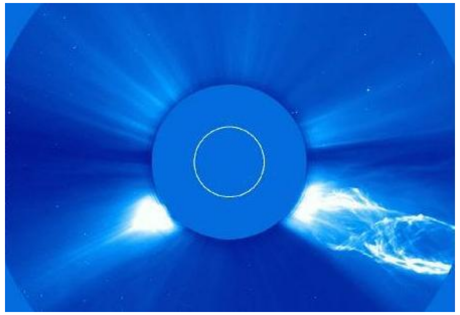
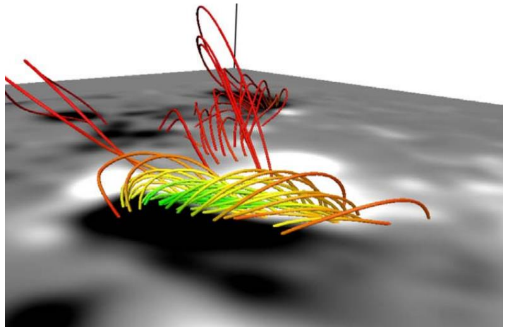
Advances in mean-field dynamo theory

Axel Brandenburg (Nordita, Stockholm)

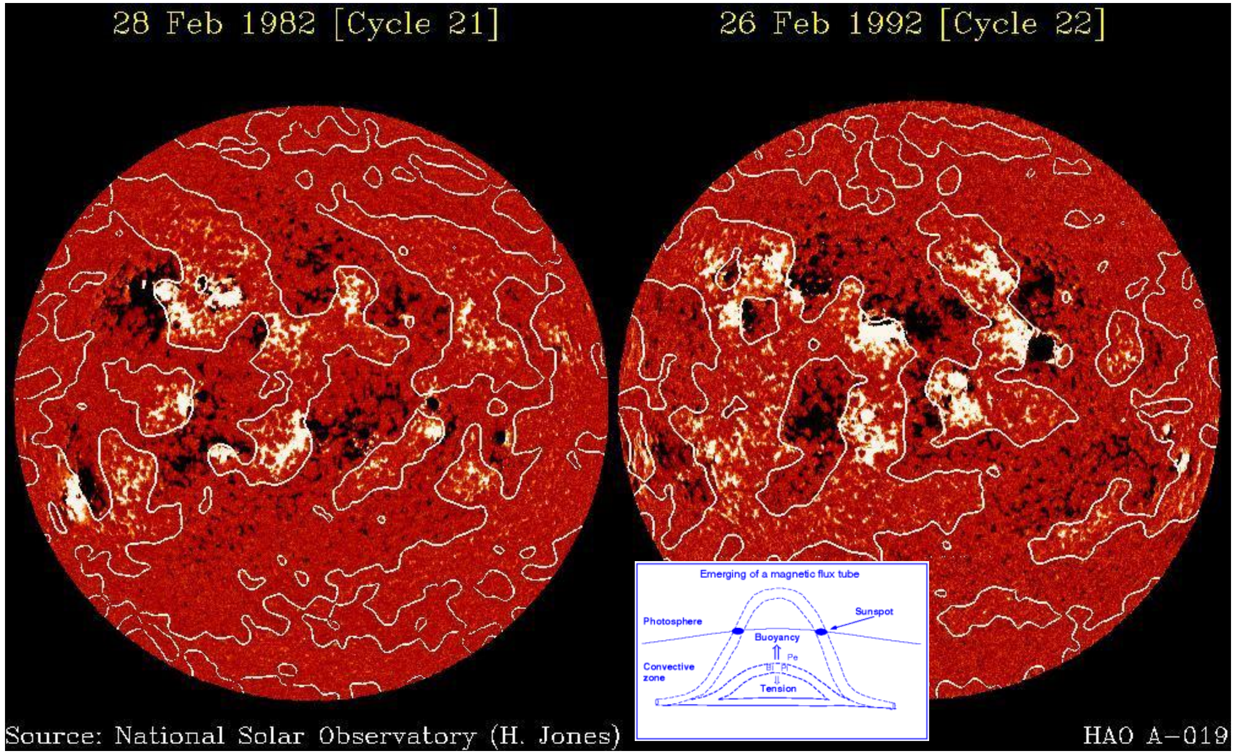
What	How	What next?
What is mean-field dynamo theory?	Large-scale field, 11 yr cycle Azimuthally averaged field	OK for the Sun, not other stars
Global simulations & mean-fields	TFM (test-field method) NSSL (near surface shear layer) Deep weakly subadiabatic Sunspots, link to the dynamo	New simulation setups Targeted simulations Detailed study of idealizations
Importance of magnetic helicity	Emerging twist in ARs Magnetic helicity in solar wind	Too much? Wrong sign? → Combined simulations

