Sub-critical dynamos and hysteresis in the Babcock-Leighton type kinematic dynamo model

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Abstract

A large-scale magnetic cycle is possible in the Sun and other stars as long as the large-scale shear and helicity of the plasma flow in their convection zones are sufficiently strong. Hence, there is a critical dynamo number for each star for the operation of a large-scale dynamo. As a star spins down, it is expected that the large-scale dynamo ceases to operate above a critical rotation period. Our study explores the possibility of the operation of the dynamo in the subcritical region using the Babcock-Leighton type kinematic dynamo model. In some parameter regimes, we find that the dynamo shows hysteresis behavior, i.e., two dynamo solutions are possible depending on the initial parameters---decaying solution if started with weak field and strong oscillatory solution (subcritical dynamo) when started with a strong field. However, under large fluctuations in the dynamo parameter, the subcritical dynamo mode is unstable in some parameter regimes. Therefore, our study supports the possible existence of subcritical dynamo in some stars which was previously shown in a mean-field dynamo model with distributed α and MHD turbulent dynamo simulations.

Introduction

• The convective flow of the ionized plasma in the solar convection zone is responsible for generation of the magnetic field and cycles through the process called the Dynamo. The important parameter here is the dynamo number (D),

$$\mathbf{D} = \frac{\alpha \Delta \Omega R^3}{\eta^2}$$

•The dependence of dynamo number and the magnetic field strength gives the hysteresis curve.

• Many Sun-like stars have magnetic field and cycles.

• Rotation plays an important role in the generation of the magnetic field.

Motivation

• Since, the rotation rate of a star decreases with the age (Skumanich 1972; Rengarajan 1984), the dynamo number is expected to decrease as the star spins down (Kitchatinov & Nepomnyashchikh 2017).

• Important question need to encounter: Will the dynamo cease immediately when $D < D_c$?

• The numerical simulation on helically forced turbulence, show the evidence of subcritical dynamo (Karak et al. 2015).

• An Idealized mean-field model with distributed α also show the evidence of subcritical dynamo (Kitchatinov & Olemskoy 2010).

• Our motivation is to study subcritical region using the Babcock–Leighton type kinematic dynamo model .

Model

Axisymmetric kinematic mean-field dynamo model

- Main Governing Equation: Induction Equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B} - \eta \nabla \times \boldsymbol{B})$$

- Axisymmetric magnetic field:

$$\boldsymbol{B} = \boldsymbol{B_P} + \boldsymbol{B_\phi} = \nabla \times [A(r,\theta,\phi)\hat{\phi}] + B(r,\theta,\phi)\hat{\phi}$$

- Velocity field:

$$v = v_P + r \sin\theta \Omega \hat{\phi} = v_r(r,\theta)\hat{r} + v_P(r,\theta)\hat{\theta} + r\sin\theta\, \Omega(r,\theta)\hat{\phi}$$

- Toroidal field evolution:

$$\frac{\partial B}{\partial t} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_r B) + \frac{\partial}{\partial \theta}(v_\theta)\right] = \eta_T\left(\nabla^2 - \frac{1}{r^2 \sin^2\theta}\right)B + r\sin\theta(B_P.\nabla)\Omega + \frac{1}{r}\frac{d\eta_T}{dr}\frac{\partial B}{\partial t}$$

- Poloidal field equation:

$$\frac{\partial A}{\partial t} + \frac{1}{r\sin\theta}(v_P.\nabla)(r\sin\theta A) = \eta_T\left(\nabla^2 - \frac{1}{r^2\sin^2\theta}\right)A + \alpha_T \overline{B}$$

Babcock-Leighton alpha

Average magnetic field in the base of convection zone (0.675R₀ – 0.725R₀)

Inclusion of quenching

Inclusion of quenching in turbulent diffusivity and alpha effect along with their non-linear behavior : magnetic quenching

$$\eta_T = \eta\phi_\eta(B) \text{ and } \alpha_T = \alpha\cos(\theta)\sin(\theta)^2\phi_\alpha(B)$$

$$\phi_\alpha(B) = \frac{15}{32B^4}\left[1 - \frac{4B^2}{3(1+B^2)^2} - \frac{1-B^2}{B} arctan(B)\right]$$

$$\phi_\eta(B) = \frac{3}{8B^2}\left[1 + \frac{4+8B^2}{(1+B^2)^2} + \frac{B^2-5}{B} arctan(B)\right]$$

