Bayesian evidence for a nonlinear damping model for coronal loop oscillations

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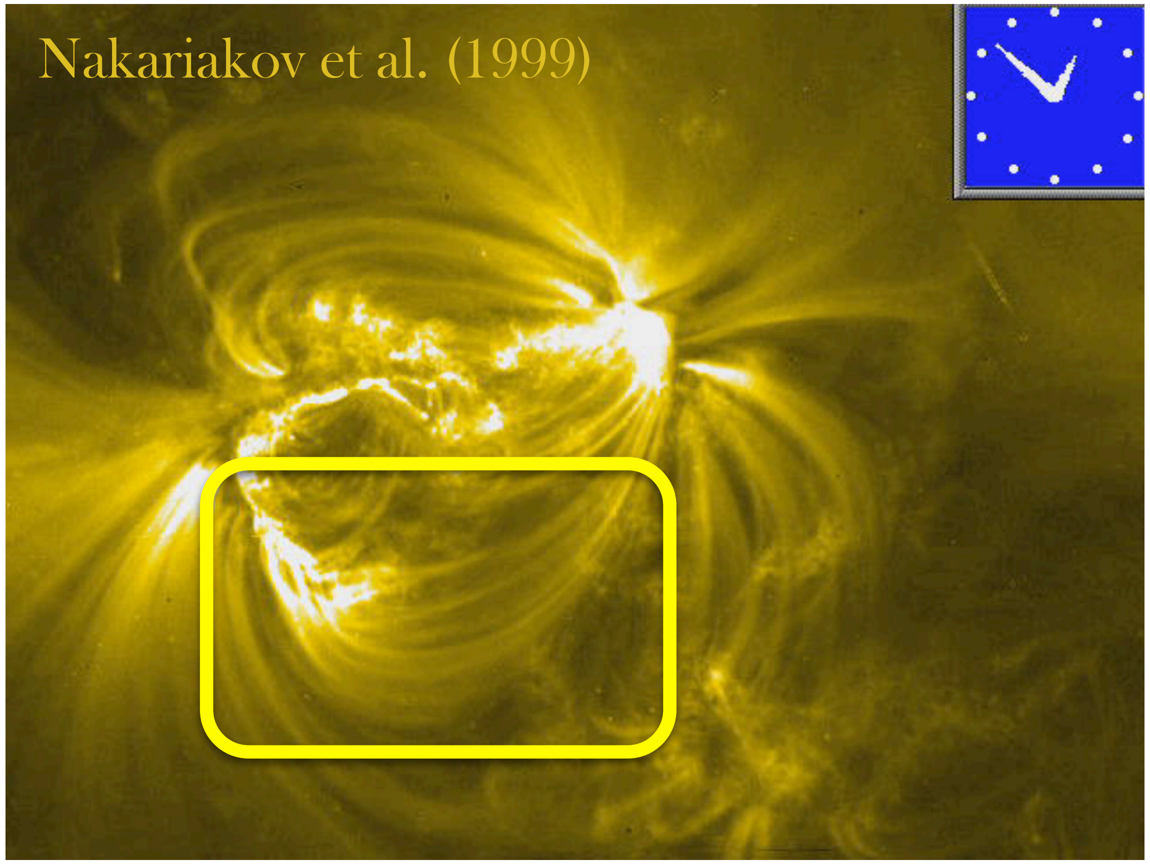
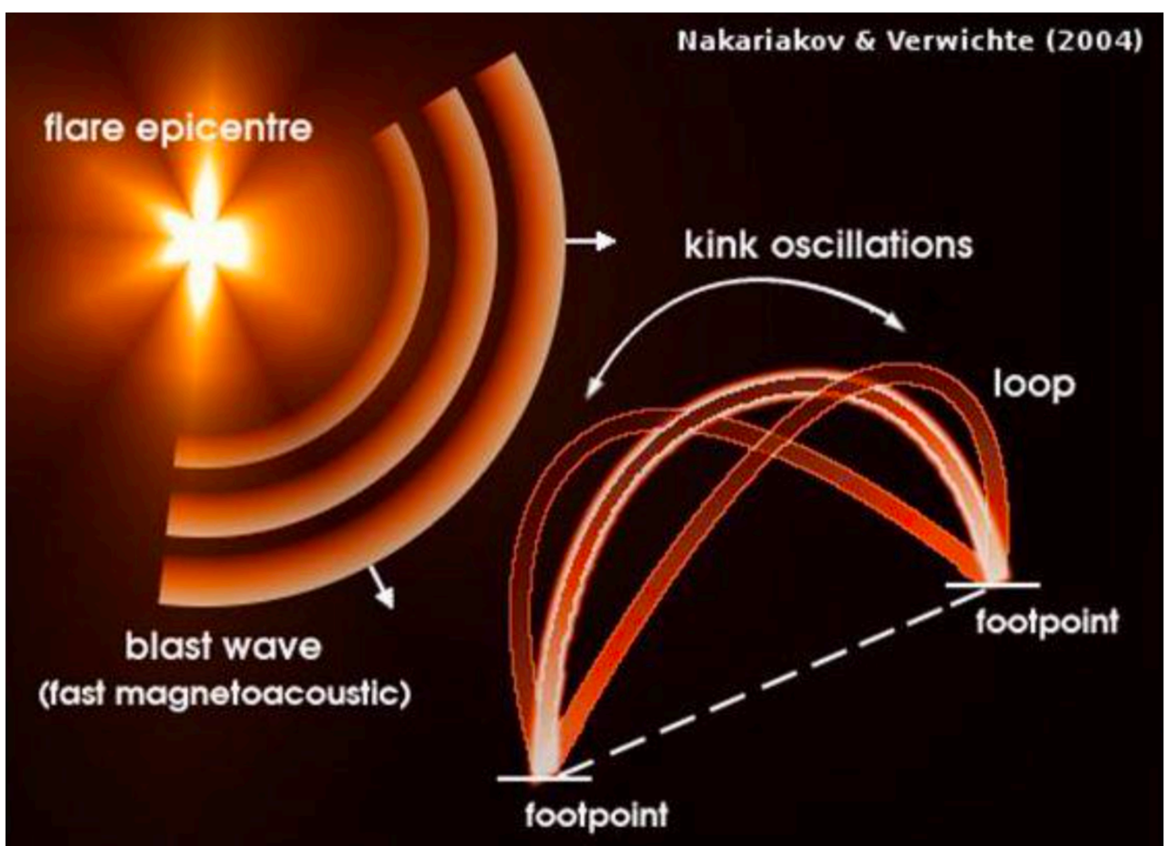
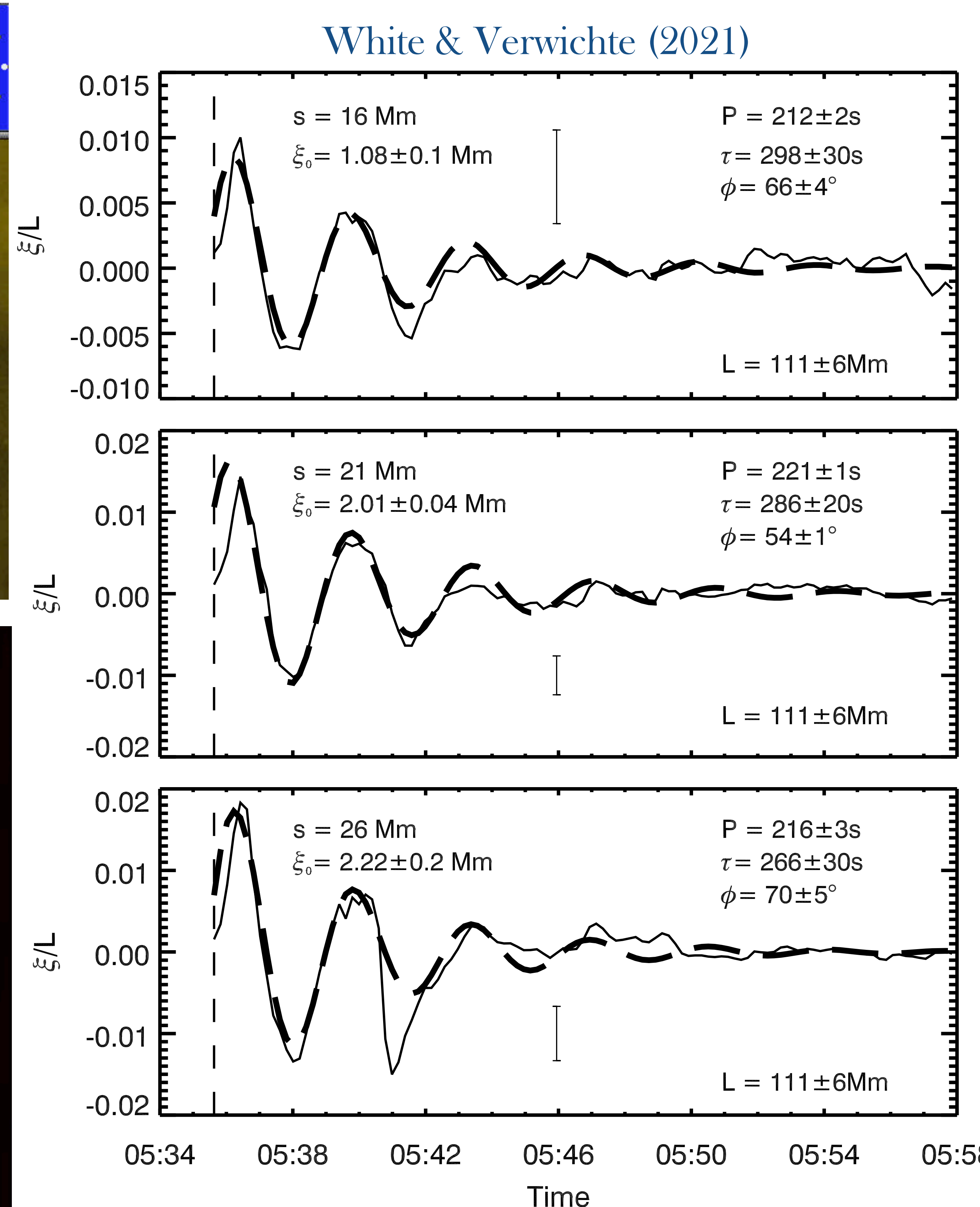
Damping of solar coronal loop oscillations







