16th European Solar Physics Meeting (6-10 September 2021)

INTRODUCTION

- Sunspot areas are strongly correlated with other solar activity indices as the Group Number or Sunspot Number
- Measures of the sunspot areas in historical drawings are very interesting in order to analyze the past solar activity
- Analysis is carried out by using computer programs that binarize the sunspot images

Sunspot area (in msh) corrected by foreshortening in the plane-parallel approximation

$$A_M = \frac{10^6}{2\pi R^2} \frac{A_S}{\cos \rho}$$

A_S → sunspot area measured directly onto the image

R → radius of the solar disk (in pixels)

ρ → angle between the direction of the centre of the solar disk and the direction of the sunspot

On the uncertainties of the sunspot area measurements

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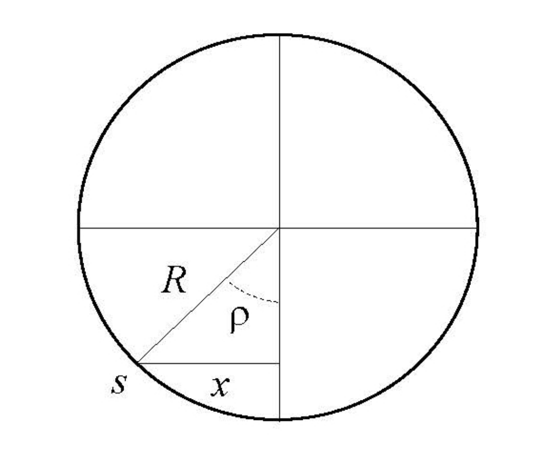
UNCERTAINTIES

1. Uncertainty associated to the sunspot position → $\sigma(A_M) = A_M \tan\rho\, \sigma(\rho) = A_M \frac{\tan\rho}{\cos\rho} \frac{\sigma(x)}{R}$



2. Uncertainty associated to the resolution of the image → $\sigma(A_M) = A_M \frac{2\sigma(R)}{R}$

3. Uncertainty associated to the error in the determination of the sunspot area (binarization) → $\sigma(A_M) = A_M \frac{\sigma(A_S)}{A_S}$

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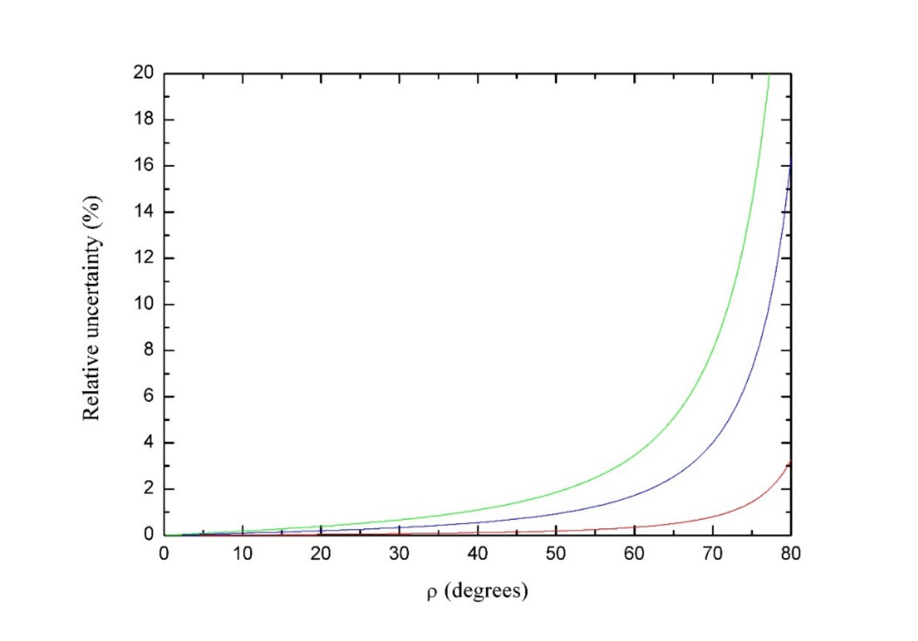
UNCERTAINTY DUE TO THE SUNSPOT POSITION

The relative sunspot area error increases with ρ → Very important in the analysis of historical drawings (inaccuracies in the sunspot positions)



Relative uncertainty of the sunspot area as function of the angle ρ for an error in x of 0.1 % (red line), 0.5 % (blue line) and 1 % (green line). The percentages are referred to the radius of the solar disk.

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UNCERTAINTY DUE TO THE BINARIZATION PROCESS (I)

Binarization → transformation of an 8-bits image (256 gray levels from 0 to 255) to a black and white image (black = 0 / white = 1)

- **Binarization threshold t ($0 < t < 1$)**
- **Cut-off intensity c → $c = minval + t(maxval - minval)$**

If intensity > c, then it is assigned the value 1 (white). Otherwise, it is assigned the value 0 (black)
Sunspot area (uncorrected by foreshortening) → sum of all black pixels

- **The sunspot area depends critically on the binarization threshold**
- **Optimal t value is unknown → binarization threshold freely selected by the user**

