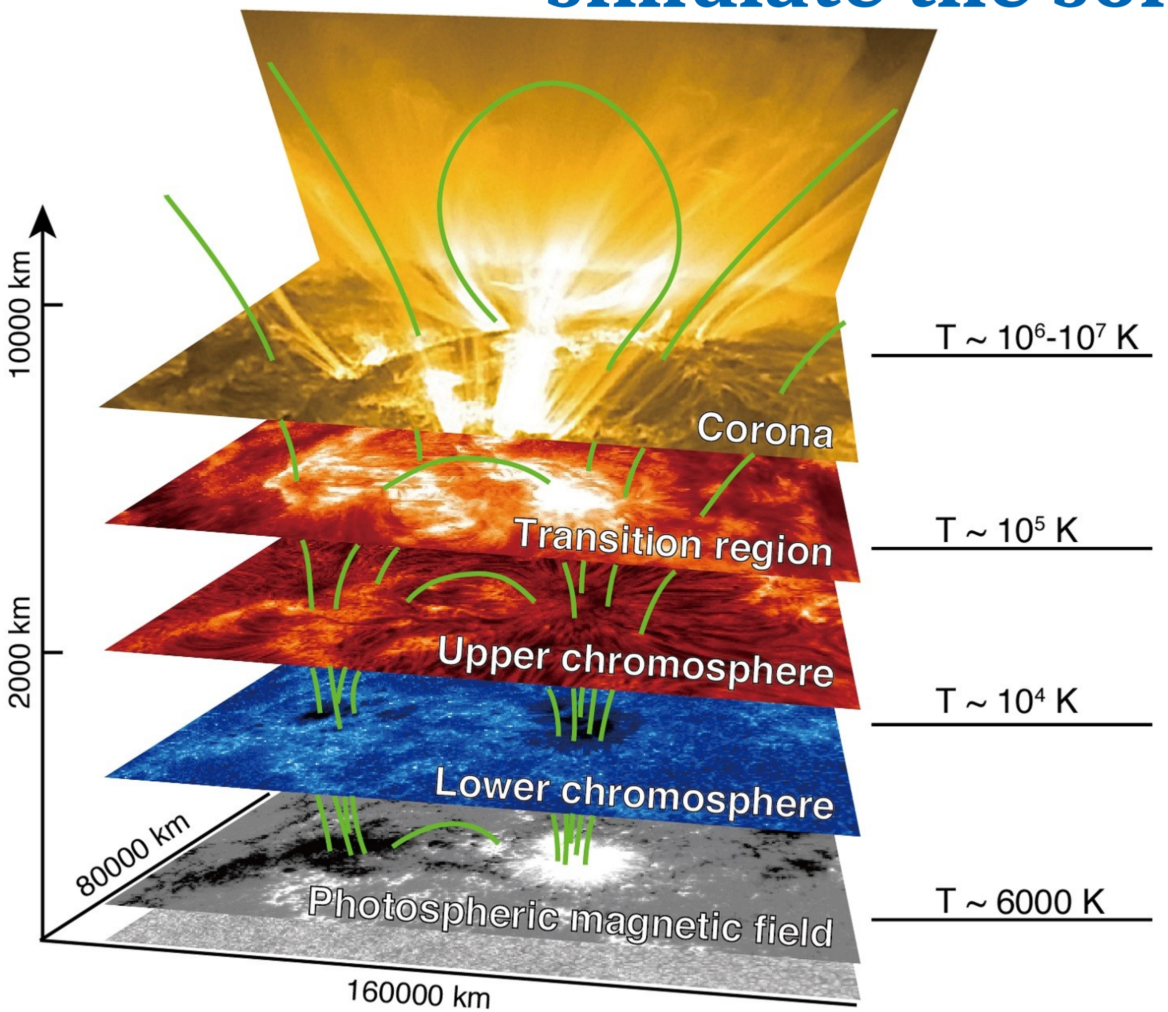
Modeling the thermal conductivity in the solar atmosphere with the code MANCHA

A. Navarro[1], E. Khomenko, M. Modestov,
N. Vitas & P. Hunana

Instituto de Astrofísica de Canarias, 38205 La Laguna, Tenerife, Spain
Dpto. de Astrofísica, Universidad de La Laguna, 38205 La Laguna, Tenerife, Spain
[1]anavarro@iac.es

Extending the code MANCHA to simulate the solar CORONA



Thermal conduction in the corona is an important transport mechanism in more realistic simulations.