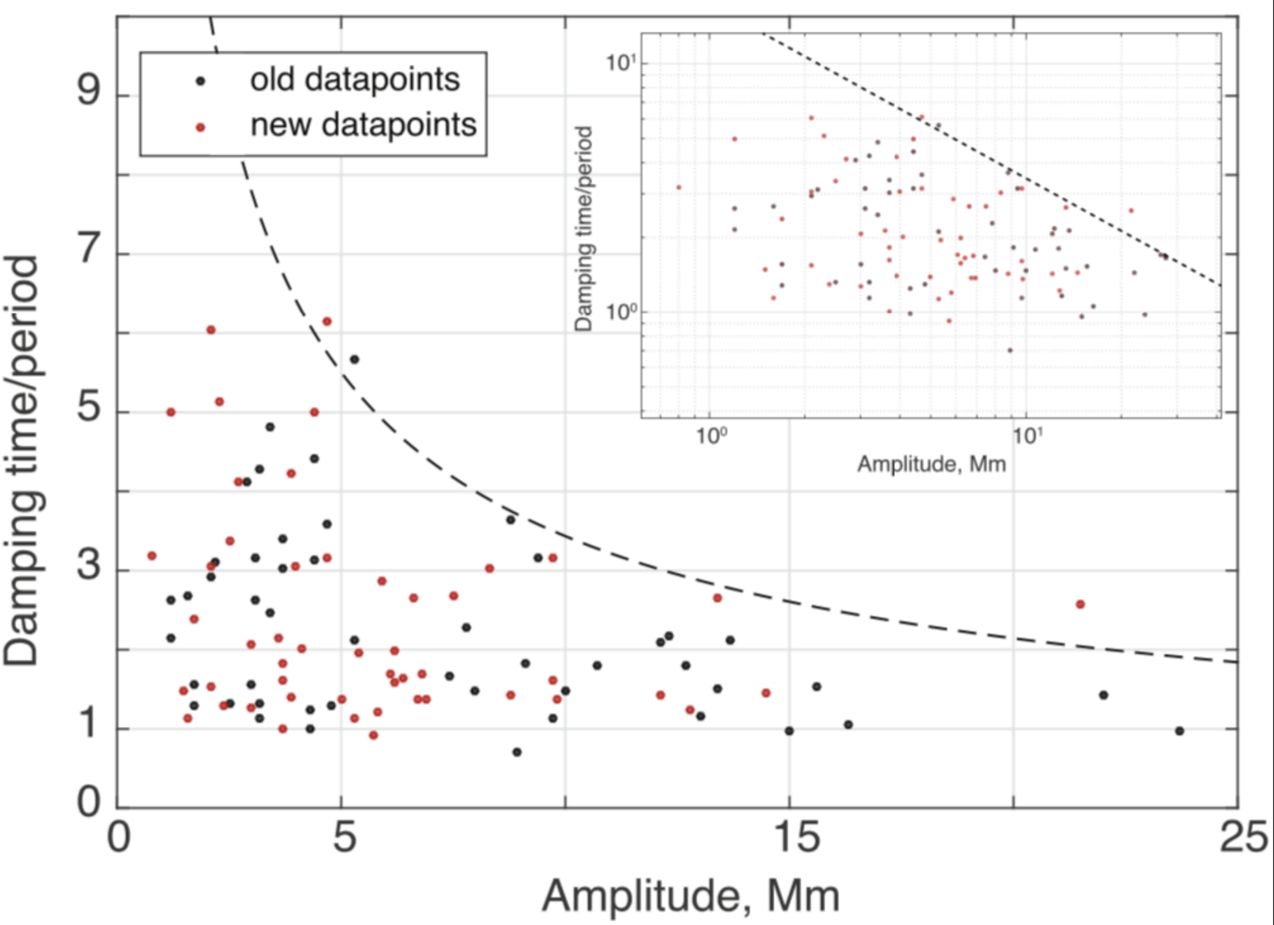
Observations

Damped transverse coronal loop oscillations

Analysis of large number of events

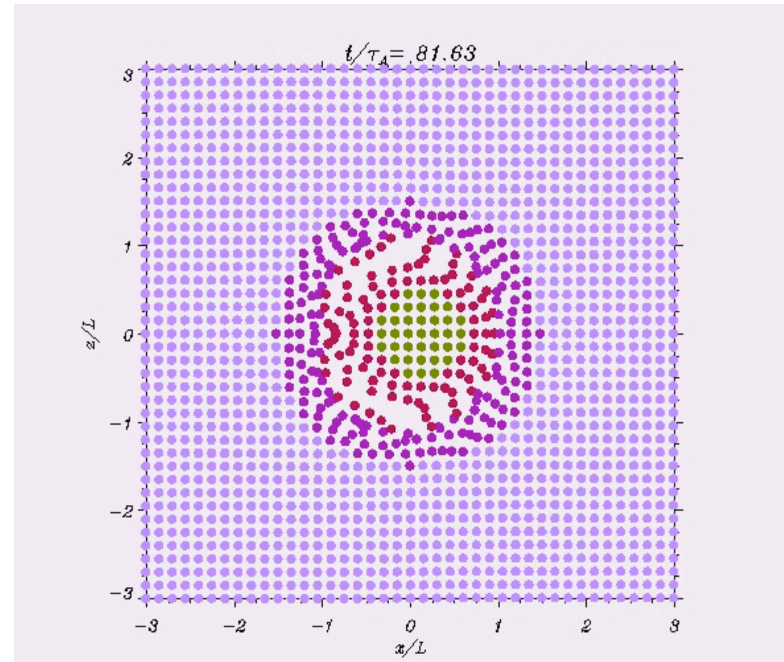
Empirical relationship between the damping and the oscillation amplitude

Nechaeva et al. (2019)



Models

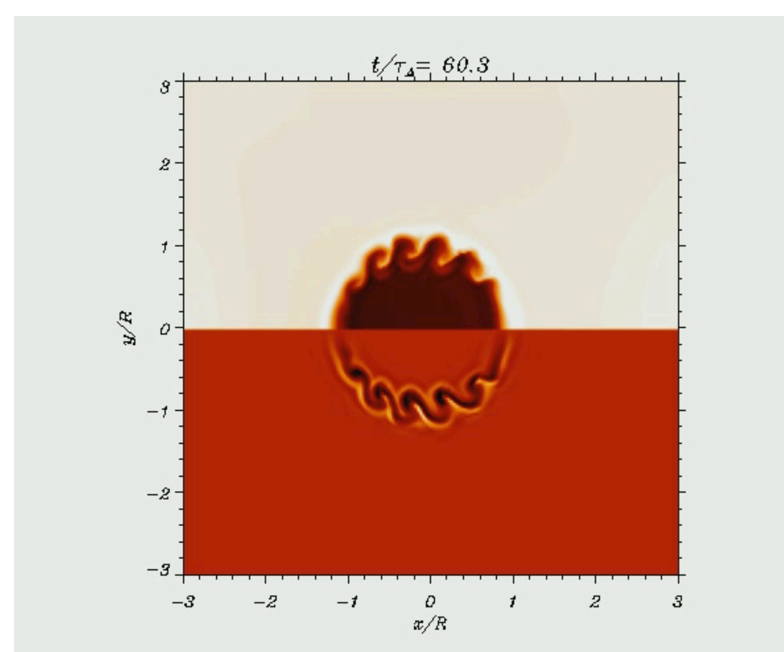
Resonant Absorption (RA) Goossens et al. (2002)



$$\frac{\tau_{\mathrm{d}}}{P}\|_{M_{\mathrm{RA}}} = \mathcal{F}\,\frac{\zeta+1}{\zeta-1}\,\frac{R}{l}$$

Nonlinear Damping (NL) Van Doorsselaere et al. (2021)

