Standing MHD waves in Magnetic Annulus Flux Tube Model

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- Magnetically confined configurations such as coronal loops and dynamic flux tubes (e.g. from spicules to prominences) are important aspects in the study of **solar magneto-seismology (SMS)** and they have been extensively studied through various theoretical models for more than a solar cycle.

- Among several models, the **cylindrically symmetric flux tube structure** having a **co-axial magnetic core and an outer shell** has been suggested and discussed in several works.

- Rudermann and Roberts (2002) also suggested the implementation of a thin inhomogeneous layer in flux tubes so as to explain the damping of observed coronal loop oscillations.

- Several analytical studies over the years have explored the different wave modes in such **annular flux tubes** and we have build upon some of these works to **study the standing MHD modes in magnetic annulus structuring**.

Theoretical Model: Basic Configuration

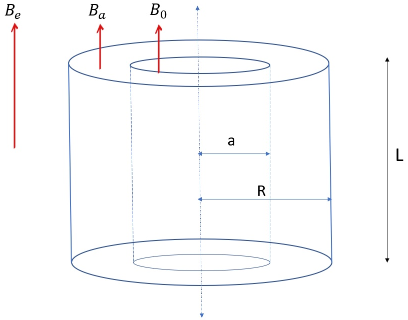
The **basic plasma configuration** in cylindrical coordinates

$$\rho_0(r) = \begin{cases} \rho_e & r > R \\ \rho_a & a < r < R \\ \rho_0 & r < a \end{cases}$$

$$\mathbf{B}_0(r) = \begin{cases} B_e\,\hat{\mathbf{e}}_z & r > R \\ B_a\,\hat{\mathbf{e}}_z & a < r < R \\ B_0\,\hat{\mathbf{e}}_z & r < a \end{cases}$$

$$p_0(r) = \begin{cases} p_e & r > R \\ p_a & a < r < R \\ p_0 & r < a \end{cases}$$

$$\mathbf{V} = 0$$



Theoretical Model: Dispersion Relation

Solving the linearized equations with the Fourier decomposition we obtain the following **dispersion relation for standing MHD waves**,

$$\boxed{\frac{Q_a^0 K_m^1(\frac{n\pi a}{L}) - \left(\frac{I_m^1(\frac{n\pi a}{L}) K_m(\frac{n\pi a}{L})}{I_m(\frac{n\pi a}{L})}\right)}{I_m^1(\frac{n\pi a}{L})(Q_a^0 - 1)} = \frac{K_m^1(\frac{n\pi R}{L})(Q_a^e - 1)}{Q_a^e I_m^1(\frac{n\pi R}{L}) - \left(\frac{(K_m^1(\frac{n\pi R}{L}) I_m(\frac{n\pi R}{L})}{K_m(\frac{n\pi R}{L})}\right)}} \quad (1)$$

$$Q_a^0 = \frac{\rho_0}{\rho_a}\frac{(\omega^2 - \omega_{A0}^2)}{(\omega^2 - \omega_{Aa}^2)} \quad , \quad Q_a^e = \frac{\rho_e}{\rho_a}\frac{(\omega^2 - \omega_{Ae}^2)}{(\omega^2 - \omega_{Aa}^2)}$$

We further define two parameters as

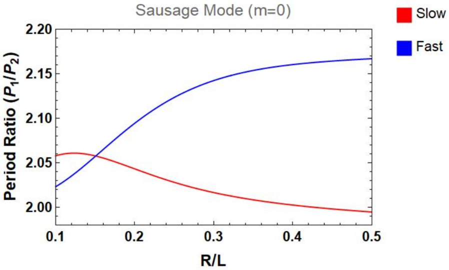
$$\epsilon = \frac{R - a}{R}, \quad (2)$$

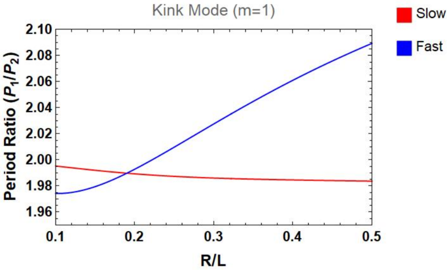
$$v_{Aa}^2 = v_{A0}^2(1 + \tau). \quad (3)$$

Under the **Weakly Homogeneous Flux Tube (WHFT) and Thin Annulus (TA)** approximations we simplify above Dispersion Relation by having $\epsilon << 1$ **and** $\tau << 1$.

