School of Physics

Free University of Tbilisi

Sound Propagation in the Ellis Wormhole

22-25 Sep. 2019

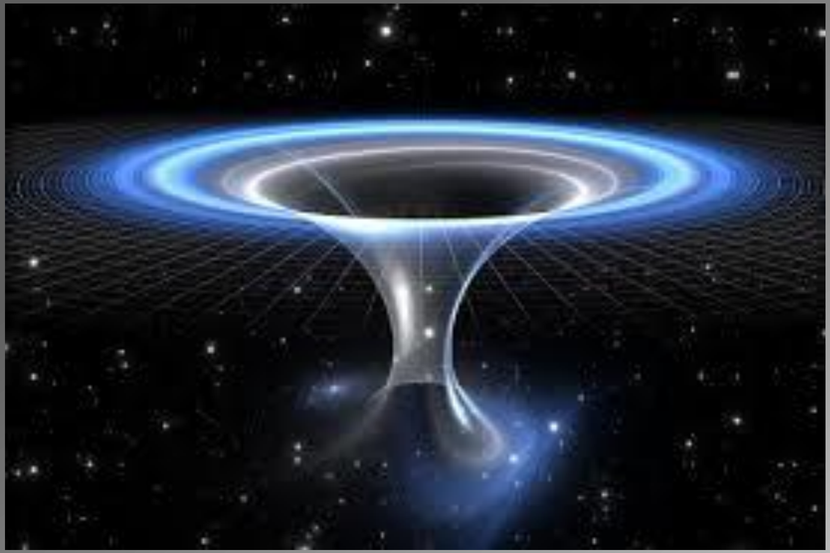
Torino, Italy

Zaza Osmanov & George Butbaia

The presentation was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [MG-TG-19-1476]

Wormhole metrics

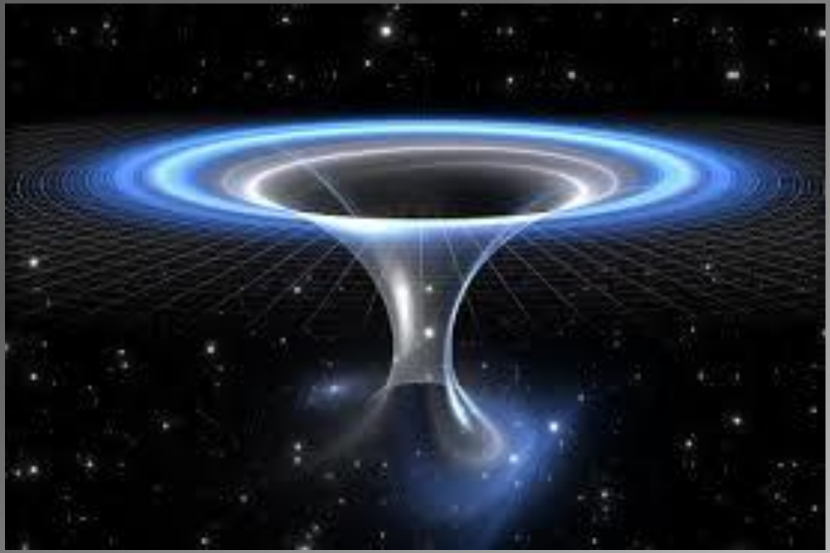
Ellis (1973)

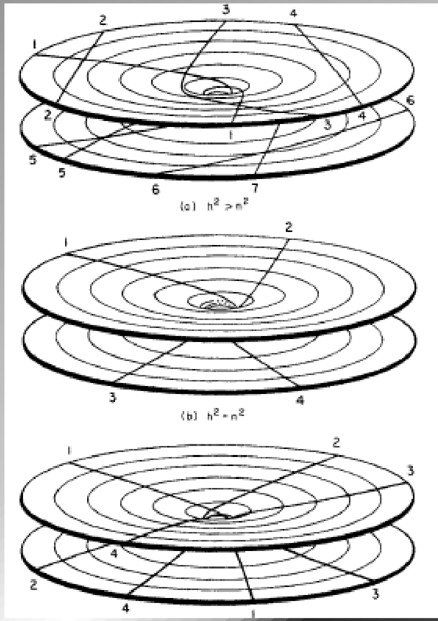


$$ds^2 = -dt^2 + dr^2 + \left(a^2 + r^2\right)\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

Wormhole metrics

Ellis (1973)



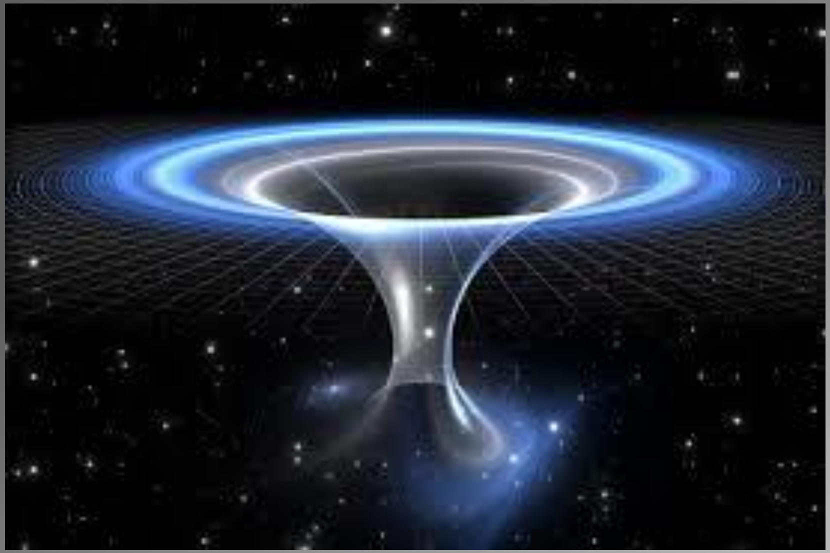


$$ds^2 = -dt^2 + dr^2 + \left(a^2 + r^2\right)\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

Flat and traversable WH

Some history: TM15

Arsenadze & Osmanov 2017



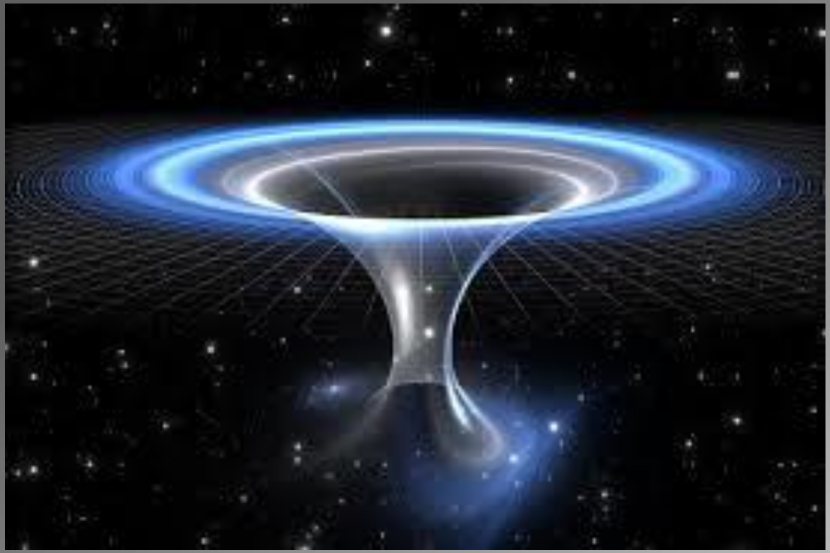
Kardashev et al. (2006,2007)

$$\frac{B^2}{8\pi} > n\varepsilon$$

Corotation

Some history: TM15

Arsenadze & Osmanov 2017



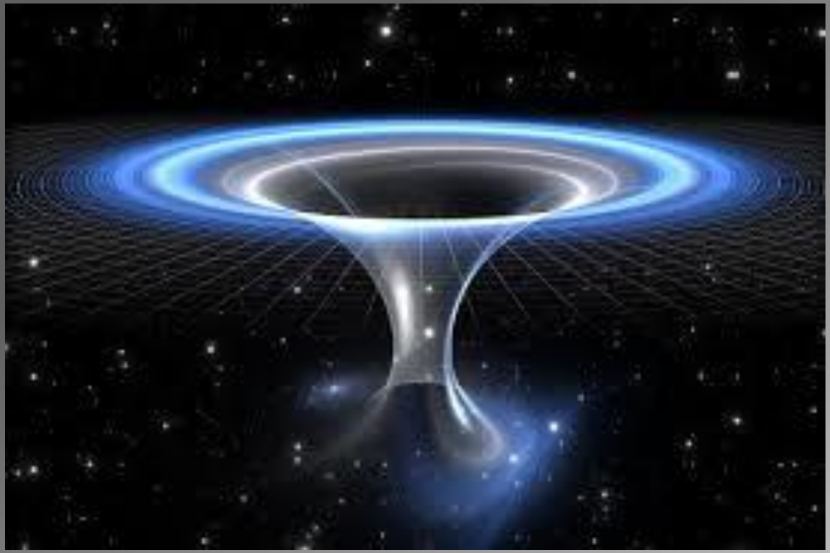
$$g_{\alpha\beta} = \begin{pmatrix} -1+\omega^2\left[b^2+r^2\right] & a\omega\left[b^2+l^2\right] \\ a\omega\left[b^2+r^2\right] & 1+a^2\left[b^2+r^2\right] \end{pmatrix}$$