$$\frac{\tau_{\mathrm{d}}}{P}\|_{M_{\mathrm{NL}}} = 20\sqrt{\pi}\,\frac{1}{2\pi a}\,\frac{1+\zeta}{\sqrt{\zeta^2-2\zeta+97}}$$



Evidence

Marginal likelihood is a measure of relational evidence

$$p(D|M) = \int_{\boldsymbol{\theta}} p(\boldsymbol{\theta}, D|M)\, d\boldsymbol{\theta} = \int_{\boldsymbol{\theta}} p(D|\boldsymbol{\theta}, M)\, p(\boldsymbol{\theta}|M)\, d\boldsymbol{\theta}$$

$p(D\,|\,M_{\mathrm{NL}})$ & $p(D\,|\,M_{\mathrm{RA}})$

Bayes Factor is a measure of relative evidence

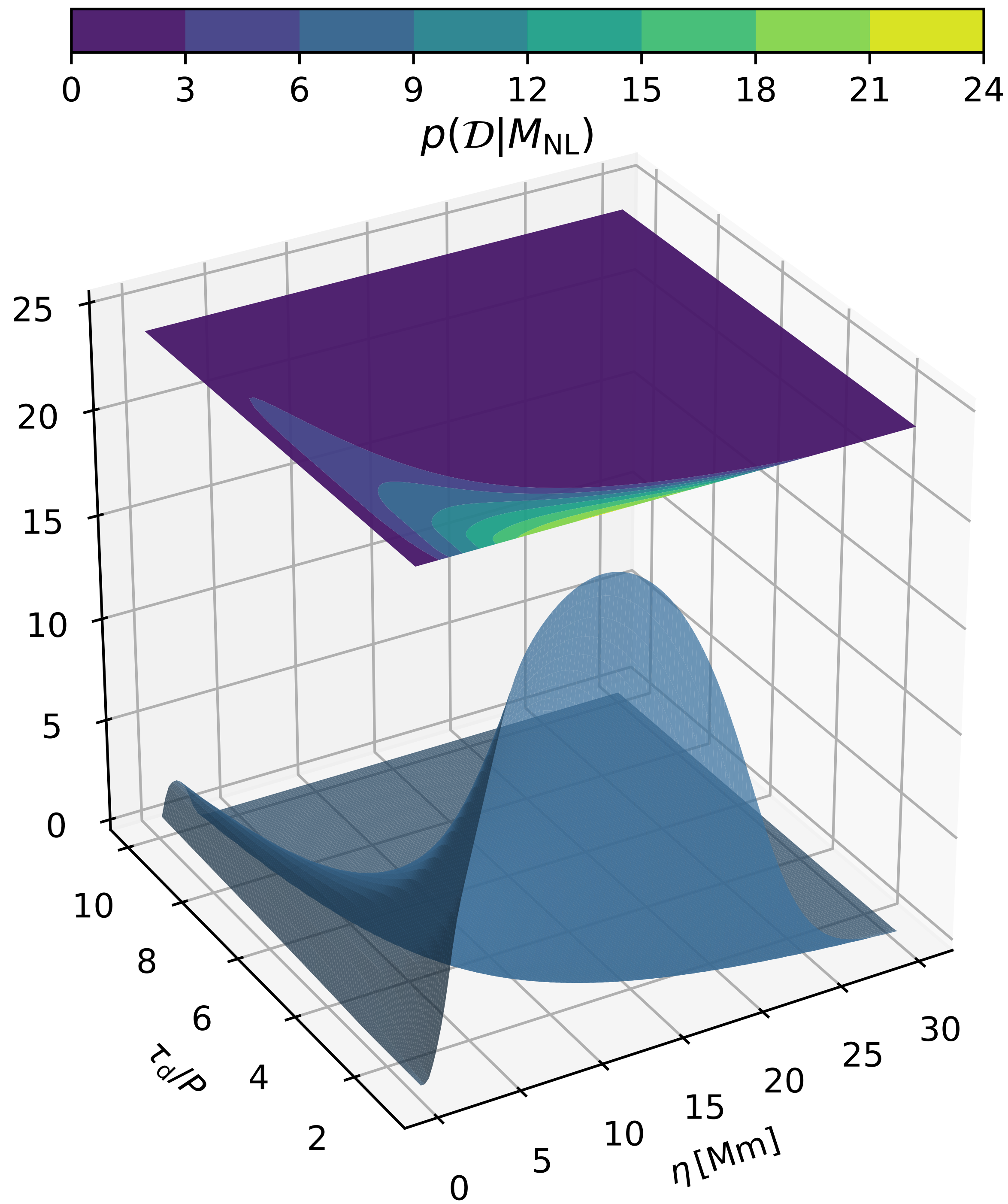
$$B_{\mathrm{NLRA}} = 2\log\frac{p(D\,|\,M_{\mathrm{NL}})}{p(D\,|\,M_{\mathrm{RA}})} = -B_{\mathrm{RANL}}$$

