ISTITUTO NAZIONALE DI ASTROFISICA
OSSERVATORIO ASTROFISICO DI TORINO

Astrometria Relativistica/Gravitazionale MA2

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con il contributo di
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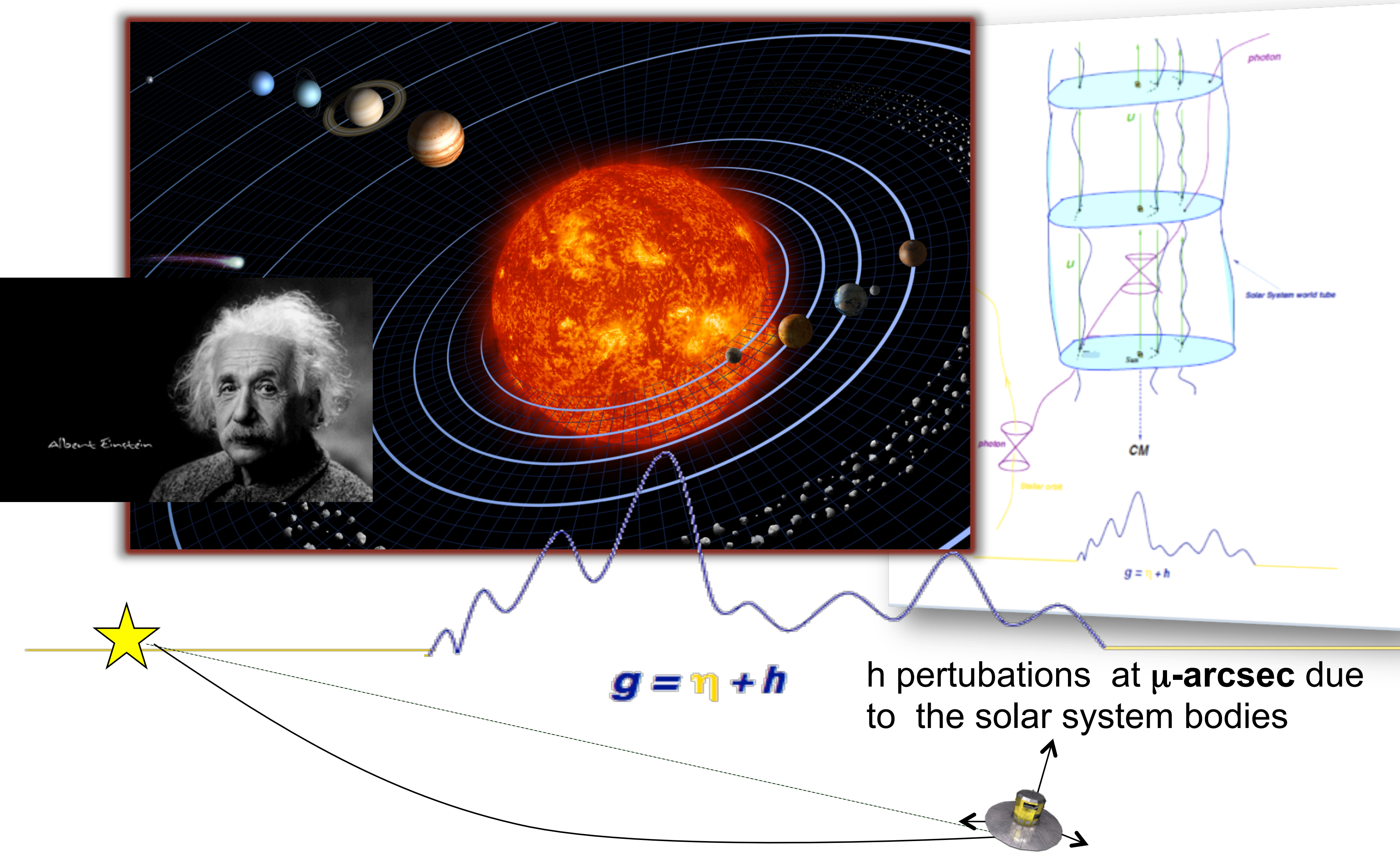
Quello che già conosciamo e stiamo per fare

$\alpha,\delta,\mu_{\alpha},\mu_{\delta},\pi, v_{rad}$, etc…..

