. Musielak, and H. Mobashi

Department of Physics, The University of Texas at Arlington, Arlington, Texas 76019, USA

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A method to determine cutoff frequencies for linear acoustic waves propagating in non-isothermal media is introduced. The developed method is based on wave variable transformations that lead to Klein-Gordon equations, and the oscillation theorem is applied to obtain the turning point frequencies. Physical arguments are used to justify the choice of the largest turning point frequency as the cutoff frequency.

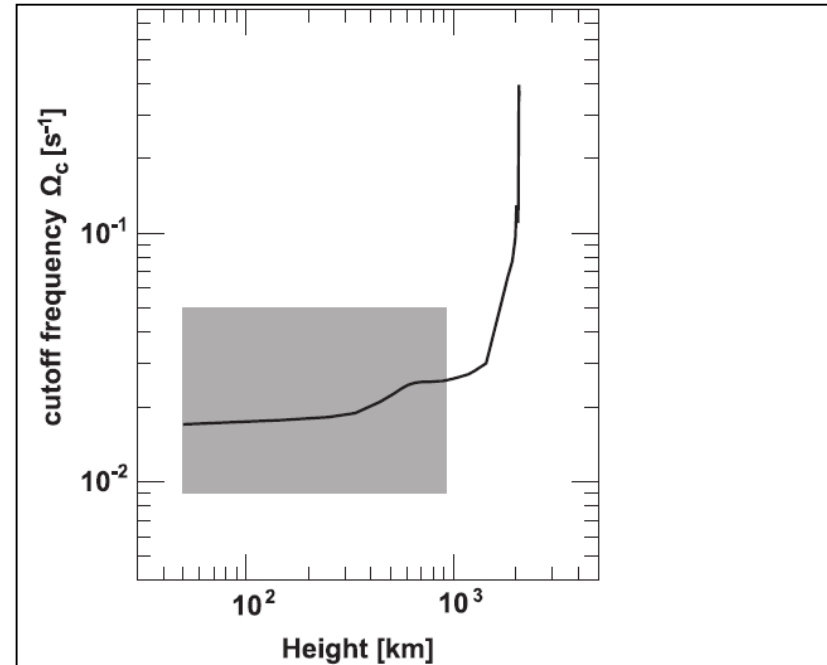


Fig. 1 (online colour at: www.an-journal.org) Cutoff frequency Ω_c of torsional waves along a thin magnetic flux tube in a VAL-C atmosphere. The shaded region indicates frequencies that have been observed as waves or as life times of vortex motions around downdrafts.

Hammer et al.(2010) – wide
or nonisothermal flux tubes

On wave equations and cut-off frequencies of plane atmospheres

F. Schmitz¹ and B. Fleck²

¹ Astronomisches Institut der Universität Würzburg, Am Hubland, D-97074 Würzburg, Germany

² ESA Space Science Department, NASA/GSFC, Mailcode 682.3, Greenbelt, MD 20771, USA

3. Kneser's (necessary and sufficient) oscillation theorem: Consider the equation $y'' + f(x)y = 0$. Then, if $0 < f(x) \leq 1/(4x^2)$ for $0 < a \leq x < \infty$, the solution $y(x)$ is nonoscillatory. If $f(x) > (1 + \epsilon)/(4x^2)$ with $\epsilon > 0$, then the solution $y(x)$ is oscillatory.

These theorems show that the critical quantities are the functions $Q_i(z)$ and $Q_i(\tau)$, but not the frequencies $\omega_i(z)$ and $\omega_i(\tau)$. So the frequencies ω_1 and ω_2 alone cannot indicate the character of the solution. However, the frequencies ω_3 and ω_4 determine the functions Q_3 and Q_4 directly. The above theorems do not give information on the ratio and the behavior of the amplitudes, the reflection, and the transmission. The theorems are presented in many text-books of differential equations (e.g. Kamke 1983, Stepanow 1963, and also in the work of Gradshteyn & Ryzhik (1980).