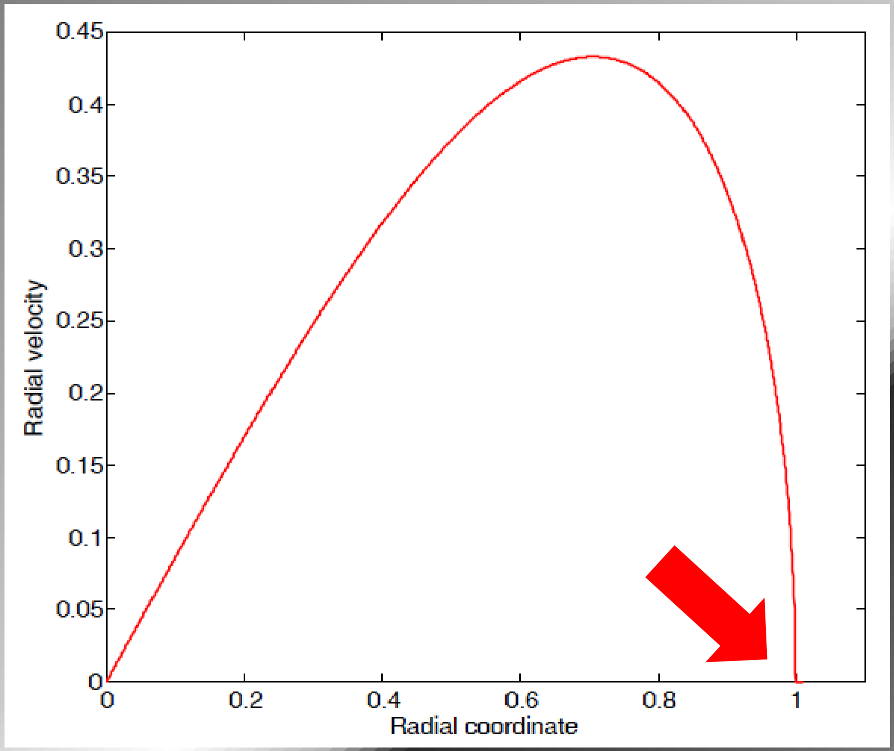
$$\theta = \frac{\pi}{2} \qquad \varphi(r) = ar$$

