Methodology for estimating the magnetic Prandtl number and application to solar surface small-scale dynamo simulations

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What are the intrinsic numerical viscosity and resistivity?

How does the action of solar dynamos depend on Pr_m?

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Introduction

Momentum equation

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v}\mathbf{v}) = -\nabla p + \rho \mathbf{g} + \mathbf{F}_{\mathrm{EM}} + \nu\rho\nabla^2\mathbf{v}$$

viscosity

Induction equation

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = \eta\nabla^2\mathbf{B}$$

resistivity

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Reynolds number

$$Re = \frac{uL}{\nu}$$

Magnetic Reynolds number

$$Re_{\mathrm{m}} = \frac{uL}{\eta}$$

Magnetic Prandtl number

$$Pr_{\mathrm{m}} = \frac{Re_{\mathrm{m}}}{Re} = \frac{\nu}{\eta}$$

For the Sun: $Re \gg 1$ $Re_{\mathrm{m}} \gg 1$ $Pr_{\mathrm{m}} \ll 1$

Is small-scale dynamo action on solar surface important?

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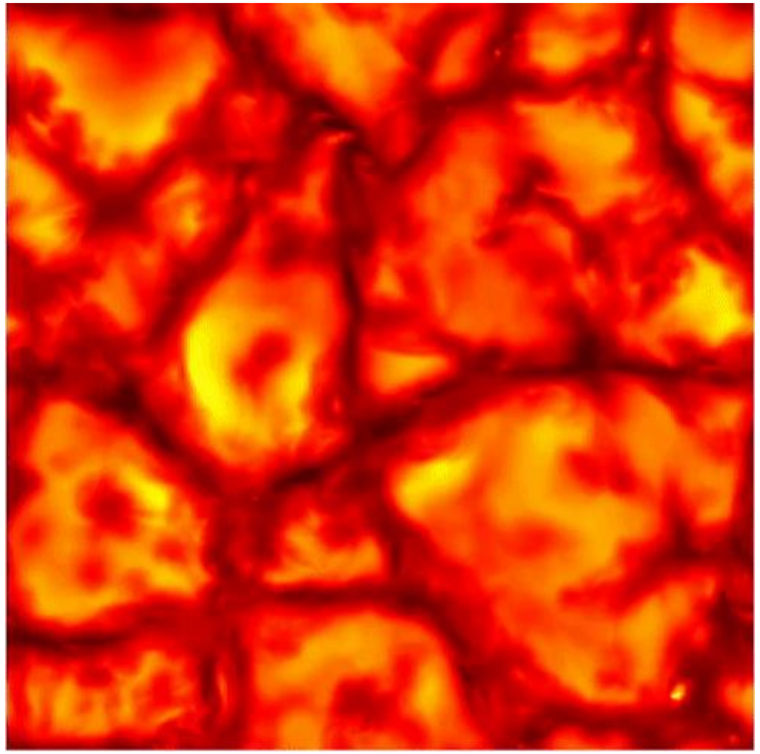
$\Rightarrow$ We need numerical simulations!

e.g., CO^5BOLD [Freytag et al., 2012]



Is small-scale dynamo action on solar surface important?

$Pr_{\rm m} \ll 1 \Rightarrow$ Cannot predict *a priori* which one wins between amplification and dissipation of magnetic fields

BUT: intrinsic numerical diffusivities!

- Make it difficult to reach realistic Re and $Re_{\rm m}$
- Re and $Re_{\rm m}$ generally unknown $\Rightarrow$ it complicates the interpretation of results

Methodology for estimating Pr_{m}

Based on method of Projection of Proper elements (PoPe) [Cartier-Michaud et al., 2016]

- Step 0

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0 \longrightarrow$$

$$\partial_t \mathbf{B} = \sum_i w_i O_i(\mathbf{B}, \mathbf{v})$$

$$\{w_i\} = \{1, -1\}$$

$$\{O_i\} = \{\nabla \cdot (\mathbf{B}\mathbf{v}), \nabla \cdot (\mathbf{v}\mathbf{B})\}$$