Characteristics of the Energy Transfer by Alfvén Waves in the Solar Atmosphere

Tsap et al. (2020), G&A, 60, 446.

The so-called turning points cannot adequately characterize the energy flux of Alfvén waves. This conclusion indicates that results based on the analysis of oscillation theorems must be revised.

$$\rho(z) = \rho_0 \exp(-z/H)$$

$$\frac{\partial^2 \delta V}{\partial t^2} = V_A^2 \frac{\partial^2 \delta V}{\partial z^2}$$

$$\delta V = C_1 H_0^{(1)}(\eta) + C_2 H_0^{(2)}(\eta), \quad \eta = \frac{2\omega H}{V_A}$$

$$F_A = -B/8\pi \operatorname{Re}(\delta V^* \delta B)$$

$$F_A = \frac{B^2}{8\pi^2 \omega H} (|C_2|^2 - |C_1|^2)$$

Initial equations

$$\varrho \frac{\partial \mathbf{V}}{\partial t} + \varrho (\mathbf{V} \cdot \nabla) \mathbf{V} = -\nabla p + \frac{1}{\mu} (\nabla \times \mathbf{B}) \times \mathbf{B} + \varrho \mathbf{g}, \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}), \quad (3)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (4)$$

$$p = \frac{k_B}{m} \varrho T, \quad (5)$$

In our case, solar atmosphere is VAL-C, the magnetic field $\mathbf{B}=\text{const}$. We used the MHD numerical code FLASH 4.5, using Adaptive Mesh Refinement (Fryxell et al., 2000) to provide the simulation. The size of the simulation region is (5 x 5) Mm, the total number of grids is 3749, which gives the maximum resolution 0.00976 Mm. Initial perturbation of the system is:

$$V_z(x, y, t = 0) = -A_0 \cdot \frac{x}{\lambda} \cdot \exp \left[-\frac{x^2 + (y - L_p)^2}{\lambda^2} \right],$$

where A_0 is the amplitude of the initial pulse, and $\lambda = 200$ km is its width. This pulse preferentially triggers Alfvén waves (Jelínek et al. 2015). The perturbation point, L_p , is located at a distance of 500 km from the photosphere. $B_0=10$ G.