$$\phi = \varphi(r) + \omega t$$

Some history: TM15

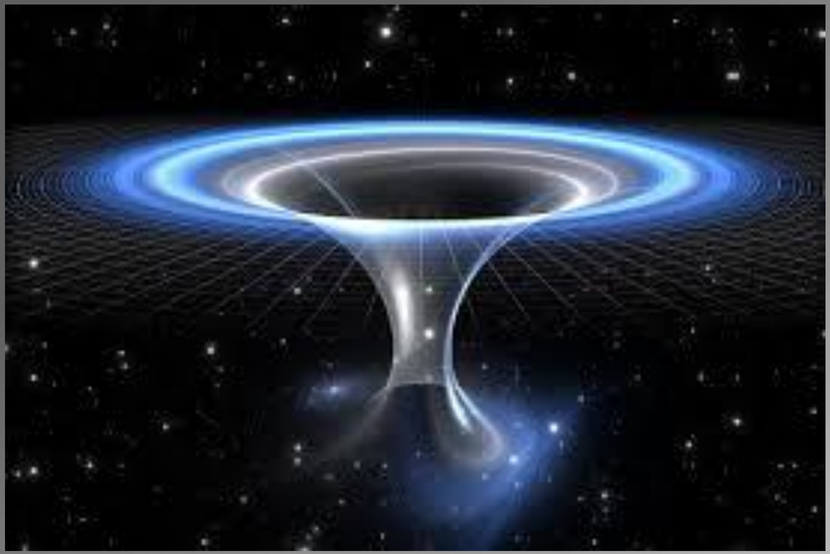
Arsenadze & Osmanov 2017

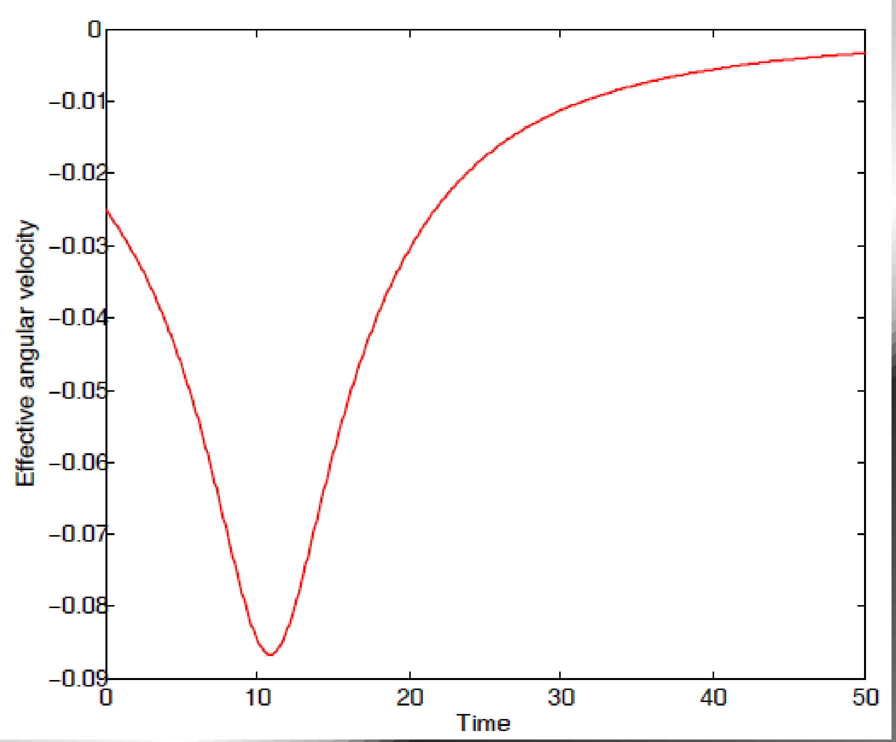




Some history: TM15

Arsenadze & Osmanov 2017

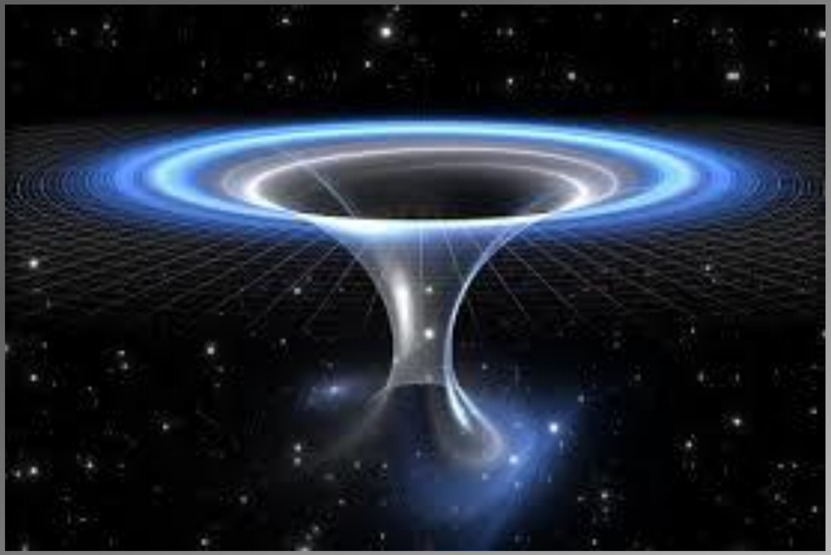


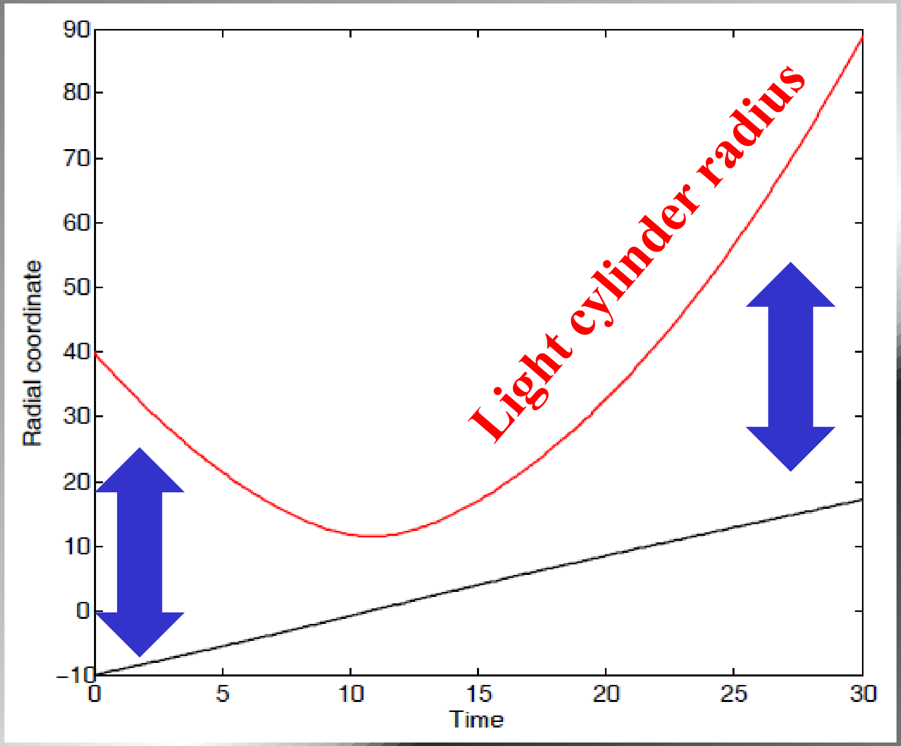


$$\Omega_{eff} \propto \frac{1}{r^2}$$

Some history: TM15

Arsenadze & Osmanov 2017

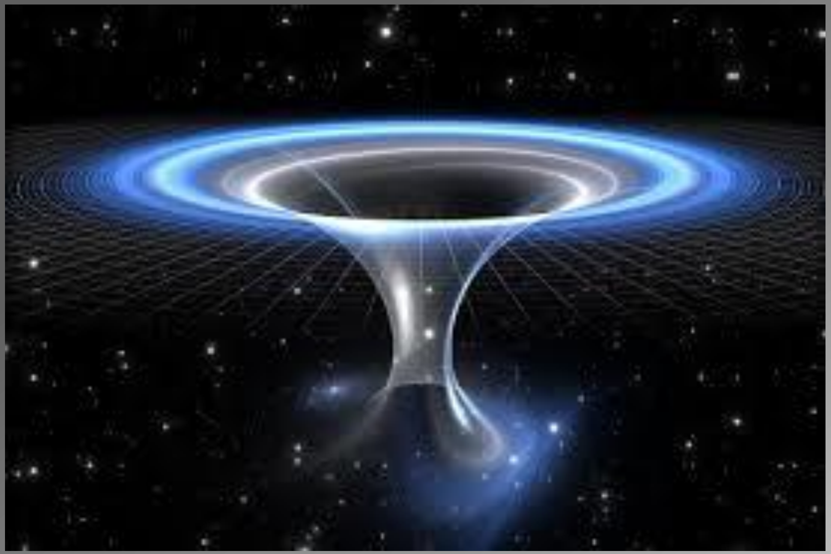


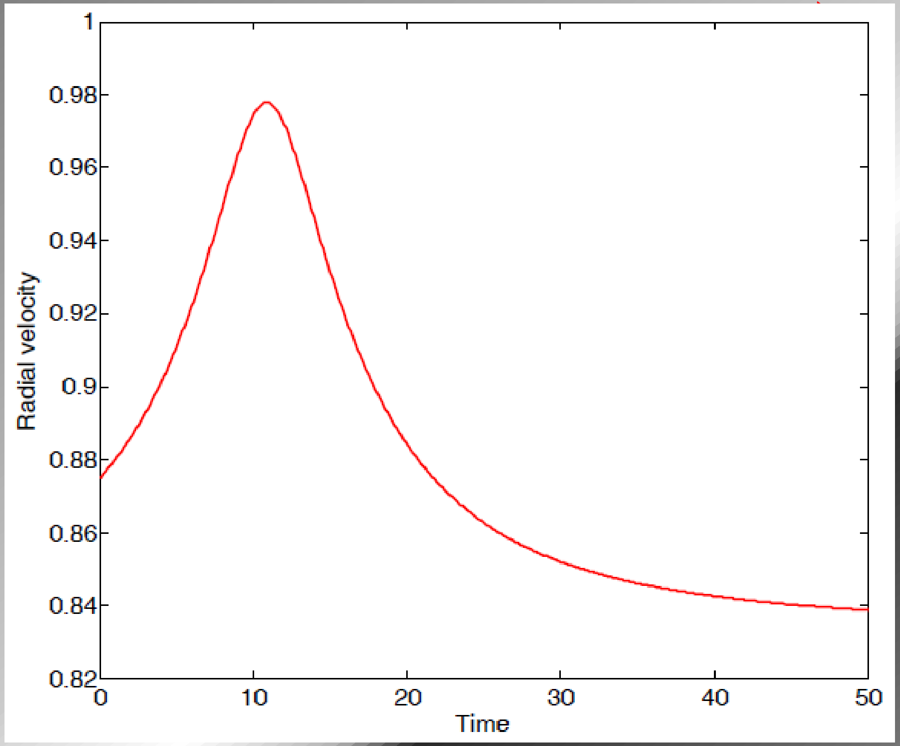


$$R_{lc} = \left(\frac{1}{\Omega^2_{eff}} - a^2\right)^{1/2}$$

Some history: TM15

Arsenadze & Osmanov 2017

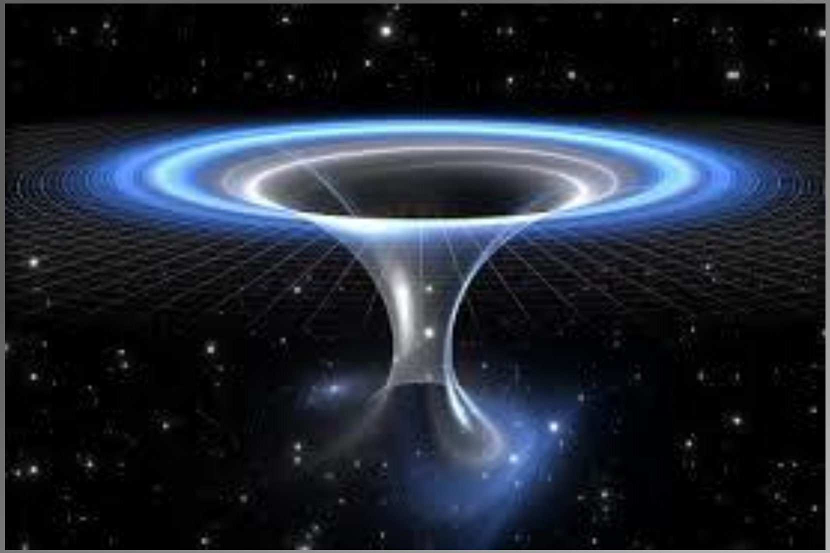


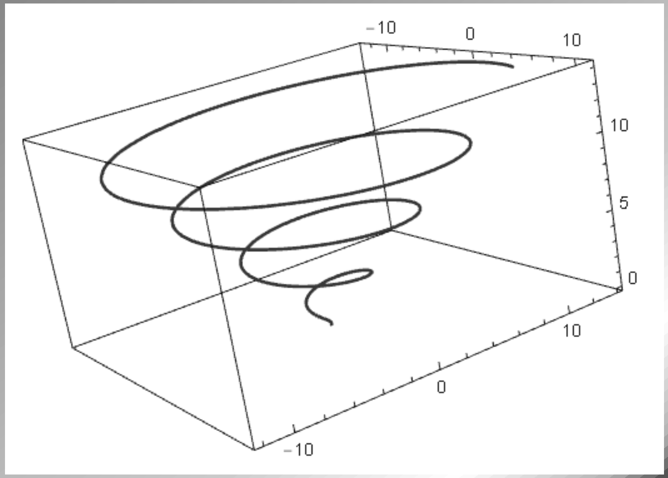


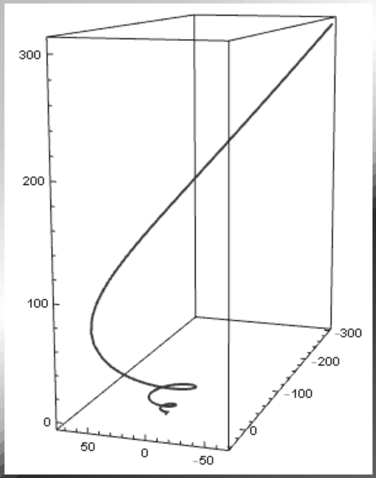
$$\upsilon = -\frac{\omega}{a} + \frac{\chi}{r^2}$$

