Relative field line helicity of active region 11158

K. Moraitis, S. Patsourakos, A. Nindos
Physics Department, University of Ioannina, Greece
k.moraitis@uoi.gr

Introduction

- **Magnetic helicity** is a geometrical measure of the twist and writhe of the magnetic field lines, and of the amount of flux linkages between pairs of lines (Gauss linking number)
- Mathematically, it is defined as

$$H=\int_V \boldsymbol{A}\cdot\boldsymbol{B}\,dV \qquad \boldsymbol{B}=\nabla\times\boldsymbol{A}$$

- Signed scalar quantity (right (+), or left (-) handed)

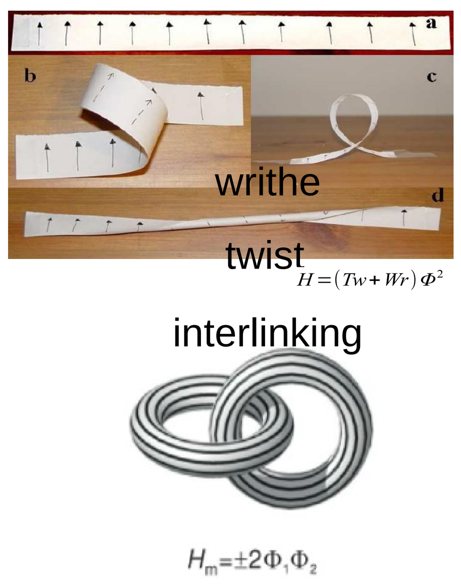


Fig. 1: The geometrical quantities that magnetic helicity quantifies.

- Units of magnetic flux squared (SI: Wb^2, cgs: Mx^2)
- Conserved in ideal MHD (Woltjer 1958), along with energy and cross helicity

$$\frac{dH_m}{dt} = \int_{\partial V}\left(\mathbf{A}\times\frac{\partial\mathbf{A}}{\partial t}\right)\cdot d\mathbf{S} - 2\int_{\partial V}(\mathbf{E}\times\mathbf{A})\cdot d\mathbf{S} - 2\int_V \mathbf{E}\cdot\mathbf{B}\;dV$$

- Topological invariant; links cannot change by 'frozen' magnetic field lines
- Even in resistive MHD (reconnection), helicity is approximately conserved (Taylor 1975; Pariat et al. 2015)
- Coronal mass ejections are caused by the need to expel the excess helicity accumulated in the corona (Rust 1994)

- Magnetic helicity depends on the gauge of the vector potential unless the volume under study is magnetically closed, and therefore, it is of limited use in Astrophysics

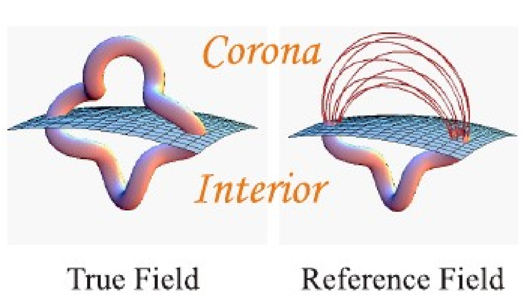


Fig. 2: Magnetic fields involved in the definition of relative magnetic helicity.

- **Relative magnetic helicity** is the appropriate form of magnetic helicity in astrophysical conditions
- It expresses the helicity of the true field with respect to a reference field. It is given by

$$H_r=\int_V(\mathbf{A}+\mathbf{A}_\mathrm{p})\cdot(\mathbf{B}-\mathbf{B}_\mathrm{p})\,dV$$

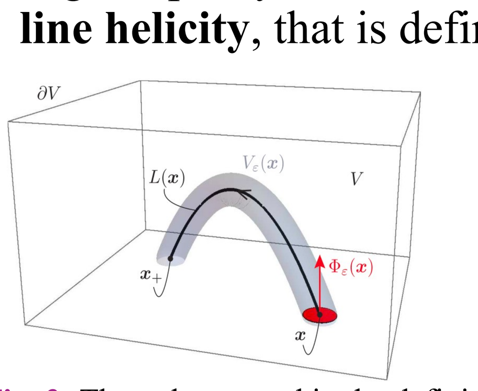
- Relative magnetic helicity is independent of the gauges chosen for the vector potentials (aka physically meaningful), as long as

$$\hat{n}\cdot\boldsymbol{B}\big|_{\partial V}=\hat{n}\cdot\boldsymbol{B}_p\big|_{\partial V} \qquad \partial V\text{: the whole boundary}$$

