The temporally nonlocal character of the quantum computational speedup

Forthcoming in Foundations of Physics

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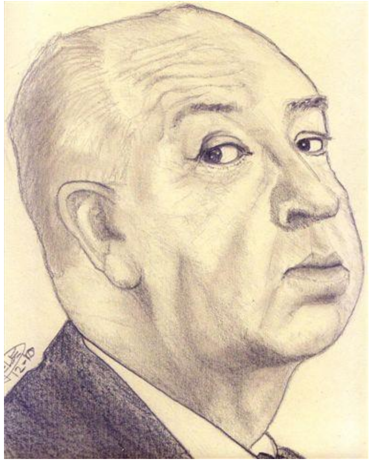
Introduction

- Usual representation of quantum algorithms: limited to the process of computing the solution of the problem
- Extending it to the process of setting the problem **highlights the relevance of time-symmetric quantum mechanics to quantum computation**
- **Problem setting and problem solution, in their quantum version, constitute pre- and post-selection:** the process between them must be affected by both boundary conditions
- Taking both into account quantitatatively accounts for the quantum computational speedup

Quantum computational speedup – an example

- Bob, the problem setter, hides a ball in one of four drawers
- Alice, the problem solver, is to locate it by opening drawers (by making *oracle queries*)
- In the classical case, Alice may need to open up to three drawers, in the quantum case, it always takes one

BOB



00
01
10
11

ALICE

Completing the representation of quantum algorithms

- Usual representation is limited to the process of computing the solution: only a **part** of the problem-solving process



- **Breaking the whole into adjacent spatio-temporal parts each with suitable surrounding conditions: application of the principle of locality**

- **In the quantum case, this can hide nonlocal effects** (temporally nonlocal character of the speedup)

- Way out: extending the usual representation to the problem-setting process

- **Bob, by the initial measurement, selects a problem setting, Alice unitarily computes the corresponding solution and reads it by the final measurement**

Three steps

1

- Extending the usual representation to the process of setting the problem: innocent operation, but with dramatic consequences

2

- extended representation must be relativized to Alice, from whom the setting of the problem must be hidden

3

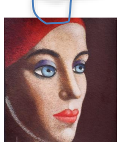
- problem-setting and problem solution, in their quantum version, constitute pre- and post-selection, hence the process between them is bound to be affected by both boundary conditions

Step 1): extending the representation

meas. $\hat{B}$		meas. $\hat{A}$
$(\lvert 00\rangle_B + \lvert 01\rangle_B + \lvert 10\rangle_B + \lvert 11\rangle_B)\lvert 00\rangle_A$		
$\Downarrow$		
$\lvert 01\rangle_B \lvert 00\rangle_A$	$\Rightarrow \mathbf{U} \Rightarrow$	$\lvert 01\rangle_B \lvert 01\rangle_A$

Step 2): relativizing it to Alice

meas. $\hat{B}$		meas. $\hat{A}$
$(\lvert 00\rangle_B + \lvert 01\rangle_B + \lvert 10\rangle_B + \lvert 11\rangle_B)\lvert 00\rangle_A$	$\Rightarrow \mathbf{U} \Rightarrow$	$\lvert 00\rangle_B \lvert 00\rangle_A + \lvert 01\rangle_B \lvert 01\rangle_A + \lvert 10\rangle_B \lvert 10\rangle_A + \lvert 11\rangle_B \lvert 11\rangle_A$
		$\Downarrow$
		$\lvert 01\rangle_B \lvert 01\rangle_A$



Alice's knowledge of the number of the drawer with the ball

Step 3): taking both boundary conditions into account

- Premise: there is a unitary transformation between the initial and final measurement outcomes, the process between them is time-reversible
- Usual picture: **the process is entirely determined by the initial measurement**, the measurement outcome propagates forward in time until becoming the outcome of the final measurement
- Time-symmetric picture: **the process is entirely determined by the final measurement**, the measurement outcome propagates backwards in time until becoming the outcome of the initial measurement
- Present picture: **the process is determined by both boundary conditions:**
- **The initial and final measurements evenly and non-redundantly contribute to the selection, in a quantum superposition of all the possible ways**

Costa de Beauregard Parisian Zigzag

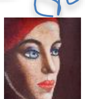
1) Initial measurement of $\hat{B}$ reduces to that of $\hat{B}_l$ and selects 0 of 01; measurement outcome propagates forward in time, then