$\overline{B} \gg 1$; $\phi_\eta \simeq 3\pi/16\,\overline{B}$, $\phi_\alpha \simeq 15\pi/64\,\overline{B}^3$ and

$\overline{B} \ll 1$; $\phi_\eta \simeq 1 - 2\overline{B}^2$, $\phi_\alpha \simeq 1 - 12\overline{B}^2/7$

From : Rüdiger and Kitchatinov 1993; Kitchatinov et al. 1994a

Results

(i) Regular dynamo solutions and Hysteresis

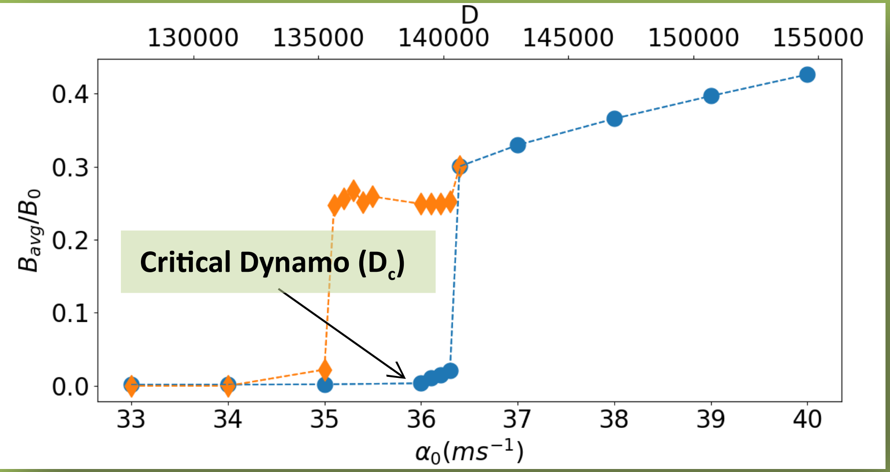

Fig. 1: Variation of the temporal average of the mean toroidal field computed at the BCZ at 14^0 latitude (B_{avg}) as a function of α_0 from simulations started with a weak field (blue) and from simulations started with strong field of previous simulation (orange).

(ii) Dynamo solution with fluctuations in the source term

$$\alpha_0=\alpha_0[1+s(\tau_{corr})\times f],$$

s = *uniform* random number in the interval $-1< s <1$

τ_{corr} = coherence time

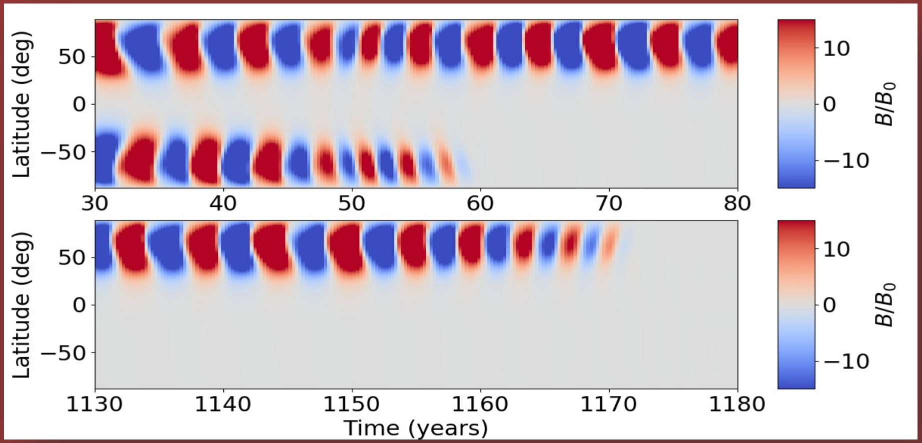

Fig. 2: Butterfly diagrams of toroidal field for subcritical dynamo at α_0= 35 m s^{-1} with 20 % fluctuations.