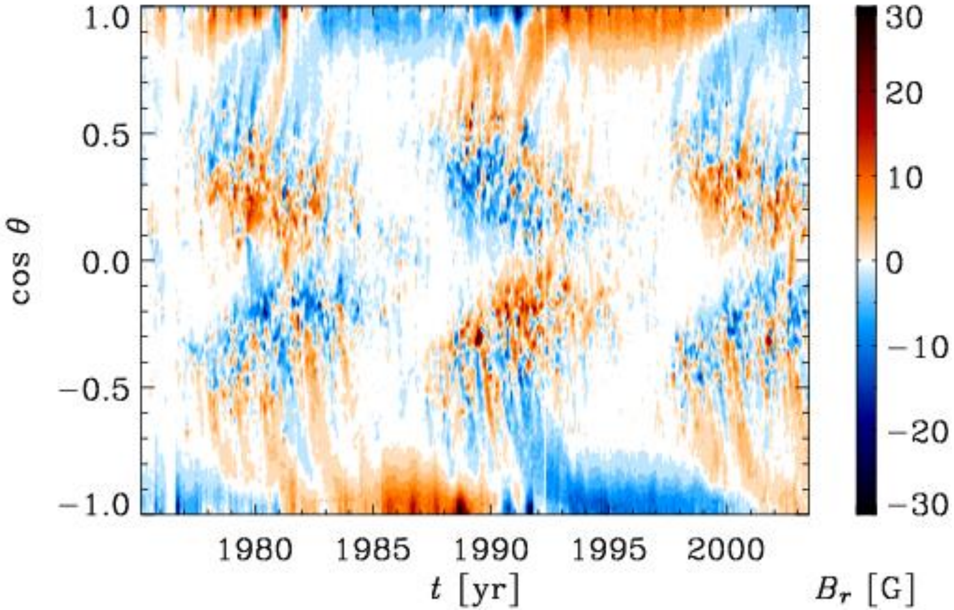


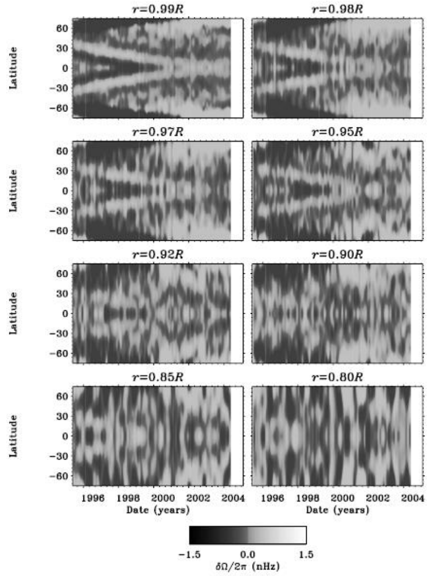


Why mean-field theory

- Spatio-temporal order
  - East-west orientation
  - Reversed 10 years later
- Captured by azimuthal averaging
  - Clear pattern in radial field
  - And in zonal flows
- To understand reasons
  - Display averaged results
  - Other stars with m=1 fields
  - Azimuthal average not useful
- Not just averaged equations!
  - i.e., not just $\alpha\Omega$ dynamos







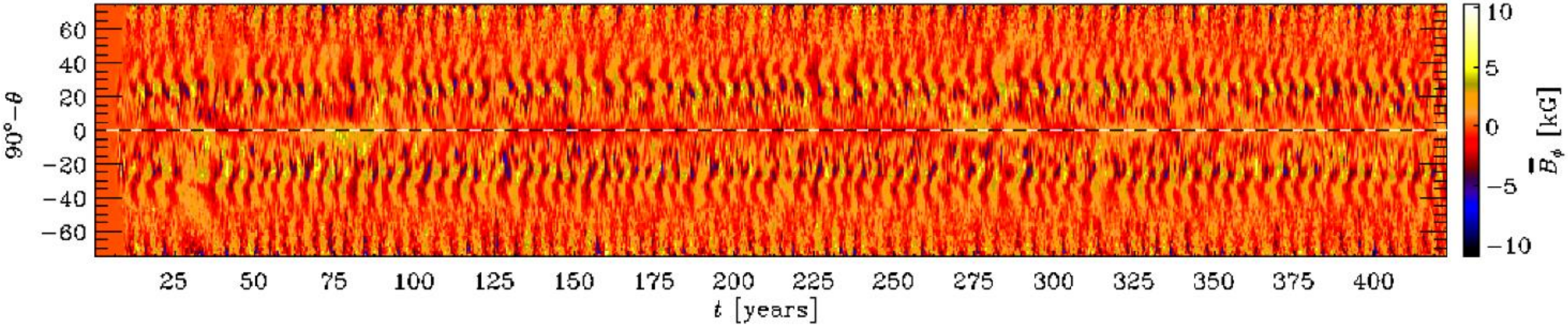
Global simulations

What	How	What next?
What is mean-field dynamo theory?	Large-scale field, 11 yr cycle	OK for the Sun, not other stars
Global simulations & mean-fields	NSSL (near surface shear layer) TFM (test-field method) Deep weakly subadiabatic Sunspots, link to the dynamo	New simulation setups Targetted simulations Detailed study of idealizations
Importance of magnetic helicity	Emerging twist in ARs Magnetic helicity in solar wind	Too much? Wrong sign? → Combined simulations

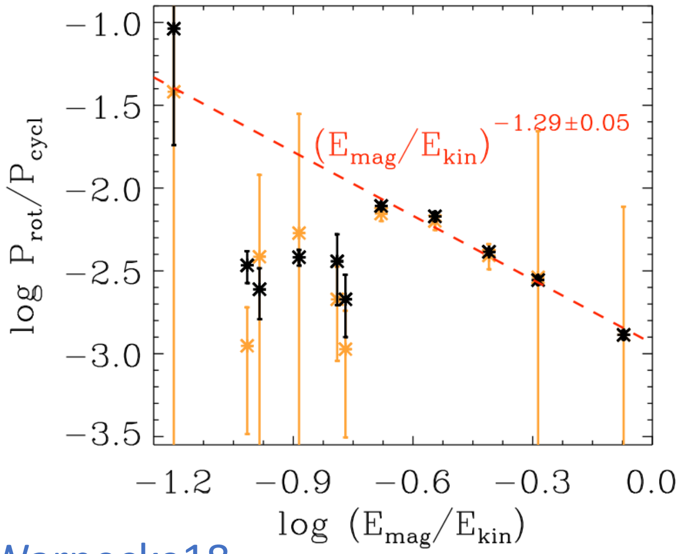
- Should eventually reproduce the Sun (& stars!)
  - Even if the truth is in Babcock-Leighton
  - Currently, there are still systematic differences
- Should already now reproduce features predicted by models
  - If model are stripped of certain complexities (surface layers)
  - Can those features be captured by averaged equations?