L'ingrediente che mancava per continuare a fare

Our laboratory: the Solar System



$g = \eta + h$

h pertubations at **μ-arcsec** due to the solar system bodies

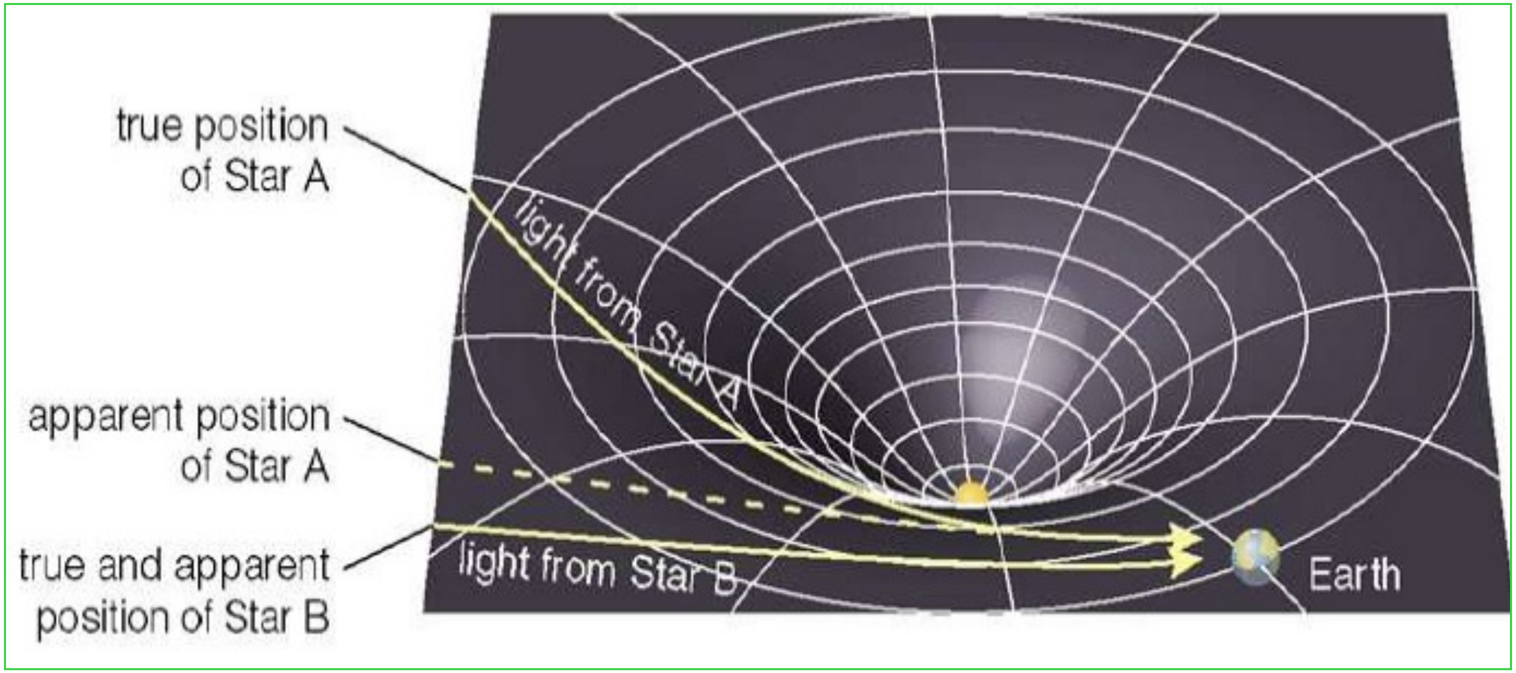
Light effects around the corner

A photon passing near the Sun at a distance d is deflected by an angle

$$\delta\theta = \frac{1}{2}(1+\gamma)\frac{4\,M_\odot}{d}\frac{1+\cos\Phi}{2}$$

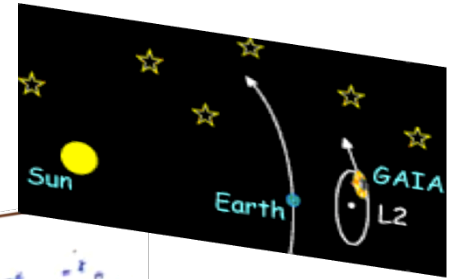
For grazing rays

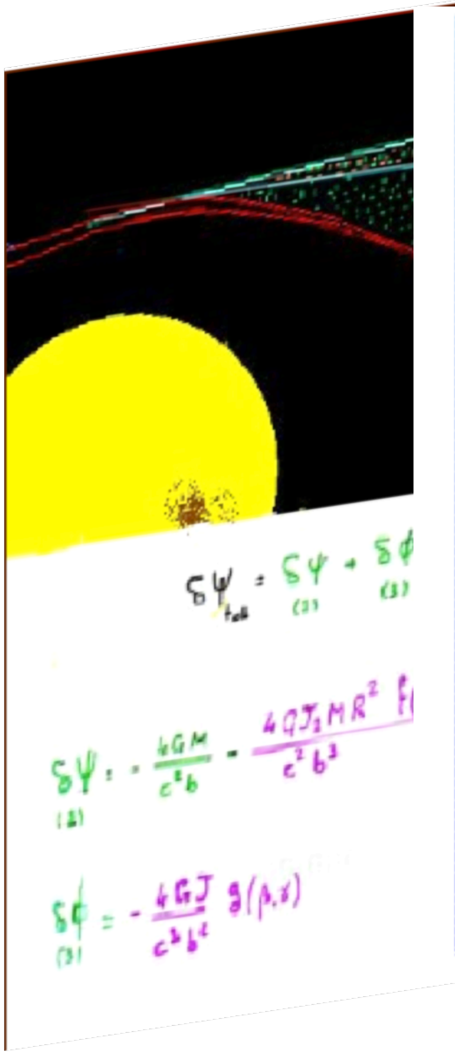
$$\delta\theta \approx \frac{1}{2}(1+\gamma)1.''7505,$$



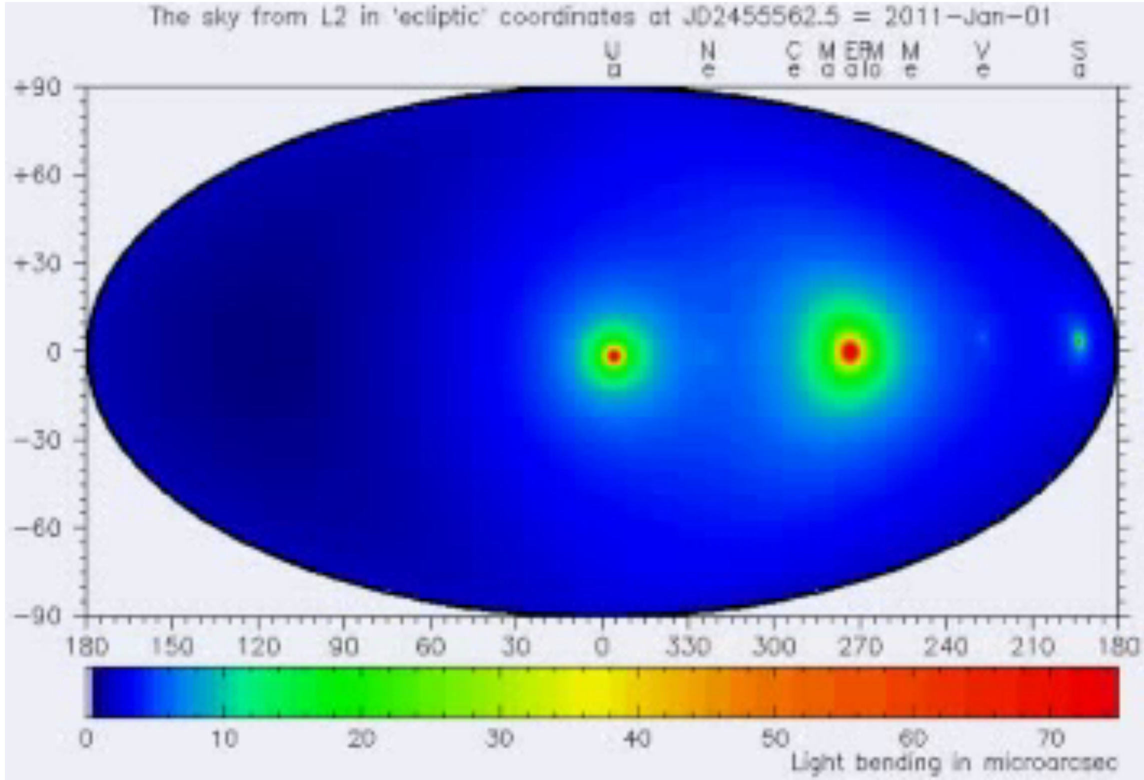
(Nowadays, VLBI accuracy ~ 100micro-arseconds)