In the corona, the heat flux is aligned with the magnetic field lines and imposes restrictions in the computational time.

$$\mathbf{q} = -\kappa \boldsymbol{\nabla} T$$

$$\kappa_{\perp}/\kappa_{\parallel} = 2 \times 10^{-31} n^2 / \left(T^3 B^2\right)$$

Related posters:

Public version of MANCHA: [...] (Mihail Modestov; session 5.4)

The opacity pipeline: from atomic data to realistic RMHD simulations (Andrea Perdomo session 8.1)

Large amplitude longitudinal oscillations [...] (Valeria Liakh; session 5.3)

From highly-collisional to collisionless fluid models [...] Peter Hunana session 5.4

MHD equations + heat conduction

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0\,, \tag{1}$$

$$\frac{\partial \rho \mathbf{V}}{\partial t} + \nabla \cdot \left[\rho \mathbf{V}\mathbf{V} + \left(P + \frac{\mathbf{B}^2}{2\mu_0}\right)\mathcal{I} - \frac{\mathbf{B}\mathbf{B}}{\mu_0}\right] = \rho \mathbf{g} \tag{2}$$

$$\frac{\partial E}{\partial t} + \nabla \cdot \left[\left(E + P + \frac{\mathbf{B^2}}{2\mu_0}\mathbf{V}\right) - \frac{\mathbf{B}}{\mu_0}(\mathbf{B}\cdot\mathbf{V})\right] = \rho \mathbf{V}\cdot\mathbf{g} - \nabla\cdot\mathbf{q} \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B})\;, \tag{4}$$

Parabolic term that restricts the time step in an explicit evolution.

Model of Spitzer [1]

$$\mathbf{q} = -\kappa T^{5/2}\,(\mathbf{B}\cdot\nabla T)\,\mathbf{B}/B^2$$

[1]Spitzer L., 1956, Physics of Fully Ionized Gases

Hyperbolic heat conduction[1]

An alternative method: hyperbolic treatment of heat conduction

$$\frac{\partial q}{\partial t} = \frac{1}{\tau}\left(-f_{\rm sat}\kappa T^{5/2}\left(\hat{\mathbf{b}}\cdot\nabla\right)T - q\right) \tag{1}$$

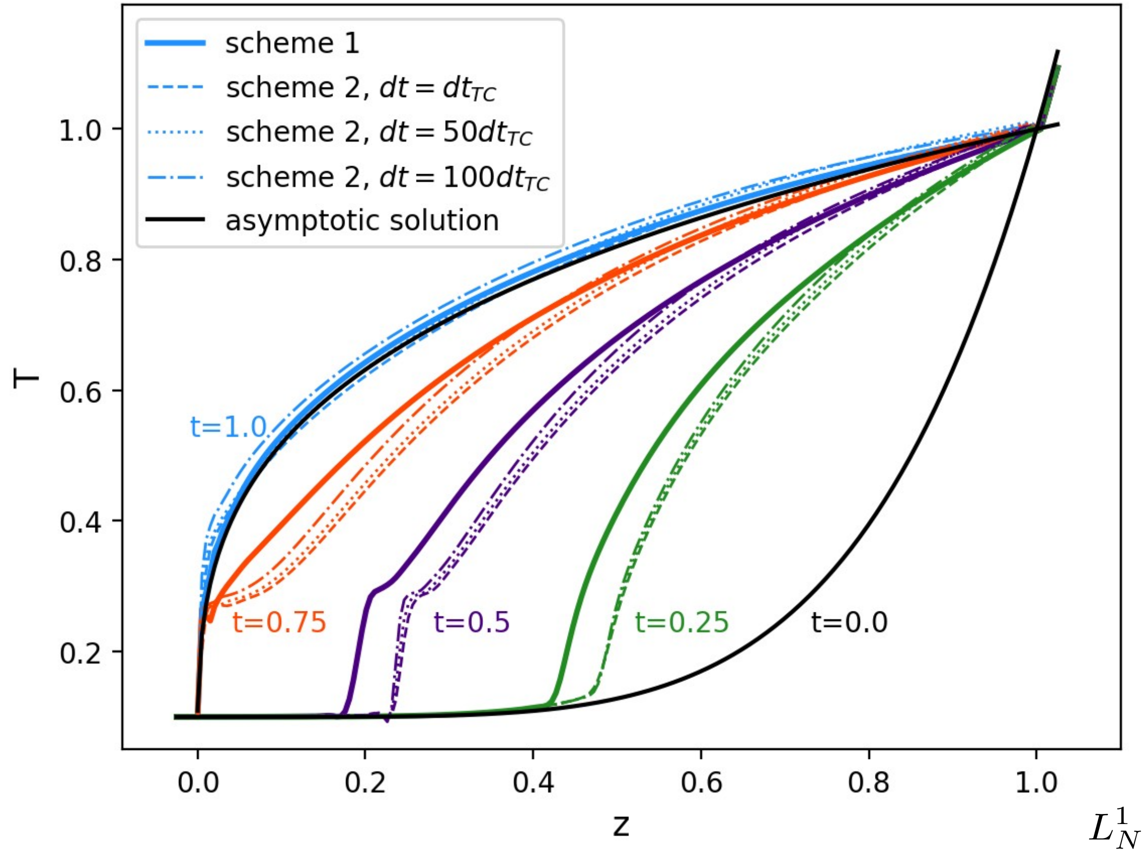
$$E_{\rm int} = [...] - \nabla\cdot\left(q\hat{\mathbf{b}}\right) \tag{2}$$

