Contrary Inferences for Classical Histories in the Consistent Histories Approach

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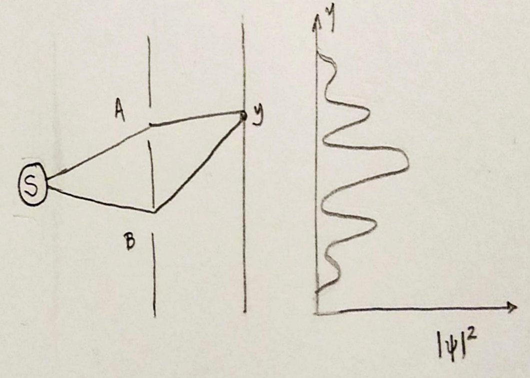
Motivation for alternative interpretations of quantum theory

- **Abandon** separation of classical and quantum world
- Quantum theory of **closed systems** such as the **universe**
- Copenhagen interpretation cannot deal with questions involving time



Overview

1. Consistent histories formalism
2. Contrary inferences
3. The semiclassical time-of-arrival problem
4. Discussion

Consistent histories formalism

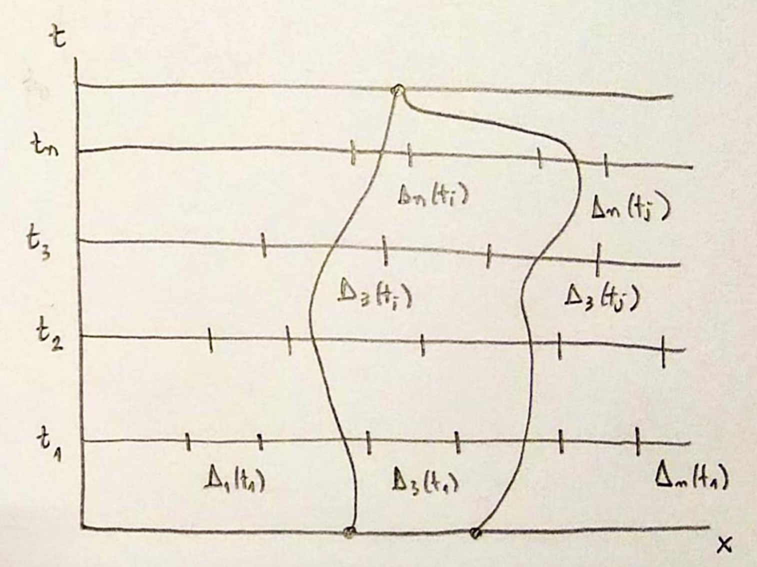
1. **Fine-grained histories** the most accurate information for the physical system, represented by a set of generalised coordinates $q^i(t)$.
2. **Coarse-grained histories** result from a smoothening of the configuration space, represented by exhaustive sets of regions of the space at different times
3. **Decoherence functional** $D(A,B) = Tr(C_A \rho C_B^\dagger)$ where

$$C_A = P_{\alpha_n} e^{-iH(t_n - t_{n-1})} ... P_{\alpha_2} e^{-iH(t_2 - t_1)} P_{\alpha_1} e^{-iH(t_1 - t_0)}$$

   (i) **Hermiticity** $D(A,B) = D^*(B,A)$
   (ii) **Positivity** $D(A,B) > 0$
   (iii) **Normalisation** $\sum_{A,B} D(A,B) = 1$
   (iv) **Superposition principle** $D(A,B) = \sum_{\alpha \in A} \sum_{\beta \in B} D(\alpha, \beta)$

4. **Candidate probability** $p(A) = D(A,A)$
5. **Consistency condition** $D(A,B) \approx 0$

Consistent histories formalism

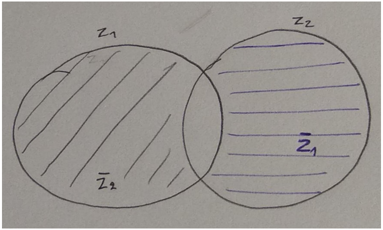


Contrary inferences

Assume two propositions with corresponding projection operators P and Q at one moment of time.

- If $[P, Q] \neq 0$, i.e. they do not commute, P, Q are called **complementary**.
- If they are orthogonal, $PQ = QP = 0$ and add to the identity, $P = 1 - Q$ they are called **contradictory**.
- If they are orthogonal and not contradictory so that $P < 1 - Q$ then they are called **contrary**.
- **Contrary inference**: when two contrary propositions are both implied with **probability one** - Not possible in classical logic.
- BUT possible in contextual logic, provided there does not exist any context containing both propositions.

