Towards measuring twist in sunspots using SDO/HMI:
A sunspot model with correlated noise

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Abstract

Measurements of the magnetic field's twist play an important role in constraining dynamo theory, models of flux emergence and the prediction of flares. We aim to characterize the sensitivity of twist measurement methods to observational noise from SDO/HMI vector magnetograms by generating Monte-Carlo synthetic data sets. By studying several example sunspots we found that the temporal fluctuations in the HMI vector magnetograms are spatially correlated. We have developed an empirical model for noise that includes these spatial correlations. We present a semi-empirical sunspot model with correlated noise based on SDO/HMI observations of NOAA AR 11072's leading sunspot and discuss implications of measuring its magnetic field's twist with the force-free parameter α.

Introduction to magnetic field twist measurements

The magnetic field in sunspots is often modelled by coherent bundles of magnetic field lines, so-called flux tubes. Measurements of their magnetic helicity, which is closely related to the magnetic field's topology, are used in solar physics to constrain theories of the solar dynamo, flux emergence or the onset of flares. A single flux tube's magnetic helicity has two components: writhe (deformation of the tube's axis) and twist.

The twist T measures how often a set of field lines turn around the tube's axis:

$$T = qL \tag{1}$$

where L is the length of the flux tube and q is the twist density (number of turns around the axis per length unit).

Under the assumption of uniformly twisted flux tubes, various methods have been proposed to measure the twist density directly from vector magnetic field observations (e.g. SDO/HMI) based on the force-free parameter α (e.g. Hagino and Sakurai, 2004; Longcope et al., 1998) or fitting the twist density q (e.g. Nandy et al., 2008).

We aim to model a sunpot and noise based on SDO/HMI observations and use it to characterize the sensitivity of these methods to observational noise.

The reference sunspot

The template sunspot for our model should closely resemble the assumption of a monolithic flux-tube, a common assumption for sunspot models. Therefore, it should be well established, roughly circular and with little influence of other strong magnetic field in its vicinity. We chose the leading spot of NOAA AR 11072 (2010.05.25 03:00:00, top row in Fig. 1) as a template for the example sunspot and noise model.

