Gravity and nonequilibrium thermodynamics: the origin of evolution equations

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in collaboration with

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Overview

- A nonequilibrium thermodynamic theory incorporating gravity
- Evolution equation and flow-frames
- Modified gravity and its applications
- Connection to quantum mechanics

Gravitation and thermodynamics

- Gravitational field coupled to thermodynamic equations of state
  - The internal energy separately contains the field and interaction energy:

$$u = Ts - pv + \mu = e - \varphi - \frac{\nabla\varphi \cdot \nabla\varphi}{8\pi G\rho}$$

- Hydrodynamic framework, extensivity is conserved
- Simple heuristic derivation or Liu's procedure for the evolution equations
- The appearance of functional derivatives without variational principles, the classical holographic property and the emergence of quantum phenomena are unexpected general consequences of the II. law of thermodynamics

VÁN and ABE, Physica A: Statistical Mechanics and its Applications, 588:126505
VÁN, Physics of Fluids 35, 057105 (2023)

Gravitation and thermodynamics

- Balances of mass, momentum and internal energy are constraints:

$$\dot{\rho} + \rho \nabla \cdot \vec{v} = 0$$

$$\rho \dot{\vec{v}} + \nabla \cdot \mathbf{P} = 0$$

$$\rho \dot{e} + \nabla \cdot \vec{q} = -\mathbf{P} : \nabla \vec{v}$$

- II. law of thermodynamics:

$$\rho \dot{s} + \nabla \cdot \vec{j} \geq 0$$

Thermodynamic derivation

$$\rho\dot{s}+\nabla\cdot\left[\frac{1}{T}\left(\mathbf{q}+\frac{1}{4\pi G}\dot{\varphi}\nabla\varphi\right)\right]=$$

$$\left(\mathbf{q}+\frac{\dot{\varphi}}{4\pi G}\nabla\varphi\right)\cdot\nabla\left(\frac{1}{T}\right)+\frac{\dot{\varphi}}{4\pi GT}\left(\Delta\varphi-4\pi G\rho\right)-$$

$$-\left[\mathbf{P}-p\mathbf{I}-\frac{1}{4\pi G}\left(\nabla\varphi\nabla\varphi-\frac{1}{2}\nabla\varphi\cdot\nabla\varphi\mathbf{I}\right)\right]:\frac{\nabla\mathbf{v}}{T}\geq 0.$$

Thermodynamic forces and fluxes

Interaction	Thermodynamic force	Thermodynamic flux
Thermal	$\nabla\left(\frac{1}{T}\right)$	$\boldsymbol{q} + \frac{\dot{\varphi}}{4\pi G}\nabla\varphi$
Mechanical	$\nabla\boldsymbol{v}$	$-\frac{1}{T}\left[\mathbf{P} - p\mathbf{I} - \frac{1}{4\pi G}\left(\nabla\varphi\nabla\varphi - \frac{1}{2}\nabla\varphi\cdot\nabla\varphi\mathbf{I}\right)\right]$
Gravitational	$\frac{1}{4\pi G}(\Delta\varphi - 4\pi G\rho)$	$\dot{\varphi}$

Constitutive relations

$$q + \frac{\dot{\varphi}}{4\pi G}\nabla\varphi = \lambda\nabla\left(\frac{1}{T}\right) = -\lambda_F \nabla T$$

$$\dot{\varphi} = l_1\left(\frac{\Delta\varphi}{4\pi G} - \rho\right) + l_{12}\nabla\cdot v$$

$$\frac{1}{3}\mathrm{Sp}(P) - p + \frac{\nabla\varphi\cdot\nabla\varphi}{24\pi G} = l_{21}\left(\frac{\Delta\varphi}{4\pi G} - \rho\right) + l_2\nabla\cdot v$$

$$P - \mathrm{Sp}(P)\frac{I}{3} - \frac{1}{4\pi G}\left(\nabla\varphi\nabla\varphi - \frac{1}{3}(\nabla\varphi)^2 I\right) = -\eta\left(\nabla v + (\nabla v)^T - \frac{2}{3}\nabla\cdot v I\right)$$

Flow-frames and evolution equations

- $N^{\alpha} = c \int_{\mathbb{R}^3} p^{\alpha} f \frac{dp}{p^0}$ $\qquad x^{\alpha} = (ct, x)$
- $T^{\alpha\beta} = c \int_{\mathbb{R}^3} p^{\alpha} p^{\beta} f \frac{dp}{p^0}$ $\qquad p^{\mu} = \left(\sqrt{(mc)^2 + |p|^2}, p\right)$
- Eckart frame – particles:

$$\mathrm{N}^{\alpha} = n_E\, u_E^{\alpha}, \qquad T^{\alpha\beta} = \frac{e_E}{c^2} u_E^{\alpha} u_E^{\beta} + P_E^{\alpha\beta} + W_E^{\alpha} u_E^{\beta} + W_E^{\beta} u_E^{\alpha}$$