Global simulations

M Kapyla + 16: multiple dynamo modes



- Cyclic variability common, even multiple cycles, but
  - No clear equatorward migration (but equatorial belts)
  - If migration: associated with shear layers
- Predictions from global simulations recovered
  - Parker-Yoshimura rule (Warnecke+14)
  - Detailed mean-field coefficients (Warnecke+21)



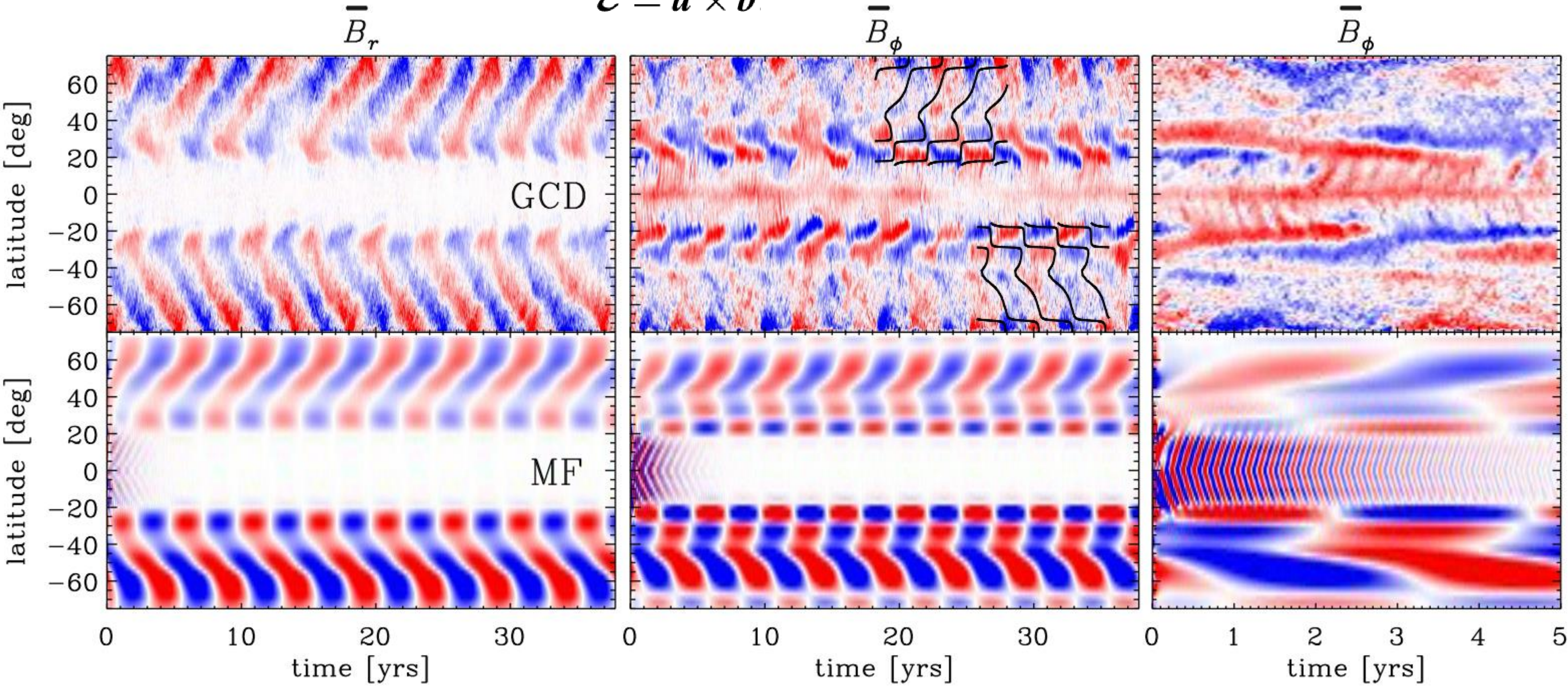
Warnecke18

Test-field results

$$\boldsymbol{\mathcal{E}} = \boldsymbol{\alpha}\cdot\overline{\boldsymbol{B}} + \boldsymbol{\gamma}\times\overline{\boldsymbol{B}} - \boldsymbol{\beta}\cdot(\boldsymbol{\nabla}\times\overline{\boldsymbol{B}}) - \boldsymbol{\delta}\times(\boldsymbol{\nabla}\times\overline{\boldsymbol{B}}) - \boldsymbol{\kappa}\cdot(\boldsymbol{\nabla}\overline{\boldsymbol{B}})^{(s)}$$

Warnecke+21

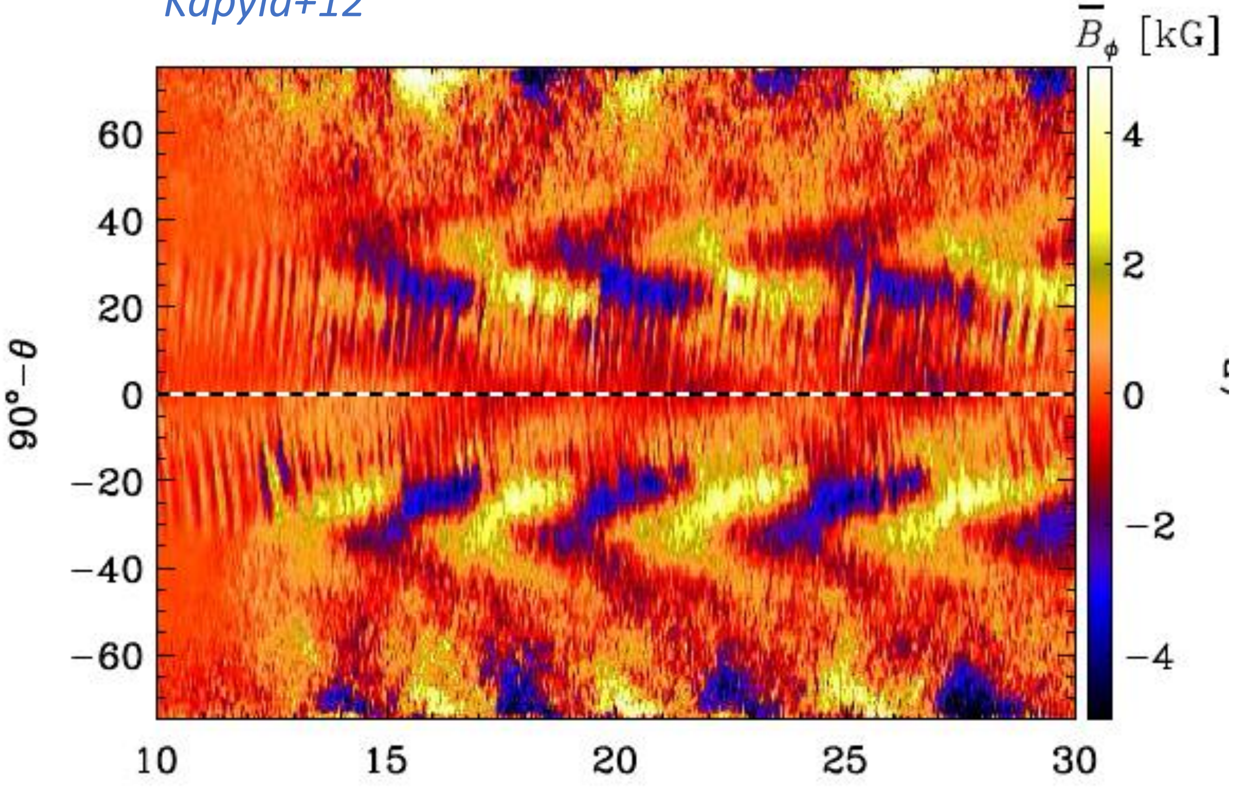
$$\overline{\boldsymbol{\mathcal{E}}} = \overline{\boldsymbol{u}\times\boldsymbol{b}}$$



