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On the uncertainties of the sunspot area measurements

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INTRODUCTION

Total sunspot area is a key parameter that it is strongly correlated with other solar activity indices. In this way, sunspot area measurements in historical solar drawings are of great interest in the reconstruction of the past solar activity. Usually, these measures are carried out by analyzing the drawings with computer programs (Hrzina et al., 2007; Cristo et al., 2011; Galaviz et al., 2020) that calculate sunspot areas by means of the binarization of the sunspot images. In regard to this, sunspot areas corrected from foreshortening (in millionths of solar hemisphere, or msh) are determined by using the well-known equation

$$A_M = \frac{10^6}{2\pi R^2}\frac{A_S}{\cos\rho}$$

where A_S is the sunspot area measured directly on the image, R, the radius of the solar disk and ρ the angle between the direction of the centre of the solar disk and the direction of the sunspot (strictly speaking, the direction of the centroid of the sunspot intensity distribution). This equation represents a plane-parallel approximation very accurate in the most cases. Note that, according above equation, the sunspot areas depends on the three indicated quantities that are independent each of other.

ASSOCIATED UNCERTAINTIES

In agreement with the aforementioned comments, the formal uncertainties arising from each parameter are:

1) Uncertainty associated to the sunspot position with respect the disk solar center (defined by ρ)

$$\sigma(A_M) = A_M \tan\rho\,\sigma(\rho)$$

This expression can be conveniently expressed in terms of the distance *x* between the sunspot and the disk solar center as

$$\sigma(A_M) = A_M \frac{\tan\rho}{\cos\rho}\frac{\sigma(x)}{R}$$

where $\sigma(x)$ is the uncertainty of this distance.

2) Uncertainty associated to the resolution of the digitized drawing or image (related to the radius *R*)

$$\sigma(A_M) = A_M \frac{2\sigma(R)}{R}$$

3) Uncertainty associated to the error in the determination of the sunspot area over the image (due to the binarization process)

$$\sigma(A_M) = A_M \frac{\sigma(A_S)}{A_S}$$

The uncertainty (2) can be minimized by using a high value of the radius in the digitized image (for example, if R = 1000 pixels, the relative error of the area is 0.2 %). By this reason only uncertainties (1) and (3) are considered here.

ANALYTICAL SUNSPOT AREAS FOR CIRCULAR AND ELLIPTICAL SHAPES

As indicated above, the equation of the sunspot area corrected from foreshortening is an approximation that ignores the size effects of the sunspot. We have considered here two ideal shapes (that approximate in many cases the irregular shapes of the sunspots): circular and spherical ellipse (Smith, 1878). This last shape is the translation of a plane ellipse to a sphere. *Circular shape*: the sunspot area for an angular radius α is $S' = 2\pi R^2(1-\cos\alpha)$.The corresponding projected sunspot area is obtained by integrating $dS = dS'\cos\rho$, where $dS' = R^2\sin\theta d\theta d\varphi$ (θ and φ being the polar and azimuthal angles). The result is $S = 0.5\,\pi R^2\cos\rho_c\,(1-\cos 2\alpha)$, with ρ_c corresponding to the center of the sunspot. *Spherical ellipse shape*: in this case $S' = 2\pi R^2 - 4R^2\sqrt{\frac{A^2-1}{A^2(1+C^2)}}\,\Pi\left[\frac{C^2}{1+C^2},\frac{\pi}{2}-\gamma\right]$ with A, C and γ functions of the semi-major (α) and semi-minor (β) angles and Π the elliptic integral of the third kind. In this case, $S = \pi R^2\cos\rho_c\,\sin\alpha\sin\beta$

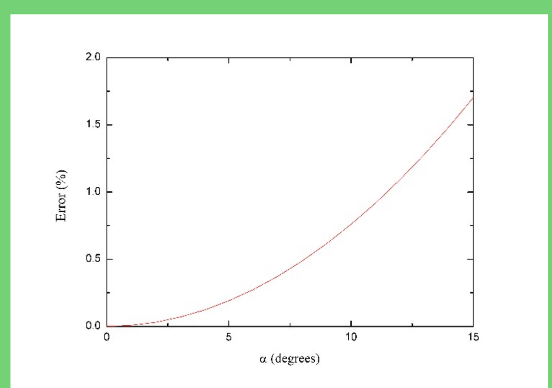

Figure 5. Error in the determination of the sunspot area for a circular shape due to the use of the plane-parallel approximation as function of the angular radius. Note that the approximation is very accurate excepting for really big sunspots