Methodology for estimating Pr_{m}

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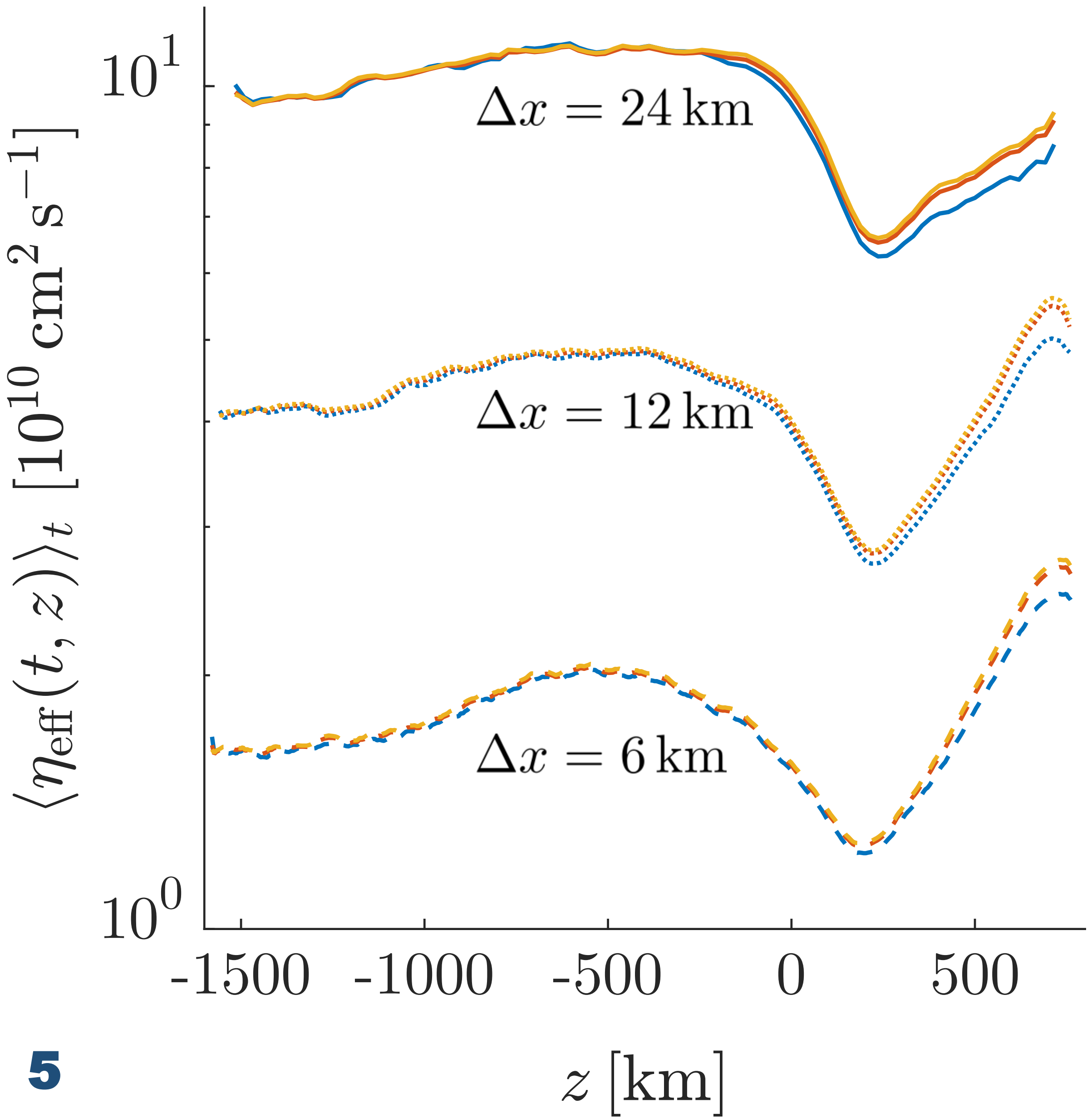
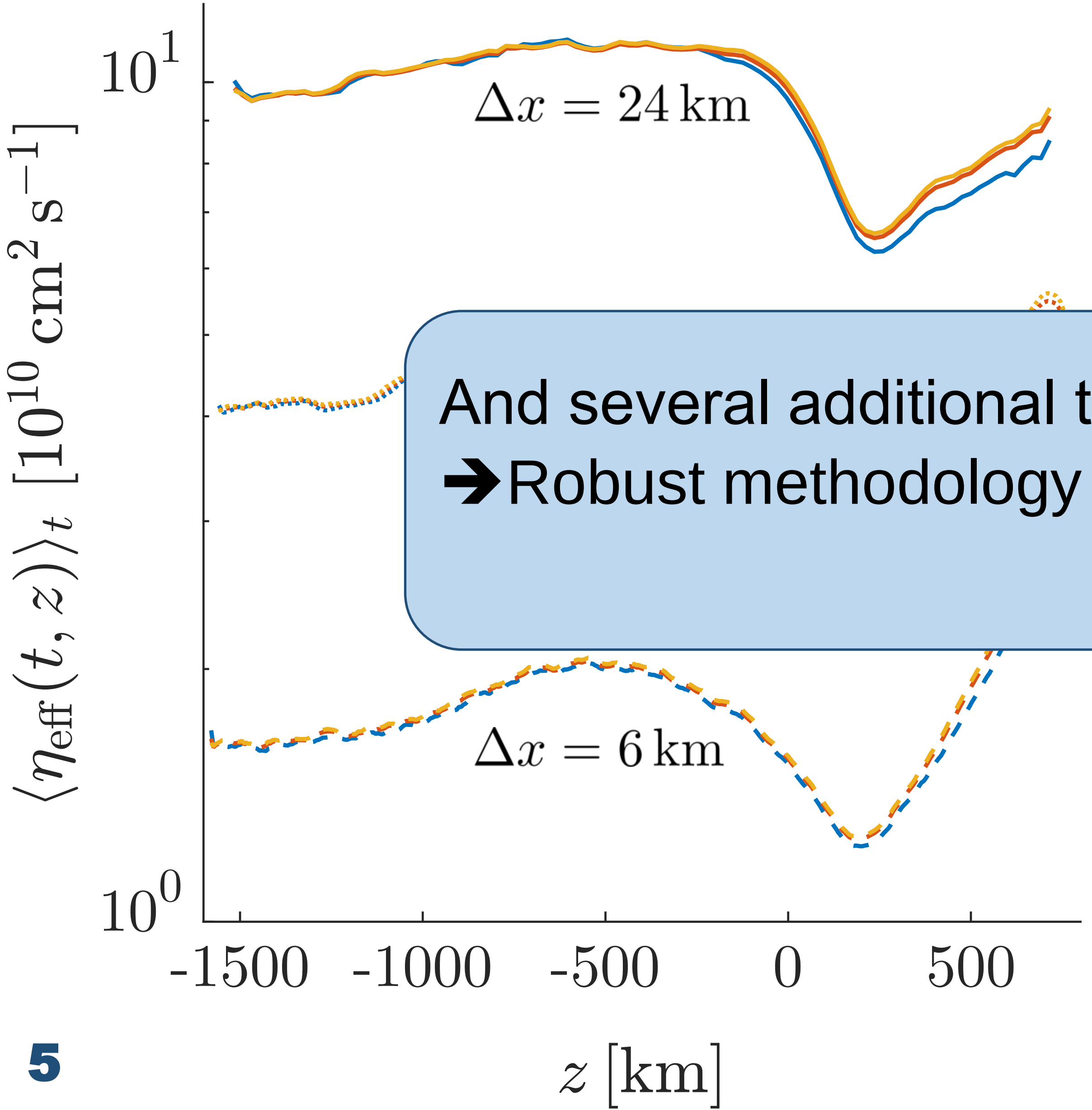
- Step 0 $\dfrac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0 \longrightarrow \partial_t \mathbf{B} = \sum_i w_i O_i(\mathbf{B}, \mathbf{v})$
- Step 1 Numerical solution from simulation code $\delta_t^{\mathrm{sc},h} \mathbf{B}^h = \sum_i w_i O_i^{\mathrm{sc},h}(\mathbf{B}^h, \mathbf{v}^h)$
- Step 2 Introduce a post-processing scheme to compute $\partial_t, \ \{O_i\}, \ \nabla^2$
  $r^h(\{\tilde{w}_i, \eta_{\mathrm{eff}}\}) \equiv \|\delta_t^{\mathrm{pp},h} \mathbf{B}^h - \sum_i \tilde{w}_i O_i^{\mathrm{pp},h}(\mathbf{B}^h, \mathbf{v}^h) - \eta_{\mathrm{eff}} (\nabla^2)^{\mathrm{pp},h} \mathbf{B}^h\|$
- Step 3 Minimize $r^h(\{\tilde{w}_i, \eta_{\mathrm{eff}}\})$ for $\tilde{w}_i, \ \eta_{\mathrm{eff}}$
- Step 4 Repeat with momentum equation for ν_{eff}
- Step 5 Compute $Pr_{\mathrm{m,eff}} = \dfrac{\nu_{\mathrm{eff}}}{\eta_{\mathrm{eff}}}$