Kass & Raftery (1995)

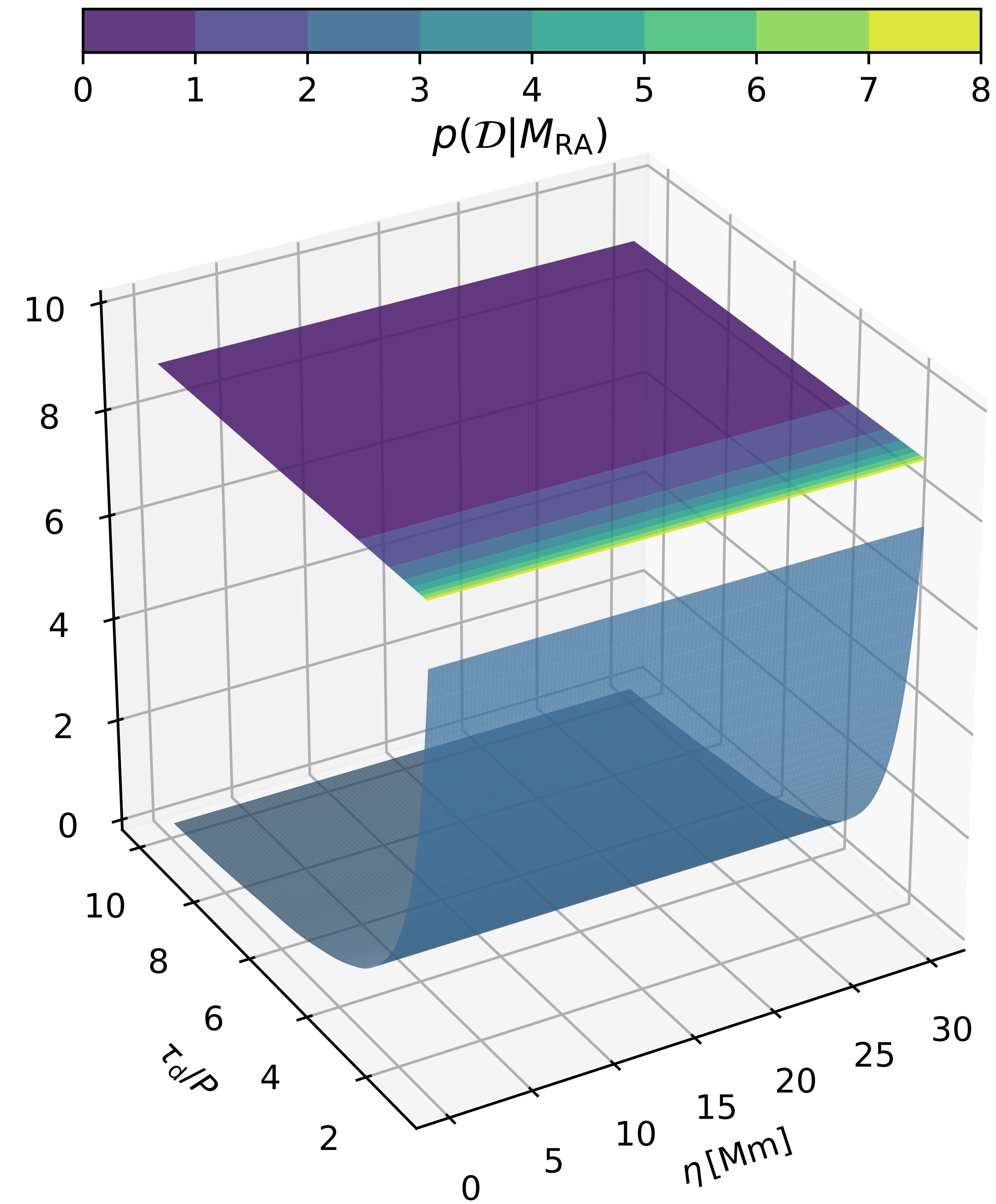
Bayes factor	Evidence
0 - 2	inconclusive
2 - 6	positive
> 6	strong
> 10	very strong