- The choice of a frame becomes a key problem in relativistic generalisations of theories involving hydrodynamics
- Stability analyses are a topic of current research avenues
- Idea: the flow is derived by assuming stability (Kovtun)

L. Gavassino, M. Antonelli, and B. Haskell, Phys. Rev. D 102, 043018 (2020)
P. Kovtun, J. High Energ. Phys. 2019, 34 (2019)

Flow-frames comparison

- Landau-Lifshitz frame – energy:

$$\mathrm{N}^{\alpha} = n_L\, u_L^{\alpha} - \nu^{\alpha}, \qquad T^{\alpha\beta} = \frac{e_L}{c^2} u_L^{\alpha} u_L^{\beta} + P_L^{\alpha\beta}$$

- Eckart frame – particles:

$$\mathrm{N}^{\alpha} = n_E\, u_E^{\alpha}, \qquad T^{\alpha\beta} = \frac{e_E}{c^2} u_E^{\alpha} u_E^{\beta} + P_E^{\alpha\beta} + W_E^{\alpha} u_E^{\beta} + W_E^{\beta} u_E^{\alpha}$$

- β-, or thermometer flow-frame – temperature four-vector:

$$\beta^{\alpha} = \beta(n_T, e_T) u^{\alpha}, \qquad T^{\alpha\beta} = \frac{e_T}{c^2} u_T^{\alpha} u_T^{\beta} + P_T^{\alpha\beta} + W_T^{\alpha} u_T^{\beta} + W_T^{\beta} u_T^{\alpha}$$

Akihiko Monnai, Phys. Rev. C 100, 014901, 2019
Byung-Hoon Hwang, Fluid Dyn. Res. 54 015501, 2022

Gravitation and thermodynamics

- Cross-effect can arise between 2 of the resulting constitutive equations:

$$\dot{\varphi} = l_1 \left(\frac{\Delta\varphi}{4\pi G} - \rho \right) + l_{12} \nabla \cdot \vec{v}$$

$$\frac{1}{3}\mathrm{Sp}(\mathbf{P}) - p + \frac{(\nabla\varphi)^2}{24\pi G} = l_{21} \left(\frac{\Delta\varphi}{4\pi G} - \rho \right) + l_2 \nabla \cdot \vec{v}$$

- Dissipative field equation for gravity:

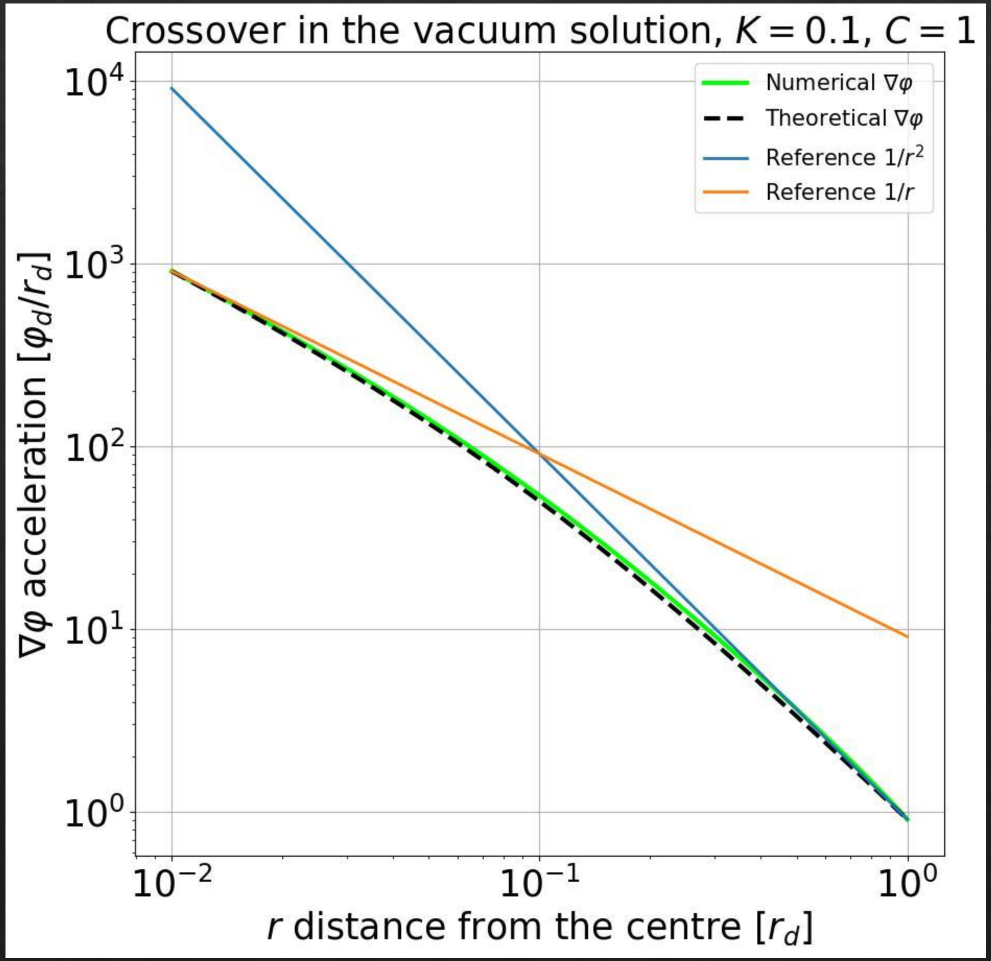
$$\frac{\partial\varphi}{\partial t} = \frac{l^2}{\tau}\left(\Delta\varphi - 4\pi G\rho + \frac{l_{12}}{6det(L)}(\nabla\varphi)^2 \right), \text{ where } K := \frac{l_{12}}{6det(L)}$$