Detectable relativistic deflections at L2





	$\delta\chi_{\rm PN}$	$\delta\chi_{J_2}$	$\delta\chi_L$	$\chi_{\max}$
Sun	$1''75$	$\sim 1\ \mu$as	$0.7\ \mu$as	$(180°)$
Mercury	$83\ \mu$as	–	–	$(7')$
Venus	493	–	–	$(4.0°)$
Earth	574	0.6	–	$(101°)$
Moon	26	–	–	$(2.3°)$
Mars	116	0.2	–	$(17')$
Jupiter	16290	240	0.2	$(87°/3')$
Saturn	5772	94	–	$(16°/51'')$
Uranus	2030	7	–	$(67'/4'')$
Neptune	2487	8	–	$(50'/3'')$
Pluto	7	–	–	$(0''3)$

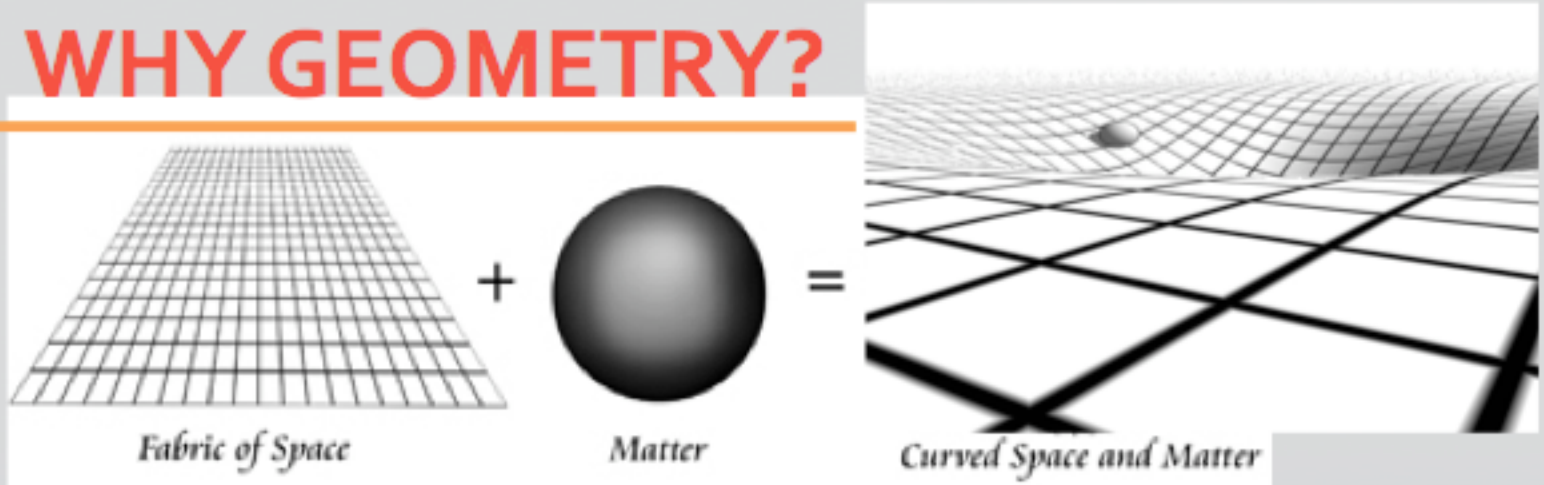


J. de Bruijne © ESA

micro-arcsecond accuracy+
dynamical gravitational fields,
relativistic models of
light propagation:

RELATIVISTIC ASTROMETRY

WHY GEOMETRY?



Brief summary of “obviousness”

- ✓ General Relativity is the theory in which geometry and physics are joined in order to explain how gravity works
- ✓ The trajectory of a photon is deduced by solving the null geodesic in a curved space-time
- ✓ The “astronomical” measurements of the light takes place in a geometrical background generated by a n-body distribution as, for example, the Solar System

RAMOD Measurement Protocol (MP)

Light trajectories& measurements protocol (MP) in General Relativity (GR)

1. Specify the phenomenon under investigation.
2. **Identify the covariant equations** which describe it.
3. **Identify the observer** who makes the measurements.
4. **Chose a frame adapted to that observer allowing the space-time splitting into the observer's space and time (e.g. Gaia, EUCLID, THEIA+, etc..)**.
5. **Understand the locality properties of the measurement under consideration** (local or non-local with respect to the background curvature).
6. **Identify the frame components of those quantities which are the observational targets**.
7. **Find a physical interpretation of the above components following a suitable criterium**.
8. Verify the degree of the residual ambiguity in the interpretation of the measurements and decide the strategy to evaluate it (i.e. comparing what already is known).

Light crossing the metric of the Solar System

(MP step 1) Specify the phenomenon under investigation.
(MP step 2) Identify the covariant equations which describe it.

