Remote Time Manipulation

David Trillo (Joint work with Ben Dive and Miguel Navascués)
arXiv:1903.10568

Time Machine Factory, September 2019
Motivation

Let $H$ be a time-independent Hamiltonian.
Motivation

Let $H$ be a time-independent Hamiltonian.

$$|\psi(t)\rangle = e^{-iHt} |\psi_0\rangle$$
Motivation

Let $H$ be a time-independent Hamiltonian.

$|\psi(t)\rangle = e^{-iHt} |\psi_0\rangle$

Figure: Time evolution of a quantum system
Motivation

Figure: Fast forward and rewind of a quantum system.
Motivation

We want to warp the evolution time of a quantum system, but
Motivation

We want to warp the evolution time of a quantum system, but

• Without resorting to relativity.
Motivation

We want to warp the evolution time of a quantum system, but

• Without resorting to relativity.
• Without knowledge of the system (i.e.: its Hamiltonian or its interaction with other systems)
Motivation

We want to warp the evolution time of a quantum system, but

- Without resorting to relativity.
- Without knowledge of the system (i.e.: its Hamiltonian or its interaction with other systems)
- With universal protocols.
Motivation

We want to warp the evolution time of a quantum system, but

- Without resorting to relativity.
- Without knowledge of the system (i.e.: its Hamiltonian or its interaction with other systems)
- With universal protocols.

We allow for probabilistic protocols, as long as they are also heralded (that is, we must know if the protocol succeeds).
**Figure:** One way to influence an unknown system.  

Example
Suppose that $n$ probes are prepared in the state $|\psi\rangle_P$ and that each one interacts with the system via the unitary $W$. 
Example

Suppose that $n$ probes are prepared in the state $|\psi\rangle_P$ and that each one interacts with the system via the unitary $W$. If, after the interaction, we post-select on the probes being in the state $|0\ldots0\rangle_P$, the final state of the system will be

$$
\langle 0\ldots0|_P W(V \otimes 1) |\phi\rangle_S |\psi\rangle_P
$$
Matrix Polynomials

Example
By writing

$$|\psi\rangle_P = \sum_{\vec{k}} c_{\vec{k}} |k_1 \cdots k_n\rangle$$
Matrix Polynomials

Example

By writing

$$|\psi\rangle_P = \sum_{\vec{k}} c_{\vec{k}} |k_1 \cdots k_n \rangle$$

and

$$U_k = \langle 0|_P W(V \otimes 1)|k\rangle_P.$$
Matrix Polynomials

Example

By writing

$$|\psi\rangle_P = \sum_{\vec{k}} c_{\vec{k}} |k_1 \cdots k_n\rangle$$

and

$$U_k = \langle 0 |_P W(V \otimes 1) |k\rangle_P .$$

We get

$$|\phi_{final}\rangle_S = \sum_{\vec{k}} c_{k_1 \cdots k_n} U_{k_1} \cdots U_{k_n} |\phi\rangle_S .$$
Central polynomials

Definition
A polynomial \( f(x_1, \ldots, x_n) \in K \langle X \rangle \) is a central polynomial for a ring \( R \) if

1. for any \( r_1, \ldots, r_n \in R \), \( f(r_1, \ldots, r_n) \) lies in the center of \( R \).
2. \( f \) is not identically zero.
3. the constant term of \( f \) is zero.
Central polynomials

**Definition**
A polynomial \( f(x_1, \cdots, x_n) \in K\langle X \rangle \) is a *central polynomial* for a ring \( R \) if

1. for any \( r_1, \cdots, r_n \in R \), \( f(r_1, \ldots, r_n) \) lies in the center of \( R \).
2. \( f \) is not identically zero.
3. the constant term of \( f \) is zero.

**Theorem (Formanek, Razmyslov)**
\( M_n(K) \) has a central polynomial.

**Remark**
*Formanek’s polynomial is of the form \( F(x, y_1, \cdots, y_n) \), homogeneous of degree \( n^2 - n \) in \( x \) and linear in \( y_i \).*
Central polynomials

Example
Let $A, B \in M_2$. Consider the polynomial

$[A, B]^2$

.
Central polynomials

Example
Let $A, B \in M_2$. Consider the polynomial

$$[A, B]^2$$

As $[A, B]$ is traceless, $[A, B] = aX + bY + cZ$.

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
Central polynomials

Example
Let $A, B \in M_2$. Consider the polynomial 

$$[A, B]^2$$

As $[A, B]$ is traceless, $[A, B] = aX + bY + cZ$.

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Therefore,

$$[A, B]^2 \propto 1.$$
Our setup

Figure: A more general interaction between systems and probes, with the targeting assumption.
Tensor polynomials

These protocols correspond to polynomials of the form

$$f(V, U_1, \cdots, U_n) = \sum_k c_k p_k(V, U_1, \cdots, U_n) \otimes \cdots \otimes q_k(V, U_1, \cdots, U_n).$$

This extra structure allows for more interesting behaviours.
Tensor polynomials

These protocols correspond to polynomials of the form

\[ f(V, U_1, \cdots, U_n) = \sum_{k} c_k p_k(V, U_1, \cdots, U_n) \otimes \cdots \otimes q_k(V, U_1, \cdots, U_n). \]

This extra structure allows for more interesting behaviours.

**Theorem**

There exist polynomials in \( M_n \otimes \cdots \otimes M_n \) which are proportional to \( P_S \), the projector onto the symmetric subspace; to \( P_A \), the projector onto the antisymmetric subspace and to permutations of the tensor factors (i.e., SWAPs).
Main result

Theorem
Let $P$ be a protocol on $n$ copies of a system of dimension $d$ with the targeting assumption. If at the end of a heralded success, system $i$ is in state $\psi(T_i)$ and the protocol took time $T'$, then it must be that

$$\sum_{i: T_i < 0} (d - 1)|T_i| + \sum_{j: T_j > 0} T_j \leq nT'.$$

Moreover, this inequality is optimal.
Take-home messages

1. Control of a system can be used to get some heralded control of another system

2. Evolution time behaves a bit like energy: it cannot be created or destroyed, but it can be transferred between systems or wasted.

3. Evolution time can be inverted at a cost depending on dimension.