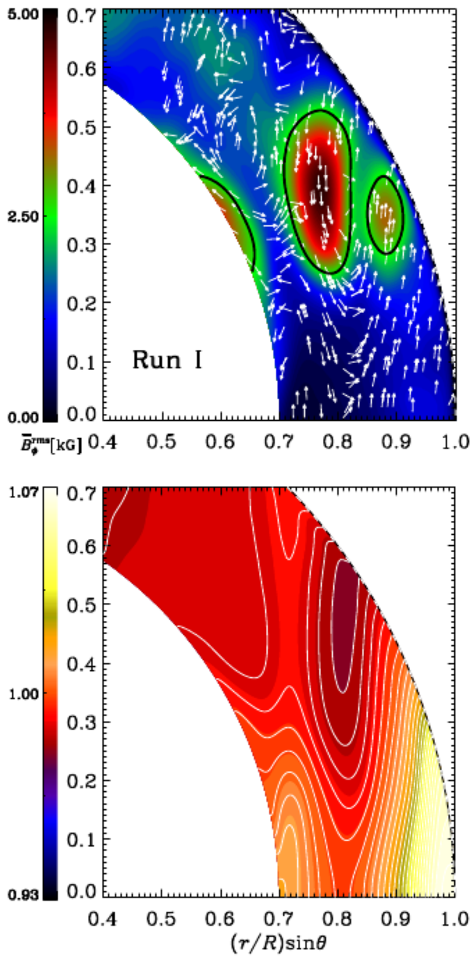
Equatorward migration

- Historically most important
    - Requires $\alpha > 0$, and
    - $d\Omega/dr < 0$
    - But: helioseismology
- Alternatives
    - Overshoot dynamo
    - Surface shear layer
    - Meridional circulation

Kapyla+12



Warnecke+14



- Caused here by negative radial shear at mid-latitudes
    - Exactly as predicted by Parker-Yoshimura: $\alpha\, d\Omega/dr < 0$
    - Not seen in helioseismology
    - Meridional circulation: multiple cylindrical shells (also not seen in seismology)

Global simulations: shortcomings:

- NSSL (near-surface shear layer) not resolved
  - Tremendous difference in time scales: 5 min vs 12 days
  - Length scales: 300 km vs 60 Mm
- Convection instability not by local Schwarzschild criterion
  - But stirring from above → drives Deardorff flux (Brandenburg16)
  - No giant cells expected (→ all global simulations flawed)
  - Stability depends on *local* opacity law

Opacity κ
Polytropic index *n*

$$\kappa = \kappa_0 \rho^a T^b \qquad n = \frac{3-b}{1+a}$$

Barekat+Brandenburg14

Set	a	b	n	Schwarzschild
A	1	−3.5	3.25	stable
B	1	0	1.5	marginally stable
C	1	1	1	unstable
D	1	5	−1	ultra unstable
E	−1	3	0/0	undefined

Gradient flux (Bohm-Vitense 1953)
Deardorff flux (Deardorff 1968)

$$\boldsymbol{F}_{\mathrm{G}} = -\frac{1}{3}\tau_{\mathrm{red}} u_{\mathrm{rms}}^2 \overline{\rho}\, \overline{T}\, \boldsymbol{\nabla}\overline{S},$$

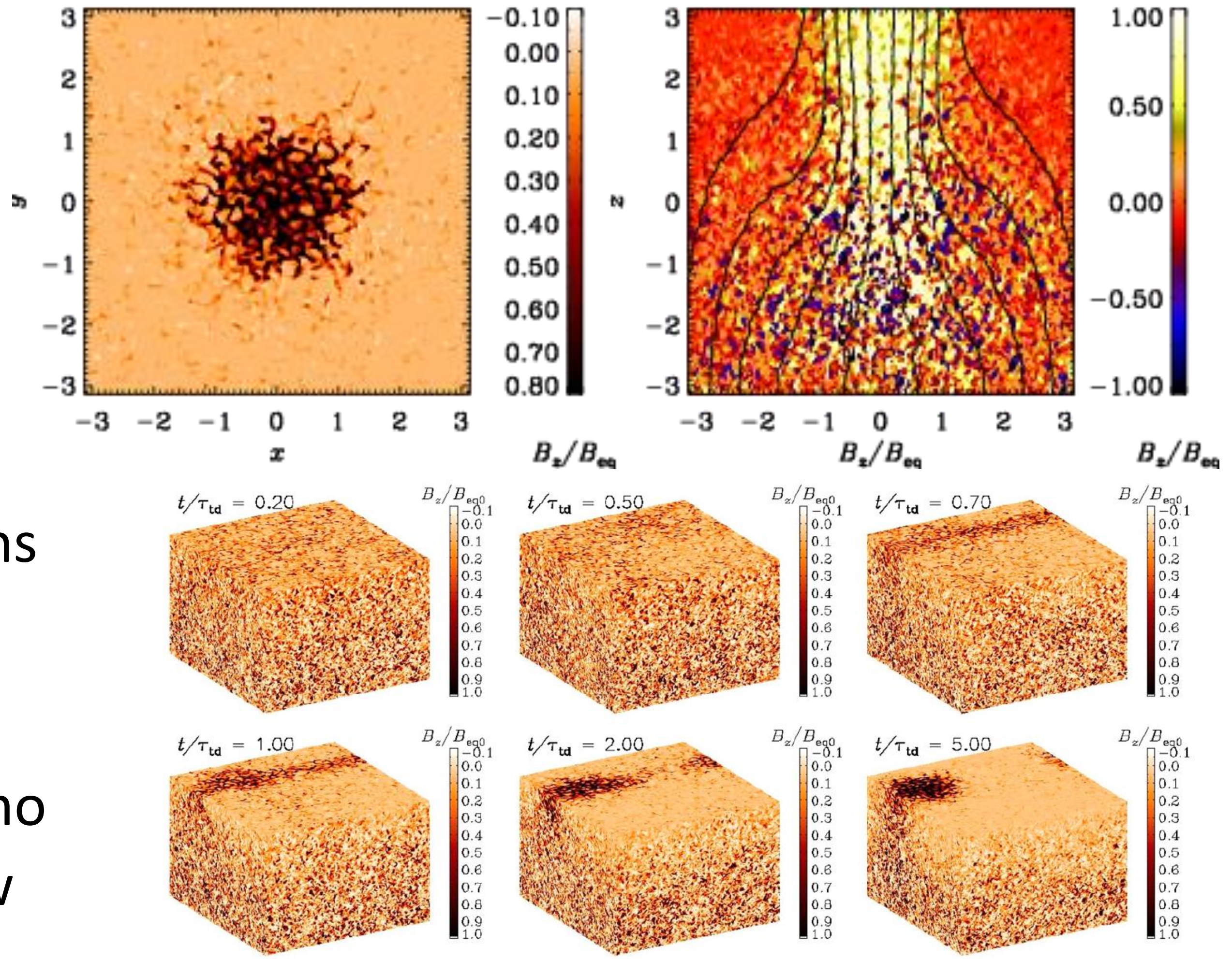
$$\boldsymbol{F}_{\mathrm{D}} = -\tau_{\mathrm{red}} \overline{s^2}\, \boldsymbol{g}\, \overline{\rho}\, \overline{T} / c_P$$