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} + O(h^2)$$

$$|h_{\alpha\beta}| \ll 1$$

according to the Virial Theorem

$$|h_{\alpha\beta}| \leq U/c^2 \sim v^2/c^2 \rightarrow \varepsilon^2$$

~ v²/c² ~ GM/rc² ~ <u>mas accuracy</u>

which requires determination of

g_{oo} even terms in ε, lowest order ε²~mas

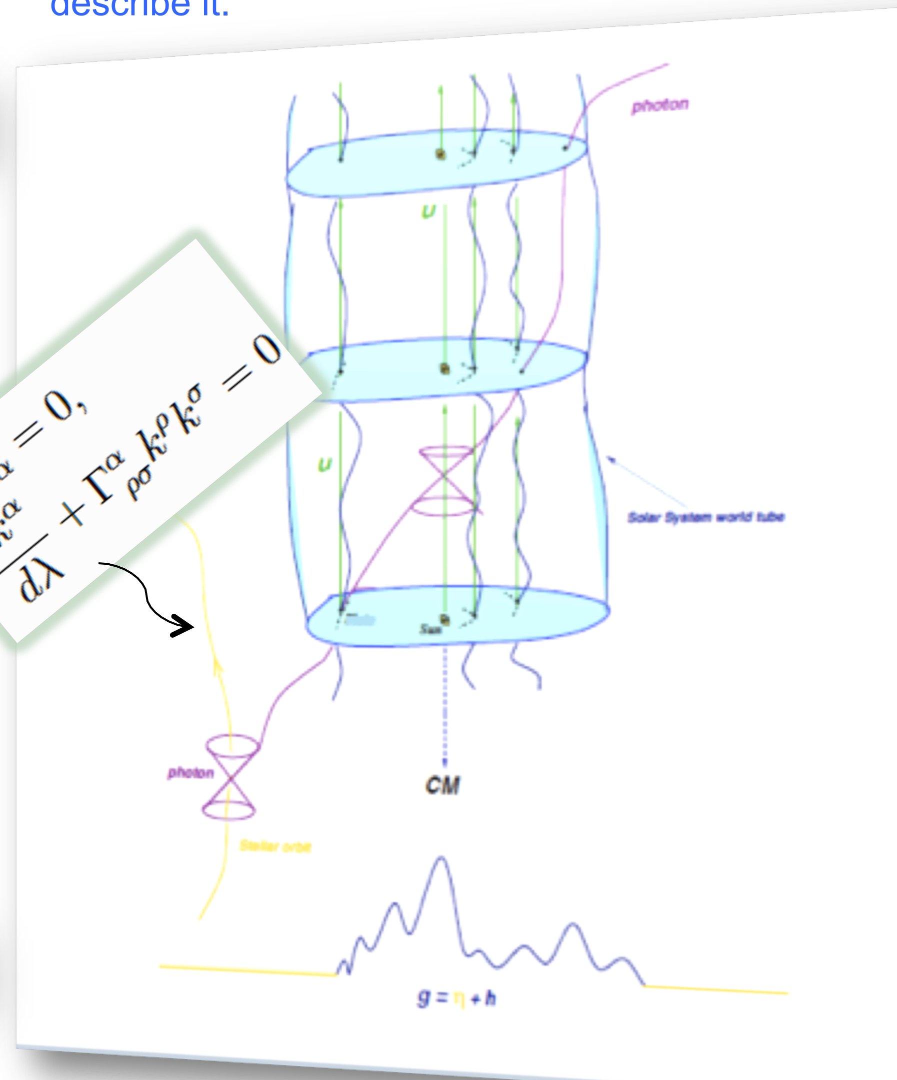
g_{oj} odd terms in ε, lowest order ε³~μ-as

g_{ij} even terms in ε, lowest order ε²~mas

Time variation of the order of $\epsilon |h_{\alpha\beta}|$

→ **IAU metric!**

$$k^\alpha k_\alpha = 0, \quad \frac{dk^\alpha}{d\lambda} + \Gamma^\alpha_{\rho\sigma} k^\rho k^\sigma = 0$$



IAU Post-Newtonian metric for the celestial reference systems

The **post-Newtonian (pN) approximation:**

✓**linear + nonlinear superimposition** (spherical fields of the Sun and all the planets, nonspherical fields to their quadrupole and higher order deformations, and fields due their momentum and angular momentum, and so on..

✓**valid inside the near zone of the Solar System (0.3 pc)**

- definition of IAU reference systems (resolution B1.3, 2000): standard astronomical reference system named **BCRS (Barycentric Celestial Reference System)**, and **GCRS (Geocentric Celestial Reference System)**
- all these reference systems are defined by the form of the corresponding metric tensors

$$g_{00} = -1 + \frac{2}{c^2} w(t,\boldsymbol{x}) - \frac{2}{c^4} w^2(t,\boldsymbol{x}) \,,$$

$$g_{0i} = -\frac{4}{c^3} w^i(t,\boldsymbol{x}) \,,$$

$$g_{ij} = \delta_{ij}\left(1 + \frac{2}{c^2} w(t,\boldsymbol{x})\right).$$

$$w(t,\mathbf{x}) = G\int d^3x' \frac{\sigma(t,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} + \frac{1}{2c^2} G \frac{\partial^2}{\partial t^2} \int d^3x' \sigma(t,\mathbf{x}')|\mathbf{x}-\mathbf{x}'|,$$

gravitational potential, gravitational mass

ric effect)

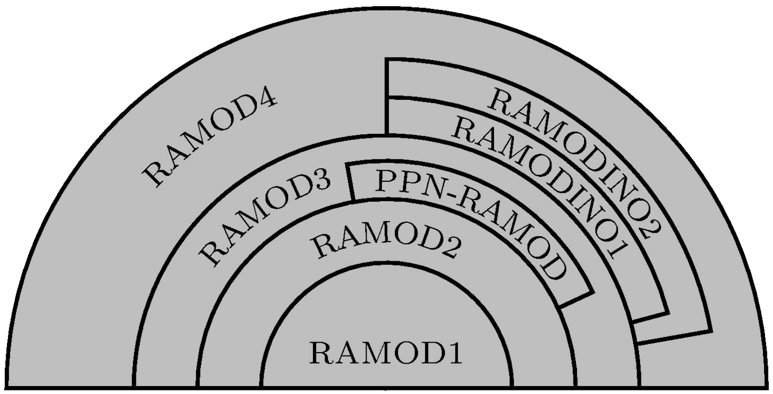
$$dT = -\frac{1}{c} g_{\alpha\beta} u'^{\alpha} dx^{\beta}$$

$$\int d^3x' \frac{\sigma^i(t,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}.$$

potential vector,current mass density
(gravitomagnetic effect)

RelativisticAstrometricMODels&Gaia

TRACING THE GEOMETRY OF THE VISIBILE UNIVERSE



RAMOD is a framework of general relativistic astrometric models

with increasing intrinsic accuracy, adapted to many different

observer's settings, interfacing numerical and analytical relativity

RAMOD applies the measurement protocol (MP) in GR

de Felice, F. & Bini, D. 2010, Classical Measurements in Curved Space-Times, Cambridge University Press

1. RAMOD1: a static non-perturbative model in the Schwarzschild metric of the Sun (de Felice et al., 1998, A&A,332,1133)
2. RAMOD2: a dynamical extension of RAMOD1 (parallaxes and proper motion, de Felice et al., 2001, A&A,373,336)
3. PPN-RAMOD: recasting RAMOD2 in the PPN Schwarzschild metric of the Sun (Vecchiato et al.,2003, A&A, 399,337)
4. RAMOD3: a perturbative model of the light propagation in the static field of the Solar System (milliarcsecond, de Felice et al., 2004, ApJ, 607, 580)
5. RAMOD4: the extension of RAMOD3 to the microarc-second level of accuracy, (de Felice et al., 2006, ApJ, 653, 1552)
6. RAMODINO1-2-3: satellite-observer model for Gaia (Bini et al., 2003, Class.Quantum Grav., 20, 2251/4695); ray tracing error budget (de Felice, F.; Preti, G. 2006CQGra..23.5467D and 2008CQGra..25p5015D)
7. RAMOD vs PM/PN approach: Crosta 2011 Class. Quantum Grav. 28 235013;
8. RAMOD analytical solutions for Gaia-like case : 2015Crosta, Vecchaito, de Felice , Lattanzi Classum Quantum Gravity