Marginal likelihood

Nonlinear damping

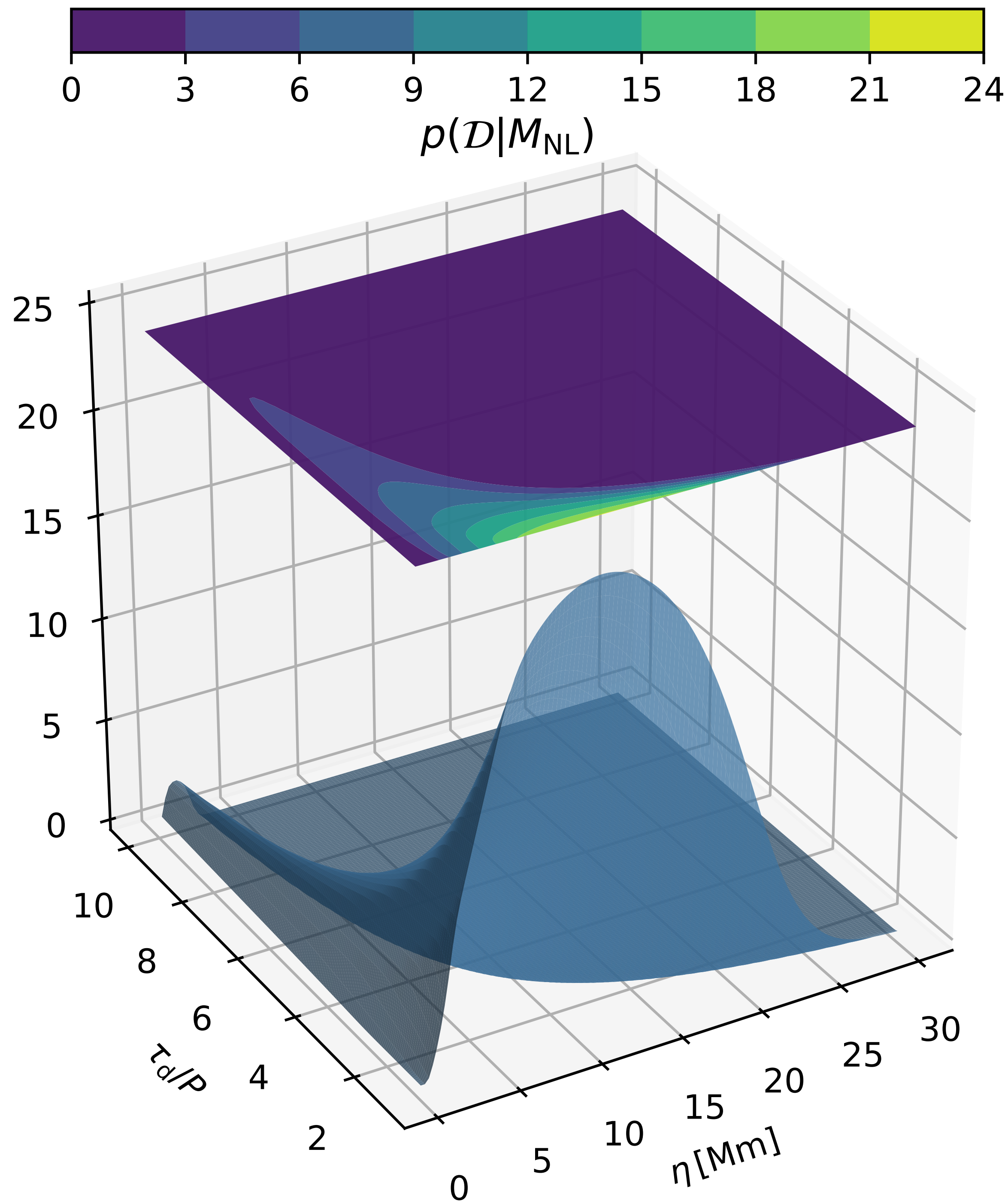


Linear resonant damping

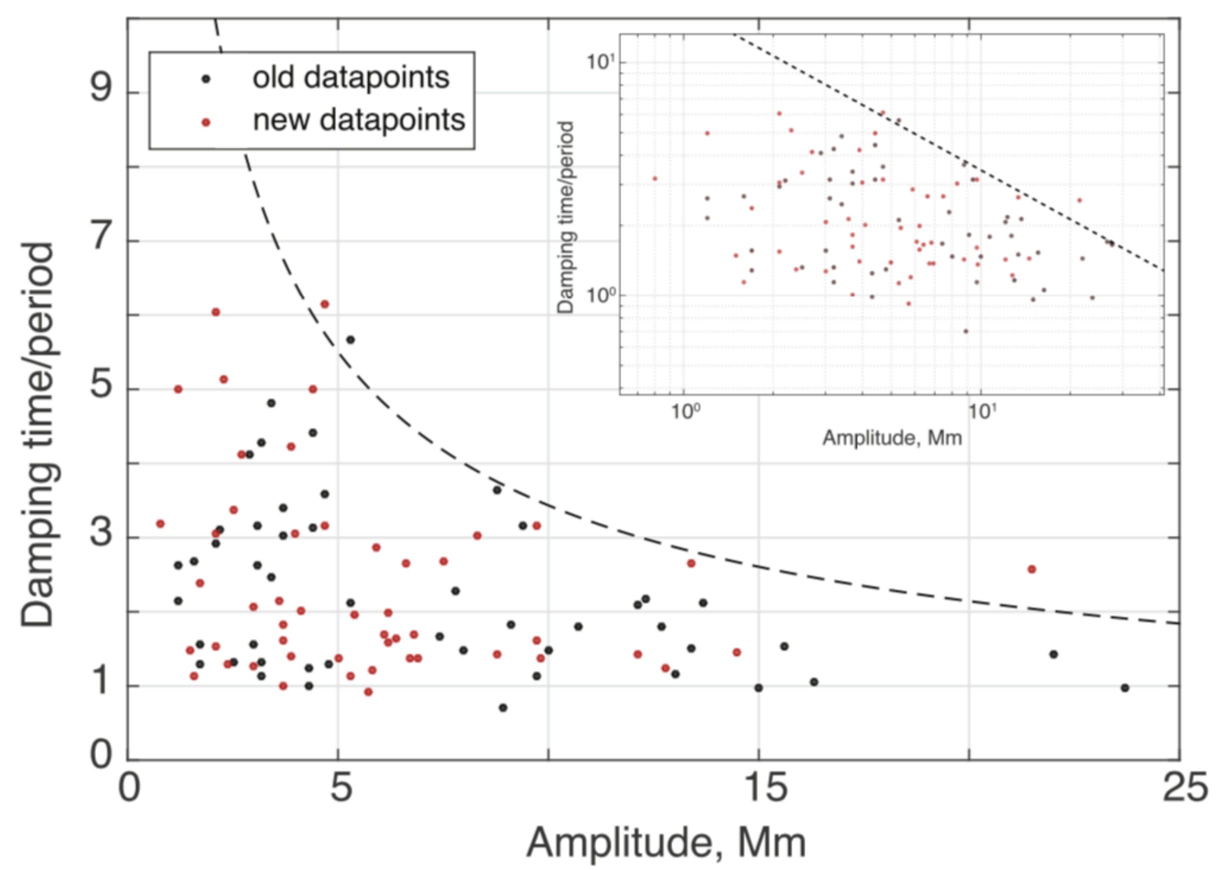


Marginal likelihood

Nonlinear damping

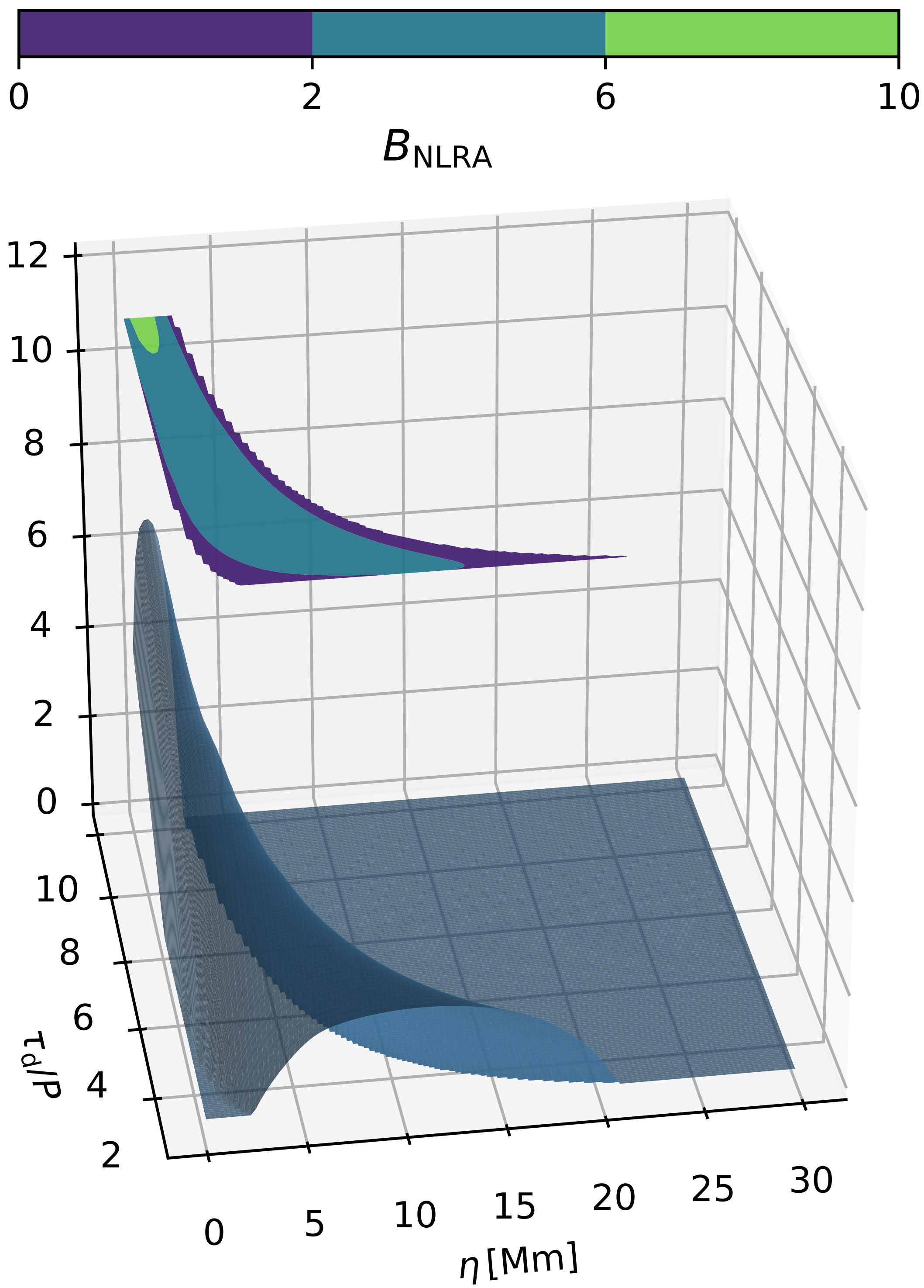


Nechaeva et al. (2019)