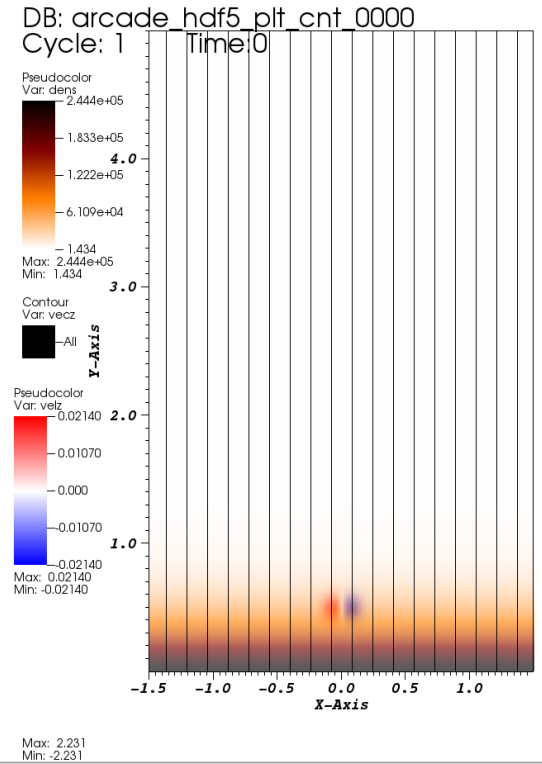
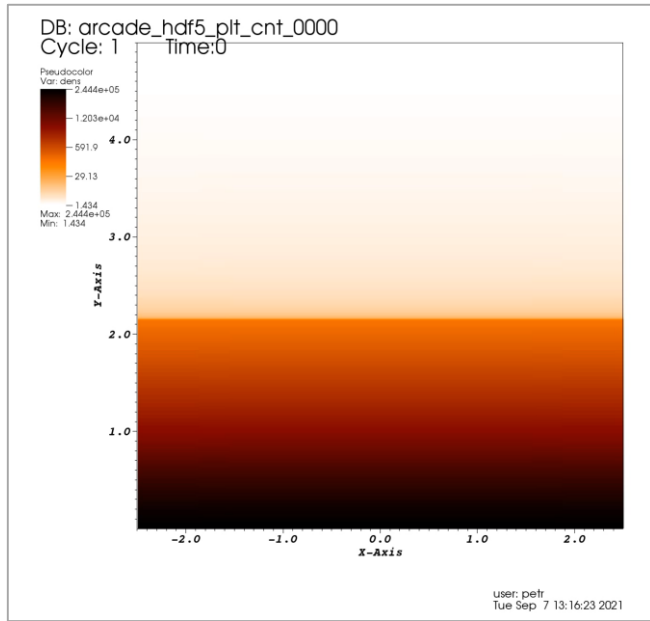
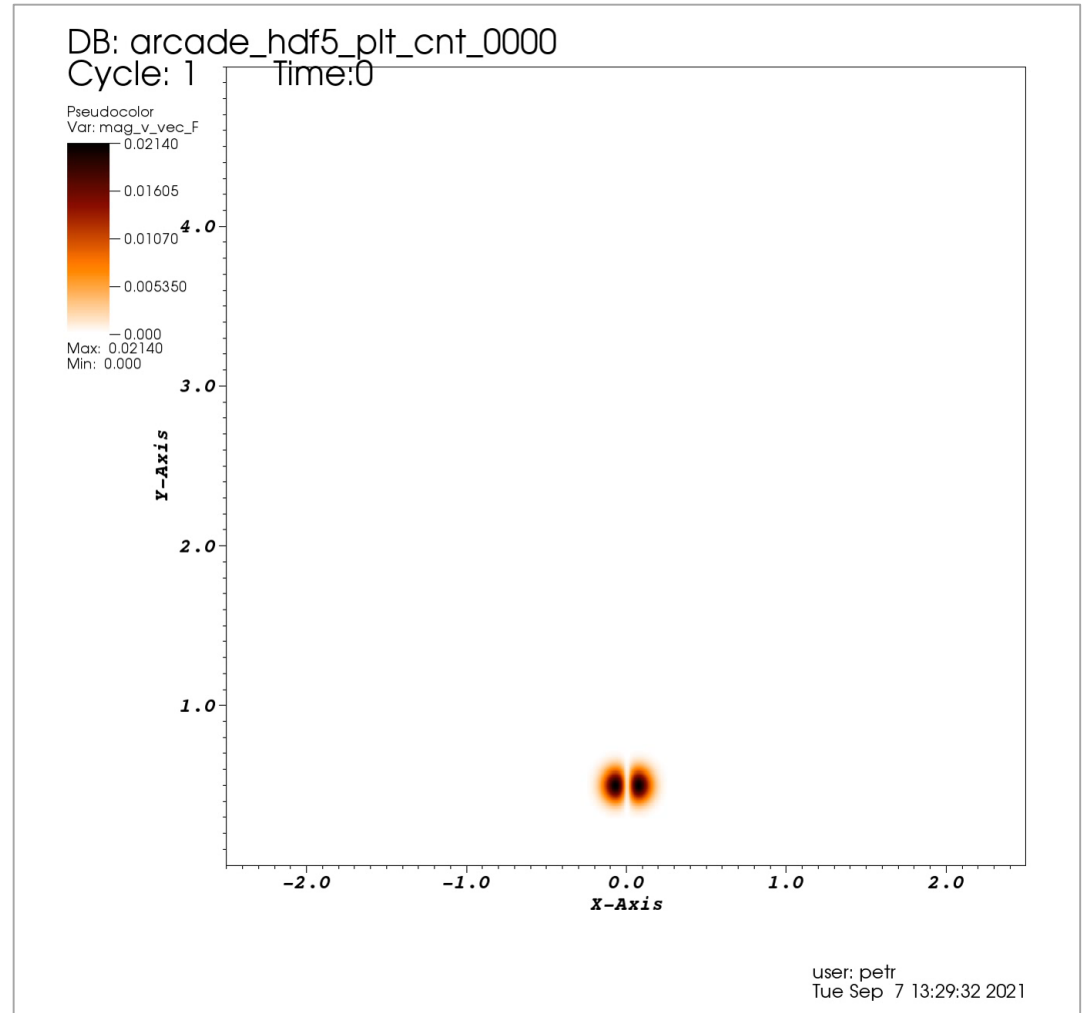