Retarded distances contributions

[Geometrized unit G=c=1]

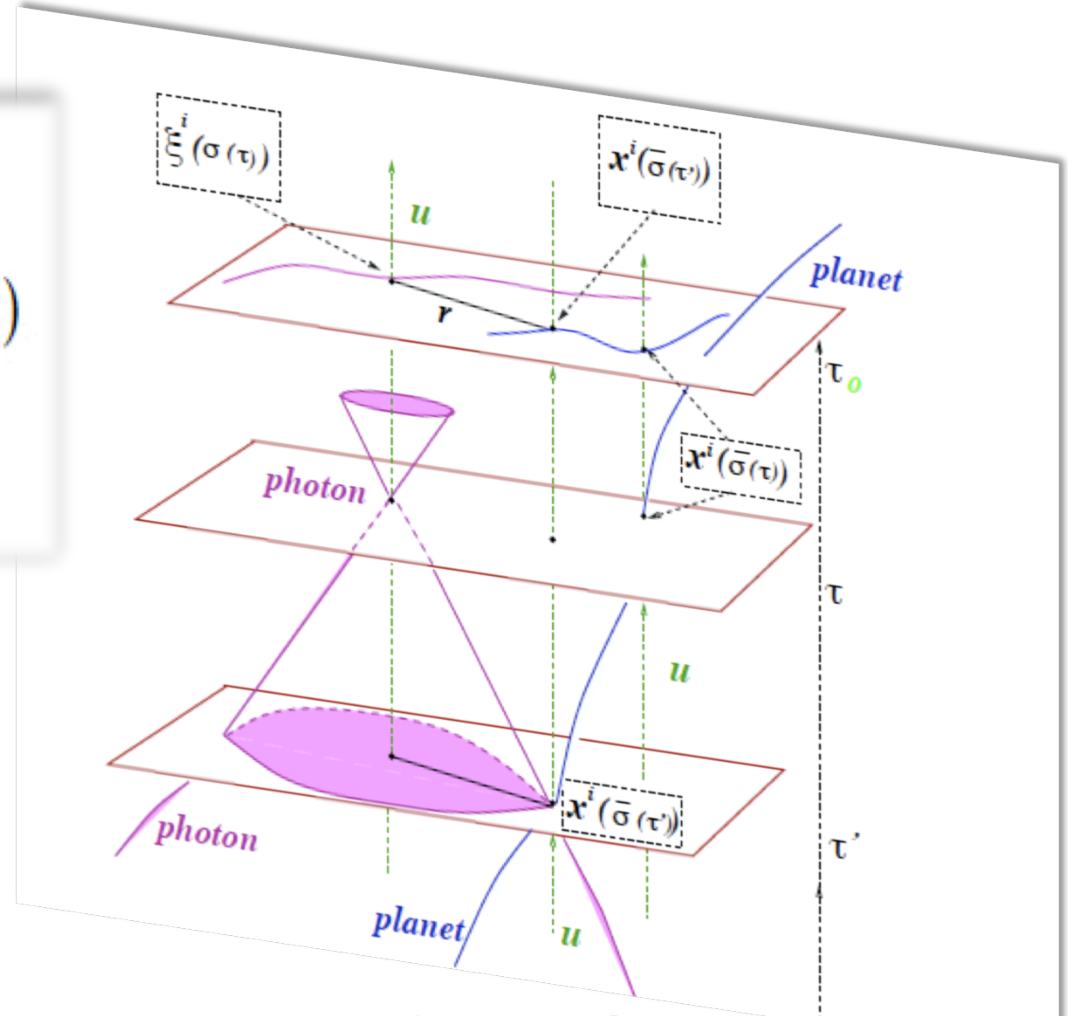
$$h_{00} = \sum_a \frac{2M_{(a)}}{r_{(a)}} + O\left(\epsilon^4\right)$$

$$h_{0i} = -\sum_a \frac{4M_{(a)}}{r_{(a)}}\tilde{\beta}_{i(a)} + O\left(\epsilon^5\right)$$

$$h_{ij} = \sum_a \frac{2M_{(a)}}{r_{(a)}}\delta_{ij} + O\left(\epsilon^4\right),$$

$$t' - t = r_{(a)}/c$$

$$r^i_{(a)}(\sigma, \tilde{\sigma}') = x^i(\sigma) - \tilde{x}^i_a(\tilde{\sigma}'),$$



$$\tilde{\beta}^j = (1 - h_{00}/2)\tilde{v}^i(\tilde{\sigma}) + O(h^2)$$

Observer & frames

(MP step 3) Identify the observer who makes the measurements.
(MP step 4) Chose a frame adapted to that observer allowing the space-time splitting into the observer's space and time.

✓Family of **fiducial observes** (FIDO) to set the reference frames:
a time-like congruences of curves *u*

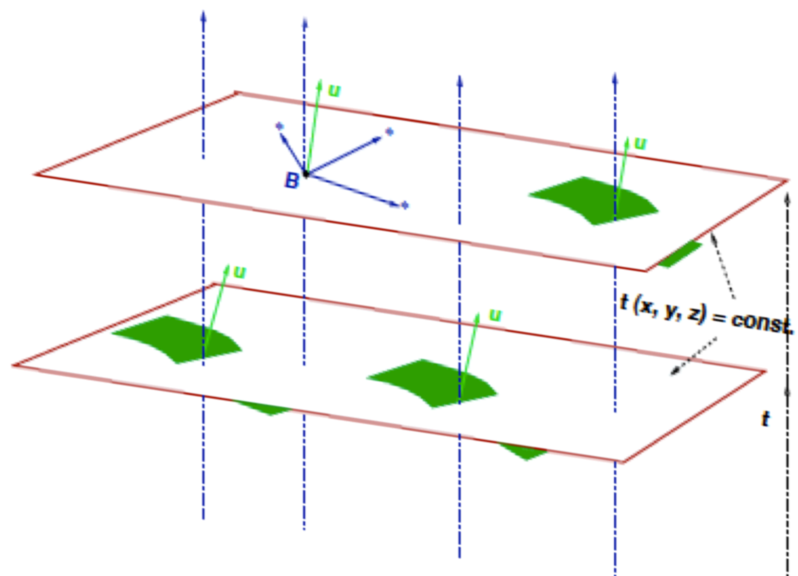
u is defined everywhere

g_{oi} ~ μ-arcsecond

the fiducial observer **u** *locally and only locally* is at rest w.r.t. the spatial coordinates