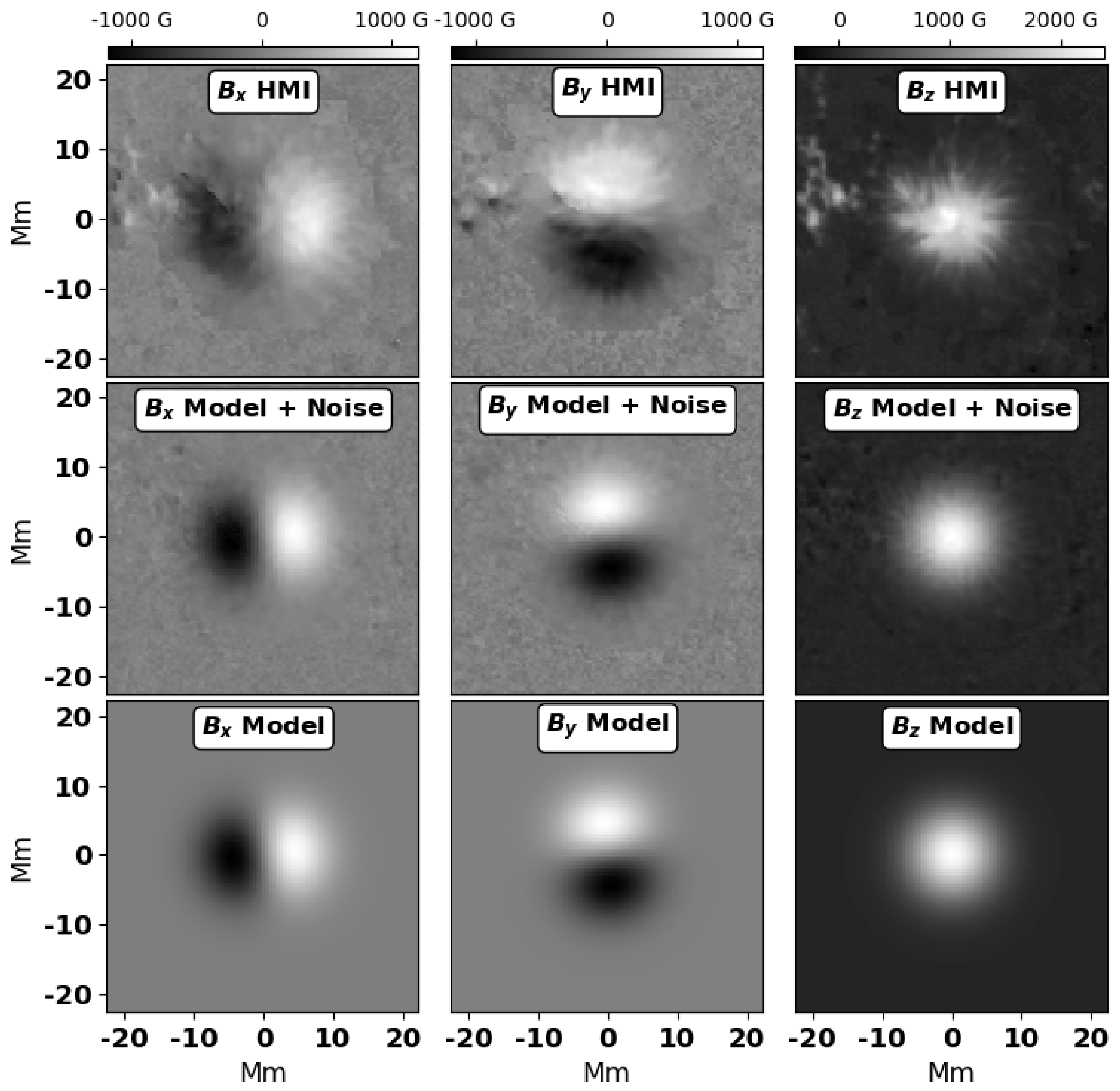

Figure 1: Comparison of the local heliographic vector components (B_x, B_y, B_z) of NOAA AR 11072's leading sunspot in HMI observations (top row) and its best fitting model with correlated noise (middle row) and without noise (bottom row).

Estimating the spatial noise covariance from the observations

We estimate the spatial noise covariance from an approximately seven hour time series of our reference sunspot. To get the temporal fluctuations of the magnetic field, we detrend the time series of each pixel by fitting a third order polynomial to the data and keeping the residuals (sketched in Fig. 2). We find that the detrended time series of neighboring pixels are correlated. We use this information to create random correlated noise maps by calculating the covariance matrix of all detrended time series and applying its Cholesky decomposition to uncorrelated Gaussian white-noise maps.