Some history: TM15

Arsenadze & Osmanov 2017

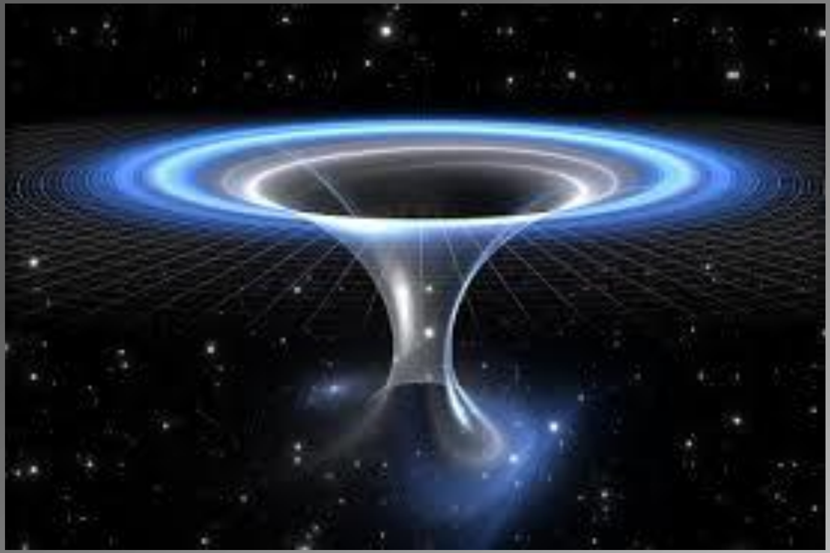


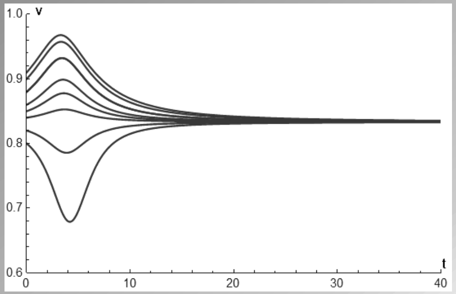


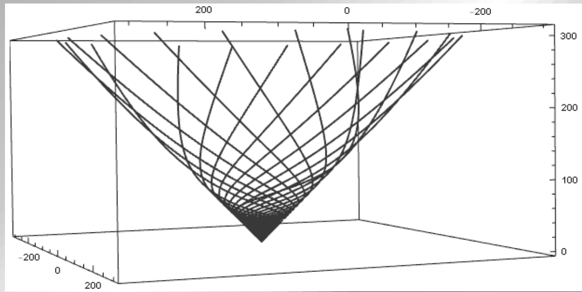


Some history: TM15

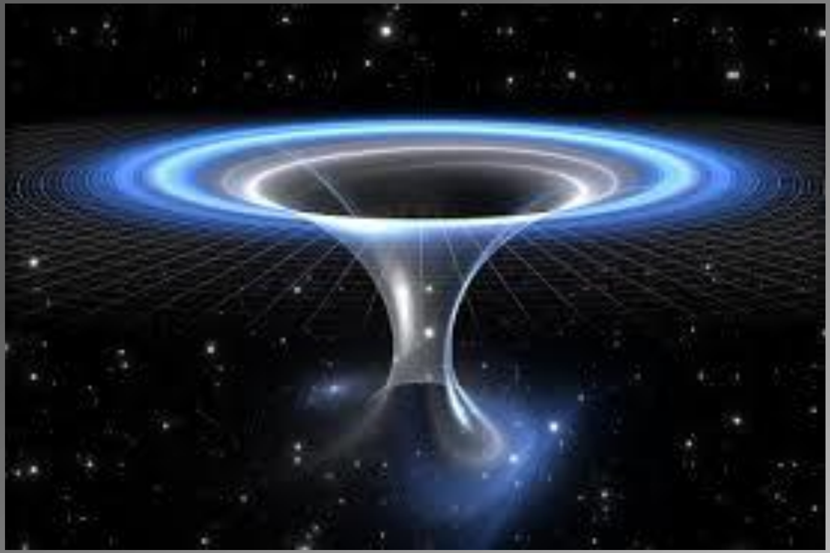
Arsenadze & Osmanov 2017







GRHD Equations



$$\nabla_\mu T^{\mu\upsilon} = 0$$

$$\nabla_\mu J^{\mu} = 0$$

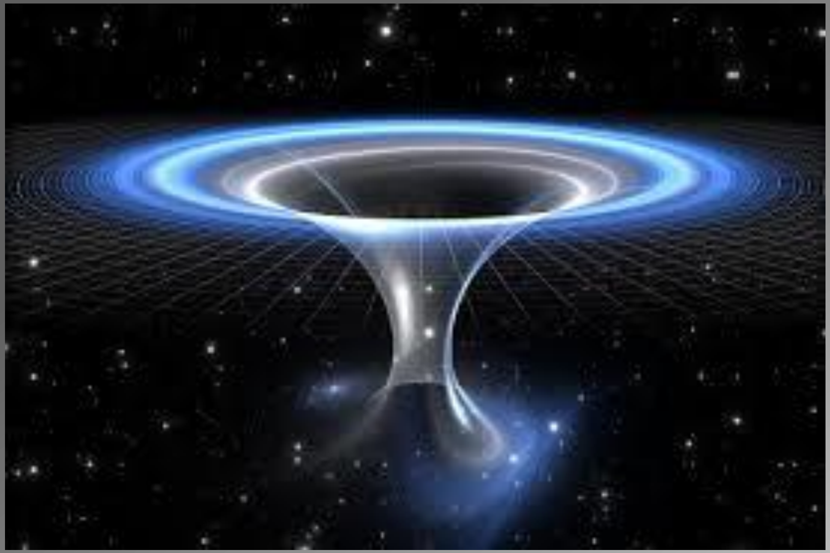
$$T^{\mu\upsilon} = \rho h u^{\mu} u^{\upsilon} + p g^{\mu\upsilon}$$

$$h = 1 + \varepsilon + p/\rho$$

$$J^{\mu} = \rho u^{\mu}$$

GRHD Equations

Linearization



$$\nabla_\mu T^{\mu\upsilon} = 0$$

$$\nabla_\mu J^\mu = 0$$

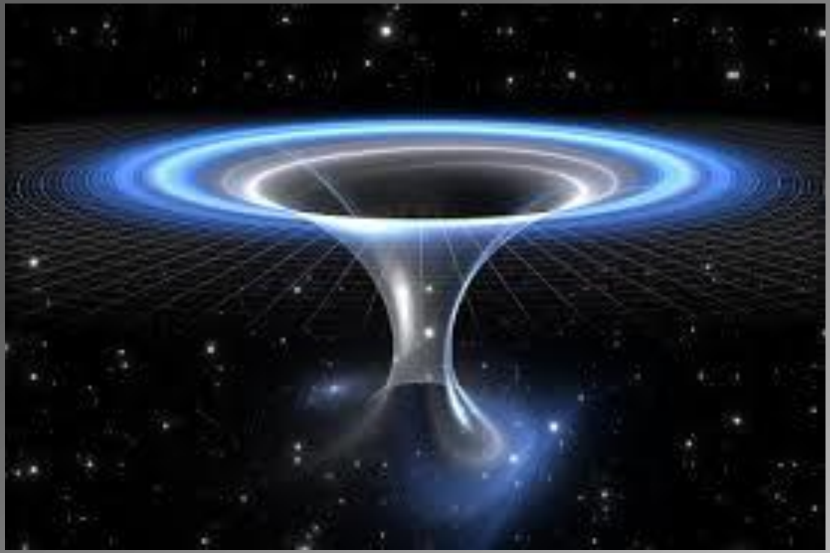
$$p \rightarrow p^{(0)} + p^{(1)}$$

$$\rho \rightarrow \rho^{(0)} + \rho^{(1)}$$

$$\upsilon \rightarrow \upsilon^{(0)} + \upsilon^{(1)}$$

GRHD Equations

Linearization



$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\mu J^{\mu} = 0$$

$$p \rightarrow p^{(0)} + p^{(1)}$$

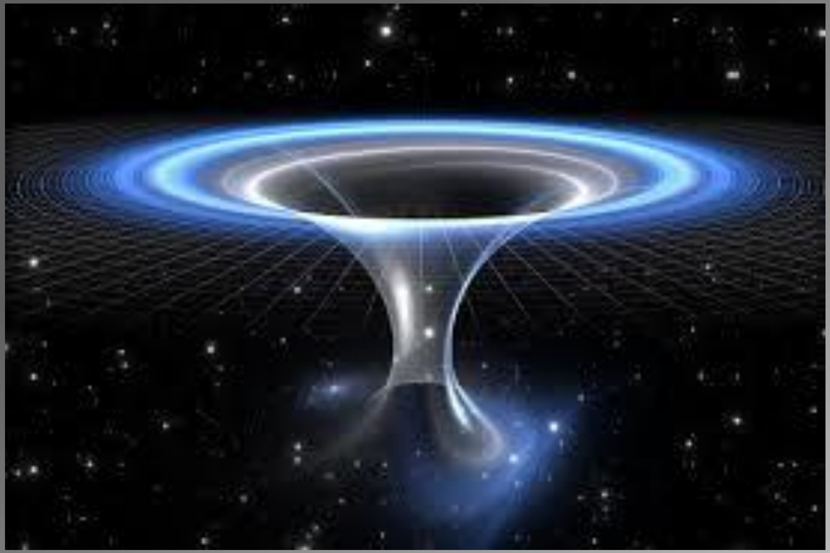
$$\rho \rightarrow \rho^{(0)} + \rho^{(1)}$$

$$\upsilon \rightarrow \upsilon^{(0)} + \upsilon^{(1)}$$

Spherical Symmetry

GRHD Equations

PDEs



$$\partial_t \rho + \alpha\left(\partial_r + \frac{2r}{a^2 + r^2}\right)\upsilon = 0$$