-> static observer at rest locally w.r.t the barycenter of the Barycentric Celestial Reference System (B) ->

u ~ $\frac{1}{\sqrt{-g_{00}}}\partial_0$



The observational target

Local line-of-sight

$$\ell^\alpha = P^\alpha_\beta(u)k^\beta(\tau)$$

Tangent to null geodesic

(MP step 6) Identify the **frame components** of those quantities which are the **observational targets**.

$$P(u')_{\alpha\beta} = g_{\alpha\beta} + u'_\alpha u'_\beta$$

Projector operator in the rest-space of **u**

$$k^\alpha k_\alpha = 0,$$
$$\frac{dk^\alpha}{d\lambda} + \Gamma^\alpha_{\rho\sigma}k^\rho k^\sigma = 0$$

Master Equations

$$\frac{d\ell^\alpha}{d\sigma} = F^\alpha(\partial_\beta h(x,y,z,t), \ell^i(\sigma(x)))$$

A general solution $\bar{\ell}^i(\sigma) = \bar{\ell}^i(\bar{\boldsymbol{\ell}}(\sigma_0), h_{\alpha\beta}(\sigma))$ depends on the observed ℓ^k_{obs}

✓**boundary condition to solve uniquely the differential equations**
✓link to the parameters of the star in the astrometric measurements (condition equation)

$$\frac{d\bar{\ell}^k}{d\sigma} + \bar{\ell}^k \left(\frac{1}{2}\bar{\ell}^i h_{00,i}\right) + \delta^{ks}\left(h_{sj,i} - \frac{1}{2}h_{ij,s}\right)\bar{\ell}^i\bar{\ell}^j - \frac{1}{2}\delta^{ks}h_{00,s} = 0.$$

STATIC CASE MODEL (enough for Gaia)

$$\frac{d\bar{\ell}^0}{d\sigma} - \bar{\ell}^i\bar{\ell}^j h_{0j,i} - \frac{1}{2}h_{00,0} = 0$$

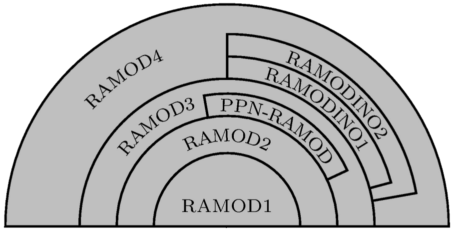
DYNAMICAL CASE MODEL

$$\begin{aligned}\frac{d\bar{\ell}^k}{d\sigma} &- \frac{1}{2}\bar{\ell}^k\bar{\ell}^i\bar{\ell}^j h_{ij,0} + \bar{\ell}^i\bar{\ell}^j\left(h_{kj,i} - \frac{1}{2}h_{ij,k}\right)\\ &+ \frac{1}{2}\bar{\ell}^k\bar{\ell}^i h_{00,i} + \bar{\ell}^i\left(h_{k0,i} + h_{ki,0} - h_{0i,k}\right) - \frac{1}{2}h_{00,k} - \bar{\ell}^k\bar{\ell}^i h_{0i,0} + h_{k0,0} = 0.\end{aligned}$$

One more equation to integrate: spatial time component!

there exists several analytical solutions

R.A.MOD. models (Crosta et al., Classum Quantum Gravity, 32 (2015) 1655008 and references therein)



(MP step 7)
Find a physical interpretation of the above components following a suitable criterium.

RAMOD3 MASTER EQUATION

n-monopoles

1. **RAMOD3a (R3a)** and , **the spatial derivatives** of the metric are considered while $\mathbf{h_{0i}}$ **are neglected**
2. **RAMOD3b** (R3b), the **spatial and time derivatives** of the metric are considered while $\mathbf{h_{0i}}$ **are neglected**

n-monopoles+ quadrupole

3. RAMOD3aQ (R3aQ)
4. RAMOD3bQ (R3bQ)

$$r(t',t) \approx r(t) - r\cdot v$$

At first order in v

Crosta 2011 Class. Quantum Grav.
Crosta et al., 2015 Class. Quantum. Grav.

RAMOD4 MASTER EQUATION

n-monopoles

1. **RAMOD4a** (R4a), the **spatial derivatives** of the metric are considered **including** $\mathbf{h_{0i}}$
2. **RAMOD4b** (R4b), the **spatial and time derivatives** of the metric are considered **including** $\mathbf{h_{0i}}$

n-monopoles+ quadrupole

1. RAMOD4aQ (R4aQ)

The implementation of RAMOD models and the need of testing them through a self-consistency check at different levels of accuracy, will benefit form this explicit classification.