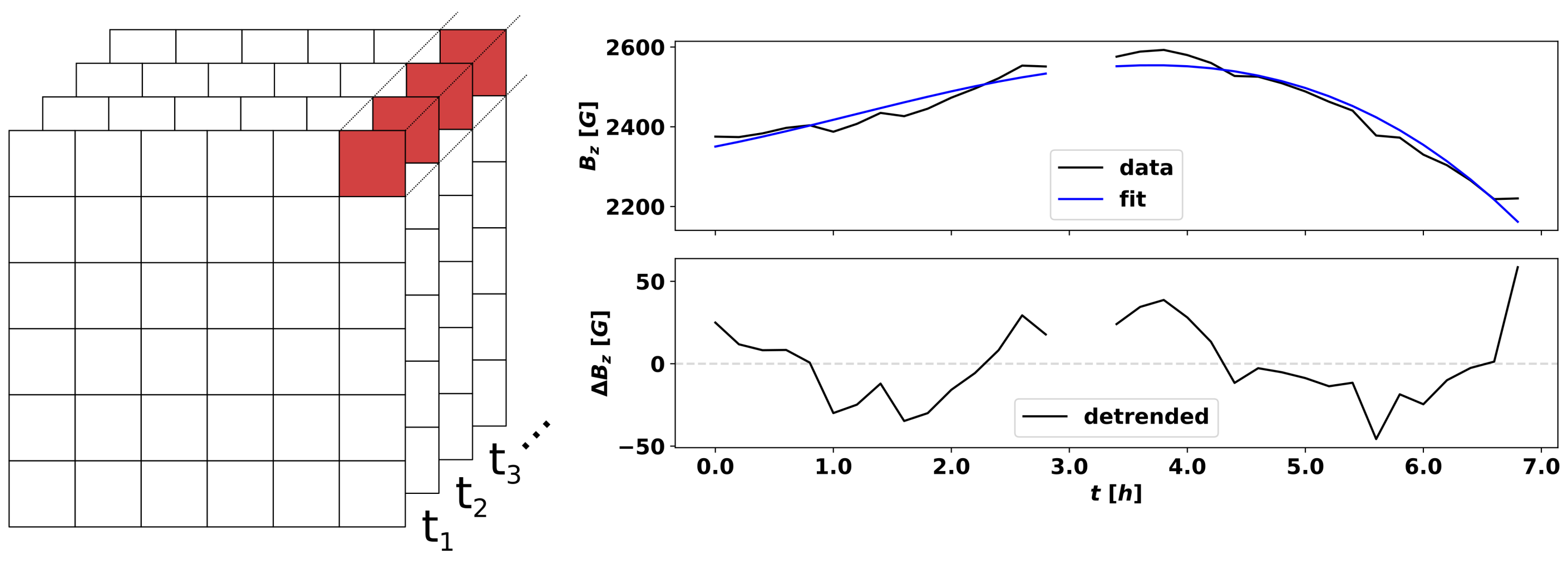

Figure 2: Sketch of the detrending process. A series of consecutive vector magnetic field observations (exemplary the B_z component), labeled with different time steps (t_1, t_2, t_3,...), is shown on the left. The black line (data) in the top right panel shows the temporal evolution of one pixel in this time series (marked in red in the left panel). The blue line represents a third order polynomial fit to the data. The bottom right panel displays the detrended time series, which shows the residuals of the data with respect to the fit.

A model for the reference sunspot

We use the semi-empirical sunspot model by Cameron et al. (2011) with added uniform twist and fit its radial profile to the observations (bottom row in Fig. 1). The middle row of Fig. 1 shows an example of the sunspot model superposed with one realization of a correlated noise map.

A proxy for measuring twist

A proxy for the magnetic field's twist is the "vertical component" of the force-free parameter

$$\alpha_z = J_z/B_z = \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)/B_z, \tag{2}$$

where J_z is the vertical current (Pevtsov et al., 2014).

Leka et al. (2005) show that α_z at a uniformly twisted tube's axis directly relates to the twist density: $\alpha_z = 2q$.

Spatial averages of α_z over sunspots or whole active regions are used to characterize their magnetic field's twist (e.g. Hagino and Sakurai, 2004; Longcope et al., 1998). It was shown that such averages typically underestimate the twist (Leka et al., 2005; Valori et al., 2005), but the chirality (sign) of the twist can be retrieved (e.g. López Fuentes et al., 2003).

Comparing twist in our sunspot model with correlated noise to HMI observations

Fig. 3 shows a comparison of α_z maps between the original HMI observations of NOAA AR 11072's leading sunspot and our best fitting sunspot model with and without superposed correlated noise.

We find that without noise the sunspot model's α_z profile changes radially, but the correct twist density (known from analytical calculations) can be retrieved at the spot's center, similar to the flux tube model used by Leka et al. (2005).

The sunspot model without noise shows opposite signs of α_z in umbral and penumbral areas. Since α_z is related to the vertical current J_z (Eq. 2), this is consistent with observations of return currents at the umbra-penumbra boundary (e.g. Tiwari et al., 2009; Zhang, 2006). The vertical current over the whole sunspot model is balanced.

Due to the presence of opposite signs of α_z in the uniformly twisted model, spatial averages of α_z that include the penumbra can yield the wrong sign of twist.

We find that applying correlated noise to the sunspot model creates α_z maps that look similar to observations. It can create complex coherent patches of opposite sign of twist even within the umbra as reported from observations (e.g. Pevtsov et al., 1994; Su et al., 2009).