Zero covers of the configuration space



- $\{Z_1, Z_2\}$ is **zero cover measure of** Ω if $\mu(Z_1) = 0, \mu(Z_2) = 0$ and $Z_1 \cup Z_2 = \Omega,\ Z_1 \cap Z_2 \neq \varnothing$
- **Contrary inferences** when two consistent sets are defined in Ω as:

$$C_1 = \{Z_1, \bar{Z}_1\}, \quad \text{where} \quad \mu(Z_1) = 0, \mu(\bar{Z}_1) = 1,$$
$$C_2 = \{Z_2, \bar{Z}_2\}, \quad \text{where} \quad \mu(Z_2) = 0, \mu(\bar{Z}_2) = 1$$

- $\bar{Z}_1 \cap \bar{Z}_2 = \varnothing$ and $\bar{Z}_1 \subsetneqq Z_2,\ \bar{Z}_2 \subsetneqq Z_1$ thus contrary propositions because e.g. $\mu(\bar{Z}_1) = 1$ and $\mu(Z_2) = 0$.

In consistent histories, every zero cover measure which contains two (coarse-grained) histories leads to contrary inferences (theorem).

The arrival time problem in quantum theory

What is the probability to find the particle at the interval Δ **at any time** t? instead, we will ask

What is the probability to find the particle at the interval $\bar{\Delta}$ **at a specific time** t*?*

1. $\mathcal{Q} = \Delta \cup \bar{\Delta}$
2. $C_{\bar{\Delta}} = g_{\bar{\Delta}}(t, t_0), \quad g(t, t_0) = e^{iHt/\hbar}, \quad C_{\Delta} = g_{\Delta}(t, t_0)$
3. $g_{\bar{\Delta}} = |\psi_r(t)\rangle$
4. $p_{\bar{\Delta}} = D(\bar{\Delta}, \bar{\Delta}) = \langle\psi| \bar{P} |\psi\rangle$,
5. $\bar{P} = g_r^{\dagger}(t, t_0) g_r(t, t_0)$ and $H_r = \bar{P} H \bar{P}$ self-adjoint in $\mathcal{H}_{\bar{\Delta}}$ when quadratic in momenta

The arrival time problem in quantum theory

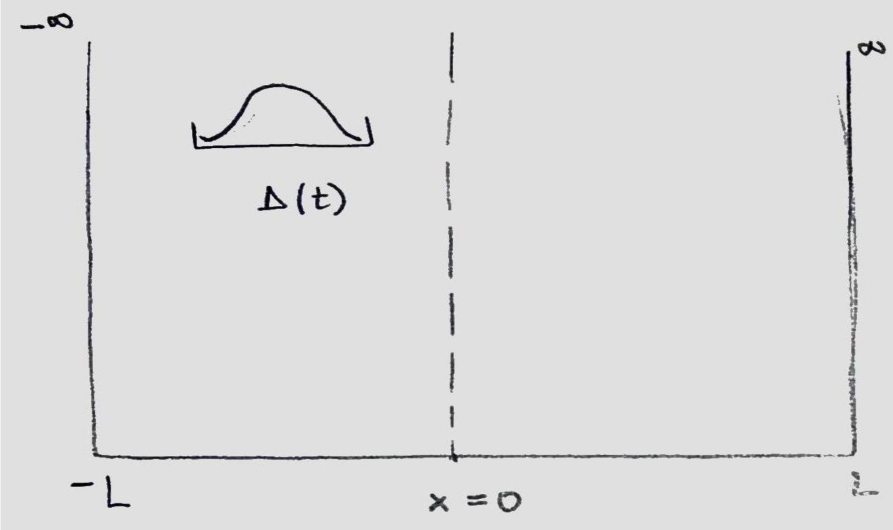
$$D(\Delta, \bar{\Delta}) = \langle\psi| C_{\Delta}^{\dagger} C_{\bar{\Delta}} |\psi\rangle \approx 0$$

This condition is satisfied when

1. boundary condition $\langle x|\psi_0\rangle_{\partial\Delta} = 0$ (so that $g_r(t, t_0) |\psi\rangle = |\psi_r(t)\rangle$)
2. $\langle\psi(t)|\psi_r(t)\rangle = \langle\psi(t_0)| \bar{P} |\psi(t_0)\rangle = 1$

We will find two sets which satisfy the above conditions and show they are consistent.