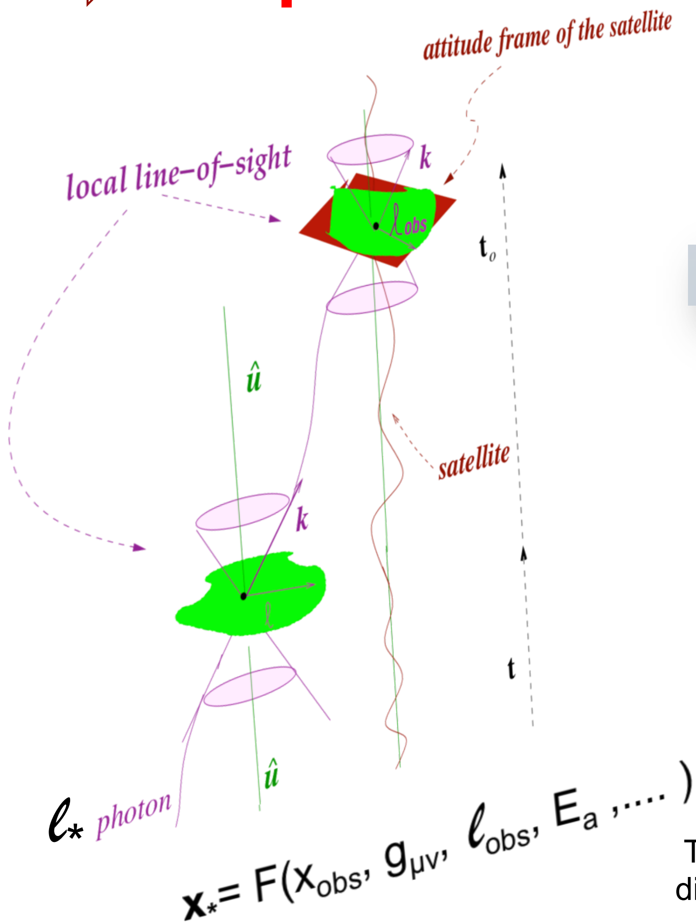
aberrated (gravitational) direction

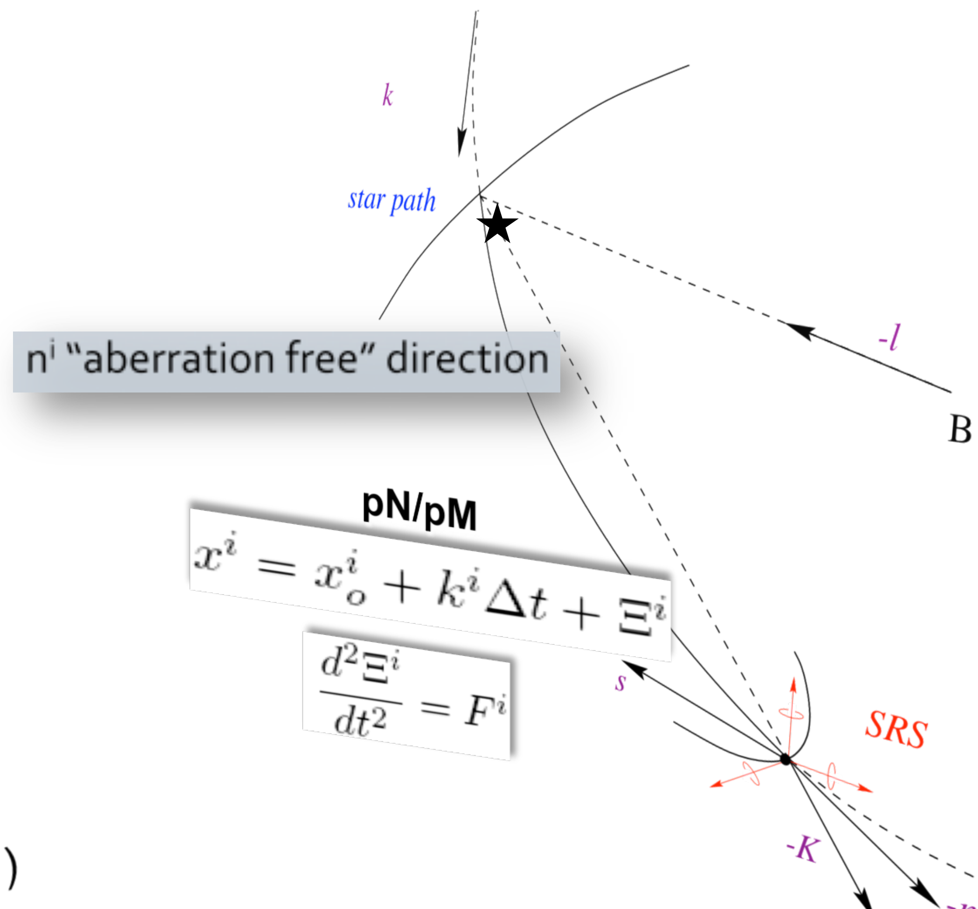
$$\bar{l}^i = n^i\left(1 - \frac{h_{00}}{2}\right) + \mathcal{O}\left(\frac{v^4}{c^4}\right)$$

$h_{00}/2 \approx U/c^2$ (local potential)[IAU solution]

100 μas!

(MP step 8) Verify the degree of the residual ambiguity in the interpretation of the measurements and decide the strategy to evaluate it (i.e. comparing what already is known).





The RAMOD local-line-of-sight is not exactly equal to the light direction used in the semi-classical approximation

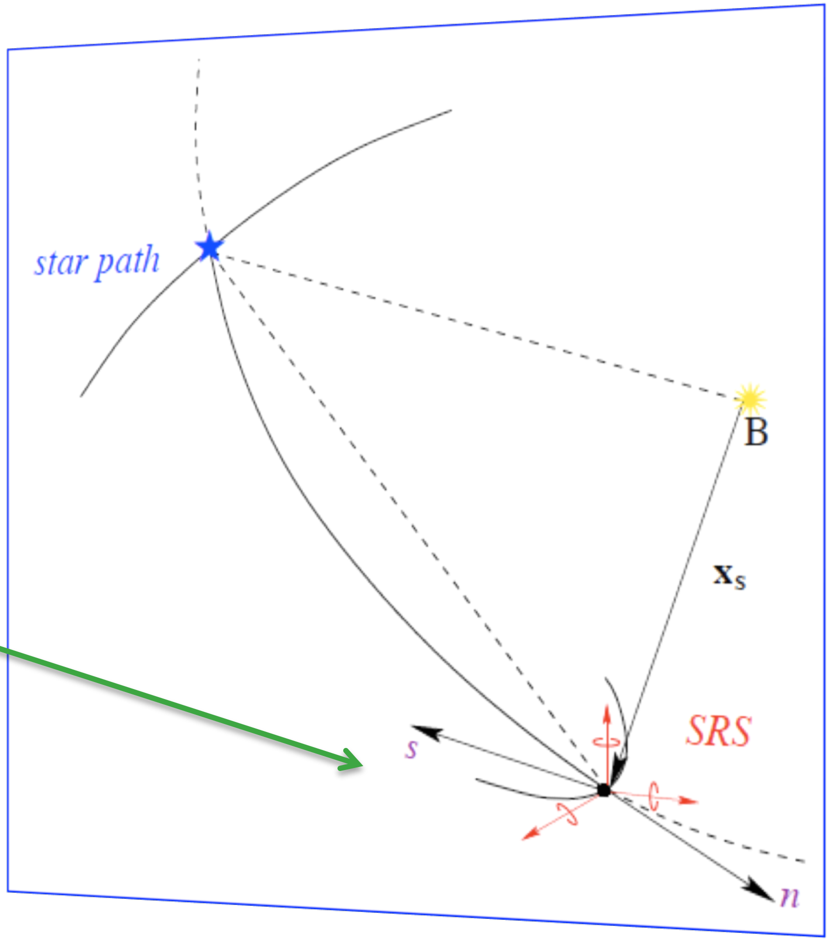
stellar direction in pN

From the null geodesic

$$x^i = x^i_o + k^i \Delta t + \Xi^i \longrightarrow \frac{d^2\Xi^i}{dt^2} = F^i$$

s^i observed direction at x_s

$$\begin{aligned} s^i = & -n^i + c^{-1}\left[\mathbf{n}\times(\mathbf{v}\times\mathbf{n})\right]^i \\ & + c^{-2}\left\{(\mathbf{n}\cdot\mathbf{v})\left[\mathbf{n}\times(\mathbf{n}\times\mathbf{v})\right]^i + \frac{1}{2}\left[\mathbf{v}\times(\mathbf{n}\times\mathbf{v})\right]^i\right\} \\ & + c^{-3}\left\{\left[(\mathbf{v}\cdot\mathbf{n})^2 + (1+\gamma_{\mathrm{PPN}})w(\mathbf{x}_s)\right]\left[\mathbf{n}\times(\mathbf{v}\times\mathbf{n})\right]^i \right. \\ & \left. + \frac{1}{2}(\mathbf{n}\cdot\mathbf{v})\left[\mathbf{v}\times(\mathbf{n}\times\mathbf{v})\right]^i\right\} + \mathcal{O}(4). \end{aligned}$$



n^i “aberration free” coord. direction = p^i/p
p^i barycentric coord. light dir. at x_s