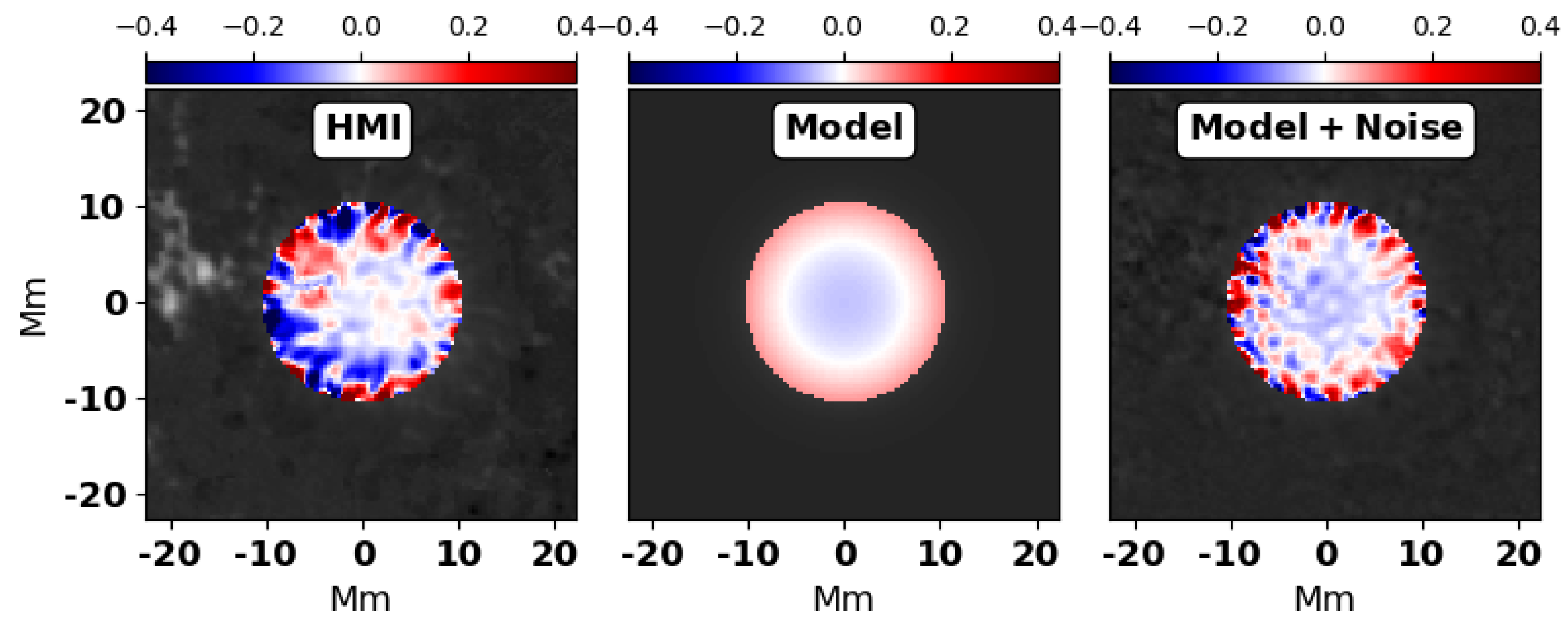

Figure 3: Comparison of α_z maps calculated from the original HMI observation (left), the model without noise (middle) and the model with correlated noise (right). The colorbar on top shows the α_z values in Mm^{-1}. The background shows the respective B_z vector field component in gray-scale.

Conclusions

- Temporal fluctuations in the magnetic field of neighboring pixels are correlated in HMI observations
- A noise model based on these correlations creates α_z maps with similar patterns to observations
- The twist in our sunspot model can be retrieved at the spot's center similar to findings by Leka et al. (2005)
- Averaging methods of α_z underestimate the twist in the model in agreement with findings by Leka et al. (2005) and Valori et al. (2005).
- Although uniformly twisted, the sunspot model shows opposite signs of α_z in the umbra and penumbra. This can be related to the presence of return currents.
- Spatial averages of α_z that include the penumbra can yield the wrong sign of twist

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