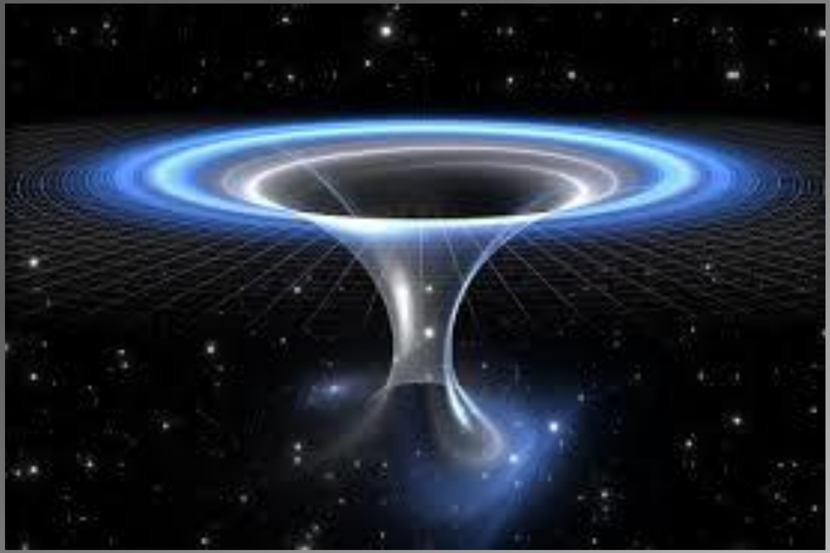
$$\alpha \partial_t \upsilon + \beta \partial_r \rho = 0$$

$$\alpha = \rho_0 + p_0$$

$$p = \beta \rho$$

GRHD Equations

PDEs



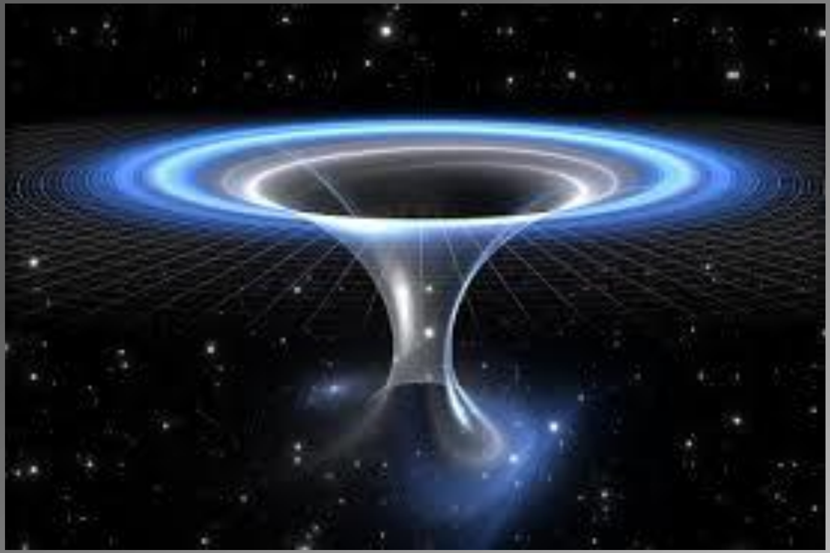
$$\partial_t \rho + \alpha\left(\partial_r + \frac{2r}{a^2 + r^2}\right)\upsilon = 0$$

$$\alpha \partial_t \upsilon + \beta \partial_r \rho = 0$$

$$f(r,a) = \frac{2r}{a^2 + r^2}$$

GRHD Equations

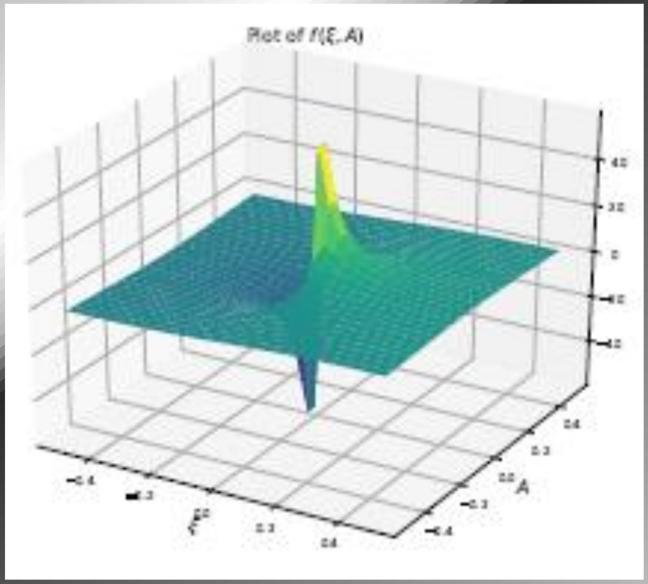
PDEs



$$\partial_t \rho + \alpha\left(\partial_r + \frac{2r}{a^2 + r^2}\right)\upsilon = 0$$

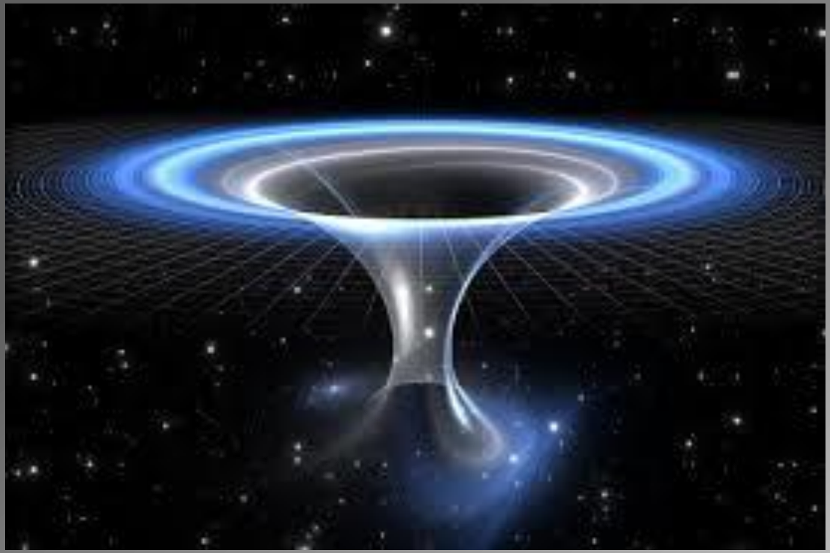
$$\alpha \partial_t \upsilon + \beta \partial_r \rho = 0$$

$$f(r,a) = \frac{2r}{a^2 + r^2}$$



GRHD Equations

PDEs



$$\partial_t \rho + \alpha \left(\partial_r + \frac{2r}{a^2 + r^2} \right) \upsilon = 0$$

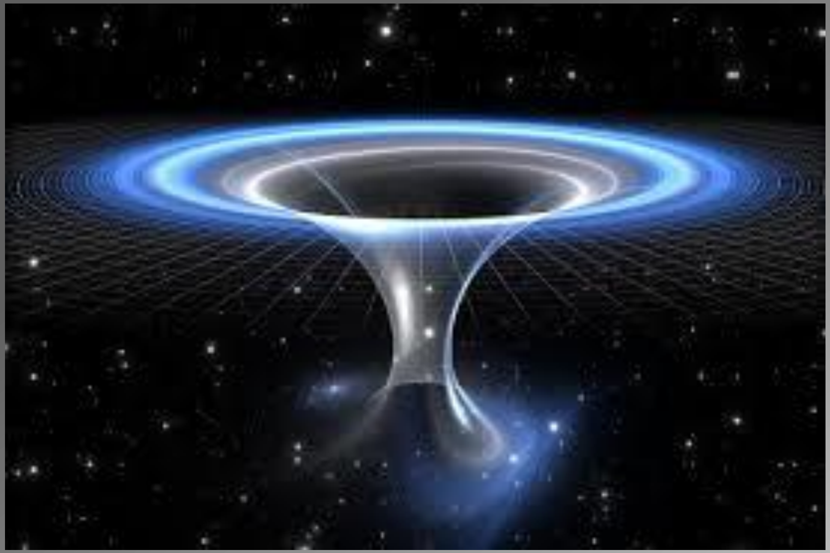
$$\alpha \partial_t \upsilon + \beta \partial_r \rho = 0$$

