Stability of a Coronal Loop Harboring a Standing Slow Wave

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Motivation

Fast kink waves have been shown to trigger Kelvin-Helmholtz instability (KHI) on the boundary of a cylindrical structure (e.g. solar coronal loop): see [1] and [2].

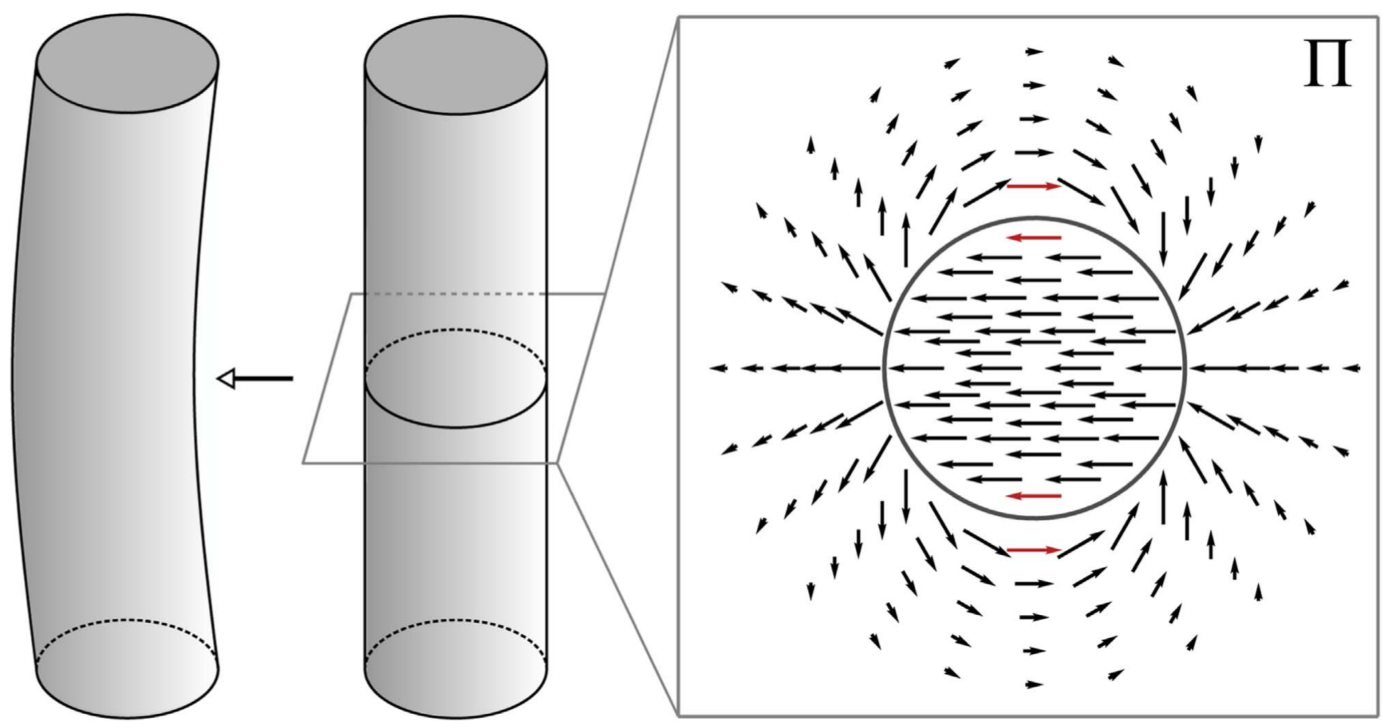
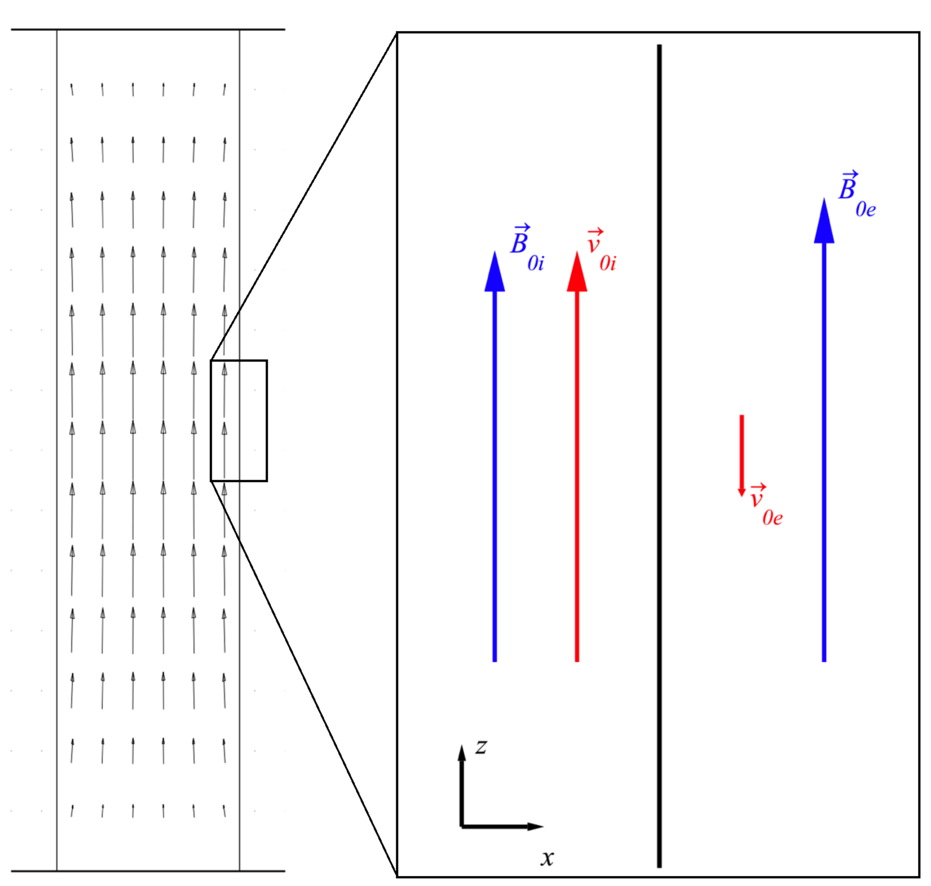