Results



- Dependence on height
- Intrinsic resistivity decreases with resolution
- Results independent of post-processing scheme (FD2, FD4, FD6)

Results



And several additional tests…
➔Robust methodology and reliable results
[Riva et al., in preparation]

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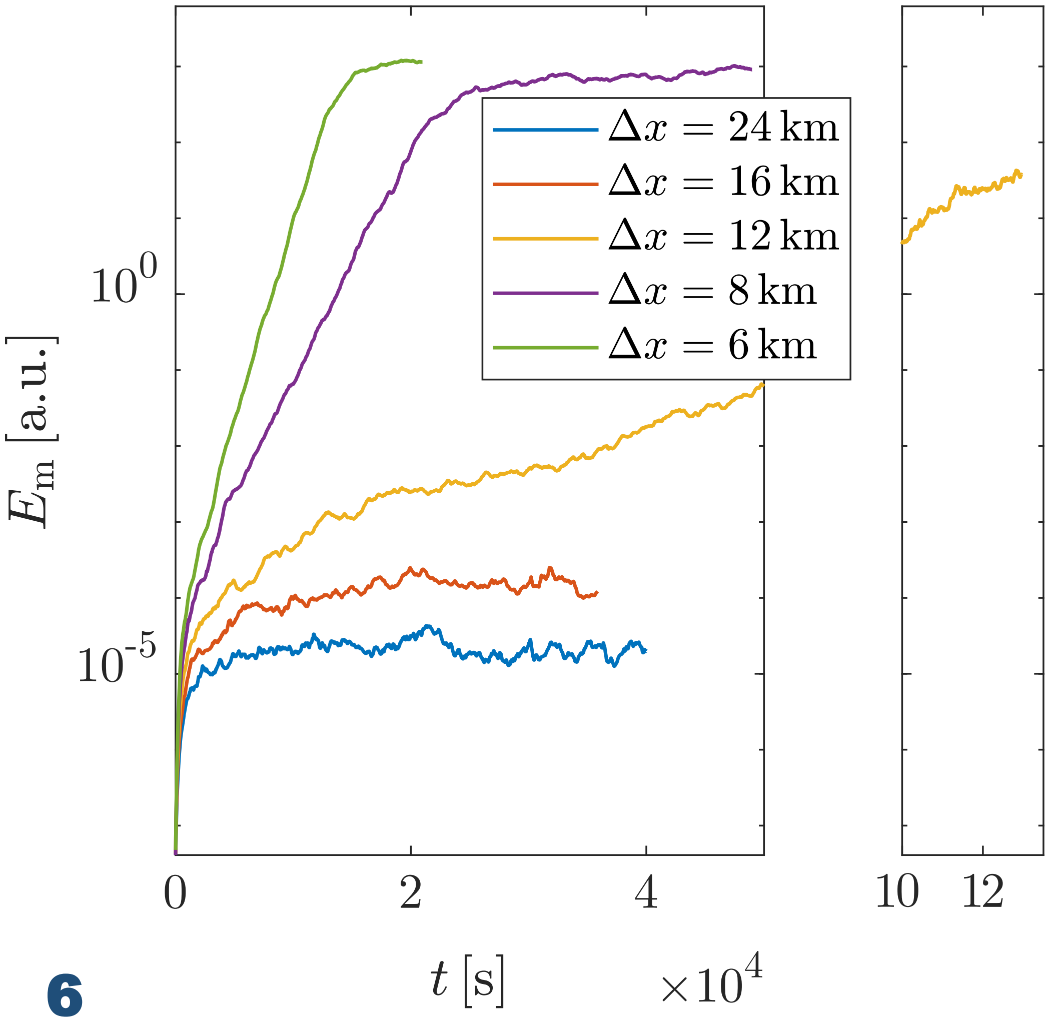
Dynamo simulations