$$f(r,a) = \frac{2r}{a^2 + r^2}$$

The function is Smooth

GRHD Equations

Separation of variables



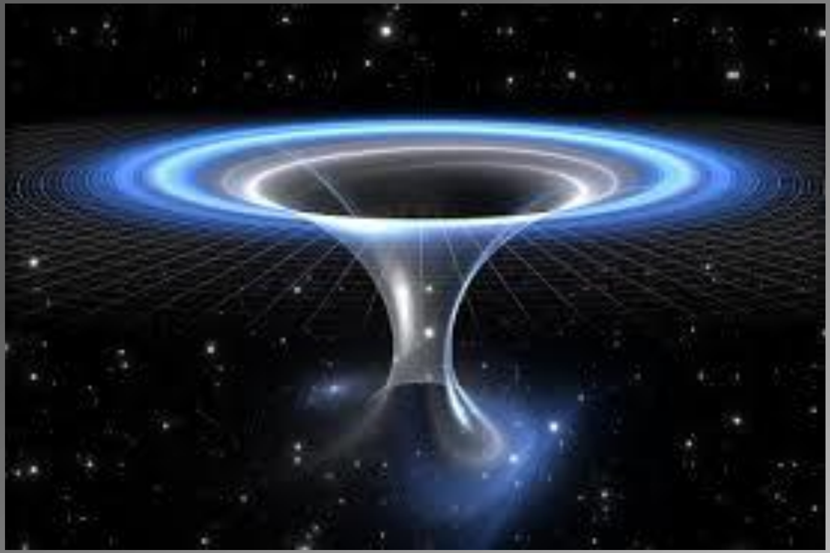
$$\partial_t \rho + \alpha\left(\partial_r + \frac{2r}{a^2 + r^2}\right)\upsilon = 0$$

$$\alpha \partial_t \upsilon + \beta \partial_r \rho = 0$$

$$f(r,a) = \sum_{i=0}^{\frac{n-1}{2}} (-1)^i \frac{2r^{2i+1}}{a^{2i+2}} + O\left(r^{2n+2}\right)$$

GRHD Equations

Near center approximation: r<<a



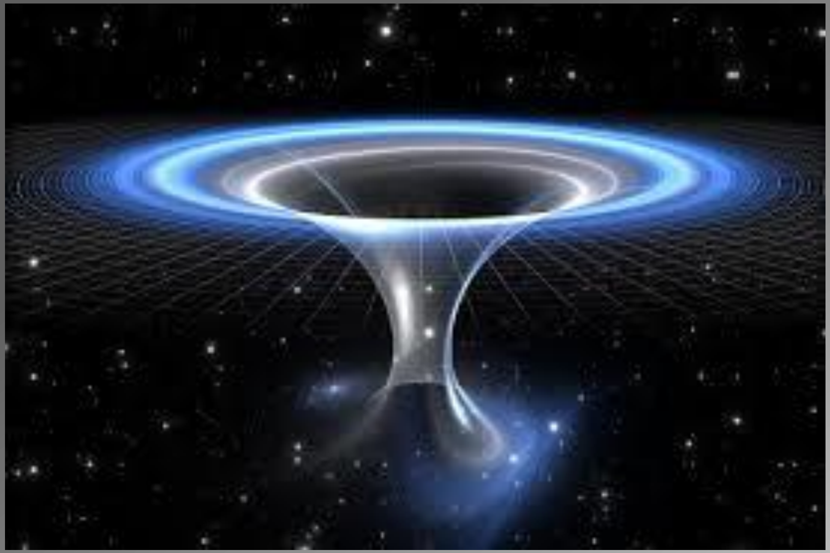
$$\rho = T(t)R(r)$$

$$\ddot{T}(t) + \omega^2 T(t) = 0$$

$$\frac{d^2R}{dr^2} + f(a,r)\frac{dR}{dr} + \omega^2 R = 0$$

GRHD Equations

Near center approximation: r<<a



$$\rho = T(t)R(r)$$

$$\ddot{T}(t) + \omega^2 T(t) = 0$$

$$\frac{d^2R}{dr^2} + f(a,r)\frac{dR}{dr} + \omega^2 R = 0$$

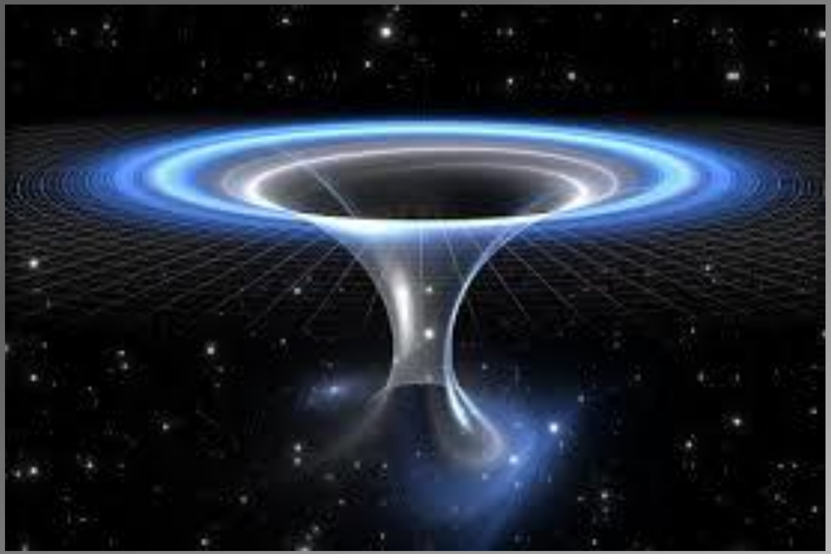
$$R(r) = C_1 e^{-\frac{r^2}{a^2}} H_k\left(\frac{r}{a}\right) + C_2 e^{-\frac{r^2}{a^2}} F_{11}\left(\frac{2-\omega^2 a^2}{4}, \frac{1}{2}, \frac{r^2}{a^2}\right)$$