- Stationary solution is a modified Poisson's equation:

$$\Delta\varphi = 4\pi G\rho + K(\nabla\varphi)^2$$

Vacuum solution

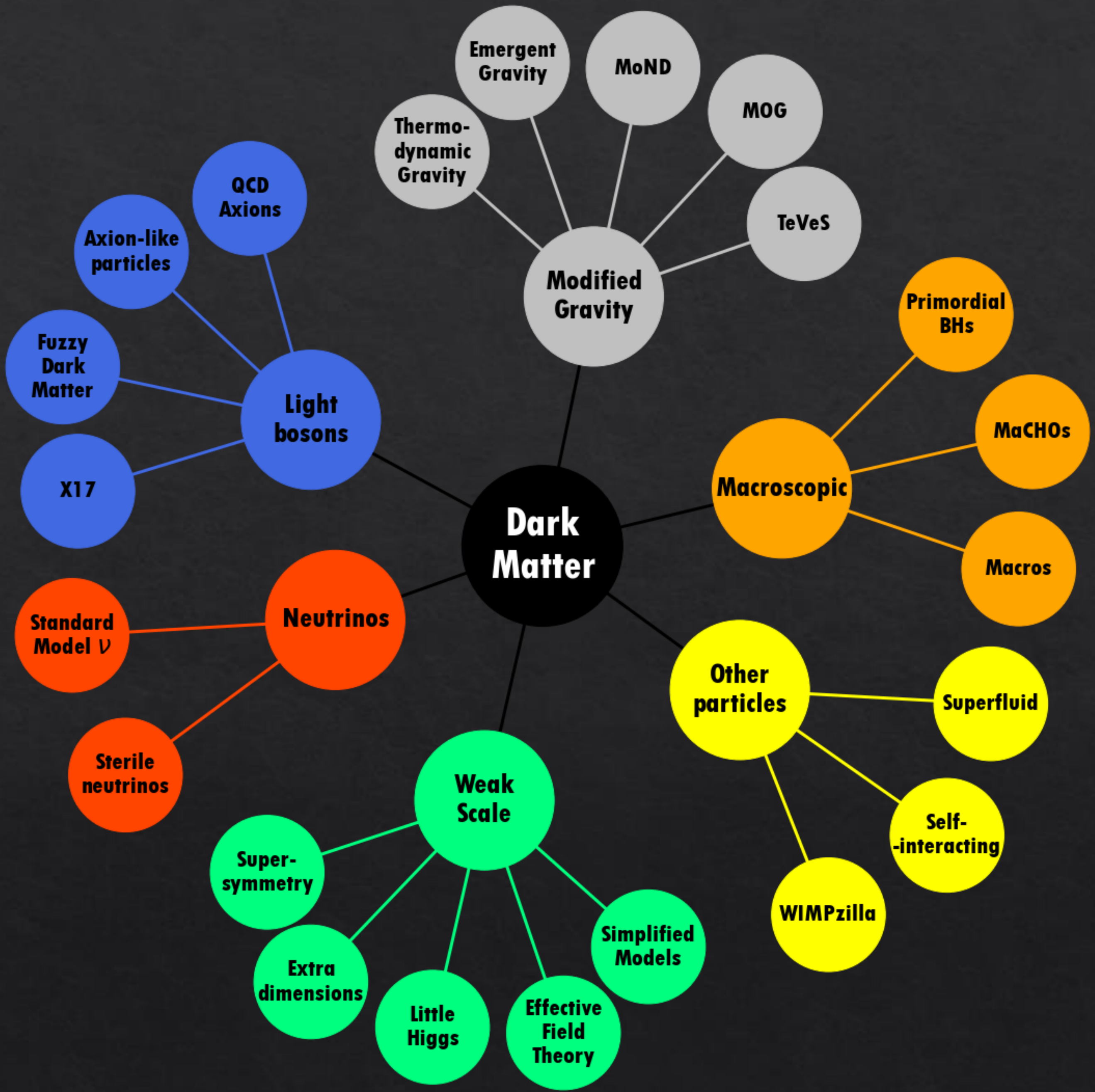
- $g(r) = -\frac{1}{Kr+Cr^2}$
- $\varphi(r) = \frac{1}{K}\ln\left(\frac{r}{K+Cr}\right) + \varphi_0$
- In Newtonian limit:
- $C = \frac{1}{GM}$
- What does M mean here in the context of thermogravity?



