Entangled histories, the two-state and the pseudo-density formalisms: Towards a better understanding of quantum temporal correlations.

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Entangled histories vs. MTS - quantum statistics

- The composite system is pre-selected at time t_1:
  $\mathcal{H}_S \otimes \mathcal{H}_A \ni |\Psi\rangle = \lambda_0 |\Psi_0 0\rangle + \lambda_1 |\Psi_1 1\rangle$
- and post-selected at time t_2:
  $|\Phi\rangle = \beta_0 |\Phi_0 0\rangle + \beta_1 |\Phi_1 1\rangle$ that leads to the entangled history $|H_{SA}) = [\Phi] \odot [\Psi]$ and to the corresponding two-time state: $|\Psi_{SA}\rangle\rangle = \langle\Phi||\Psi\rangle$.
- A measurement performed only on the system S:

$$\begin{aligned} P(A = a_n) &= \\ &= |(\lambda_0^* \langle\Phi_0 0| + \lambda_1^* \langle\Phi_1 1|)[U_2 \otimes I_A][P_N \otimes I_A][U_1 \otimes I_A] x \\ &\quad x(\beta_0 |\Psi_0 0\rangle + \beta_1 |\Psi_1 1\rangle)|^2 \\ &= |\lambda_0^* \beta_0 \langle\Phi_0 | U_2 P_n U_1 | \Psi_0\rangle + \lambda_1^* \beta_1 \langle\Phi_1 | U_2 P_n U_1 | \Psi_1\rangle|^2 \end{aligned} \tag{1}$$

exhibits a destructive interference for the ancillary system's orthogonal states at times t_2 and t_1.

E. Cohen, M. Nowakowski, Phys. Rev. **D** 97, 088501.

Quantum entanglement in time

Intrinsically consistent history on times $\{t_3, t_2, t_1, t_0\}$:

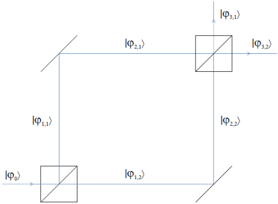
$$|\Lambda) = \alpha([\varphi_{3,1}] \odot I_{t_2} \odot [\varphi_{1,1}] + [\varphi_{3,2}] \odot I_{t_2} \odot [\varphi_{1,2}]) \odot [\varphi_0]$$

After tracing out the time t_2:

$$|\Lambda_1) = \tilde{\alpha}([\varphi_{3,1}] \odot [\varphi_{1,1}] + [\varphi_{3,2}] \odot [\varphi_{1,2}])$$

displays entanglement in time apparently.
The history $|\tau GHZ)$-like state $|\Psi)$:

$$|\Psi) = \gamma([\varphi_{3,1}] \odot [\varphi_{2,1}] \odot [\varphi_{1,1}] + [\varphi_{3,2}] \odot [\varphi_{2,2}] \odot [\varphi_{1,2}])$$



One can analyze the interferometer via four-times histories on times $t_0 < t_1 < t_2 < t_3$:

$|\varphi_0\rangle \rightarrow \frac{1}{\sqrt{2}}(|\varphi_{1,1}\rangle + |\varphi_{1,2}\rangle) \rightarrow \frac{1}{\sqrt{2}}(|\varphi_{2,1}\rangle + |\varphi_{2,2}\rangle) \rightarrow \frac{1}{\sqrt{2}}(|\varphi_{3,1}\rangle + |\varphi_{3,2}\rangle)$.