$$\tau = \left(f_{\rm CFL}\frac{\min(\Delta x, \Delta y, \Delta z)}{\Delta t} - |v|\right)^{-2}\frac{f_{\rm sat}\kappa T^{5/2}}{E_{\rm int}} \tag{3}$$

$$f_{\rm sat} = \left(1 + \frac{|\kappa T^{5/2}\left(\hat{\mathbf{b}}\cdot\nabla\right)T|}{1.5\rho c_s^3}\right)^{-1} ;\ \hat{\mathbf{b}} = \mathbf{B}/B \tag{4}$$

[1] M. Rempel 2017 ApJ 834 10

1D Heat conduction test[1]



$$L_N^1 = \frac{1}{N} \sum_{i=1,N} \left| T_i - T_i^{\mathrm{exact}} \right|$$

$$\rho = 1, \kappa = 0.01$$
$$T_i(z) = 0.1 + 0.9z^5, T(0) = 0.1, T(1) = 1.0$$
$$T_s(z) = \left[0.1^{3.5} + \left(1 - 0.1^{3.5}\right) z\right]^{2/7}$$

Scheme	L^1_N
Parabolic equation	1.05×10^{-2}
Hyperbolic equation $dt=dt_{TC}$	8.6×10^{-3}
Hyperbolic equation $dt=50dt_{TC}$	1.51×10^{-2}
Hyperbolic equation $dt=100dt_{TC}$	2.3×10^{-2}

[1] M. Rempel 2017 ApJ 834 10

Static Ring 2D [1,2,3,4]

$$T = \begin{cases} 12 & \text{if } 0.5 < r < 0.7 \\ & \frac{11}{12}\pi < \theta < \frac{13}{12}\pi, \\ 10 & \text{otherwise,} \end{cases}$$