ABE and VÁN, Symmetry 2022, 14(5), 1048

Why do we need Dark Matter?

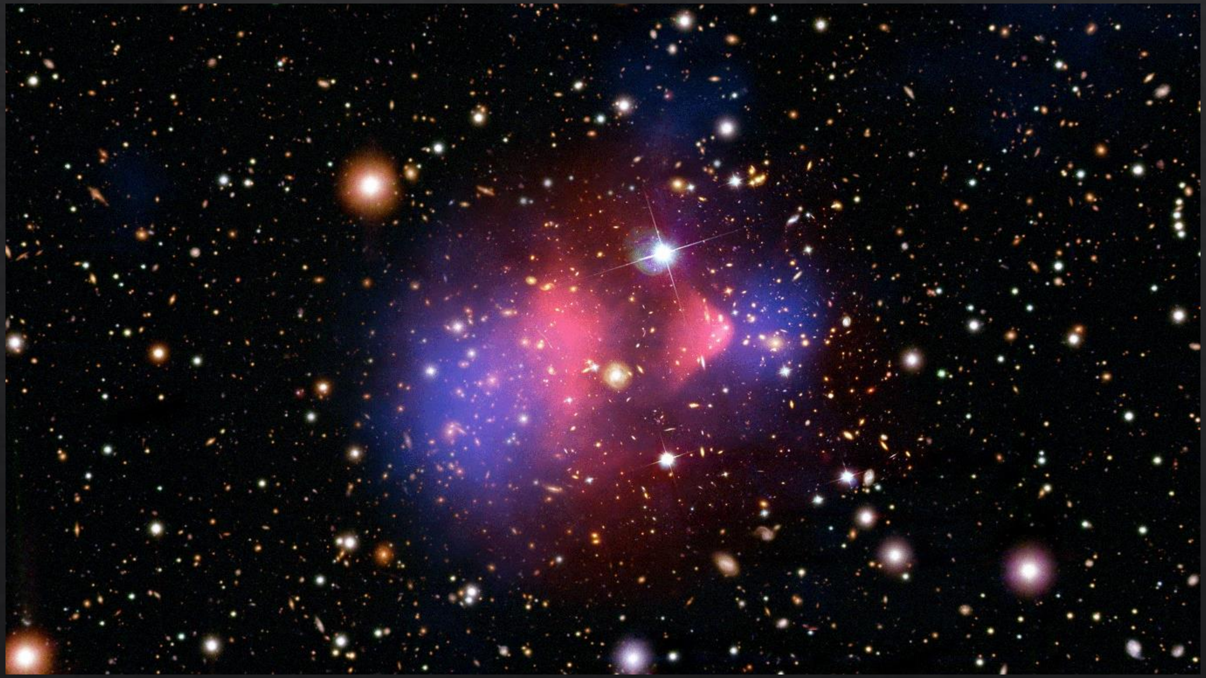
- Astronomical observations suggest an abundance of gravitating matter, more than what can be accounted for:
  - The dynamics of galaxies and galaxy clusters
  - Large-scale structure of the universe and CMB-anisotropies
  - Gravitational lensing