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UNCERTAINTY DUE TO THE SUNSPOT POSITION

The analysis of the corresponding equation shows that the relative error in the sunspot area increases with the angle ρ (figure 1), that is, when the sunspot is closer to the limb (Meadows, 2002). This is especially important in historical drawings due to the unavoidable inaccuracies introduced in the registered position of the sunspots, in contrast with the more precise positions provided by the images taken with the current digital devices.

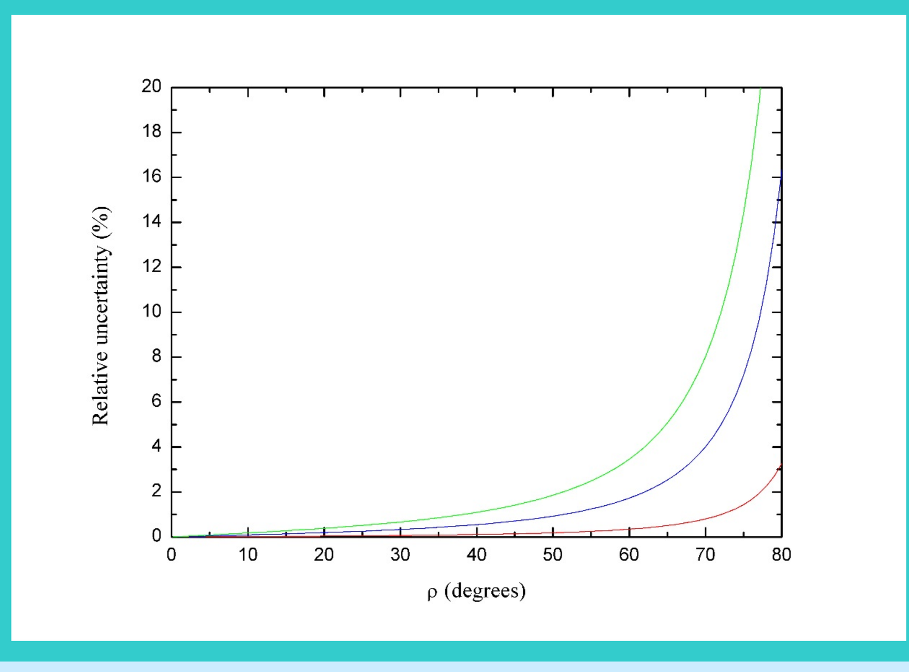

Figure 1. Relative uncertainty of the sunspot area as function of the angle ρ for an error in *x* of 0.1 % (red line), 0.5 % (blue line) and 1 % (green line). The percentages are referred to the radius of the solar disk.

Relative uncertainties increase dramatically from ρ = 60º even with modest *x* errors. Area sunspot measurements with ρ > 60º should be carefully analysed.

UNCERTAINTY DUE TO THE BINARIZATION PROCESS

The determination of the sunspot area (uncorrected by foreshortening) in historical drawings or modern images is performed by binarizing them. In this way, an 8-bits (usually) gray levels image, where each pixel has an intensity between 0 and 255 is transformed to a black and white image (black = 0 and white = 1). The parameter controlling this process is the so-called binarization threshold, *t*, that defines an intensity cut-off value, in the form

$$c = minval + t(maxval - minval)$$

where *minval* and *maxval* are, respectively, the minimum and maximum gray levels in the image. If the intensity of a pixel is greater than *c*, is considered as white; otherwise, is taken as black. The sunspot area is then calculated by adding all black pixels. Obviously, this means that the binarization threshold plays a crucial role in the determination of the sunspot area (note that a high value of *t* implies a darker binarized image). As the optimal binarization threshold cannot be evaluated *a priori* (its choice is on the hands of the program user) we have carried out a procedure to estimate the best possible values, from a simulated sunspot. This procedure has the following steps:

1) **Choosing a real sunspot in a digital image (figure 2), in order to simulate a realistic sunspot, and measuring of the intensity distribution along the sunspot (in an 8-bits image)**
2) **Building a circular spot (our simulated sunspot) by fitting a Cauchy function to the sunspot intensity data (figure 3)**
3) **Introducing gaussian noise in order to simulate a real sunspot, with fluctuations in the intensity**
4) **Binarizing the noisy image**
5) **Comparing the number of black pixels in the binarized image with those of the original simulated image, using a convenient reference value to define the sunspot limits (figure 4)**

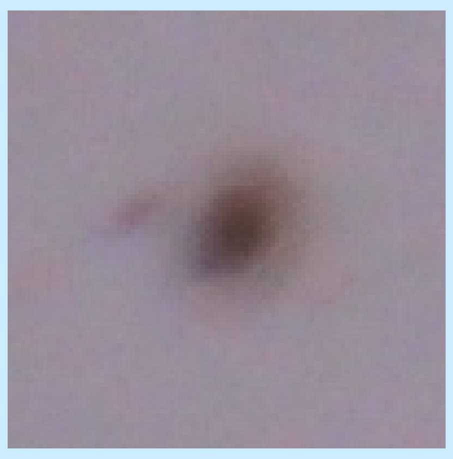

Figure 2. Sunspot image (1-7-2012, 8h 30m U.T., Maksutov 10 cm + Canon EOS 600D, Exposure = 1/800 s)

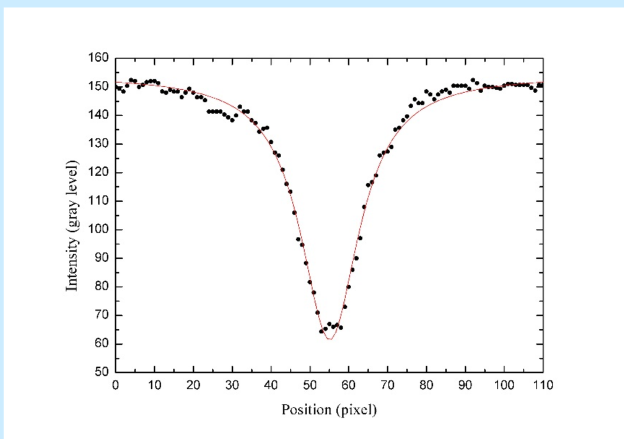

Figure 3. Fit of a Cauchy function to the intensity distribution –along an horizontal line crossing its center- of the sunspot of figure 2 (R^2 = 0.99)

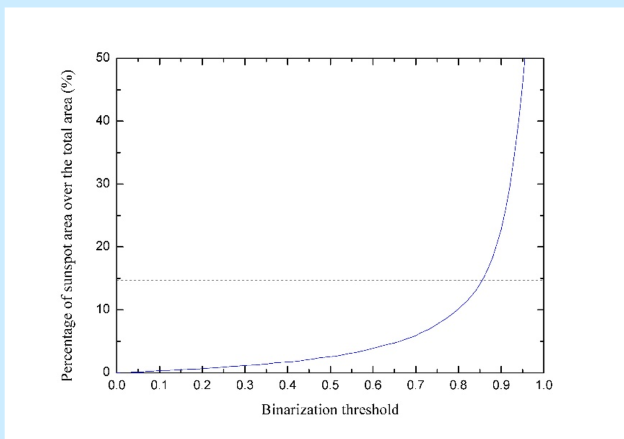

Figure 4. Percentage of the sunspot area with respect the total area as function of the binarization threshold (using the gray level of 140 as the intensity cut-off value). Horizontal dashed line represents the percentage of the original modelled sunspot

This preliminar example leads to an optimal value of *t* about 0.85, with noteworthy errors around 0.8 or 0.9. Note that high values are useful to measure total sunspot areas (Galaviz et al., 2020).

CONCLUSIONS

Several factors that influence the measures of the sunspot areas in drawings or digital images have been analyzed, being the accuracy of the position and the selected binarization threshold the most important. The effect of the use of the plane-parallel approximation has been analyzed for sunspots of circular and elliptical shapes, showing that this approximation is very accurate excepting for big sunspots

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