- When the reference field is a potential field, then no current→no helicity
- Relative magnetic helicity is a single number that characterizes the whole volume

Field line helicity

- Magnetic helicity provides no spatial information about the locations where helicity is more important
- A density for magnetic helicity cannot be defined since the vector potential is a non-local quantity
- A good proxy for the density of magnetic helicity is **field line helicity**, that is defined by



$$\mathcal{A}(C) = \lim_{\epsilon\to0}\left(\frac{1}{\Phi_\epsilon}\int_{V_\epsilon}\mathbf{A}\cdot\mathbf{B}\,\mathrm{d}V\right) = \lim_{\epsilon\to0}\left(\frac{1}{\Phi_\epsilon}\int_{V_\epsilon}(\mathbf{A}\cdot\mathrm{d}\boldsymbol{l})\,(\mathbf{B}\cdot\mathrm{d}\boldsymbol{S})\right) = \int_C\mathbf{A}\cdot\mathrm{d}\boldsymbol{l}$$

Fig. 3: The volume used in the definition of field line helicity (figure from Yeates & Page 2018).

✔ Magnetic helicity then reduces to a surface integral along the boundary

$$H=\int_{\partial V}\mathcal{A}\,d\Phi$$

✗ Field line helicity is gauge-dependent for open field lines
For closed field lines, it is the magnetic flux through the surface bounded by the field line (SI/cgs unit: Wb/Mx)

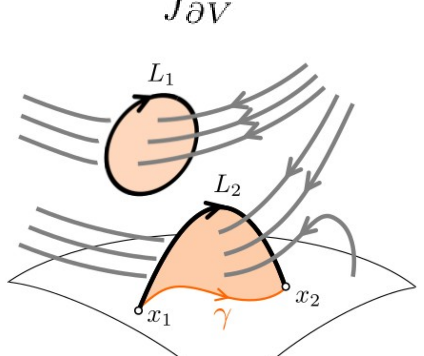


Fig. 4: Physical meaning of field line helicity for open and closed field lines (figure from Yeates & Hornig 2016).

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No 893489.

A **relative field line helicity** can be similarly defined for relative magnetic helicity, as

$$H_r=\int_{\partial V}\mathcal{A}_r\,\mathrm{d}\Phi$$

Generic, unconstrained gauge

$$\mathcal{A}_r^+=\int_{\alpha_+}^{\alpha_-}(\mathbf{A}+\mathbf{A}_\mathrm{p})\cdot\mathrm{d}\boldsymbol{l}-\int_{\alpha_+}^{\alpha_{p-}}(\mathbf{A}+\mathbf{A}_\mathrm{p})\cdot\mathrm{d}\boldsymbol{l}_\mathrm{p}$$

$$\mathcal{A}_r^-=\int_{\alpha_+}^{\alpha_-}(\mathbf{A}+\mathbf{A}_\mathrm{p})\cdot\mathrm{d}\boldsymbol{l}-\int_{\alpha_{p+}}^{\alpha_-}(\mathbf{A}+\mathbf{A}_\mathrm{p})\cdot\mathrm{d}\boldsymbol{l}_\mathrm{p}$$

$$\mathcal{A}_r^0=\frac{1}{2}\left(\mathcal{A}_r^++\mathcal{A}_r^-\right)$$

Moraitis et al. 2019

Berger & Field gauge

$$\mathcal{A}_r^{\mathrm{YP},+}=\int_{\alpha_+}^{\alpha_-}\mathbf{A}\cdot\mathrm{d}\boldsymbol{l}-\int_{\alpha_+}^{\alpha_{p-}}\mathbf{A}_\mathrm{p}\cdot\mathrm{d}\boldsymbol{l}_\mathrm{p}$$