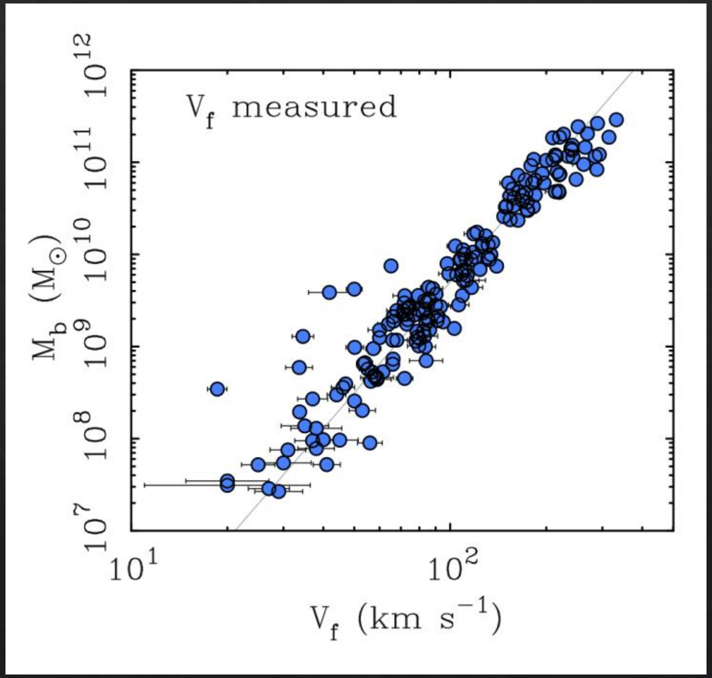


Own work, based on G. Bertone and T. M. P. Tait. Nature, 562(7725):51–56

Is modified gravity better than dark matter?

- Regularities in local scaling relations between baryons and dynamics in galaxies pose a problem for dark matter descriptions, but explain a broad range of observations





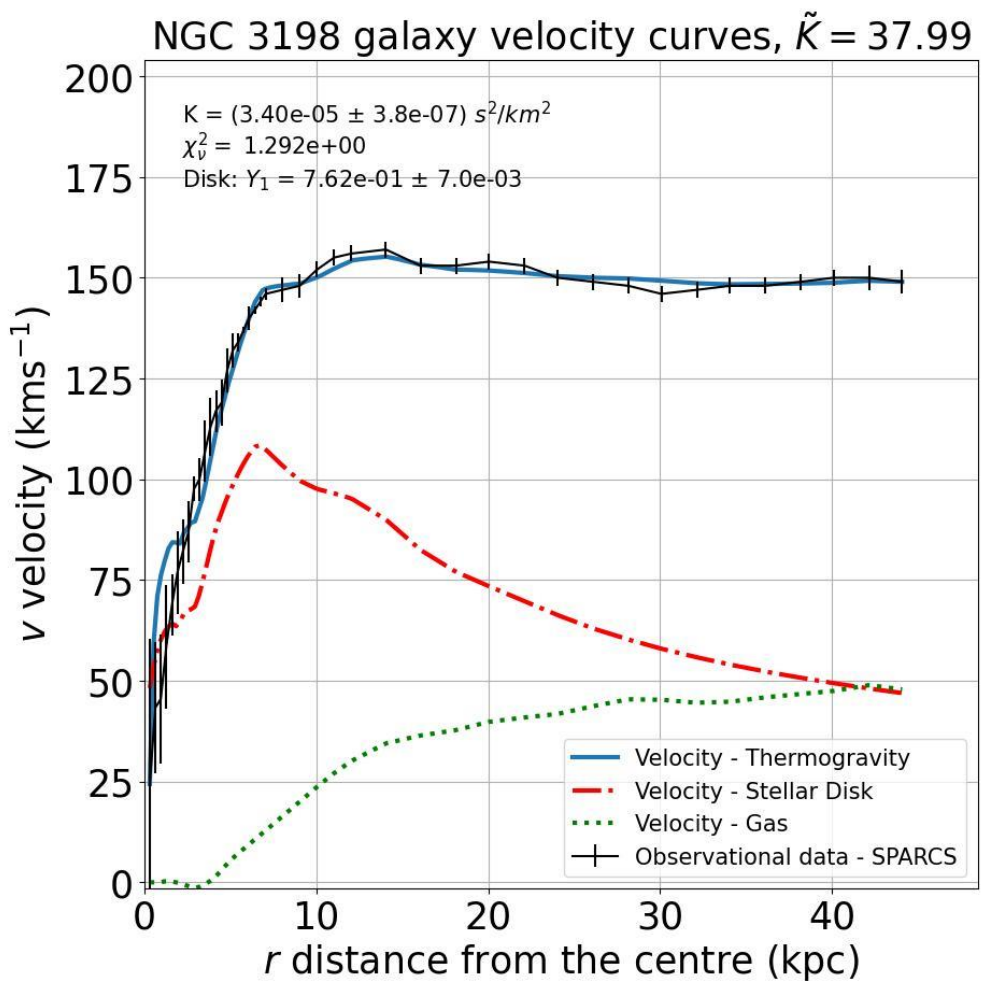
Left: Bullet Cluster (1E 0657-56),
X-ray (pink): NASA/CXC/CfA/M.Markevitch et al.;
Optical: NASA/STScI; Magellan/U.Arizona/D.Clowe et al.;
Lensing Map (blue): NASA/STScI; ESO WFI; Magellan/U.Arizona/D.Clowe et al.