H – Hermite polynomial

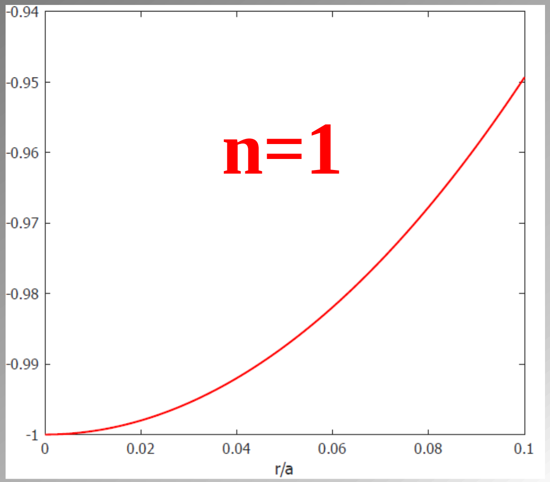
F – Kummer's function of the first kind

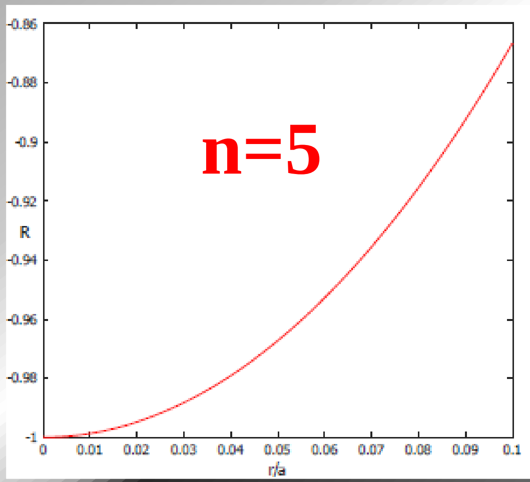
GRHD Equations

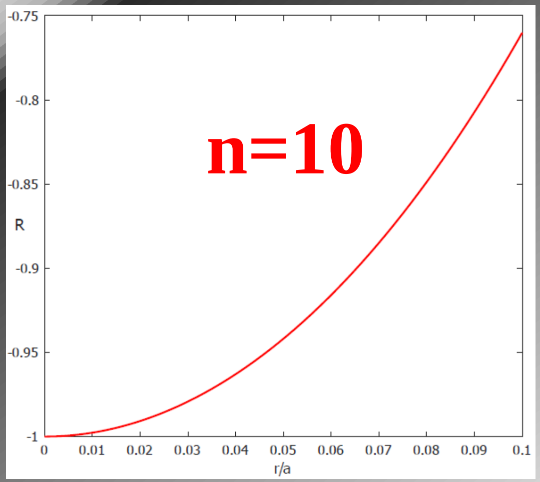
Near center approximation: r<<a



$$\omega_n = \left(\frac{1+2n}{a^2} \right)^{\frac{1}{2}}$$

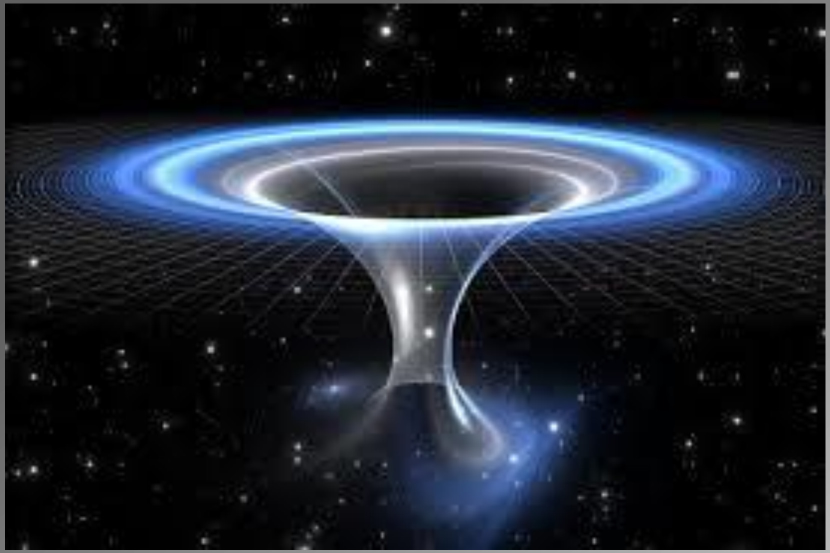




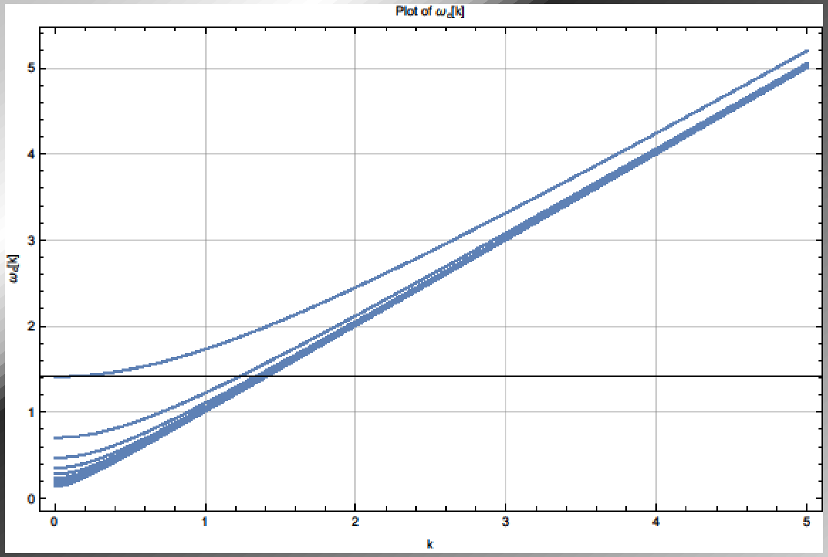


GRHD Equations

Dispersion Relation

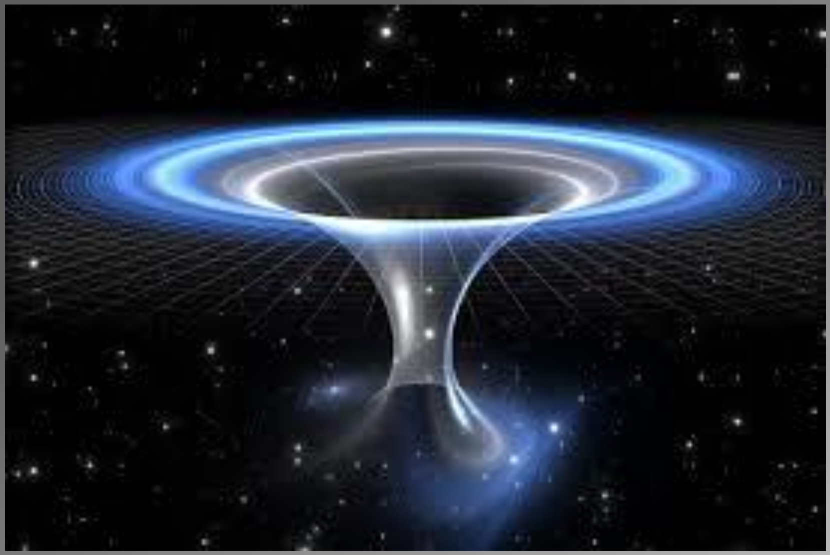


$$\omega = \left(\alpha \frac{2 + a^2 k^2}{a^2} \right)^{\frac{1}{2}}$$

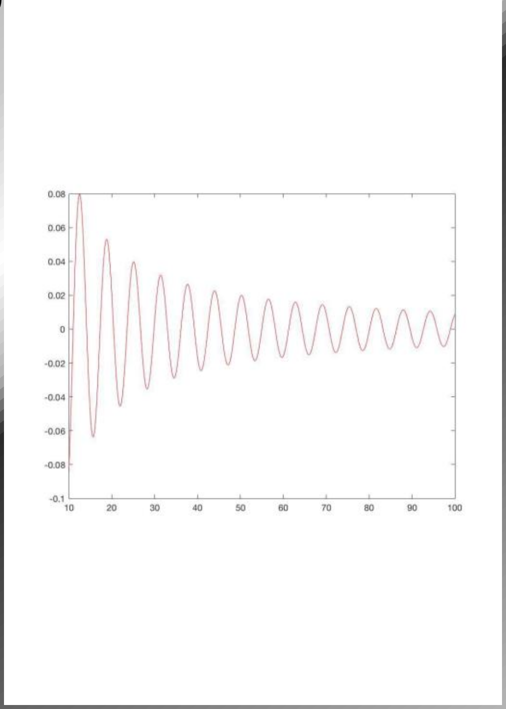


GRHD Equations

Far center approximation: r>>a



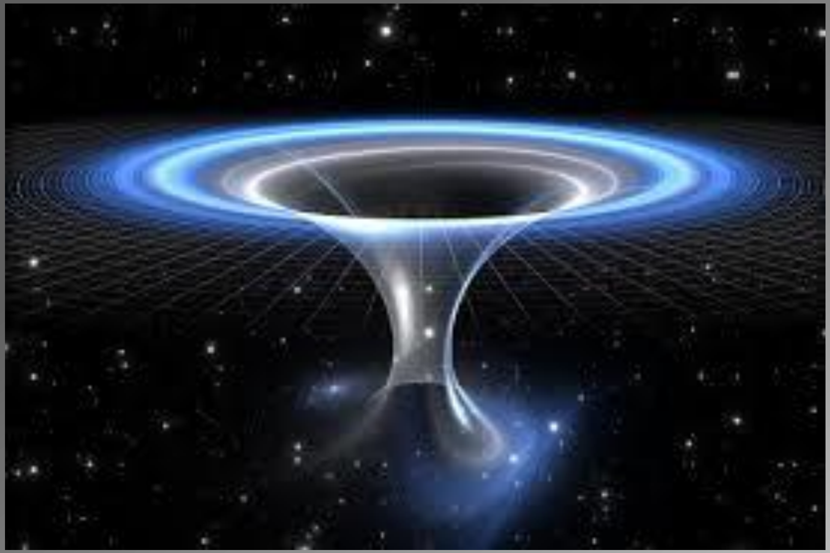
$$f(r,a)=\sum_{i=0}^{\frac{n-1}{2}}(-1)^i\frac{2a^{2i}}{r^{2i+1}}+O\left(r^{-n-2}\right)$$



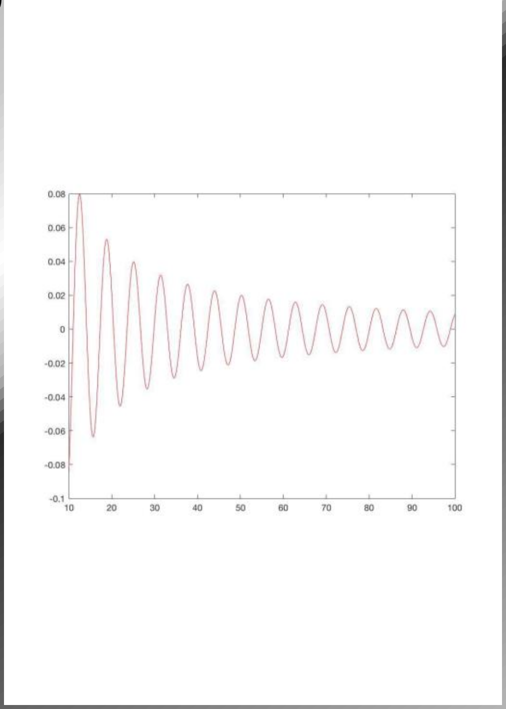
$$R(r)=\frac{1}{r}\left(C_1e^{-i\omega r}-iC_2e^{i\omega r}\right)$$

GRHD Equations

Far center approximation: r>>a



$$f(r,a)=\sum_{i=0}^{\frac{n-1}{2}}(-1)^i\frac{2a^{2i}}{r^{2i+1}}+O\left(r^{-n-2}\right)$$