$$Pr_{\mathrm{m}} = \frac{\nu}{\eta} \simeq 0.9 - 1$$



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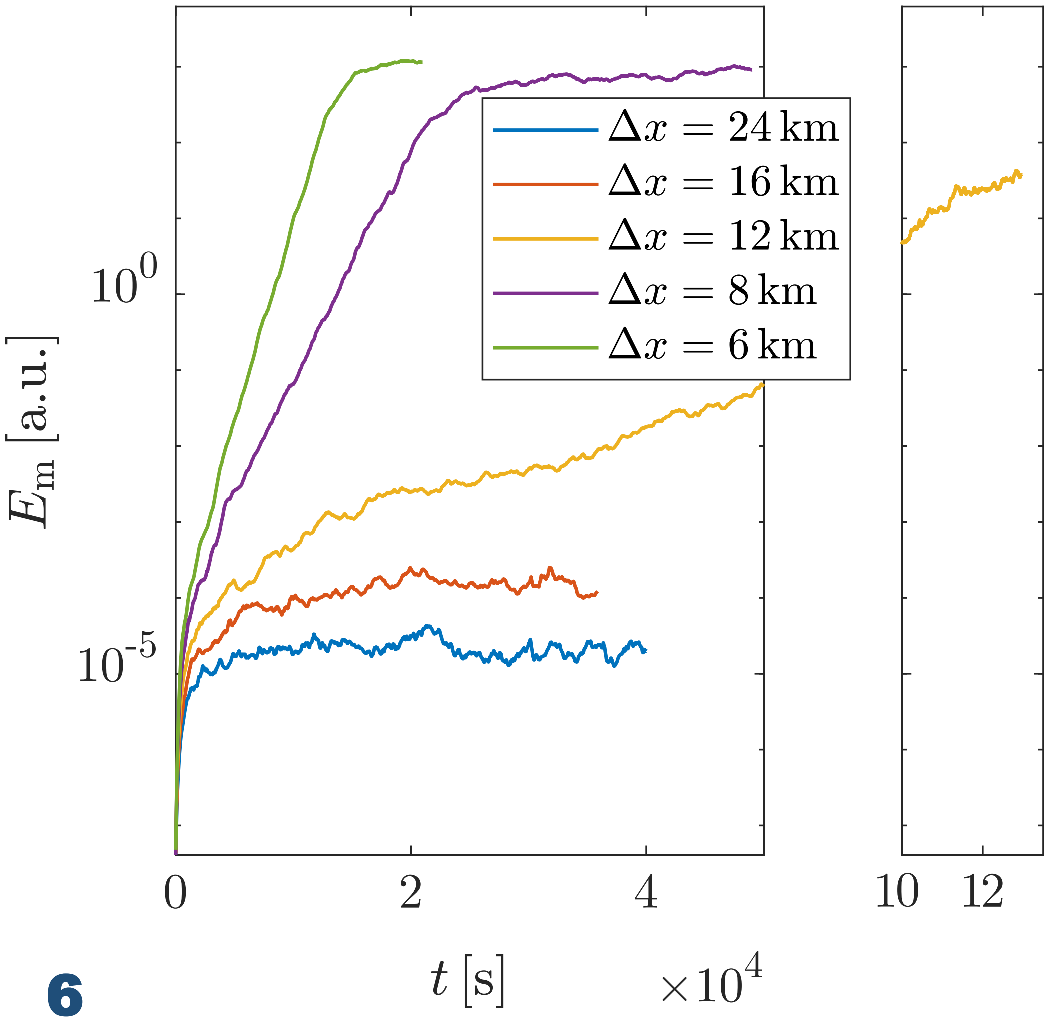
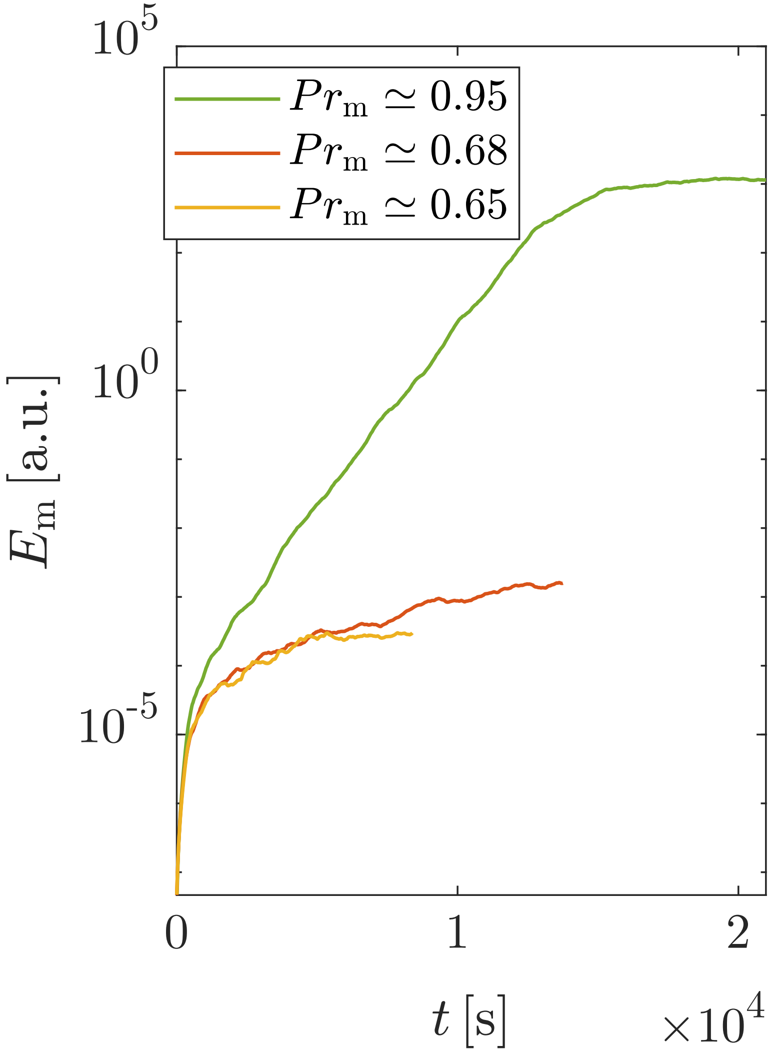


$$Re \simeq 1900 - 2400$$



Conclusions and future perspectives

Conclusions:

- Extended (i)PoPe methodology to estimating viscosity and resistivity in radiative MHD codes
- Applied methodology to CO5BOLD simulations and validated the procedure
- Demonstrated possibility of simulating self-generated magnetic fields with CO5BOLD, even at $Pr_\mathrm{m} \simeq 0.68$

Future work:

- Test with hyper-viscosity and hyper-resistivity
- Investigate impact of domain size and boundary conditions
- Investigate smaller Pr_m