Right: Baryonic Tully-Fisher relation
Stacy McGaugh

Galactic rotation curves

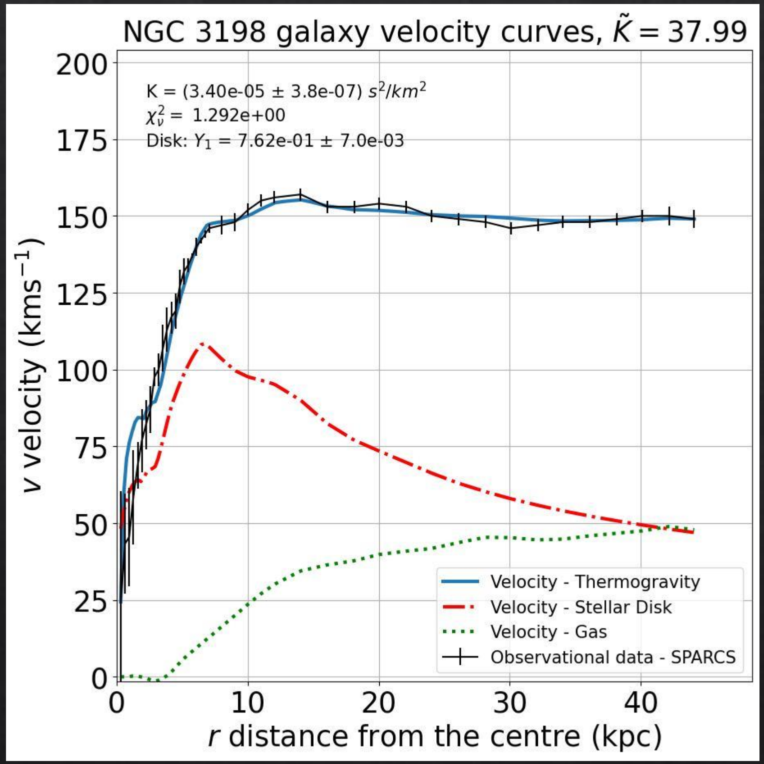
- Thermodynamic gravity applied to observed density distributions
- Comparable results to Dark Matter or MOND approaches (DOI 10.1016/j.dark.2024.101660):

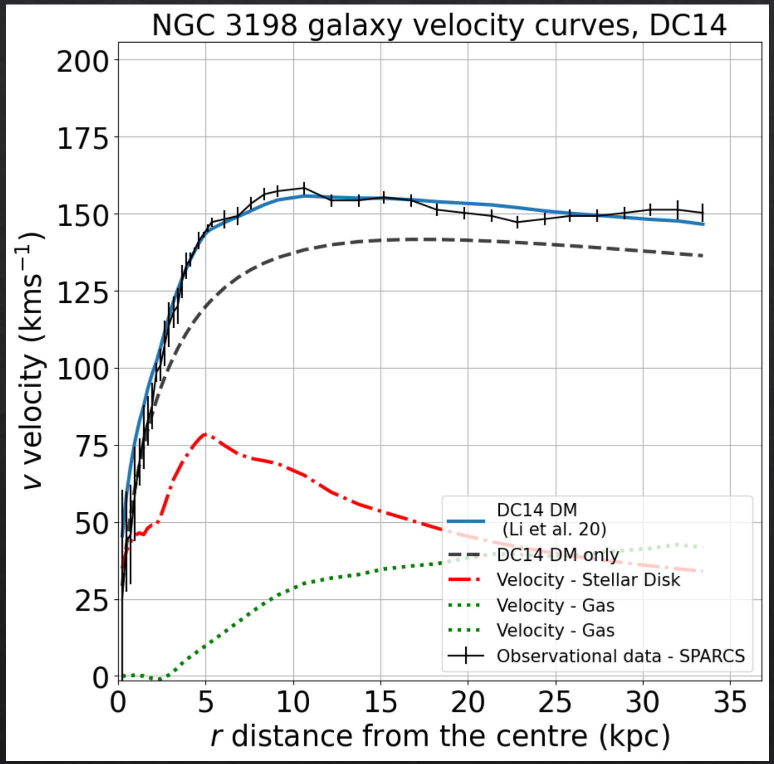
Model	χ^2_ν
Thermodynamic gravity	1.29
Dark matter – DC14	1.26
MOND – EFE realisation	1.62