Brandenburg16

Another missing piece: surface appearence

- Stratified MHD turbulence produces spots
  - Even without convection
  - Can form + disappear in days
  - Strong scale separation required
  - Best in forced turbulence
  - Unclear how important for the Sun
- Buoyant rise picture questionable
  - Expansion during ascent
  - Slender tubes not seen in simulations
  - Anticipated role of tachocline?
- Link between dynamo & butterfly
  - Must be integral part of solar dynamo
  - Surface appearance possibly shallow

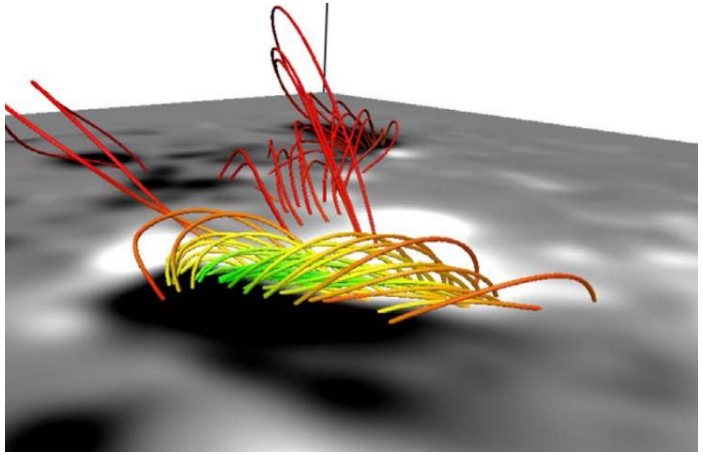
Brandenburg+13



Magnetic helicity

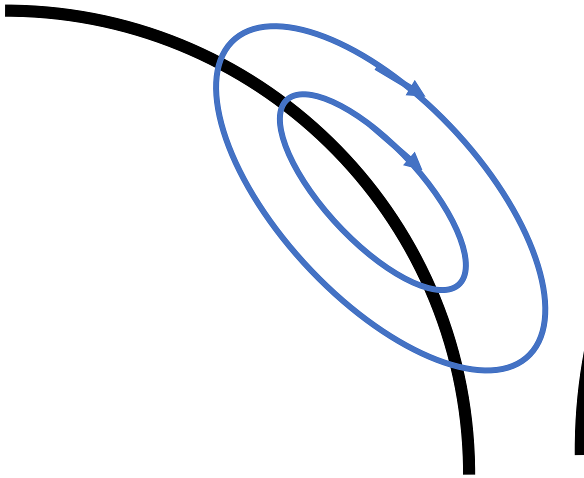
What	How	What next?
What is mean-field dynamo theory?	Large-scale field, 11 yr cycle	OK for the Sun, not other stars
Global simulations & mean-fields	NSSL (near surface shear layer) TFM (test-field method) Deep weakly subadiabatic Sunspots, link to the dynamo	New simulation setups Targetted simulations Detailed study of idealizations
→ Importance of magnetic helicity	Emerging twist in ARs Magnetic helicity in solar wind	Too much? Wrong sign? → Combined simulations

- Emergent twist seems very strong
  - Where from?
- Theoretically expected from dynamos
  - But maybe not so much
  - Alleviates catastrophic quenching

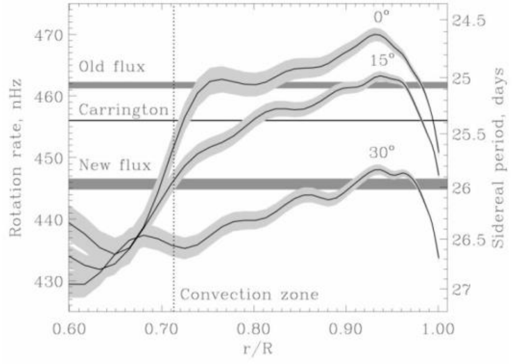


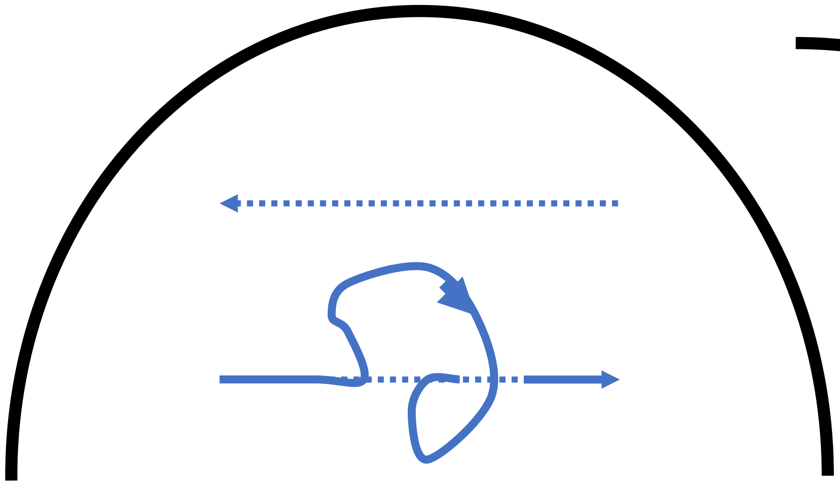
Vemareddy 2019 (NLFFF for AR12673)