Yeates & Page 2018

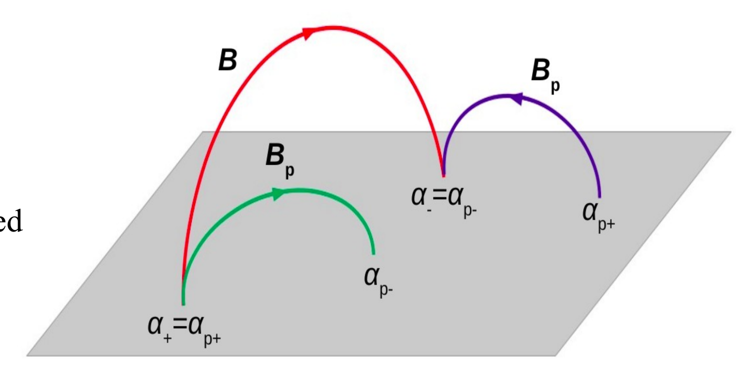


Fig. 5: The field lines involved in the definition of relative field line helicity.

Purpose of this work: To examine the behaviour of relative field line helicity in a solar active region for the first time – so far, only MHD/semi-analytic fields have been considered.
Target active region: the well-studied AR 11158

Coronal magnetic field modelling

To compute helicity and RFLH we need to know the coronal magnetic field of the AR. We reconstruct it from the SDO/HMI observed photospheric magnetic field with a **NLFF extrapolation** (Thalmann et al. 2019), with the characteristics:
- 215 Mm x 130 Mm x 185 Mm
- 148 x 92 x 128 grid points
- resolution 2'' per pixel
- 115 snapshots in total during the interval 12-16 Feb 2011
- 1 hr cadence + 12 min around the M6.6 and the X2.2 flares

The NLFF field is of high-quality with respect to its solenoidality, as the low values of the following metrics indicate:
- f_i=2.2 x 10^{-4} (Wheatland et al. 2000)
- E_{div}/E=0.006 (Valori et al. 2013)

which is essential for reliable helicity values (Valori et al. 2016)

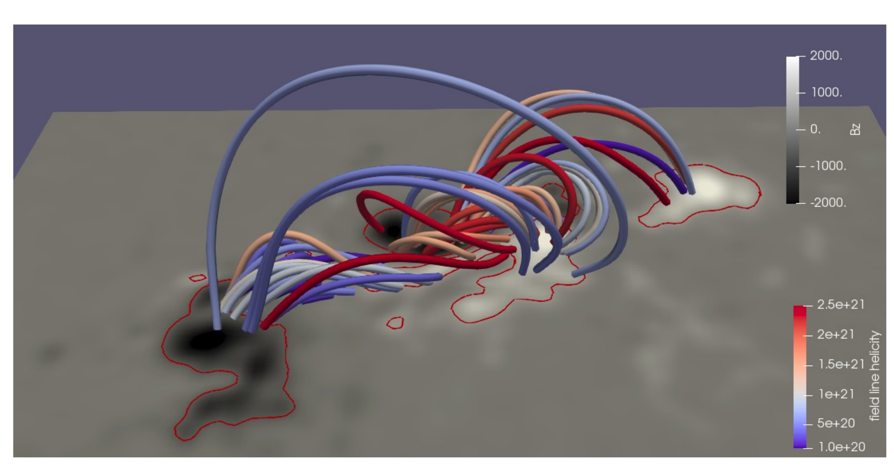


Fig. 6: Morphology of the 3D model coronal magnetic field used for AR 11158.

From the 3D coronal field we compute RFLH and show its morphology in Fig. 7. We note that RFLH is different than B_z or j_z, and that it provides new information. Fig. 8 depicts two successful tests for RFLH; a) it reproduces the correct relative helicity to <5%, and b) its morphology is insensitive to the gauge used in its computation.