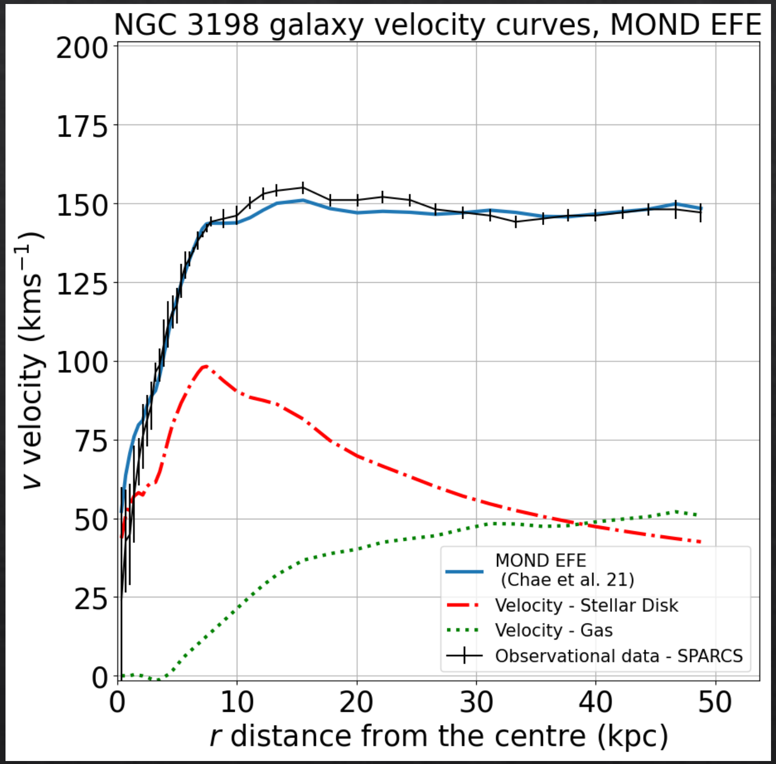


NGC 3198
Source: Sloan
Digital Sky Survey

NGC 3198 rotation curves







Connection to quantum mechanics

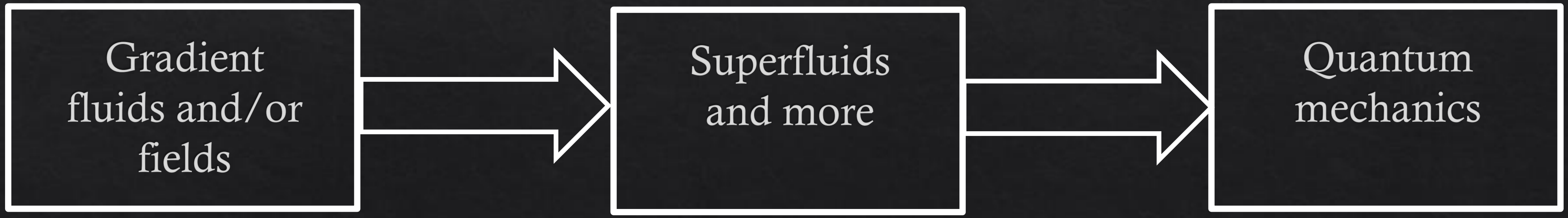
- Korteweg fluids – constitutive state variables: $(e, \nabla e, \rho, \nabla\rho, \nabla^2\rho, \mathbf{v}, \nabla\mathbf{v})$
- Entropy production: $\rho\dot{s} + \nabla\cdot\boldsymbol{J} = \boldsymbol{q}\cdot\nabla\left(\frac{1}{T}\right) - \left[\boldsymbol{P} - p\boldsymbol{I} - \frac{\rho^2}{2}\left(\nabla\cdot\frac{\partial s}{\partial\nabla\rho}\boldsymbol{I} + \nabla\frac{\partial s}{\partial\nabla\rho}\right)\right] : \frac{\nabla\mathbf{v}}{T} \geq 0$
- Rigorous methods are needed: Liu's procedure, as before
- The resulting pressure has classical holographic property:

$$\nabla\cdot\boldsymbol{P}_K = \rho(\nabla\phi + T\nabla s), \qquad \text{where} \quad \phi = \frac{\partial\rho u}{\partial\rho} - \nabla\cdot\frac{\partial(\rho u)}{\partial\nabla\rho} = \delta_\rho(\rho u)\Big|_{\rho s}$$

- Additivity leads to Bohm potential: $\phi_Q(\rho, \nabla\rho, \nabla^2\rho) = -\frac{\hbar^2}{2m^2}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}$, recover the Schrödinger equation
- These fluids thus allow the modelling of quantum-classical transition

Connection to quantum mechanics

- The derivation of evolution equations can be inverted: QM equations can be obtained using this hydrodynamic connection from thermodynamic principles
- Using Liu's procedure, the II. law alone produces holography and dynamics
- QFT, GR can have fluid interpretation: Jackiw et al. (JP A, 2004), Biró and Ván (Foundation of Phys, 2015)
- The connection of thermodynamics and gravity has also been explored by Jacobson (PRL 1995) and Verlinde (JHEP 2011)



Summary

- Particle approach to DM has been a long and unfruitful search, while alternatives keep being developed
- Models using hydrodynamic descriptions have potential for describing galactic dynamics
- Thermodynamic gravity within the hydrodynamic framework offers a potentially powerful method for the description of dynamics on cosmological scales
- Hydrodynamic description greatly benefits from a rigorous foundation and can be a thermodynamic link between QM and gravity