$$B_x = 10^{-5}\cos(\theta + \pi/2)/r$$

$$B_z = 10^{-5}\sin(\theta + \pi/2)/r$$

$$\rho = 1, \kappa = 0.01$$

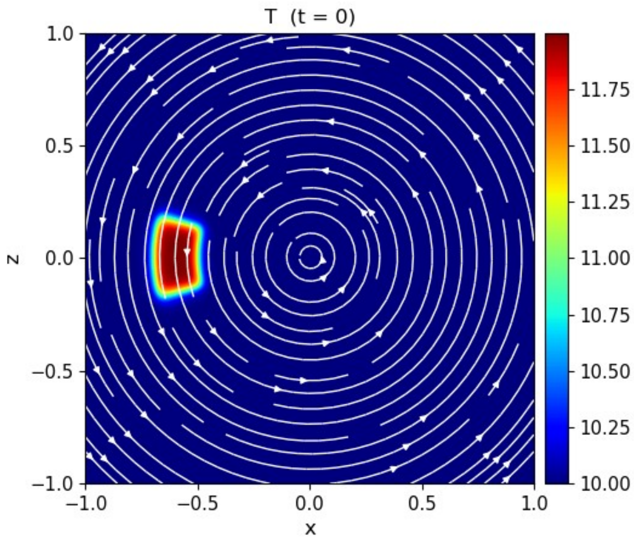


Fig 1: Initial State

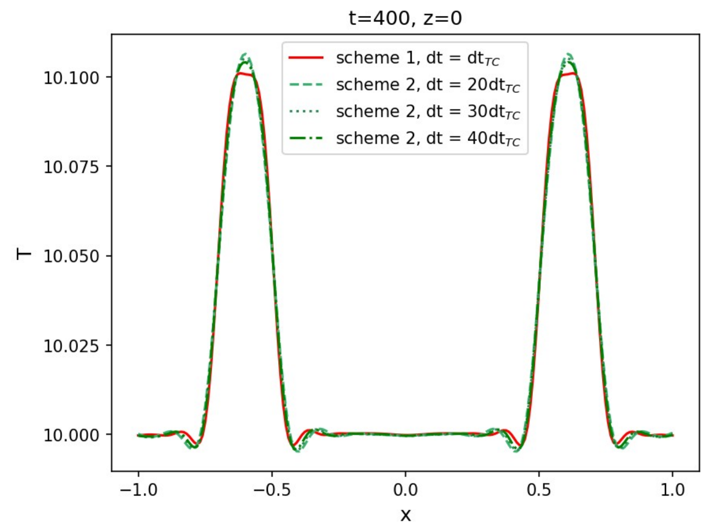


Fig 2: Comparisons

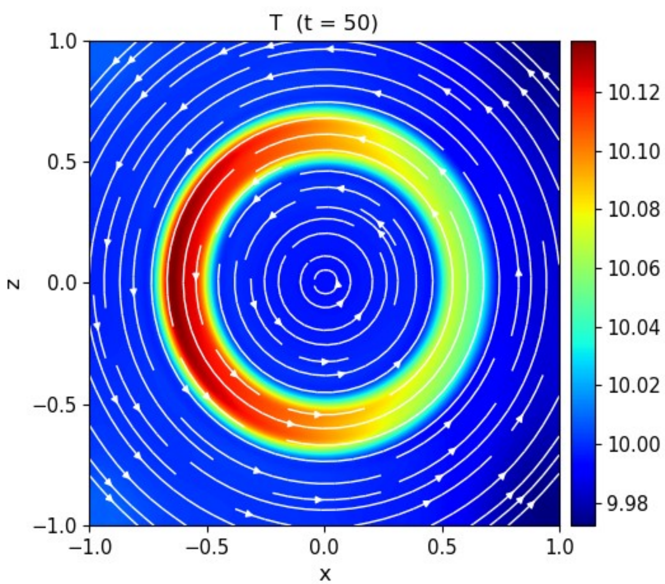


Fig 3: Final State, t = 50

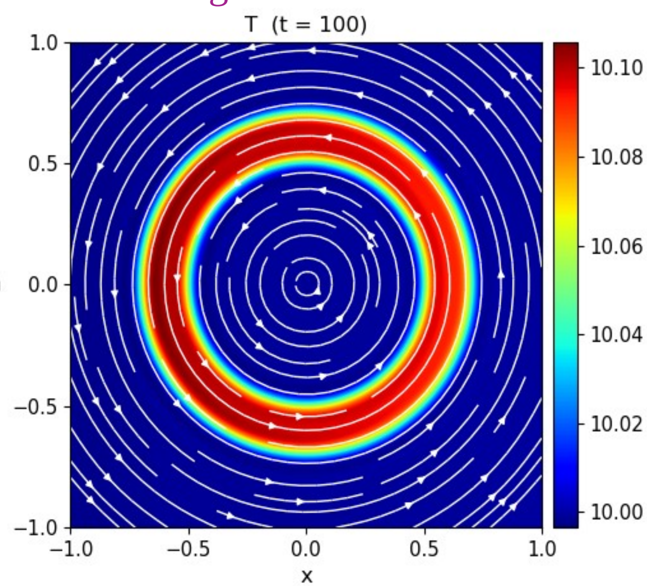


Fig 4: Final State, t = 400

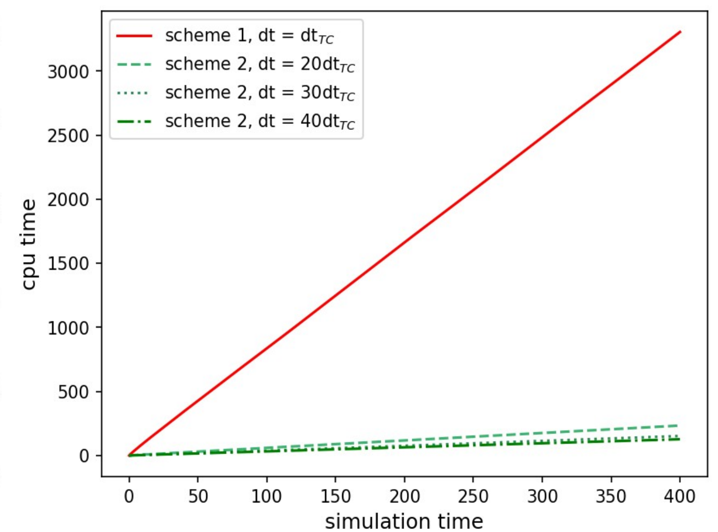


Fig 5: Computational times

[1] Parrish, I. J. & Stone, J. M. 2005, ApJ, 633, 334
[2] Sharma, P. & Hammett, G. W. 2007, J. Comput. Phys, 227, 123
[3] Meyer, C. D., Balsara, D. S., & Aslam, T. D. 2012, MNRAS, 422, 2102
[4] C. Xia et al 2018 ApJS 234 30

Static Ring 3D [1]