Figure taken from [1].

- Oscillating velocity field creates shear, which is greatest at "top" and "bottom" ends in transverse cut
- Numerical simulations have shown transverse wave-induced KHI to develop
- Analytical Cartesian models of local stability of interface point out it is always unstable, either to KHI if shear is great enough or to parametric resonance instability between background driver and produced perturbations

Goal: What about (standing) slow waves? Background magnetic field and velocity are then parallel. Can they similarly induce KHI or parametric instability? Or is magnetic field too strong in a realistic environment?

Model



Local model for the boundary of a flux tube harboring a standing slow wave, as a straight interface in Cartesian coordinates separating two different homogeneous plasmas with oscillating background velocity fields $\boldsymbol{v}_0 = V_0 \cos(\omega_0 t)\mathbf{1}_z$ and aligned constant background magnetic fields $\boldsymbol{B}_0 = B_{0z}\mathbf{1}_z$.

Analytical derivations

- Started from compressible linearized ideal MHD equations
- Derived equation for compression term $\nabla \cdot \boldsymbol{\xi}$ (with $\boldsymbol{\xi}$ Lagrangian displacement) in a homogeneous plasma of infinite extent with an oscillating $\boldsymbol{v}_0$ aligned with $\boldsymbol{B}_0$
- Ratio $\overline{m} := m_i/m_e$ between internal and external "normal wavenumbers" depends on k_y and k_z, contrary to incompressible plasma where $m_i/m_e \to 1$
- Derived equation governing evolution of normal displacement component ξ_r over time: Mathieu equation with normalized parameters

$$a = \kappa_z^2 \left[\frac{\overline{m} + r\overline{v}_A^2}{\overline{m} + r} - \frac{r\overline{m}M_A^2}{2\,(\overline{m}+r)^2} \right]$$

$$q = \kappa_z^2 \frac{r\overline{m}M_A^2}{4\,(\overline{m}+r)^2}$$

where $\kappa_z = k_z v_{Ai}/\omega_0$ is longitudinal wavenumber, $\overline{v}_A = v_{Ae}/v_{Ai}$ is ratio of Alfvén speeds, $r = \rho_{0e}/\rho_{0i}$ is ratio of densities and $M_A = (V_{0i} - V_{0e})/v_{Ai}$ is Alfvén Mach number.