Kinetic helicity in the solar dynamo



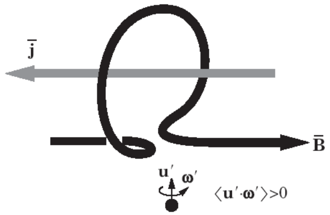
Differential rotation
(faster inside)



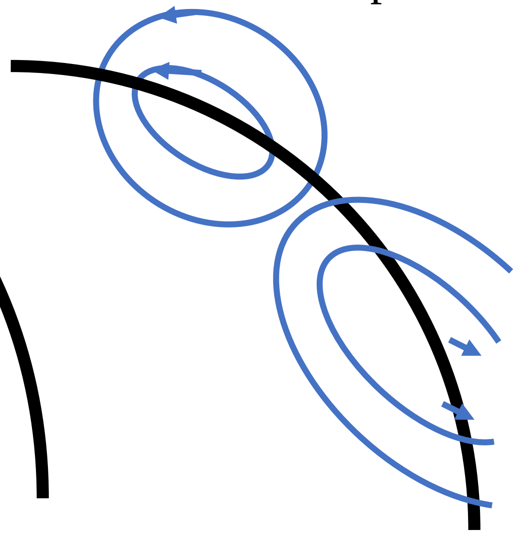


Cyclonic convection;
Buoyant flux tubes

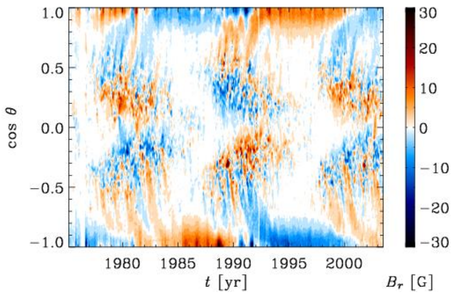
→ α-effect



New loop



Equatorward
migration



Crafoord prize in Astronomy 2020

"for pioneering and fundamental studies of the solar wind and magnetic fields from stellar to galactic scales"

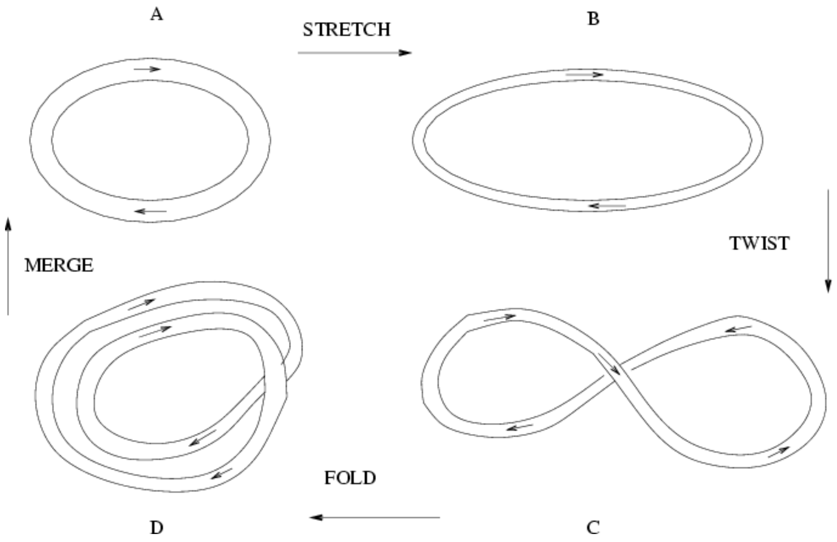
- Kinetic helicity
  - <**ω**.*u*>, **ω**=curl***u***
  - Alpha effect in dynamo theory
  - <***u***x***b***>=α<***B***>

Magnetic helicity conservation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \tfrac{1}{2}\mathbf{B}^2 \right\rangle = -\left\langle \mathbf{u}\cdot\left(\mathbf{J}\times\mathbf{B}\right)\right\rangle - \eta\left\langle \mathbf{J}^2 \right\rangle$$