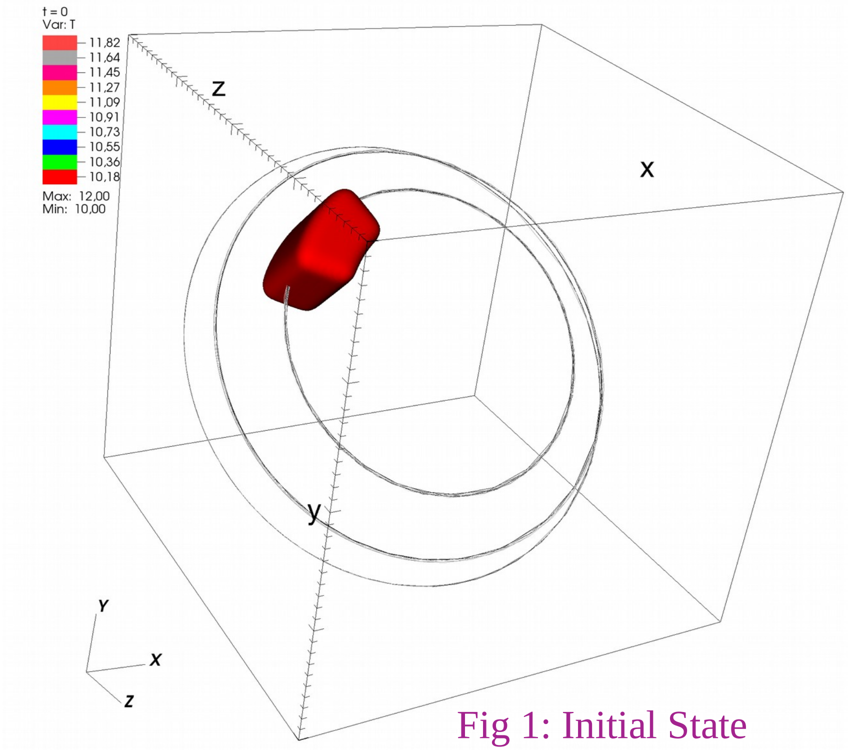


Fig 1: Initial State

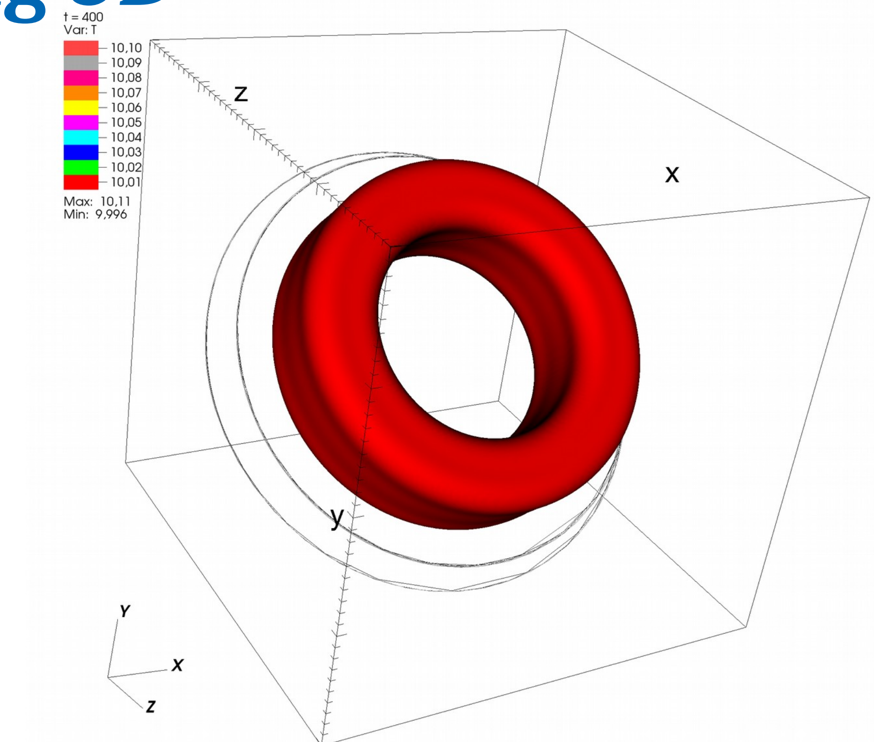


Fig 2: Final State

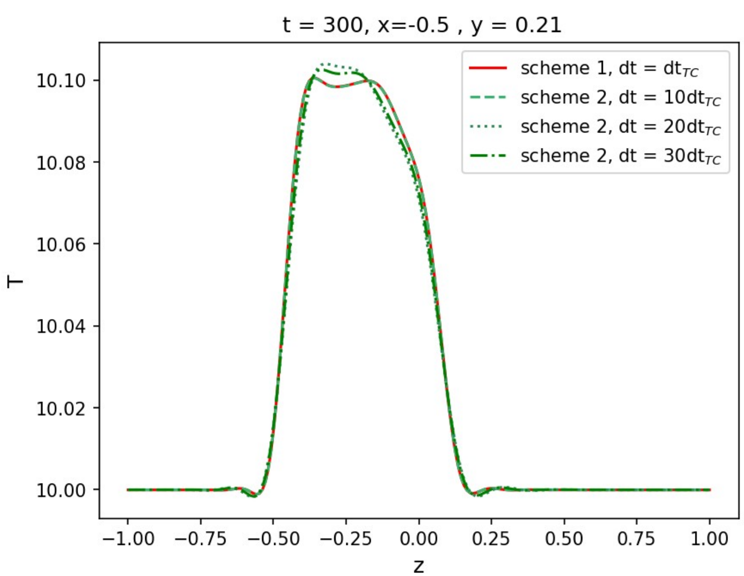


Fig 3: Comparisons

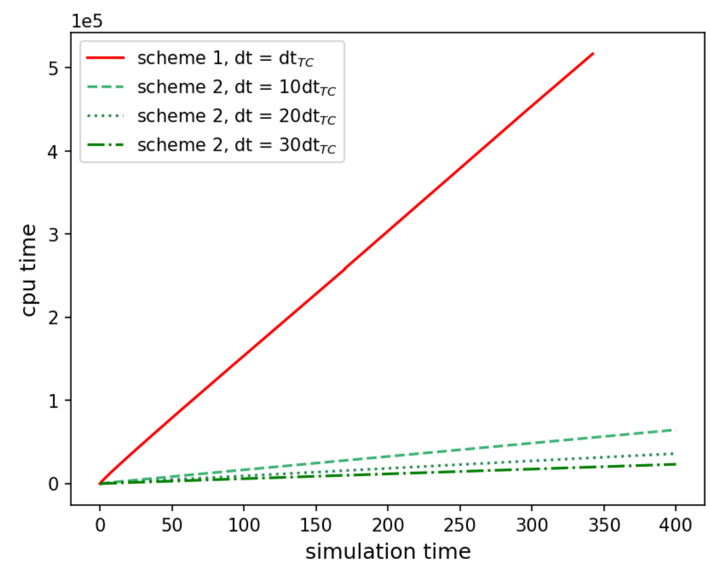


Fig 4: Computational times

[1] C. Xia et al 2018 ApJS 234 30

Conclusions

- We have implemented two numerical schemes for field aligned heat conduction in the code Mancha:

  The explicit evolution for the parabolic term.

  An hyperbolic treatment of heat conduction that imposes a maximum characteristic speed for heat conduction speed.

- The thermal conductivity implemented in the code Mancha reproduces standard test in 1D, 2D and 3D.

- Hyperbolic treatment for the heat conduction exhibits an enormous improvement in the computational speed while keeping good accuracy.

Thank you