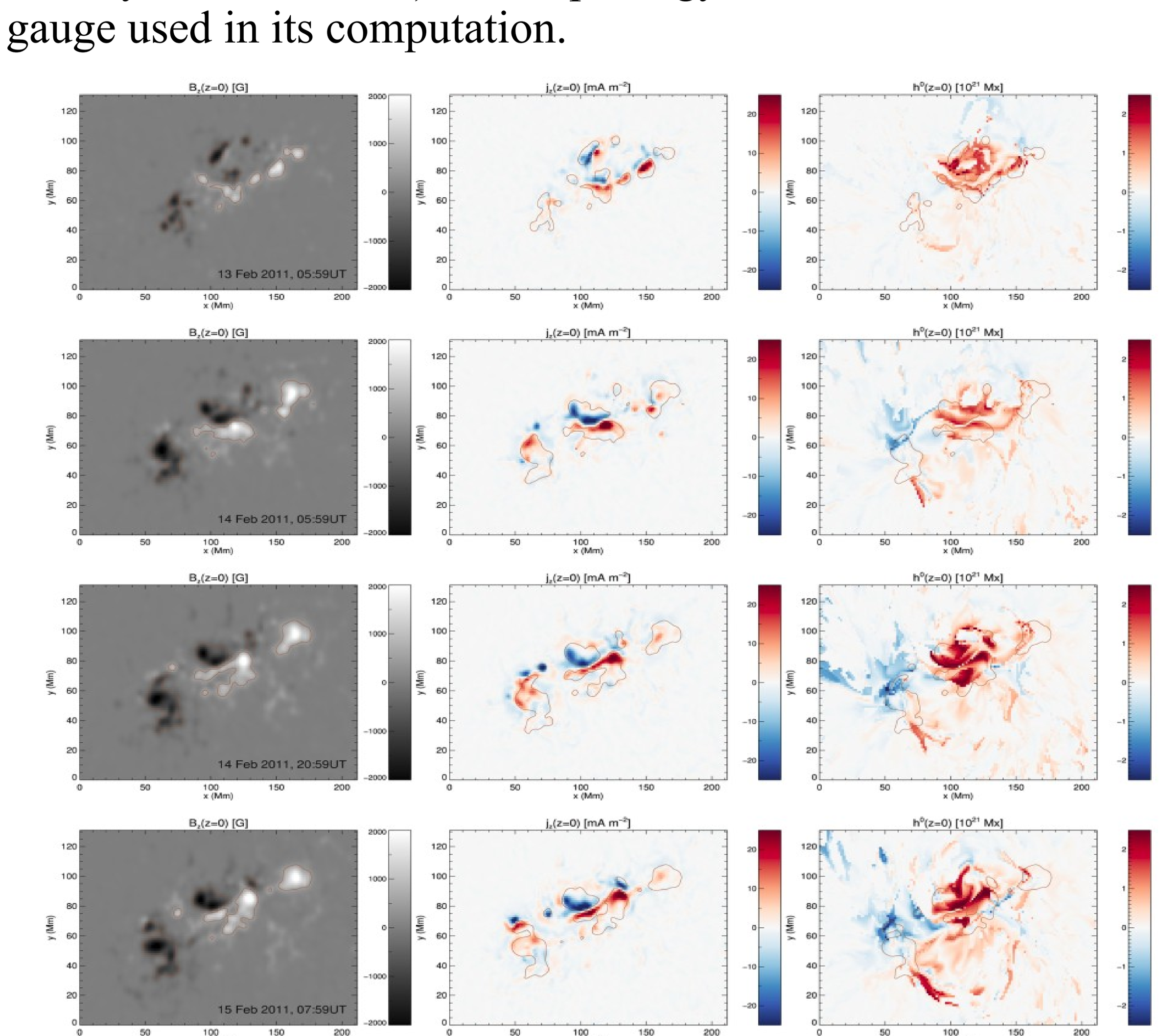


Fig. 7: Morphology of the normal component of the magnetic field (left), normal component of the electrical current (middle), and of relative field line helicity (right), on the photospheric plane.