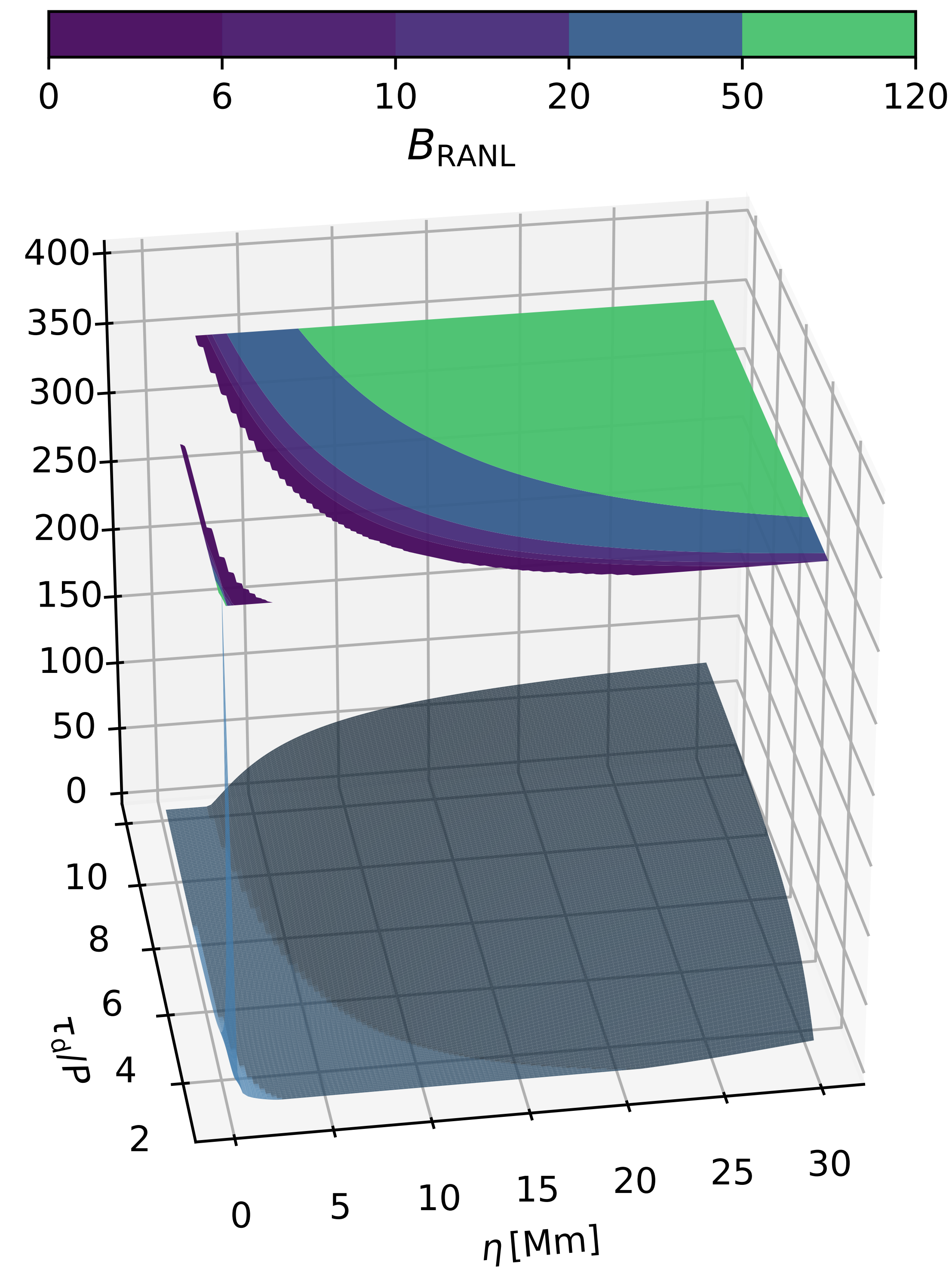


Bayes factor

Nonlinear damping

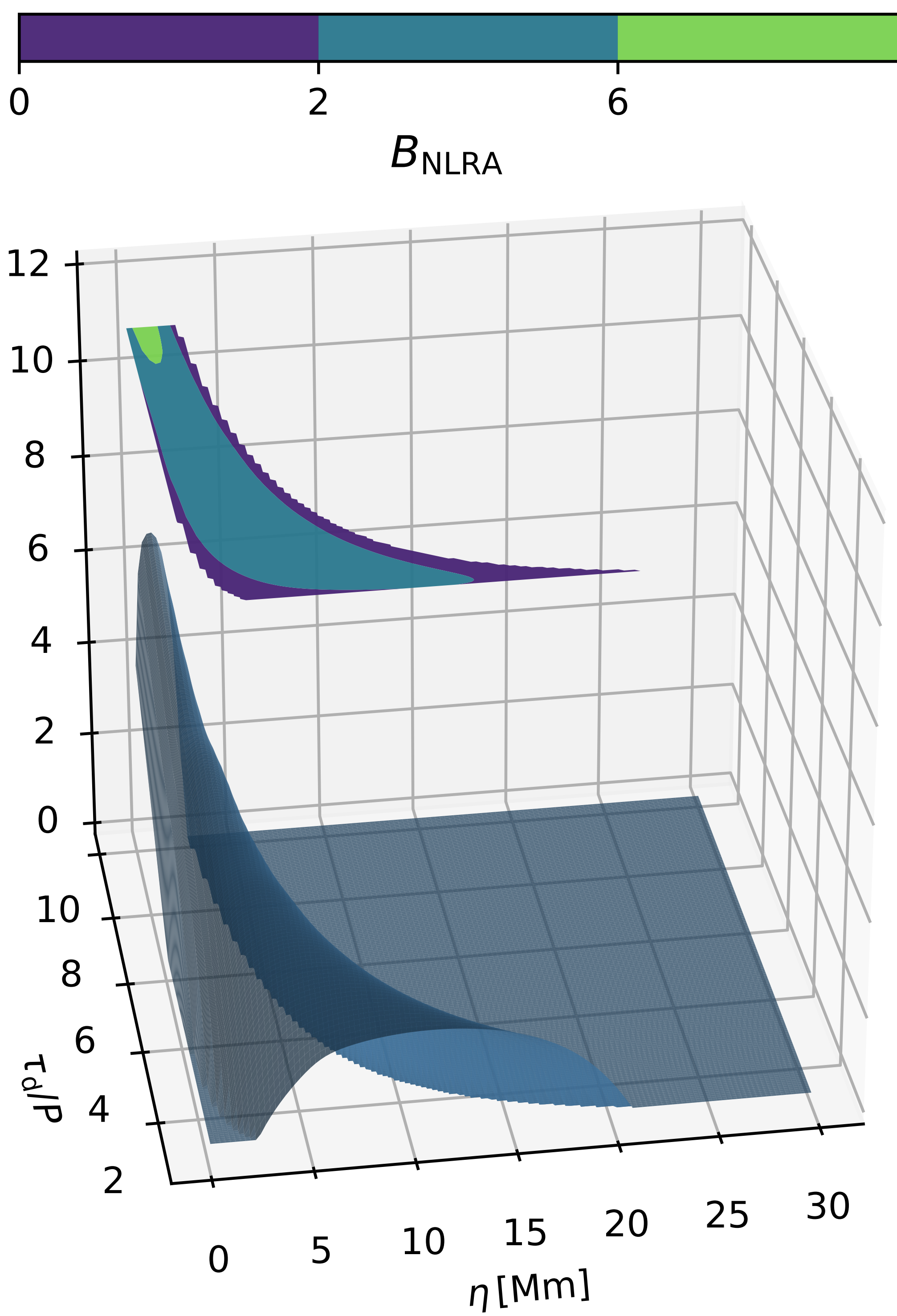