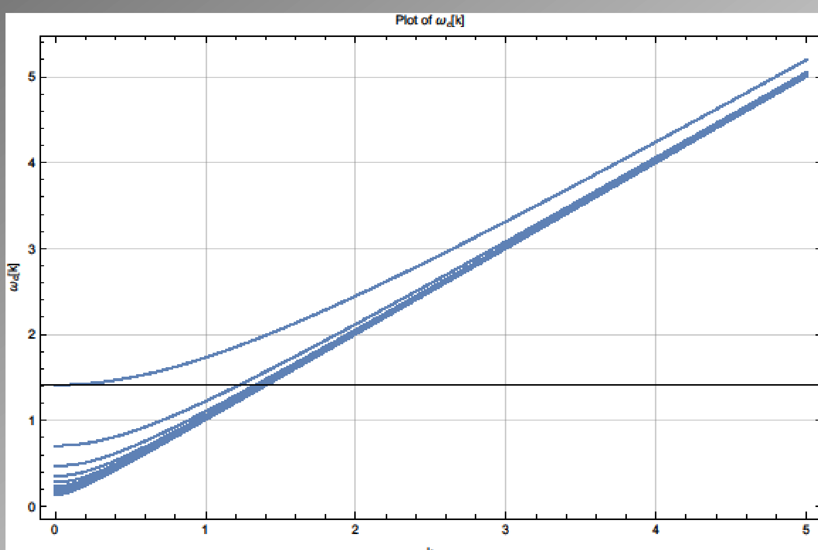
$$R(r)=\frac{1}{r}\left(C_1e^{-i\omega r}-iC_2e^{i\omega r}\right)$$

Nothing interesting

GRHD Equations

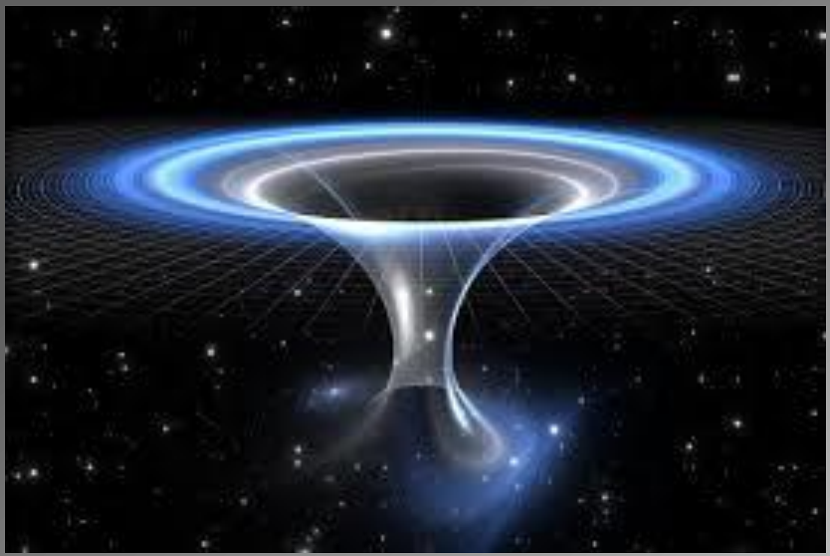
Near center approximation: r<<a

$$\omega_n = \left(\frac{1+2n}{a^2} \right)^{\frac{1}{2}}$$

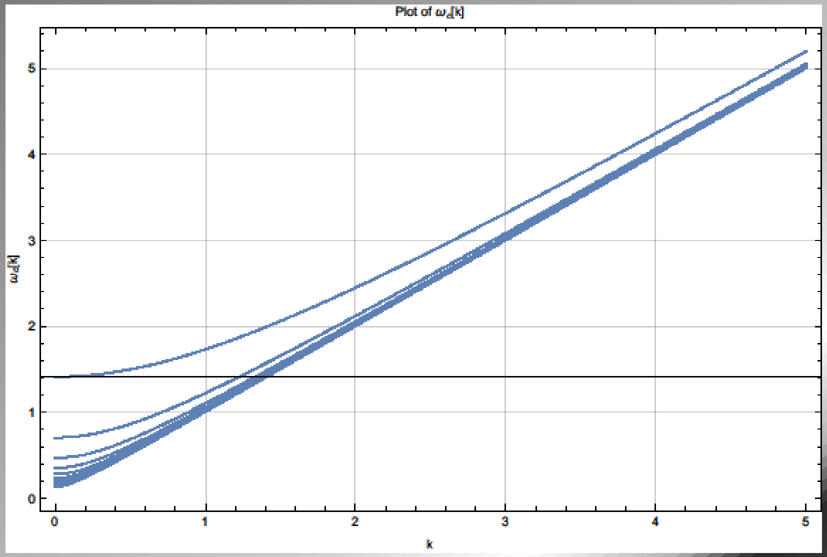


GRHD Equations

Near center approximation: r<<a



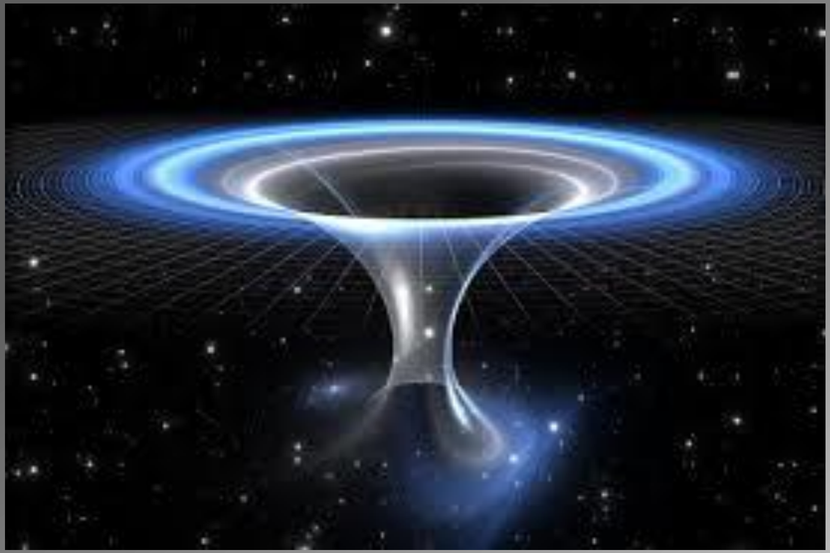
$$\omega_n = \left(\frac{1+2n}{a^2}\right)^{\frac{1}{2}}$$



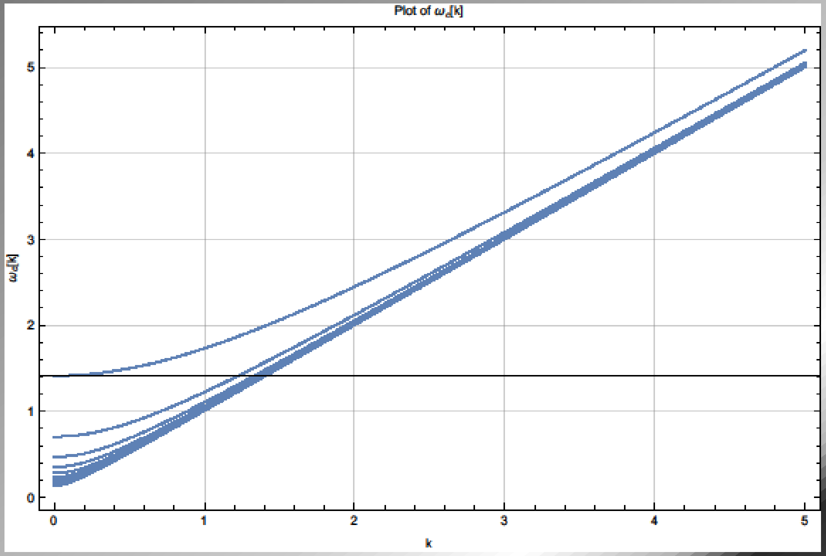
But what we observe in space comes from radiation

GRHD Equations

Near center approximation: r<<a



$$\omega_n = \left(\frac{1+2n}{a^2} \right)^{\frac{1}{2}}$$



The next step is to understand influence of sound on an emission pattern

Conclusions

- For both cases we have found the solutions by separation of variables

- We have found that for the near center approximation the fundamental frequencies might exist

- We have derived dispersion relation in near center

Conclusions

- For both cases we have found the solutions by separation of variables

- **We have found that for the near center approximation the fundamental frequencies might exist**

- We have derived dispersion relation in near center

Conclusions

- For both cases we have found the solutions by separation of variables
- We have found that for the near center approximation the fundamental frequencies might exist
- **We have derived dispersion relation in near center**

References

- Ellis, H.G., 1973, JMP, 14, 104
- Kardashev N.S. et al., 2006, Astr.Rep. 50, 8
- Kardashev N.S. et al., 2007, IJMPD, 16, 5
- Arsenadze G. & Osmanov Z., 2017, IJMPD, 26, 1750153

MANY THANKS