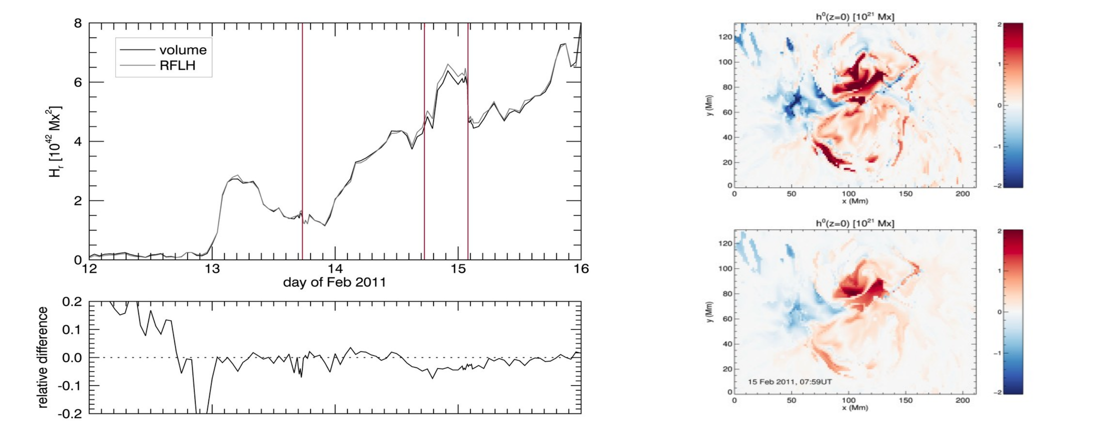


Fig. 8: (left) Relative helicity as computed by the volume, and the RFLH methods. The agreement is to within 5%. (right) The morphologies of RFLH as computed by the unconstrained gauge (top) and the Berger & Field gauge (bottom) agree qualitatively.

Flare-related changes during the X2.2 flare

We focus on the X2.2 flare of 15 Feb 2011, peak time 01:56 UT. We identify two ROIs based on the SDO/AIA 1600Å images (top Fig. 9). The RFLH difference images (bottom Fig. 9) reveal a large helicity decrease during the flare, spatially coincident with the flux rope of the AR.