2) Final measurement of $\hat{A}$ reduces to that of $\hat{A}_r$ and selects 1 of 01, measurement outcome propagates backwards in time

meas. of $\hat{B}_l$		meas. of $\hat{A}_r$
$(\|00\rangle_B + \|01\rangle_B + \|10\rangle_B + \|11\rangle_B)\|00\rangle_A$		
$\Downarrow$		
$(\|00\rangle_B + \|01\rangle_B)\|00\rangle_A$	$\Rightarrow \mathbf{U} \Rightarrow$	$\|00\rangle_B\|00\rangle_A + \|01\rangle_B\|01\rangle_A$
		$\Downarrow$
$\|01\rangle_B\|00\rangle_A$	$\Leftarrow \mathbf{U}^\dagger \Leftarrow$	$\|01\rangle_B\|01\rangle_A$

$$|01\rangle_B|00\rangle_A \Rightarrow \mathbf{U} \Rightarrow |01\rangle_B|01\rangle_A$$

meas. of $\hat{B}_l$		meas. of $\hat{A}_r$
$(\|00\rangle_B + \|01\rangle_B + \|10\rangle_B + \|11\rangle_B)\|00\rangle_A$	$\Rightarrow \mathbf{U} \Rightarrow$	$\|00\rangle_B\|00\rangle_A + \|01\rangle_B\|01\rangle_A + \|10\rangle_B\|10\rangle_A + \|11\rangle_B\|11\rangle_A$
		$\Downarrow$
$(\|01\rangle_B + \|11\rangle_B)\|00\rangle_A$	$\Leftarrow \mathbf{U}^\dagger \Leftarrow$	$\|01\rangle_B\|01\rangle_A + \|11\rangle_B\|11\rangle_A$



$$(|01\rangle_B + |11\rangle_B)|00\rangle_A \Rightarrow \mathbf{U} \Rightarrow |01\rangle_B|01\rangle_A + |11\rangle_B|11\rangle_A$$

$$(|01\rangle_B + |11\rangle_B)|00\rangle_A \Rightarrow \mathbf{U} \Rightarrow |01\rangle_B|01\rangle_A + |11\rangle_B|11\rangle_A$$

- **Computational complexity of the problem to be solved by Alice is reduced**

- **All is as if she knew in advance half of the information that specifies the setting and thus the solution of the problem and could use this information to reach the solution by opening fewer drawers (by fewer *oracle queries*)**

- **Quantitatively accounts for the speedup**

In general

- **Advanced knowledge rule**: *the number oracle queries required to solve an oracle problem in an optimal quantum way is that logically required to find the solution given the advanced knowledge of half of the information that specifies the problem setting (and thus the solution)*

Major quantum algorithms

- Grover (problem setting = number of the drawer with the ball): knowing in advance half of the n bits that specify it allows to search in $2^{n/2}$ drawers instead of 2^n : quadratic speedup

- Simon, Shor (problem setting = table of the periodic function). The two complementary measurements should select respectively two consecutive periods of the table of the function
  Alice knows in advance a period of the table: one evaluation of the function for an argument immediately outside it identifies the solution: exponential speedup

Half causal loop

- In each instance, all is as if Alice knew in advance half of the information about the solution she will read in the future and could use this information to find the solution with fewer oracle queries
- Can this be physical?
- Apparently, information goes backwards in time from the final to the initial measurement: temporal nonlocality
- However, taking the superposition of all instances yields the ordinary unitary transformation of a quantum superposition

meas. of $\hat{B}_l$		meas. of $\hat{A}_r$
$(\|00\rangle_B + \|01\rangle_B + \|10\rangle_B + \|11\rangle_B)\|00\rangle_A$	$\Rightarrow \mathbf{U} \Rightarrow$	$\|00\rangle_B\|00\rangle_A + \|01\rangle_B\|01\rangle_A + \|10\rangle_B\|10\rangle_A + \|11\rangle_B\|11\rangle_A$
		$\Downarrow$
$(\|01\rangle_B + \|11\rangle_B)\|00\rangle_A$	$\Leftarrow \mathbf{U}^\dagger \Leftarrow$	$\|01\rangle_B\|01\rangle_A + \|11\rangle_B\|11\rangle_A$

$$(|01\rangle_B + |11\rangle_B)|00\rangle_A \Rightarrow \mathbf{U} \Rightarrow |01\rangle_B|01\rangle_A + |11\rangle_B|11\rangle_A$$

Conclusion

- Details should not obscure the simplicity of the argument
- Problem setting and problem solution, in their quantum version, constitute pre- and post-selection, hence the process in between must be affected by both the initial and final boundary conditions
- The selection of the process (of either the initial or the final measurement outcome) should be evenly shared between the initial and final measurements in a quantum superposition of all the possible ways
- In each of them, Alice, the problem solver, remains shielded from the half information coming to her from the initial measurement, not from the half coming to her from the final measurement
- All is as if she knew in advance half of the information about the solution she will read in the future and could use this information to reach the solution with fewer oracle queries
- Hopefully lays the foundations of a new line of research, merging quantum information and the foundations of quantum mechanics

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