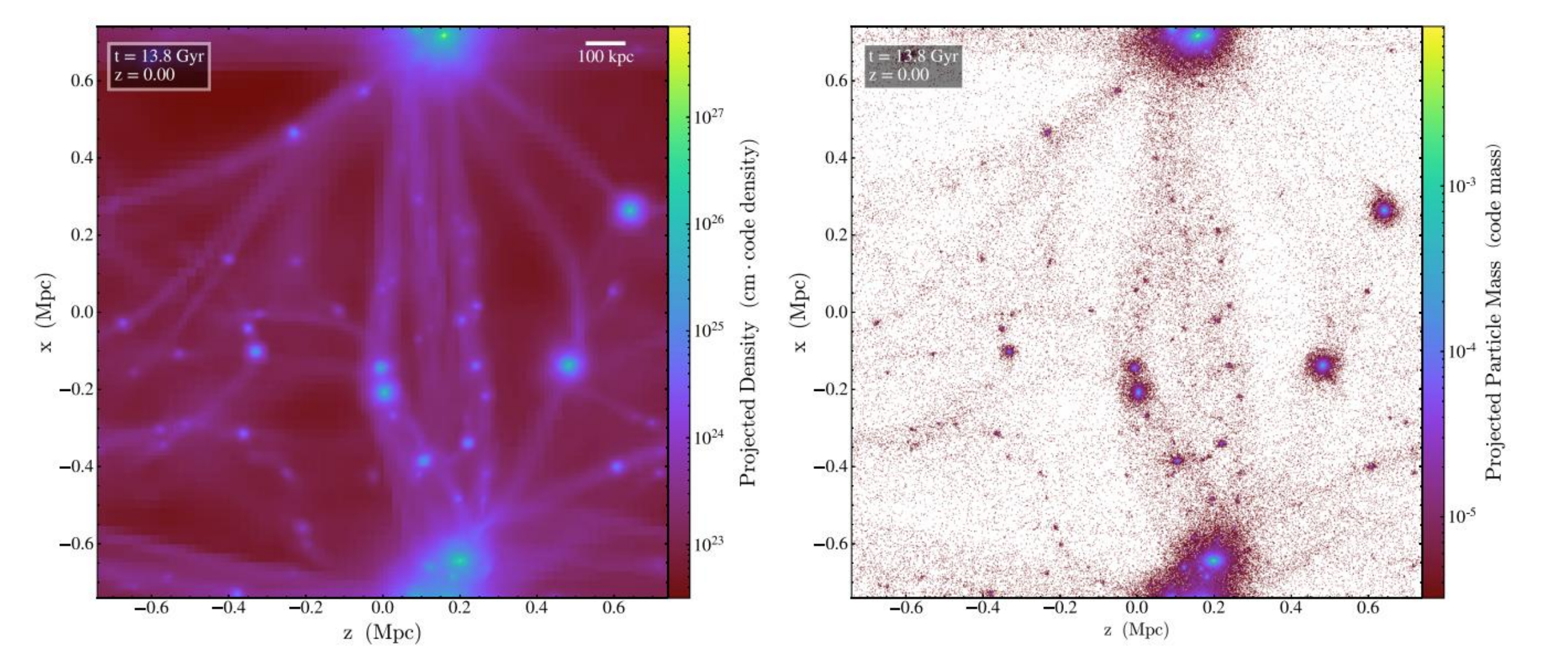
Thank you for your attention!

- Main references:

  - Peter Ván and Sumiyoshi Abe. Emergence of extended Newtonian gravity from thermodynamics. Physica A: Statistical Mechanics and its Applications, 588:126505, 2022
  - P. Ván, Holographic fluids: A thermodynamic road to quantum physics, Physics of Fluids 35, 057105, 2023
  - L. Gavassino, M. Antonelli, and B. Haskell, When the entropy has no maximum: A new perspective on the instability of the first-order theories of dissipation, Phys. Rev. D 102, 043018, 2020
  - S. Abe and P. Ván. Crossover in Extended Newtonian Gravity Emerging from Thermodynamics. Symmetry, 14(5), 2022
  - M. Pszota and P. Ván. Field equation of thermodynamic gravity and galactic rotational curves. DOI 10.1016/j.dark.2024.101660, accepted in Physics of the Dark Universe, (arXiv:2306.01825v3)

Cosmological and galactic simulations

- Evolution of ΛCDM halos in the fluid approximation (left)
- N-body simulation (right)
- Periodic boundary conditions
- Scalar Field DM, Bose-Einstein condensate (superfluid) description



FOIDL, RINDLER-DALLER, and ZEILINGER, PHYS. REV. D 108, 043012 (2023)