Weakly Homogeneous Flux Tube and Thin Annulus Approximations

The panels are plotted for $B_0 = B_a = B_e$, $\rho_0 = 4\rho_e$ and $\tau = 0.3$, $\epsilon = 0.3$





Thin Tube Approximation (TTA)

Considering that $ka \ll 1$ **and** $kR \ll 1$, we can now approximate and simplify the general dispersion relation as below,

Sausage Modes (m=0):

$$\boxed{Q_a^0 Q_a^e \left(\frac{(\frac{n\pi R}{L})^2 K_0(\frac{n\pi R}{L})}{2}\right) + Q_a^0 + \frac{(\frac{n\pi a}{L})^2 K_0(\frac{n\pi a}{L})}{2} = \frac{(\frac{n\pi a}{L})^2 K_0(\frac{n\pi R}{L})}{2}(Q_a^0 - 1)(Q_a^e - 1)} \tag{4}$$

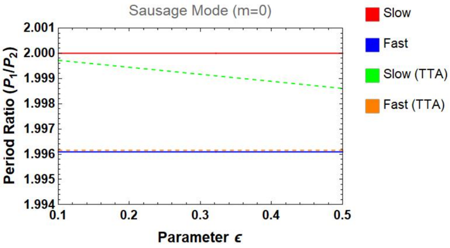
Kink Modes (m=1):

$$\boxed{\frac{(Q_a^0 - 1)(Q_a^e - 1)}{(Q_a^0 + 1)(Q_a^e + 1)} = \frac{R^2}{a^2}} \tag{5}$$

We further apply the Weakly Homogeneous Flux Tube (WHFT) and Thin Annulus (TA) Approximations on the above dispersion relations to find their analytical solutions.

Thin Tube Approximation (TTA) - Sausage Modes

$\rho_0 = \rho_a = \rho_e$ and $v_{A0} = 10 v_{Ae}$



$B_0 = B_a = B_e$ and $\rho_0 = 4\rho_e$

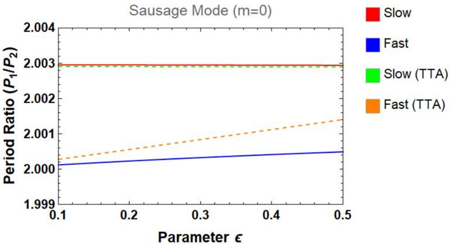


Figure: In the left panel $\tau = -0.3$, $R/L = 0.01$ while in the right panel $\tau = 0.3$, $R/L = 0.01$.

Wide Tube Approximation

Considering that $ka \gg 1$ **and** $kR \gg 1$, we can approximate the general dispersion relation as,

$$\boxed{e^{\frac{2n\pi\epsilon R}{L}} = \frac{(Q_a^e - 1)(Q_a^0 - 1)}{(Q_a^e + 1)(Q_a^0 + 1)} \times \left(1 + \left(\frac{LQ_a^e}{(Q_a^e + 1)n\pi R}\right) - \left(\frac{LQ_a^0}{(Q_a^0 + 1)n\pi a}\right)\right)} \tag{6}$$

Further considering the Thin Annulus (TA) and Weakly Homogeneous Flux Tube (WHFT) approximations and ignoring terms of higher order we get an 8th order polynomial in ω after some cumbersome algebra,

$$C_1\omega^8 + C_2\omega^6 + C_3\omega^4 + C_4\omega^2 + C_5 = 0. \tag{7}$$

The coefficients C_1 to C_5 are **algebraically solved** using Wolfram Mathematica (2016).

$\rho_0 = \rho_a = \rho_e$ and $v_{A0} = 10v_{Ae}$

$B_0 = B_a = B_e$ and $\rho_0 = 4\rho_e$

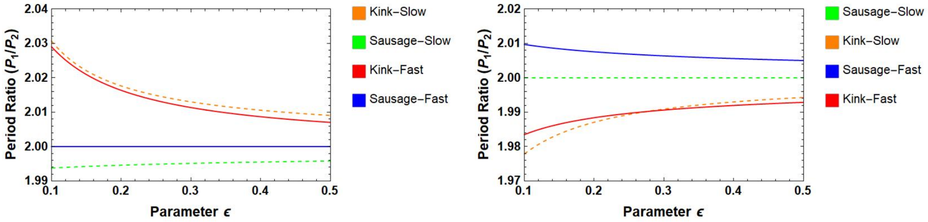


Figure: In the left panel $\tau = -0.3$, $R/L = 10$ while in the right panel $\tau = 0.3$, $R/L = 10$.

Discussion and Conclusion

- We have developed a **model for the standing MHD waves in annular flux tubes** using line tying boundary conditions and further considered the **thin annulus** and **weakly homogeneous conditions** to find approximate solutions.

- The presented model (with ϵ **and** τ **parameters**) provides a good mathematical structure for analysing the **deviation in period ratio from the case of monolithic cylindrical tube** due to annular structuring and inhomogeneity.

- We plan to further develop this study and analyze the period ratio of fundamental and first harmonic modes for different coronal and photospheric conditions so as to understand the role of annular structuring in flux tubes and how it can lead to better explanation for the observed results.

Thank You