Linear resonant damping

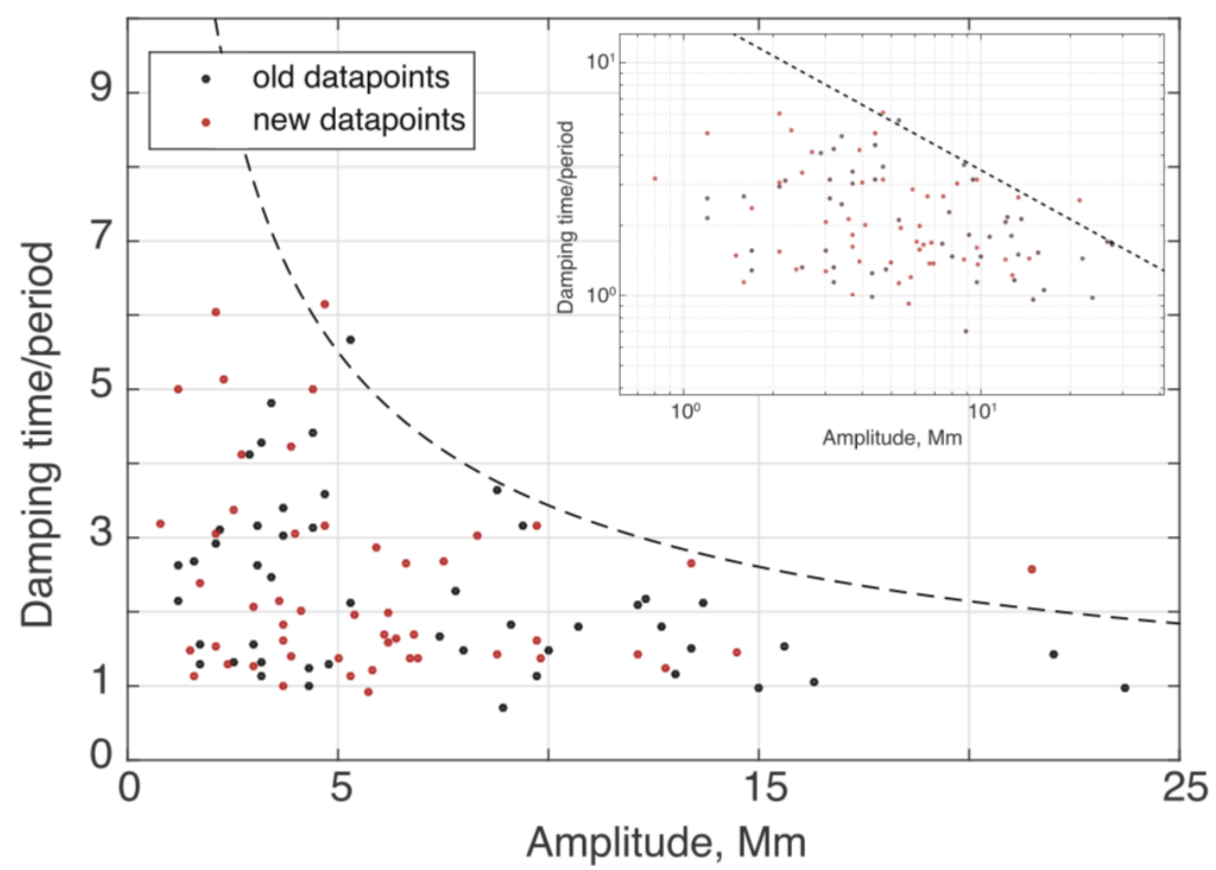


Bayes factor

Nonlinear damping



Nechaeva et al. (2019)