Generating quantum entanglement in time

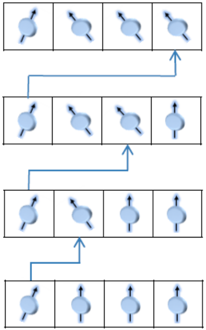
$$|\tau GHZ) = \frac{1}{\sqrt{2}}([z^+] \odot [z^+] \odot [z^+] - [z^-] \odot [z^-] \odot [z^-])$$

$$t_3 : \; |\Psi_{t_3}\rangle_{PR} = CNOT_{PR_3} \otimes \mathbb{I}_{R_1R_2} |\Psi_{t_2}\rangle_{PR} = \frac{1}{\sqrt{2}}|z^+\rangle|000\rangle + \frac{1}{\sqrt{2}}|z^-\rangle|111\rangle$$

$$t_2 : \; |\Psi_{t_2}\rangle_{PR} = CNOT_{PR_2} \otimes \mathbb{I}_{R_1R_3} |\Psi_{t_1}\rangle_{PR} = \frac{1}{\sqrt{2}}|z^+\rangle|000\rangle + \frac{1}{\sqrt{2}}|z^-\rangle|110\rangle$$

$$t_1 : \; |\Psi_{t_1}\rangle_{PR} = CNOT_{PR_1} \otimes \mathbb{I}_{R_2R_3} |\Psi_{t_0}\rangle_{PR} = \frac{1}{\sqrt{2}}|z^+\rangle|000\rangle + \frac{1}{\sqrt{2}}|z^-\rangle|100\rangle$$

$$t_0 : \; |\Psi_{t_0}\rangle_{PR} = \frac{1}{\sqrt{2}}[(|z^+\rangle + |z^-\rangle)]|000\rangle$$



- One projects the reference system onto $\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$, i.e. the external observer correlates with an arbitrary history branch.

Entangled histories from global separable histories?

Can we derive quantum entanglement in time from the following history not breaking the concept of monogamy of entanglement and the general rules of contracting tensored spaces of quantum states?

$$|H_{SA}) = \gamma[|\Phi\rangle\langle\Phi|] \odot [P_N \otimes I_A] \odot [|\Psi\rangle\langle\Psi|]$$

with bridging operators $B(t_2, t) = U_2 \otimes I_A$ and $B(t, t_1) = U_1 \otimes I_A$.

- Components of the history of this type $|h_{SA}) = [|\Phi_1 1\rangle\langle\Phi_1 1|] \odot [P_N \otimes I_A] \odot [|\Psi_0 0\rangle\langle\Psi_0 0|]$ cannot be realized (a zero-weight history $Pr(|h_{SA})) = 0$).
- Naive spatial tracing out of A leads to: $|h_S) = [|\Phi_1\rangle\langle\Phi_1|] \odot [P_N] \odot [|\Psi_0\rangle\langle\Psi_0|]$

A correct reduced entangled history:
$|H_S) = \gamma[|\Phi_0\rangle\langle\Phi_0|] \odot [P_N] \odot [|\Psi_0\rangle\langle\Psi_0|] + \gamma[|\Phi_1\rangle\langle\Phi_1|] \odot [P_N] \odot [|\Psi_1\rangle\langle\Psi_1|]$

M. Nowakowski, E. Cohen, P. Horodecki, Phys. Rev. A **98**, 032312 (2018).

M. Nowakowski, External vs. internal observer of the realized paths, In preparation.

Entangled histories vs. MTS

...is there any isomorphism?

- For the TSVF, a scalar product of a pair of vectors $|\Psi\rangle\rangle$ and $|\Phi\rangle\rangle$ in a space $\mathcal{M} = \mathcal{H}_{t_2}^{\dagger} \otimes \mathcal{H}_{t_1}$ with a basis $\mathcal{B} = \{\langle\phi_i^2||\phi_j^1\rangle\}$ is:

$$\langle\langle\Phi|\Psi\rangle\rangle = \sum_{ijkl} \alpha_{ij}\alpha_{kl}^{*}\langle\phi_i^2|\phi_k^2\rangle\langle\phi_l^1|\phi_j^1\rangle = \sum_{ijkl} \alpha_{ij}\alpha_{kl}^{*}\delta_{ik}\delta_{jl}$$

- A scalar product of a pair of history states $|\Psi)$ and $|\Phi)$ can be:

$$(\Phi|\Psi)_s \equiv Tr[|\Phi)^{\dagger}|\Psi)]$$

- An inner semi-definite product (which is not a scalar product) for history vectors $|\Psi)$ and $|\Phi)$:

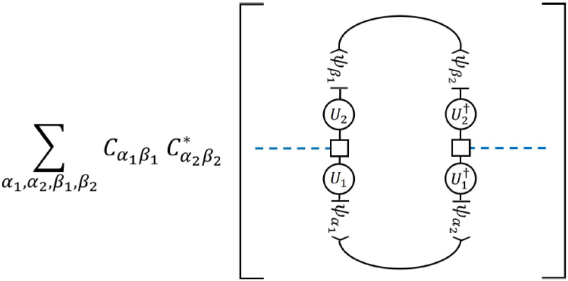
$$(\Phi|\Psi)_K = Tr[K^{\dagger}(|\Phi))K(|\Psi))]$$

- In general $\langle\langle\Phi|\Psi\rangle\rangle \neq (\Phi|\Psi)_K$. A space $\mathcal{M}$ of multi-time state vectors equipped with a scalar product $\langle\langle\cdot|\cdot\rangle\rangle$ is isomorphic to a space $\mathcal{E}$ of entangled histories equipped with a scalar product $(\cdot|\cdot)_s$.

M. Nowakowski et al., Phys. Rev. A **98**, 032312 (2018).

Superdensity formalism and what else?

- We can define unnormalized superdensity operator:
  $\rho[\mathcal{O}_1, \mathcal{O}_2^\dagger] = \sum_{\alpha_1,\alpha_2,\beta_1,\beta_2} C_{\alpha_1\beta_1} C^*_{\alpha_2\beta_2} tr(\langle\psi_{\beta 1}|U_2\mathcal{O}_1 U_1|\psi_{\alpha 1}\rangle\langle\psi_{\alpha 1}|U_1^\dagger\mathcal{O}_2^\dagger U_2^\dagger|\psi_{\beta 2}\rangle)$



Diagrammatic representation of the unnormalized super-density operator corresponding to a two-state vector, $U_1 = U(t_1; t)$ and $U_2 = U(t; t_2)$.

J. Cotler, F. Wilczek, Phys. Scripta T168, 014004 (2016).

Monogamy of quantum entanglement in time?

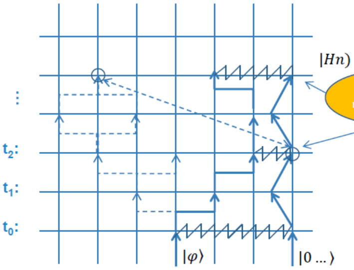
- Tsirelson's bound hold for quantum temporal correlations. For any history density matrix W and Hermitian history dichotomic observables $A_i = I \odot A_i^{(1)}$ and $B_j = B_j^{(2)} \odot I$ where $i, j \in \{1, 2\}$ the following bound holds:

$$
\begin{aligned}
S_{LGI} &= c_{11} + c_{12} + c_{21} - c_{22} \\
&= Tr((A_1 B_1 + A_1 B_2 + A_2 B_1 - A_2 B_2) W) \\
&\leq 2\sqrt{2}
\end{aligned}
$$

- Spatial monogamy relations do not hold for temporal correlations but the entanglement structure of temporal correlations seems to be similar to the spatial entanglement.Invasive measurements can destroy it.

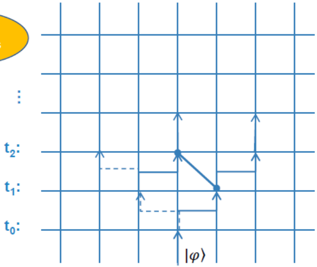
Computation on history branches

Multipartite entanglement in time and ancilla steering the evolution



Trivial CNOT on two branches (with the same bridging operators):

$$\tau\mathrm{CNOT}|H_1H_2) = |H_1H_1 \oplus H_2)$$



$\mathrm{CNOT}_{t_i t_j}$

- A history branch is a quantum state on which nature performs quantum operations.
- No-signaling and causality holds for the network.

Marcin Nowakowski, Quantum computaton in time, In preparation.

ThankYou!