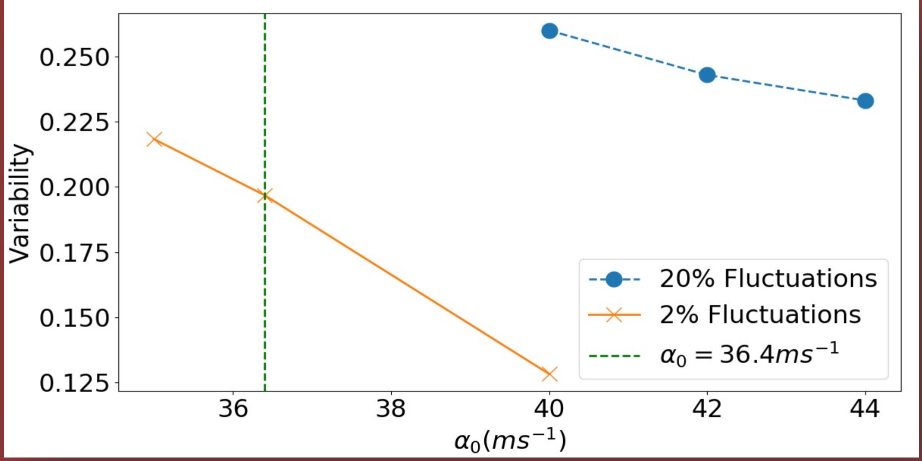

Fig. 3: Variation of variability with respect to the increase in α_0 with 2% fluctuations (orange solid line) and with 20% fluctuations (blue dashed line).

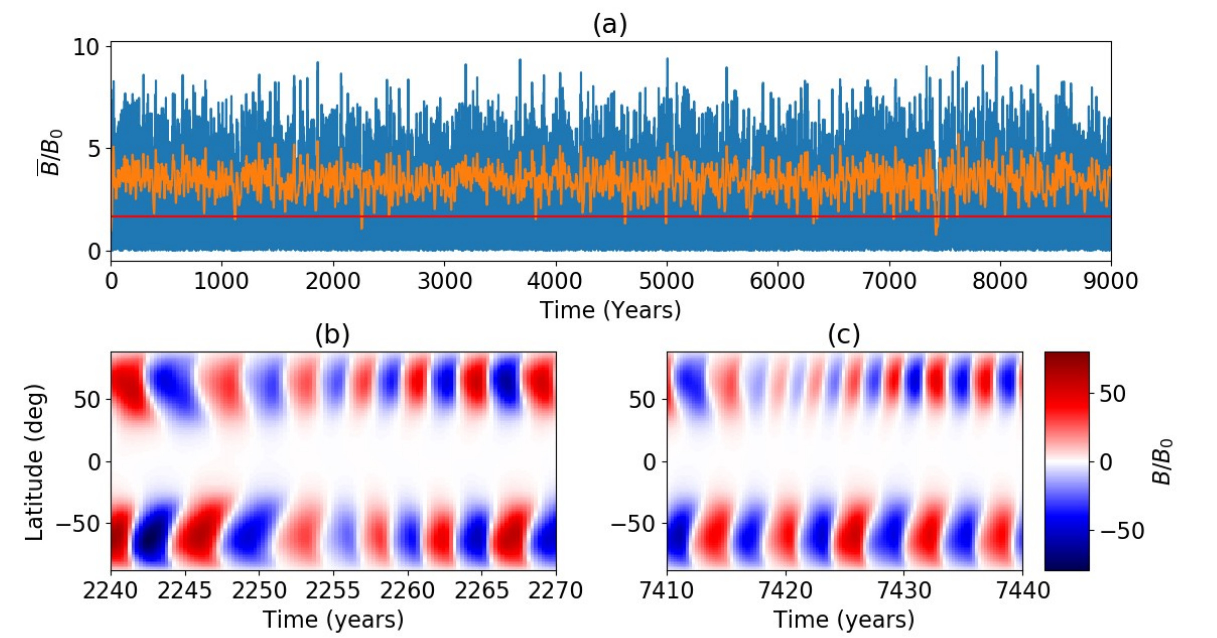

Fig. 4: (a) Time series plot along with its smoothed curve (yellow) of toroidal magnetic flux. Red horizontal line shows the half of the mean of this smooth curve. (b) & (c) Butterfly diagram of toroidal field for the two grand minima. These are obtained from a supercritical dynamo.

Conclusions

- An axisymmetric kinematic solar dynamo model with a Babcock–Leighton α_{BL} as the source of the poloidal field produces dynamo hysteresis.
- Our study provides a possible existence of subcritical dynamo for the execution of large-scale magnetic cycles in sun-like stars.
- The subcritical branch decays with fluctuations and the supercritical branch is always stable as well as produces some grand minima.
- We also studied the variability and observed its relation with the dynamo number. We conclude that as the dynamo number increases the number of grand minima also decreases.