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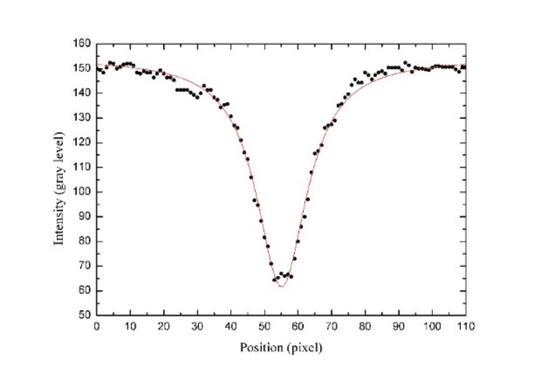
UNCERTAINTY DUE TO THE BINARIZATION PROCESS (II)

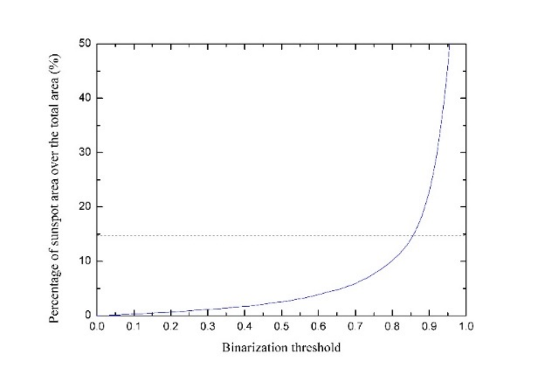
Estimation of the best binarization threshold values:

- Selection of a real sunspot (a) from a digital image (8-bits) and measuring the intensity along the sunspot
- Fit a Cauchy function to the intensity data (b). This Cauchy function represents a simulated sunspot
- Introduction of Gaussian noise to simulate a real sunspot, with fluctuations in the intensity
- Binarization of the noise image
- Comparison the number of black pixels in the binarized image with those of the original simulated image, using a convenient reference value to define the sunspot limits (c)

a



b



c

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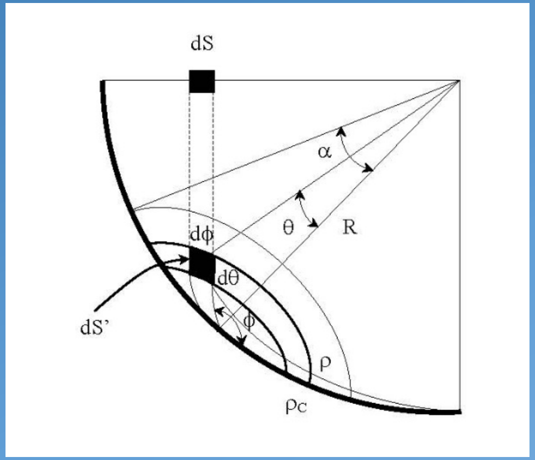
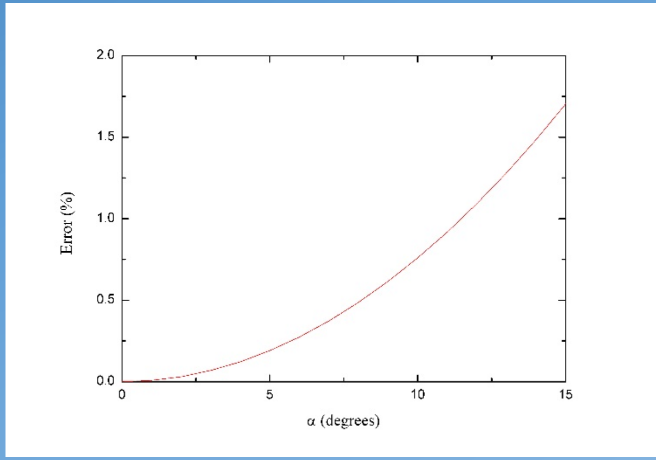
ANALYTICAL SUNSPOT AREA FOR CIRCULAR SHAPE

The size effects are ignored in the plane-parallel approximations

- **Sunspot area of a circular sunspot with radius α →** $S' = 2\pi R^2(1 - \cos\alpha)$
- **Projected sunspot area (measured over the image) →** $S = \int dS = \int dS' \cos\rho = R^2 \int \cos\rho \sin\theta d\theta d\varphi = 0.5\, \pi R^2 \cos\rho_c\ (1 - \cos 2\alpha)$ ρ_c **= angle between the direction of the centre of the sunspot and the direction of the centre of the disk**





The error due to the use of the classical plane-parallel approximation is very small excepting for large sunspots

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ANALYTICAL SUNSPOT AREA FOR ELLIPTICAL SHAPE

The spherical ellipse is the translation of the plane ellipse to a sphere, with semi-major angle α and semi-minor angle β

- **Sunspot area of the elliptical sunspot →** $S' = 2\pi R^2 - 4R^2\sqrt{\frac{A^2-1}{A^2(1+C^2)}}\,\Pi\left[\frac{C^2}{1+C^2},\frac{\pi}{2}-\gamma\right]$

Π→ Elliptic integral of the third kind

$$A = \frac{1}{\sin\alpha} \quad B = \frac{1}{\sin\beta} \quad C = \sqrt{\frac{B^2-A^2}{A^2-1}} \quad D = \sqrt{\frac{B^2-A^2}{A^2}} \quad \gamma = \sin^{-1}\sqrt{\frac{1+D^2}{1+C^2}}$$

- **Projected sunspot area (measured over the image) →** $S = \int dS = \int dS' \cos\rho = R^2 \int \cos\rho \sin\theta d\theta d\varphi = \pi R^2 \cos\rho_c \sin\alpha \sin\beta = \pi\, ab \cos\rho_c$ **(projected plane ellipse)**

$$a = R \sin\alpha$$
$$b = R \sin\beta$$

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