- Limit of no background flow ($\boldsymbol{v}_0 = \mathbf{0}$): recover known dispersion relation of surface modes (with frequency ν):

$$\nu^2 = \kappa_z^2 \frac{\overline{m} + r\overline{v}_A^2}{\overline{m} + r}$$

- Limit of constant background flow: dispersion relation

$$\nu^2 = \kappa_z^2 \left[\frac{\overline{m} + r\overline{v}_A^2}{\overline{m} + r} - \frac{r\overline{m}M_A^2}{(\overline{m}+r)^2} \right]$$

$\hookrightarrow$ can be unstable to KHI, if $\nu^2 < 0$, i.e. if $M_A^2 > \dfrac{(\overline{m}+r)\left(\overline{m}+r\overline{v}_A^2\right)}{r\overline{m}}$

- Limit of weak shear in oscillating flow (corresponds to $q \ll 1$): dispersion relation

$$\nu^2 \approx \kappa_z^2 \left[\frac{\overline{m} + r\overline{v}_A^2}{\overline{m} + r} - \frac{r\overline{m}M_A^2}{2\,(\overline{m}+r)^2} \right]$$

$\hookrightarrow$ can be unstable to KHI, if $\nu^2 < 0$, i.e. if $M_A^2 > 2\dfrac{(\overline{m}+r)\left(\overline{m}+r\overline{v}_A^2\right)}{r\overline{m}}$, i.e. if $a < 0$

$\hookrightarrow$ can also be unstable to parametric instability, when there is a resonance with the driving frequency: this happens when $a = j^2$ for a $j \in \mathbb{Z}_0$

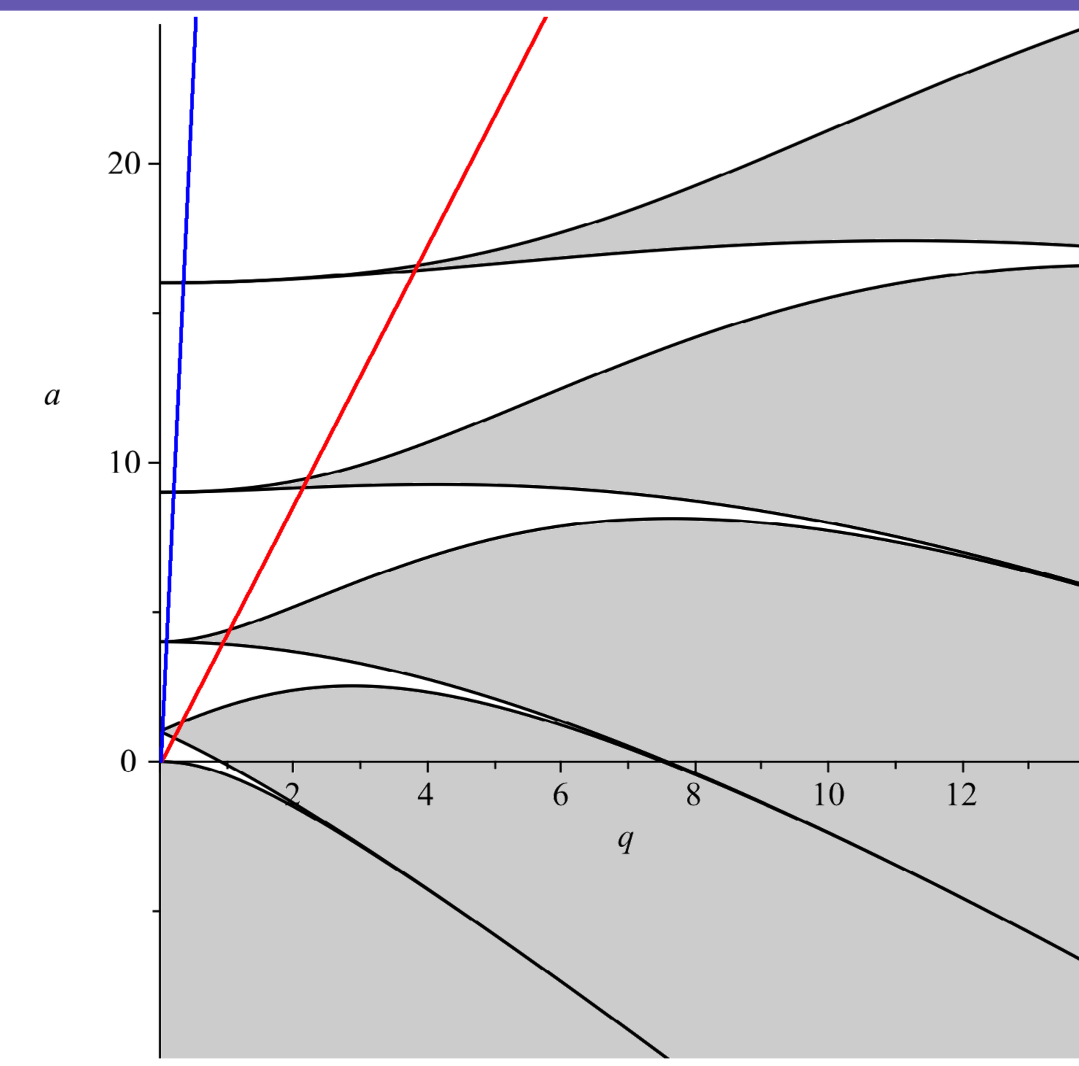
Stability diagram of Mathieu equation (see figure on the right) shows stability of its solution with respect to the parameters a and q: white is stable, grey is unstable