Figure 1. Initial state of the system in time = 0 s. Colors represent the mass density distribution (logarithmic scale). Black solid lines – the magnetic field lines. Blue and red circles represent the location of the initial velocity pulse.

Simulation results



Movie 1. Mass density evolution (units in kg/m³). Axis in Mm.



Movie 2. Time evolution of the velocity. Axis in Mm.

Simulation results

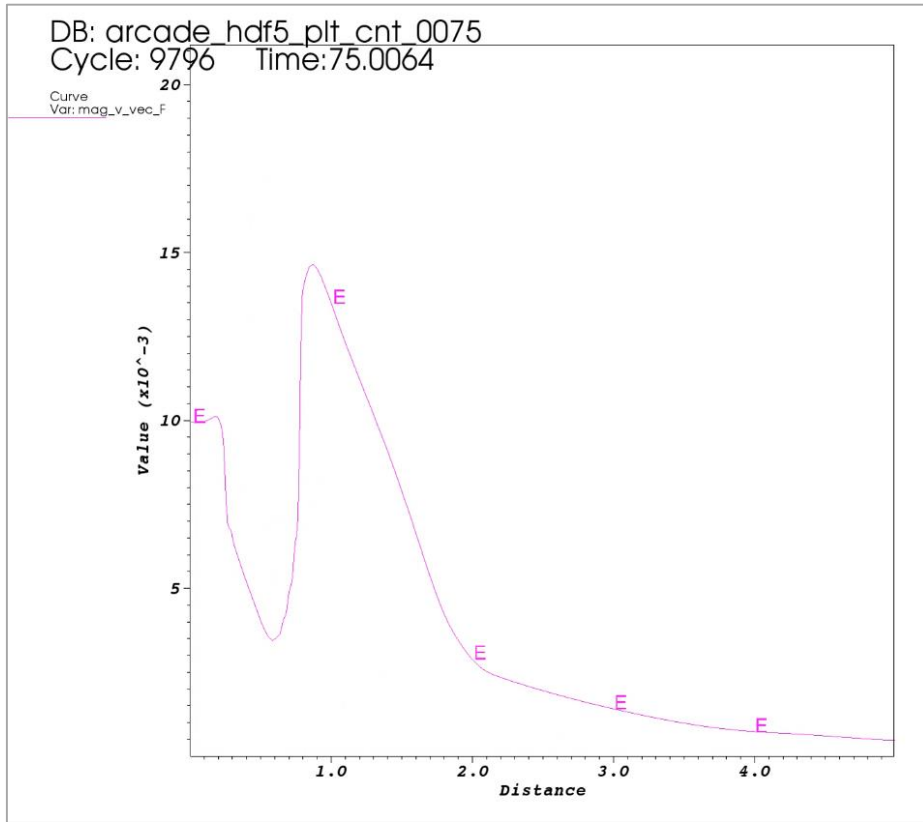


Figure 2. Velocity profile in time = 75 s, after the wave is reached the transition region. It is seen that the amplitude of the wave decreased. Distance in Mm.

