Quantum field measurements with(out) superluminal signalling

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work with José de Ramón, Eduardo Martín-Martínez

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Theory of measurements/operations for QFT

How to do quantum operations in a relativistic spacetime?
- i.e. how to introduce agents
- finite times (push beyond S-matrix)

Contexts:
- Time machine paradoxes, BH entropy, …
- QI experiments in relativistic regime (q. optics, cQED, …)
- Foundational question: What can be measured in principle?
QM operations → QFT?

Naively, can use standard idealised measurements from QM

- PVMs: projectors \[ \rho \to P_a \rho P_a \]
e.g., from self-adjoint \[ A = \sum_a a P_a \to \langle A \rangle \]

- POVMs: \[ \rho \to M_i \rho M_i^\dagger \]

- Channels (CPTP maps): \[ \rho \to \sum_i M_i \rho M_i^\dagger \]
  \[ \text{s.t.} \quad \sum_i M_i^\dagger M_i = 1 \]

Does relativity impose further requirements?
Impossible Measurements on Quantum Fields*

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Abstract
It is shown that the attempt to extend the notion of ideal measurement to quantum field theory leads to a conflict with locality, because (for most observables) the state vector reduction associated with an ideal measurement acts to transmit information faster than light. Two examples of such information-transfer are given, first in the quantum mechanics of a pair of coupled subsystems, and then for
Sorkin’s Impossible Measurement

- **Initial state** $\rho_0 = |0\rangle\langle 0|$
- **Operation in A**
  $$U = e^{i\lambda \phi(y)} \quad (\lambda \in \mathbb{R})$$
  $$\rho = U \rho_0 U^\dagger$$
- **Measurement in B (unconditional)**
  $$P = |\psi\rangle\langle \psi|$$
  with $|\psi\rangle := \alpha|0\rangle + \beta|1\rangle \quad (|1\rangle \in \mathcal{H}_1)$
  $$\rho' = P \rho P + (1 - P) \rho (1 - P)$$
- **Expectation value in C**
  $$\langle \phi(x) \rangle_{\rho'} \quad \text{depends on } \lambda$$
Sorkin’s Impossible Measurement

- Initial state \( \rho_0 = |0\rangle \langle 0| \)
- Operation in A
  \[ U = e^{i\lambda \phi(y)} \quad (\lambda \in \mathbb{R}) \]
  \[ \rho = U \rho_0 U^\dagger \]
- Measurement in B (unconditional)
  \[ P = |\psi\rangle \langle \psi| \]
  with \( |\psi\rangle := \alpha |0\rangle + \beta |1\rangle \quad (|1\rangle \in \mathcal{H}_1) \)
  \[ \rho' = P \rho P + (1 - P) \rho (1 - P) \]
- Expectation value in C
  \[ \langle \phi(x) \rangle_{\rho'} \text{ depends on } \lambda ! \]
Implications

Assuming theory should not admit superluminal signalling of this kind…

- Sorkin’s quantum measure (path integral); OR
- (Operator framework): not every idealized quantum operation is admissible
  - Measurement postulates vs. Relativistic spacetime structure
    \[
    A = \sum_a a P_a \\
p_a = \text{tr}(\rho P_a) \quad \text{vs.} \quad [\phi(x), \phi(y)] = 0, \\
\rho \rightarrow P_a \rho P_a \\
\text{for } (x - y)^2 > 0 \text{ (spacelike)}
    \]
  - Counterexample to: self-adjoint \(\implies\) observable
Aim

- What measurements are/are not allowed?
  - What precisely is the issue with Sorkin’s measurement?

- Would like:
  - List of observables, e.g., $H, P, N, \phi(f), \phi(f)^2, \ldots$?
  - Characterization of allowable PVMs/POVMs, channels
How would these measurements be performed?

- Measurements via interactions
  - Idealized von Neumann measurement

\[
\text{probe: } |0\rangle_P \quad \text{system: } \sum_a \psi_a |a\rangle_S \quad U_I \quad \sum_a \psi_a |a\rangle_S |a\rangle_P
\]
How would these measurements be performed?

- Measurements via interactions
  - Idealized von Neumann measurement

channel on system \[ \mathcal{U} : \rho \mapsto \sum_a P_a \rho P_a \]
Causality condition

Formulate condition that operations should satisfy:

- For any map in (any) \( \mathcal{O}_1 \)
  \[ \Lambda : \mathcal{A}(\mathcal{O}_1) \to \mathcal{A}(\mathcal{O}_1) \]
- Operation (CP, unital) in \( \mathcal{O}_2 \)
  \[ \mathcal{U} : \mathcal{A}(\mathcal{O}_2) \to \mathcal{A}(\mathcal{O}_2) \]

should be s.t.
- any state \( \omega : \mathcal{A}(\mathcal{M}) \to \mathbb{C} \)
- any \( A \in \mathcal{A}(\mathcal{O}_1^\perp) \)

\[ \omega(\Lambda \circ \mathcal{U}(A)) = \omega(\mathcal{U}(A)) \]
Measurements via interactions with probes

• Sorkin-type impossible measurement

\[ H_I(t) = \lambda \mu(t) \otimes \phi(f)^2 \]

\[ = \lambda \mu(t) \otimes \int dx dy f(t, x) f(t, y) \phi(t, x) \phi(t, y) \]

• Allowed, e.g., Weyl channels

\[ \rho \rightarrow \sum_a p_a e^{-ia\phi(f)} \rho e^{ia\phi(f)} \]
Issues with internal dynamics

- Non-relativistic probes highly constrained

\[ H = H_F \otimes 1 + 1 \otimes H_D + H_I \]

Unruh-deWitt-like model

\[ H_I = \lambda p(t) \otimes \phi(f) \]
Towards a general characterization

- Non-relativistic probes generate examples of allowable measurements/operations
- May not exhaust set of allowable idealized operations
  - (If not, some kind of dilation theorem?)
- Source in the classical interaction?
- Eventually should take axiomatic approach to avoid proving existence of general classes of models
Summary

• Gained insight into issues causing superluminal signalling for idealized measurements
  – Non-local interaction terms, internal probe dynamics, …

• Simple class (Weyl channels) of allowable operations generated by non-relativistic probes

• **Message:** be careful when performing quantum measurements/operations on relativistic spacetime!