First consistent set

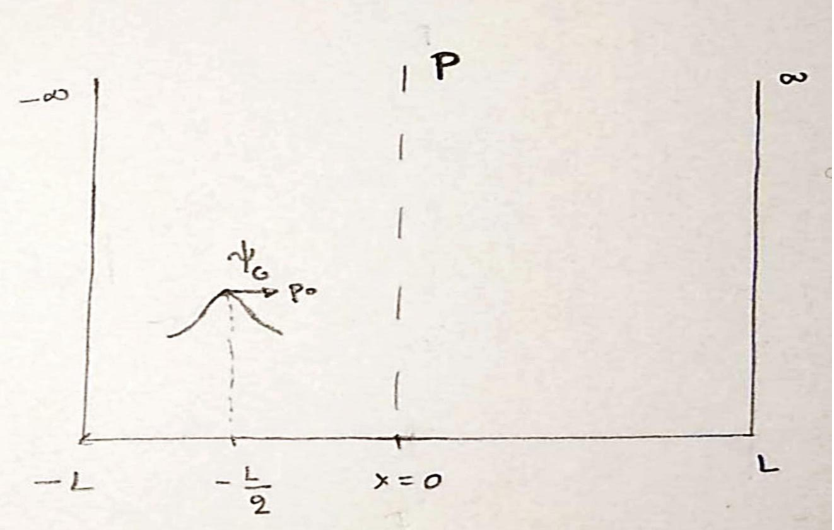


First consistent set

- **Gaussian wave packet** $\Psi_G(x,0) = Ae^{-(x+L/2)^2/4\sigma^2}e^{ip_0(x+L/2)/\hbar}$
- **Coarse-graining** $\Delta(t) = [-L+ut, 0+ut]$
- $A = \frac{2\sqrt{5}}{\sqrt{\sqrt{2\pi}\,\text{erf}\,\frac{5(L-2ut)}{\sqrt{2}}+\sqrt{2\pi}\,\text{erf}\,\frac{5(L+2ut)}{\sqrt{2}}}}$
- For every t, the overlap is $\langle\Psi(x,t)|\Psi_r(x,t)\rangle \approx 1$

Thus, when we ask if the particle has passed to the positive axis, the answer is YES!

Second consistent set



Second consistent set

- $\Psi_r(x,0) = \frac{1}{\sqrt{\sigma\sqrt{2\pi}\,\text{erf}(\frac{L}{2\sqrt{2}\sigma})}} e^{-(x+L/2)^2/4\sigma^2} e^{ip_0(x+L/2)/\hbar}$
- $\Psi(x,0) = \frac{1}{\sqrt{\sigma\sqrt{2\pi}\frac{1}{2}\left(\text{erf}(\frac{L}{2\sqrt{2}\sigma})+\text{erf}(\frac{3L}{2\sqrt{2}\sigma})\right)}} e^{-(x+L/2)^2/4\sigma^2} e^{ip_0(x+L/2)/\hbar}$
- $\langle\Psi_r(t_0)|\Psi(t_0)\rangle \approx 1$ because of the initial condition
- $\langle\Psi_r(t_i)|\Psi(t_i)\rangle \approx 1$
- $\Psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n e^{-iE_n t/\hbar}$
- $u_n = \sin\frac{n\pi x}{L}$, $v_m = \sin\frac{m\pi(x+L)}{2L}$
- **Overlap**

$$A(t) = \langle\Psi_r(x,t)|\Psi(x,t)\rangle = \sum_{n,m=1}^{\infty} (c_n^L)^* c_m^{2L} e^{iEt/\hbar(n^2-m^2/4)} \int_{-L}^{0} u_n(x) v_m(x) dx$$
$$= 1 - \epsilon(t)$$

- Classical limit ($L = 2, p_0 = 1, m = 1, \hbar = 1, \sigma = L/10$)

$$A(T_{rev} \approx 8) = 0,947 + 0,001i, \quad |A(T_{rev})|^2 = 0,897$$

The answer to whether the particle passed the $x = 0$ is NO!

Contrary inferences in the classical arrival time problem

Histories in the configuration space $\Omega = \{h_1, h_2, h_3, h_4\}$

$h_1 =$ paths which follow the particle

$h_2 =$ paths which do not follow the particle

$h_3 =$ paths that remained in the negative axis all time from $t = 0$ till time t

$h_4 =$ paths that at some time within $[0, t]$ crossed to the positive axis

Consistent sets $C_1 = \{\{h_1\}, \{h_2\}\},\ C_2 = \{\{h_3\}, \{h_4\}\}$

$$h_1 \subset h_4,\ h_1 \cup h_4 = \Omega,\ h_1 \cap h_4 \neq \varnothing$$
$$\mu(h_1) = 1,\ \mu(h_4) = 0$$

Thus there are contrary inferences!

Discussion

1. Approximate decoherence
2. This problem exists even in quantum histories, even though it can be avoided.
3. At the classical limit it is more severe, since we cannot even make predictions for the semiclassical states, even when have already intuition.
4. CH without **selection criterion** cannot recover classical intuition in the classical limit.

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