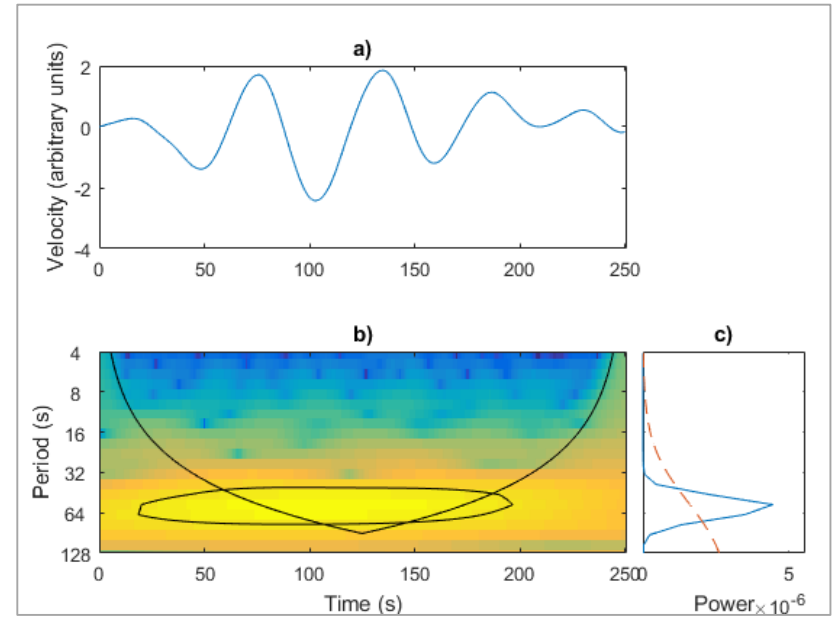
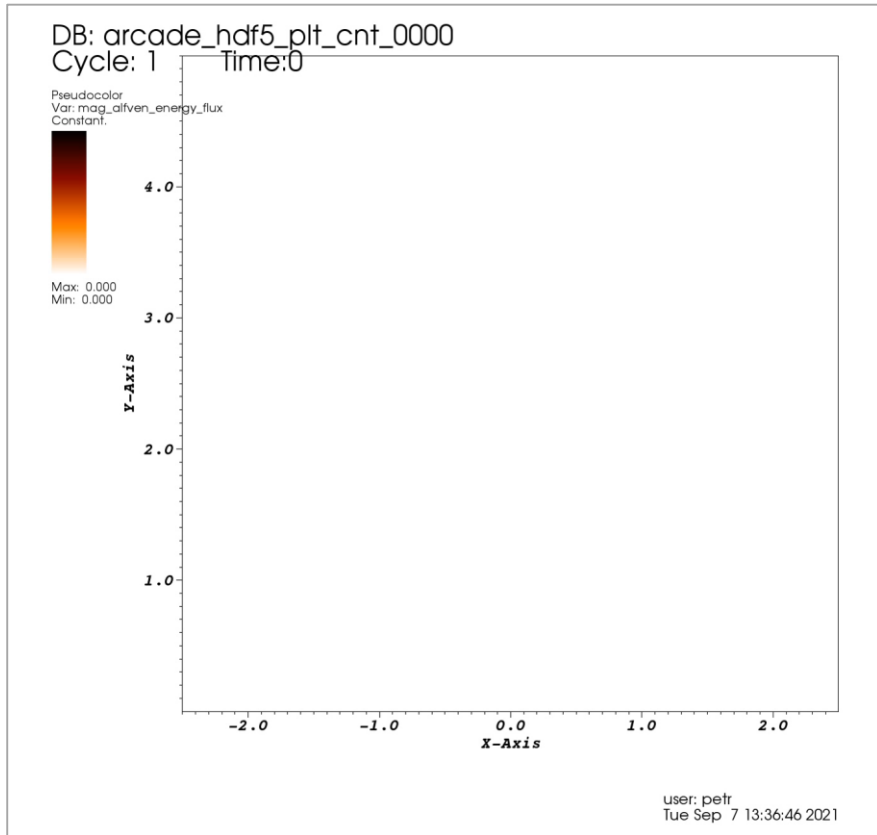