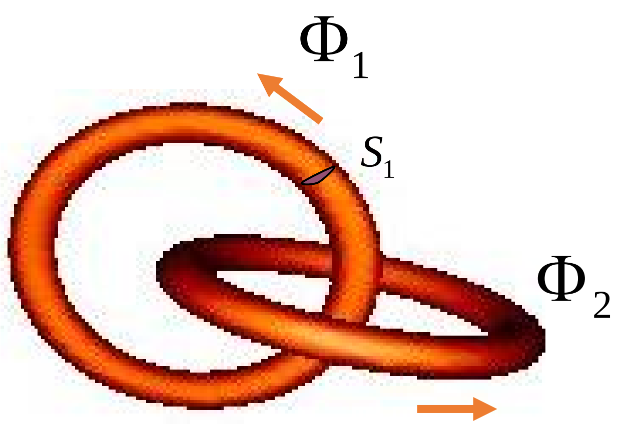
J diverges as η→0

$$J \propto \eta^{-1/2} \propto kB \propto k$$

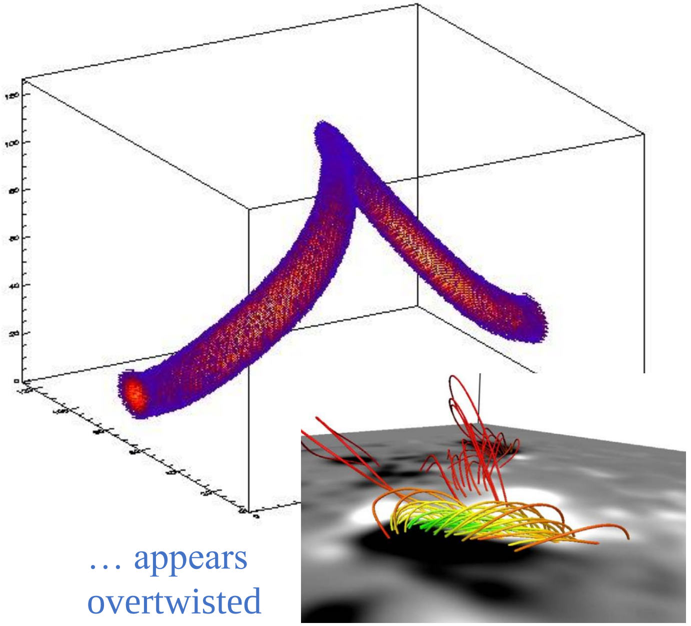


$$\tfrac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \mathbf{A}\cdot\mathbf{B} \right\rangle = -\left\langle \mathbf{u}\cdot\left(\mathbf{B}\times\mathbf{B}\right)\right\rangle - \eta\left\langle \mathbf{J}\cdot\mathbf{B} \right\rangle \to \eta\eta^{-1/2} = \eta^{1/2} \to 0$$

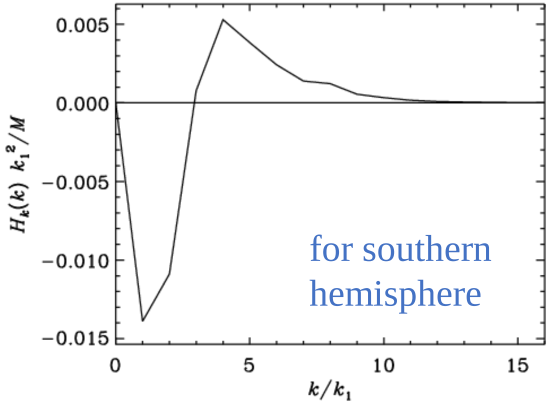
$$\int H(k)dk = \left\langle \mathbf{A}\cdot\mathbf{B} \right\rangle = \left\langle \overline{\mathbf{A}}\cdot\overline{\mathbf{B}} \right\rangle + \left\langle \mathbf{a}\cdot\mathbf{b} \right\rangle$$



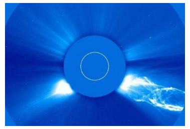
$$H = \pm 2\Phi_1\Phi_2$$



… appears overtwisted

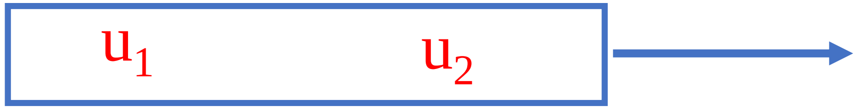


Expell as waste



Ulysses in situ measurements → wrong sign

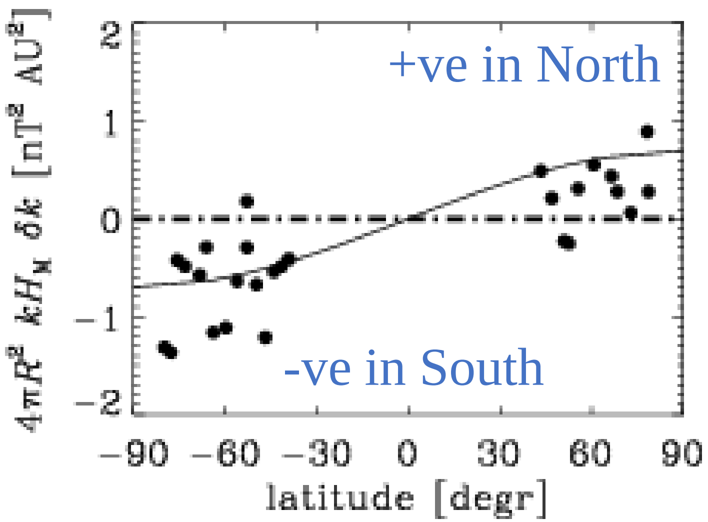
Taylor hypothesis:

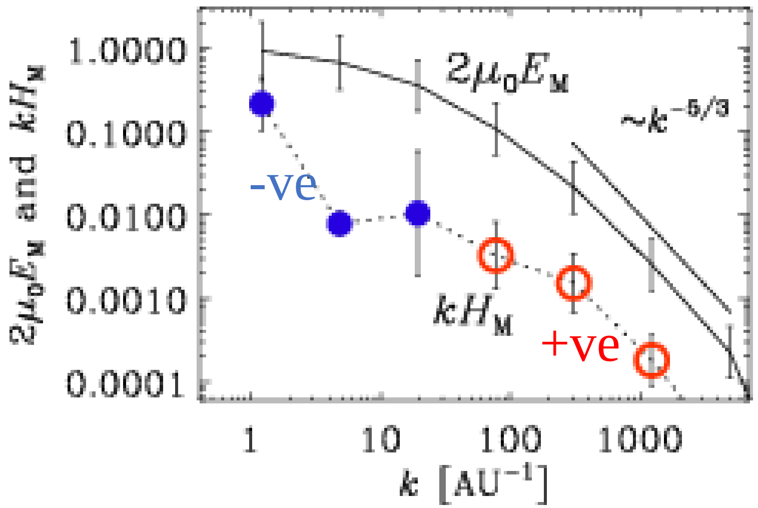


Chunk of turbulence passing by space craft

$$R = R_0 - u_R t$$

$$H_M^{1D}(k_R) = 4\,\mathrm{Im}(\hat{B}_T \hat{B}_N^{\star})/k_R.$$





- Broad *k* bins
- Southern latitude with opposite sign
- Small/large distances
- Positive *H* at large *k*
- Break point with distance to larger *k*