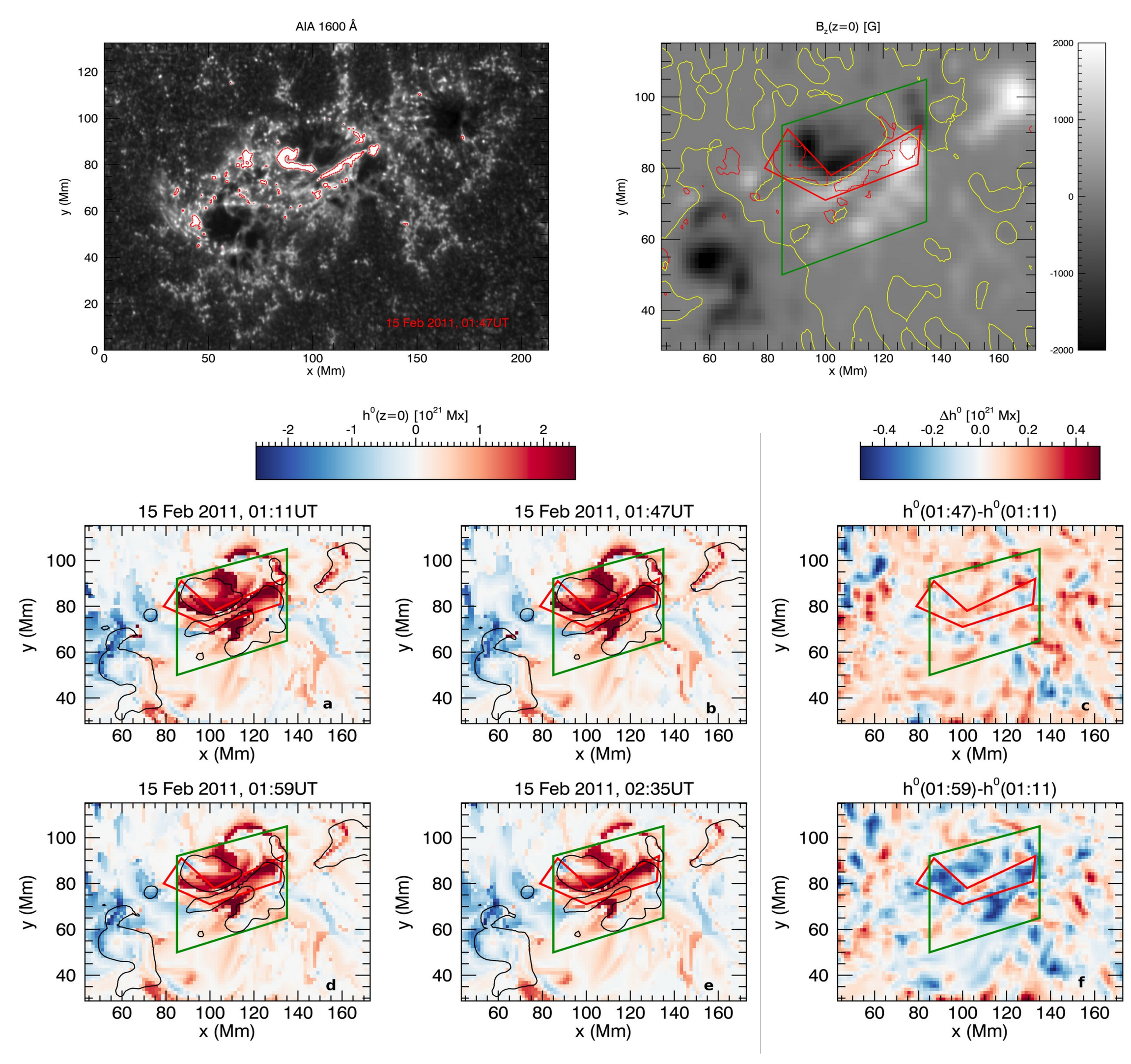


Fig. 9: (top) AIA image at 1600Å at the start of the X2.2 flare (left), and zoom of the B_z map overplotted with the AIA intensity contours (right).
(bottom) Zoom of the RFLH morphology around the X2.2 flare (panels a, b, d, e), and RFLH difference images before (panel c), and after the flare (panel f).

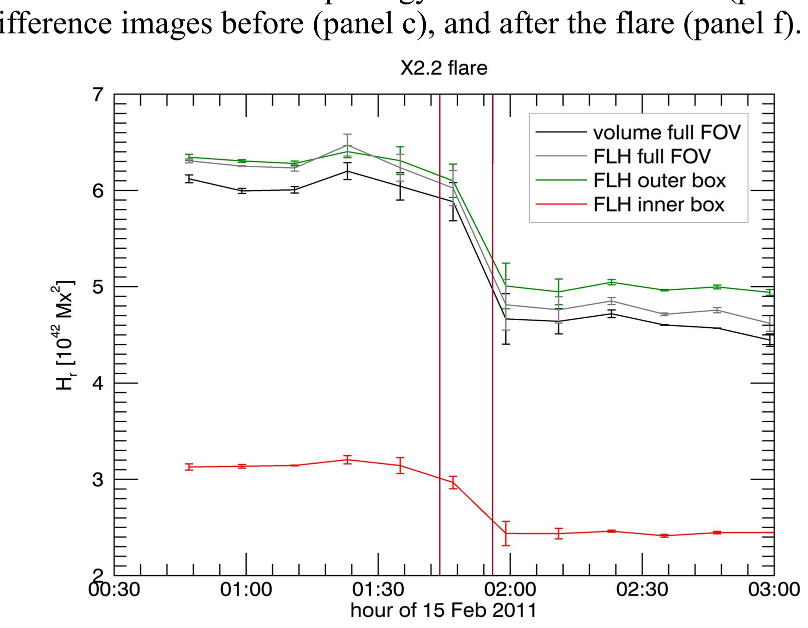


Fig. 10: Evolution of relative helicity of the various ROIs shown in Fig. 9.

From the evolution of the various ROIs relative helicities in Fig. 10 we note:
- Volume + RFLH methods agree to <5%
- Green box contains almost the same amount of helicity as whole FOV, more before the flare
- All curves drop by 20-25% (beyond errors) during flare, by ~1.5x10^{42} Mx^2
- Red box contains half helicity, and drops by 7x10^{41} Mx^2
- Unfortunately, no relation with the detected ICME possible, with the reported 2x10^{41} Mx^2

Conclusions

- Relative field line helicity is a good proxy for the density of relative helicity
- First application of RFLH in a solar active region – Moraitis, Patsourakos & Nindos 2021, Astronomy & Astrophysics, 649, A107
- RFLH has important potential in highlighting locations of intense helicity
- Main disadvantage of RFLH is its gauge dependence
- With RFLH we can compute the helicity, or the helicity difference between two instances, in an arbitrarily-shaped photospheric ROI