Figure 3. Time series of the velocity variations in a reference point $y = 1.8$ Mm (a). Wavelet power spectrum (b). $P = 50$ s.

Simulation results



Movie 4. Evolution of the Alfvén energy flux magnitude. Axis in Mm.

The energy flux could be estimated as:

$$S = -1/\mu_0[\mathbf{B}^2 \mathbf{V} - (\mathbf{B}\mathbf{V})\mathbf{B}] \text{ (W/m}^2\text{)}.$$

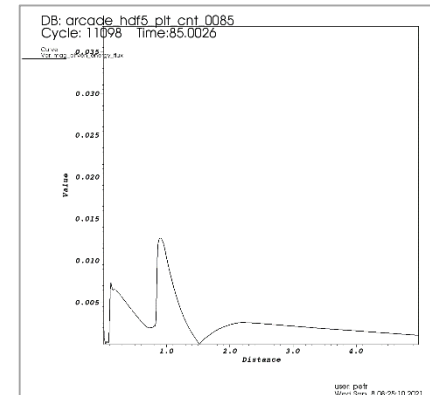
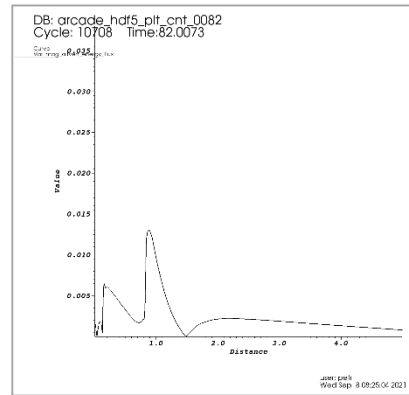
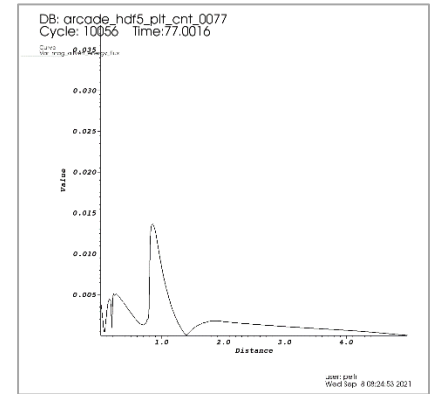
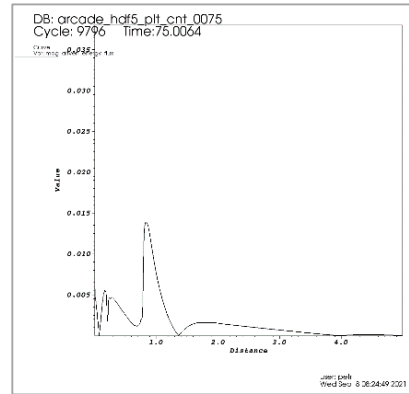


Figure 4. Profile of the Alfvén energy flux magnitude in times = 75, 77, 82 and 85 s, after the wave is reached the transition region. It is seen that the wave amplitude is not sufficiently changed. Distance in Mm. Flux: $\text{W/m}^2 \times 10^6$.

Conclusions

1. We produced numerical simulation of propagation of Alfvén perturbations in stratified solar atmosphere.
2. The results of simulations clearly show the Alfvén waves with a period of 50 s.
3. The energy flux of Alfvén waves does not feel the turning points in the lower solar atmosphere.