1500 AU^{-1} ~ 0.01 Mm^{-1}

1 AU^{-1} ~ 1 μHz

Change of sign: (i) in latitude, (ii) in scale

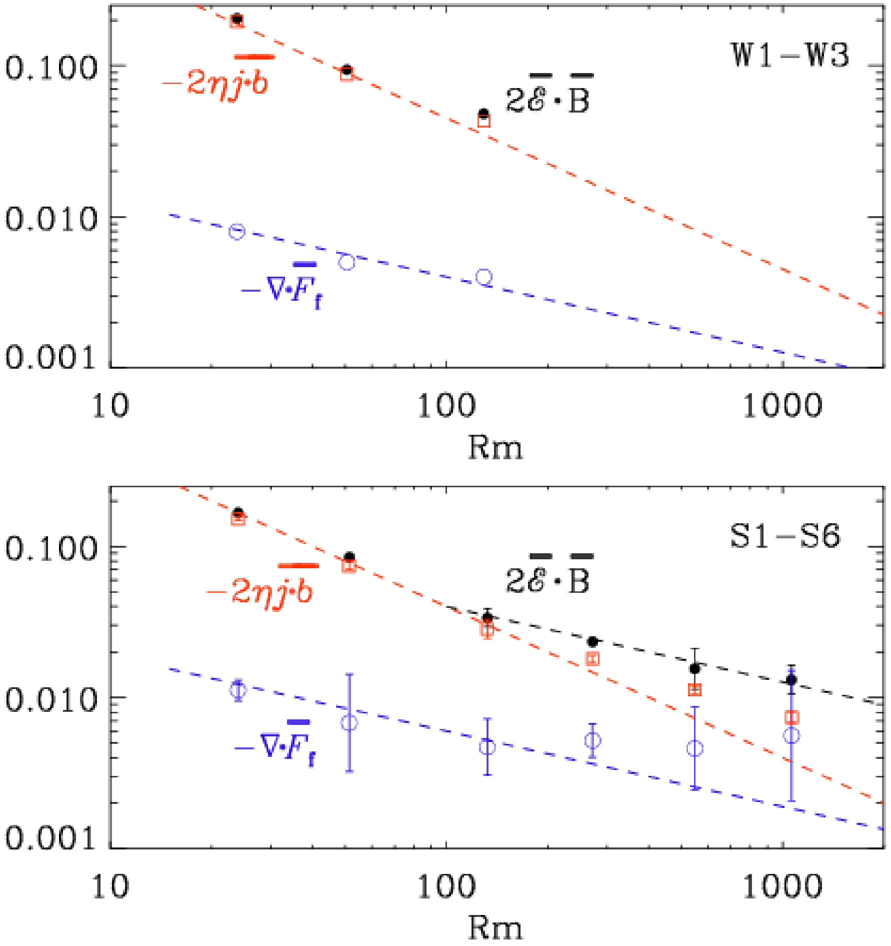
Magnetic helicity flux in simulations

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \overline{\mathbf{A}}\cdot\overline{\mathbf{B}}\rangle = +2\langle \overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}}\rangle - 2\eta\langle\overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle - \nabla\cdot\overline{\boldsymbol{\mathcal{F}}}_{\mathrm{m}}$$

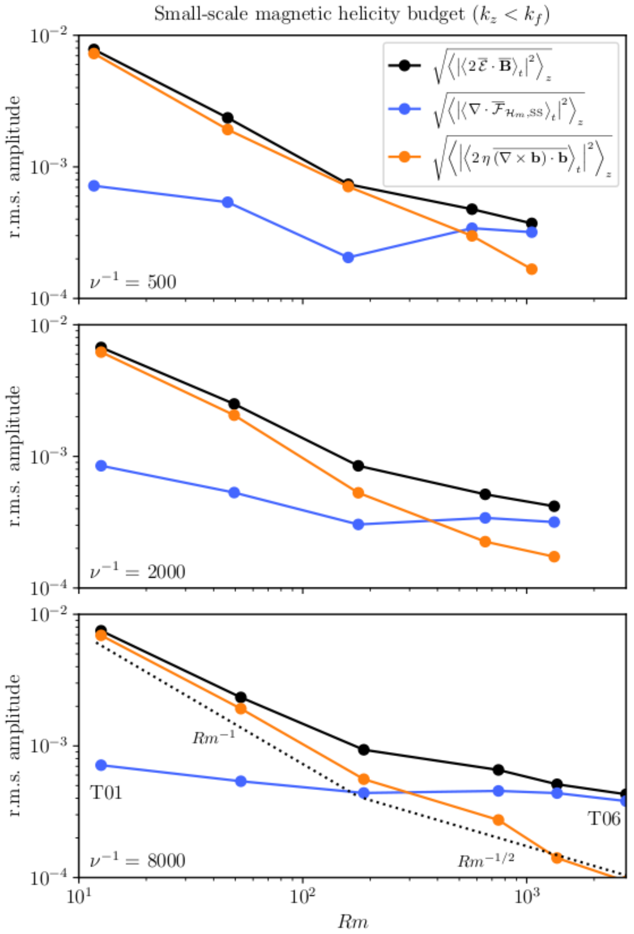
$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \mathbf{a}\cdot\mathbf{b}\rangle = -2\langle \overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}}\rangle - 2\eta\langle\mathbf{j}\cdot\mathbf{b}\rangle - \nabla\cdot\overline{\boldsymbol{\mathcal{F}}}_{\mathrm{f}}$$

- EMF and resistive terms still dominant
  - Fluxes import at large Rm ~ 1000
  - Rm based on k_f
  - Smaller by 2π
- Here for ``galactic'' wind
  - Not yet done for CMEs
  - Need 10^{46} Mx^2/cycle
  - Also through equator

Gauge-invariant in steady state!



Rincon21: equatorial fluxes



Del Sordo, Guerrero, Brandenburg (2013, MNRAS 429, 1686)

Conclusions

- Sun's spatio-temporal order remarkable, even by comparison with other stars (e.g., from Wilson sample; Baliunas+95)
- Global simulations not there yet (NSSL, deep subadiabatic layer)
- Inconsistencies/puzzles regarding magnetic helicity

What	How	What next?
What is mean-field dynamo theory?	Large-scale field, 11 yr cycle	OK for the Sun, not other stars
Global simulations & mean-fields	NSSL (near surface shear layer) TFM (test-field method) Deep weakly subadiabatic Sunspots, link to the dynamo	New simulation setups Targetted simulations Detailed study of idealizations
Importance of magnetic helicity	Emerging twist in ARs Magnetic helicity in solar wind	Too much? Wrong sign? → Combined simulations