$\Rightarrow$ From expressions of a and q above, one can derive that all modes have to lie between positive a-axis and "minimum slope"-line with equation

$$a = \left\{ \frac{4\left(1+\overline{v}_A\right)^2}{M_A^2} - 2 \right\} q.$$

conditions	M_A	minimum slope
coronal	0.066	11415
photospheric	0.066	1450
photospheric	0.36	46
coronal	1	47
photospheric	1	4.25

Results



To the left is a table summarizing the value of the slope of the "minimum slope"-line given by previous equation, for different conditions: standing slow wave in a coronal loop ($M_A = 0.066$), standing slow wave in a photospheric pore ($M_A = 0.066$), and resonant slow waves in photospheric pore ($M_A = 0.36$). The blue line in the diagram above has slope 46. As comparison, values for the minimum slope for the unrealistically high value of $M_A = 1$ have been added in both coronal and photospheric conditions. The red line in the diagram has slope 4.25.

Conclusion We see the modes lie in a region of almost uniform stability, at least with respect to KHI. From this local model, we conclude cylindrical flux tubes (e.g. coronal loop, photospheric pore) are stable with respect to KHI in both coronal and photospheric conditions, even for resonant slow waves: the alignment of magnetic field with velocity prevents KHI from starting, and the oscillation doesn't help destabilizing the setup with respect to KHI. Only the parametric instability seems possible, but the question is whether these happen in reality: indeed, modes are discrete in the diagram and might therefore avoid the tiny parametrically unstable regions.

References & acknowledgements

[1] Mihai Barbulescu et al. "An Analytical Model of the Kelvin-Helmholtz Instability of Transverse Coronal Loop Oscillations". In: 870.2, 108 (Jan. 2019), p. 108. DOI: 10.3847/1538-4357/aaf506. arXiv: 1901.06132 [astro-ph.SR].

[2] Andrew Hillier et al. "On Kelvin-Helmholtz and parametric instabilities driven by coronal waves". In: 482.1 (Jan. 2019), pp. 1143–1153. DOI: 10.1093/mnras/sty2742. arXiv: 1810.02773 [astro-ph.SR].

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