Application to SDO/AIA loop oscillations

Bayes factor for 101 oscillation events in the catalog by Nechaeva et al. (2019)

i	Event ID	Loop ID	P [min]	τ_D [min]	τ_D/P	η [Mm]	$B_{\rm NLRA}$
1	1	1	3.42 ± 0.06	5.34 ± 1.12	1.56 ± 0.33	1.7	0.4
2	1	2	4.11 ± 0.05	10.76 ± 2.79	2.62 ± 0.68	1.2	1.4
3	3	1	2.46 ± 0.03	8.8 ± 1.8	3.58 ± 0.73	4.7	3.4
4	3	2	3.62 ± 0.08	4.12 ± 0.47	1.14 ± 0.13	9.7	2.2
5	4	1	2.29 ± 0.03	7.18 ± 1.5	3.14 ± 0.66	4.4	3.5
6	4	2	3.47 ± 0.03	$7.44 \pm 1.$	2.14 ± 0.29	1.2	1.0
7	7	1	1.69 ± 0.02	7.23 ± 1.3	4.28 ± 0.77	3.2	4.1
8	8	1	3.74 ± 0.07	$10. \pm 1.$	2.67 ± 0.27	1.6	2.5
9	9	1	5.14 ± 0.17	5.09 ± 0.98	0.99 ± 0.19	4.3	0.9
...							
94	84	1	3.78 ± 0.33	15.6 ± 8.3	4.13 ± 2.23	2.7 ± 1.0	2.6
95	87	1	5.94 ± 0.32	25.1 ± 10.8	4.23 ± 1.83	3.9 ± 1.0	2.7
96	87	2	9.24 ± 0.62	12.7 ± 4.6	1.37 ± 0.51	6.9 ± 2.2	2.1
97	88	1	9.38 ± 0.19	13.0 ± 5.4	1.39 ± 0.58	5.0 ± 0.4	1.8
98	88	3	13.75 ± 1.18	15.5 ± 6.5	1.13 ± 0.48	5.3 ± 1.8	1.5
99	90	1	9.32 ± 0.31	8.6 ± 2.4	0.92 ± 0.26	5.7 ± 1.4	1.2
100	92	1	6.51 ± 0.31	9.1 ± 2.4	1.40 ± 0.37	3.9 ± 1.3	1.6
101	93	1	8.32 ± 0.10	21.4 ± 4.9	2.57 ± 0.59	21.5 ± 2.4	-6.2

$B_{\rm NLRA} > 0$

Evidence if favour of nonlinear damping

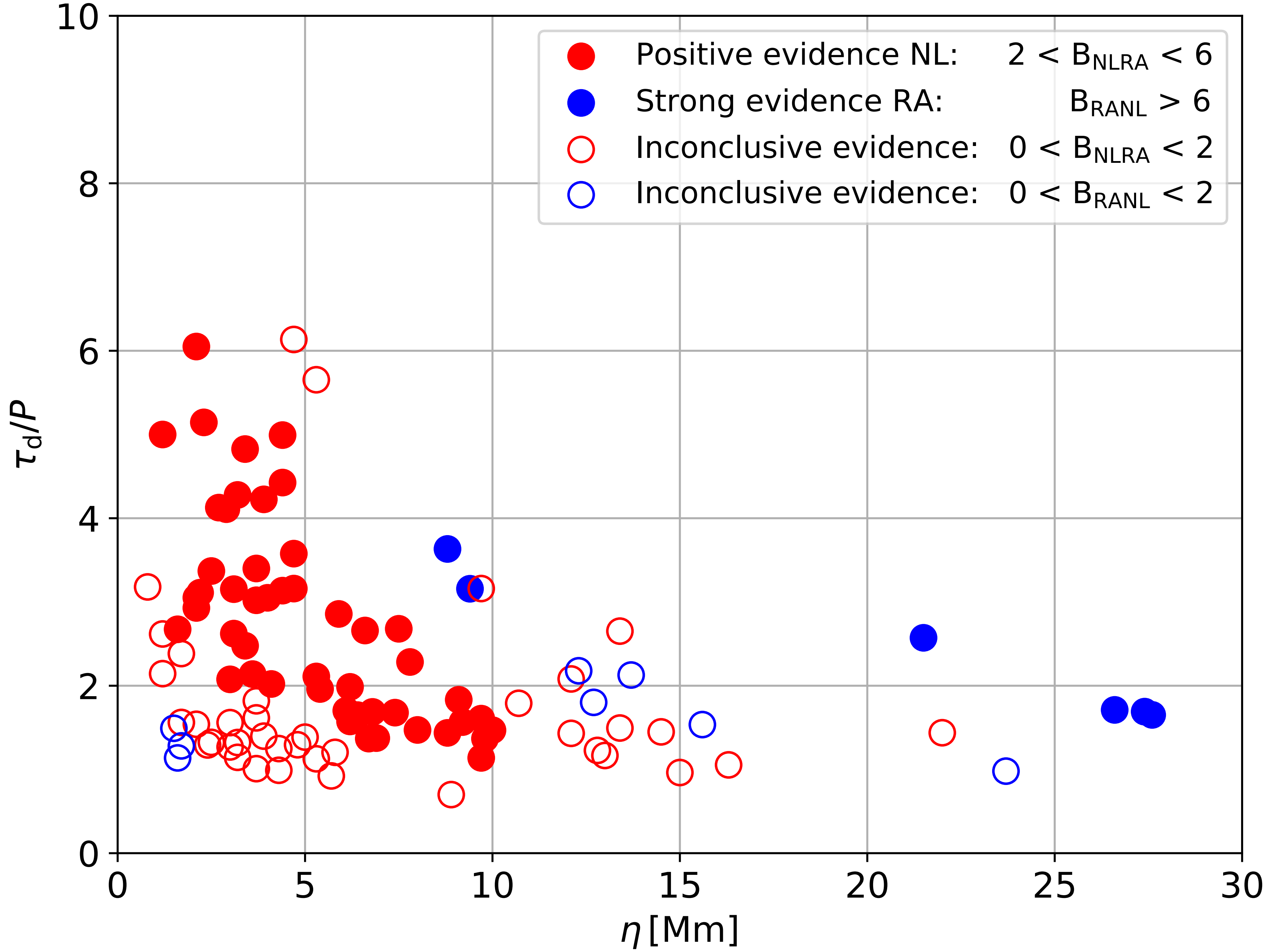
Bayes factor	Evidence
0 - 2	inconclusive
2 - 6	positive
> 6	strong
> 10	very strong

$B_{\rm NLRA} < 0$

Evidence if favour of resonant damping

Application to SDO/AIA loop oscillations

(bayesian) results for all 101 oscillating loops



(frequentist) statistics

cases

49
38
} 87

6
8
} 14

Nonlinear damping is a plausible explanation for the observed properties of damped transverse coronal loop oscillations