SRS=Satellite Reference System,
B =Solar System Barycenter
v= velocity of the satellite

Orders of approximation in parallax and proper motion

reference value: Barnard star

$$\pi \sim 1 \text{ arcsecond}$$

$$\mu \sim 10 \text{ arcsecond/year}$$

ε^2 ~ mas

ε^3 ~ μ-as

Accounted term	order of approximation
π	$0.5\ \epsilon \cdot 10^{-1}$
μ	$0.5\ \epsilon$ year^{-1}
π^2	$0.3\ \epsilon^2 \cdot 10^{-2}$
μ^2	$0.3\ \epsilon^2$ year^{-2}
$\mu^2\epsilon$	$0.3\ \epsilon^3$ year^{-2}
$\epsilon^2\mu$	$0.5\ \epsilon^3$ year^{-1}
Neglected term	**order of approximation**
π^3	$1.3\ \epsilon^3 \cdot 10^{-4}$
μ^3	$1.3\ \epsilon^3 \cdot 10^{-1}$ year^{-3}
$\pi^2\epsilon$	$0.3\ \epsilon^3 \cdot 10^{-2}$
$\pi\mu\epsilon$	$0.3\ \epsilon^3 \cdot 10^{-1}$ year^{-1}
$\epsilon^2\pi$	$0.5\ \epsilon^3 \cdot 10^{-1}$
$\mu\pi^2$	$1.3\ \epsilon^3 \cdot 10^{-3}$ year^{-1}
$\mu^2\pi$	$1.3\ \epsilon^3 \cdot 10^{-2}$ year^{-2}

The deflection and proper motion contribution

$$n^i \equiv -\frac{\dot{x}^i(\tau_o)}{|\dot{\vec{x}}(\tau_o)|} \rightarrow n^i \;=\; -\left(k^i + c^{-1}\dot{\Xi}^i(\tau_o)\right)\cdot\left\{1 - c^{-2}\left[(\dot{\vec{\Xi}}\cdot\vec{k}) - \frac{1}{2}(\dot{\vec{\Xi}}\cdot\dot{\vec{\Xi}}) + \frac{3}{2}(\vec{k}\cdot\dot{\vec{\Xi}})^2\right]\right\}$$

$$+O(\Xi^3\cdot c^{-3}),$$

$$k^i = -K^i + O(c^{-2}) \longrightarrow K^i = -\frac{x^i_o(t) - x^i_*(T)}{|\vec{x}_o(t) - \vec{x}_*(T)|} \longrightarrow |\vec{x}_*(T)| \equiv R(T)$$

t= observer's BCRS coordinate time

T= BCRS stellar coordinate time, T_0 emission time

barycentric coordinate direction

$$\boxed{l^i(T) \equiv -\frac{x^i_*(T)}{R(T)}}$$

parallactic shift

$$\boxed{\pi^i \equiv \frac{1}{R}[\vec{l}_0 \times (\vec{x} \times \vec{l}_0)]^i}$$

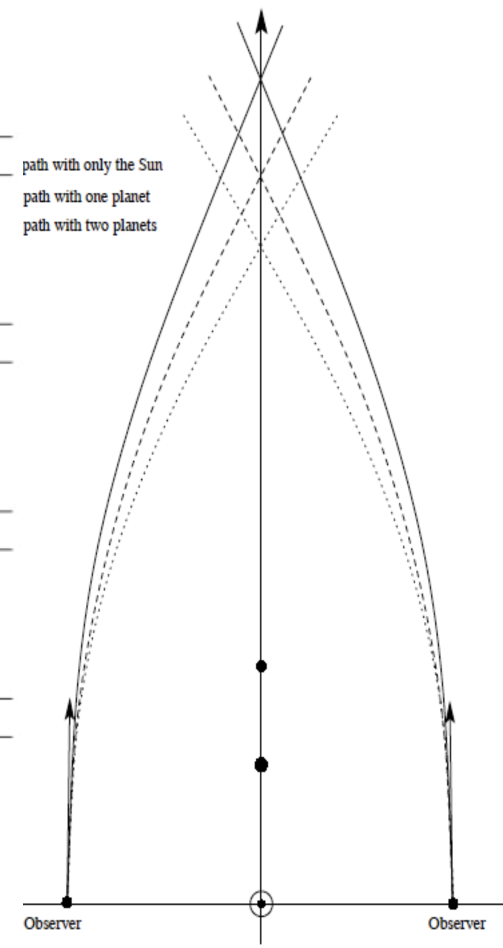
proper motion

$$\boxed{\mu^i \equiv -\dot{l}^i_0} \longrightarrow \dot{l}^i_0 = \left[\frac{d}{dT}\left(-\frac{x^i_*(T)}{R(T)}\right)\right]_{T=T_0}$$

from the local line-of-sight to the star "position"

$$\mathbf{x}_* = F(x_{obs}, g_{\mu\nu}, \ell_{obs}, E_a, \ldots)$$

	⊙	⊙+♃	⊙+♃+♄	⊙+♃+♄+♅	⊙+♃+♄+♅+☿
			distance $\sim$ 1 pc		
d (AU)	206261.338	206259.698	206259.216	206259.142	206259.054
p_i ($''$)	1.00001775	1.00002571	1.00002804	1.00002840	1.00002883
$p_\odot - p_i$ (μarcsec)		-7.95	-10.29	-10.65	-11.07
$\Delta\psi_i$ (μarcsec)		-8.00	-10.30	-10.60	-11.10
			distance $\sim$ 10 pc		
d (AU)	2062283.55	2062120.14	2062072.27	2062064.94	2062056.3
p_i ($''$)	0.10001777	0.10002569	0.10002802	0.10002837	0.10002879
$p_\odot - p_i$ (μarcsec)		-7.93	-10.25	-10.60	-11.02
$\Delta\psi_i$ (μarcsec)		-8.00	-10.30	-10.60	-11.00
			distance $\sim$ 100pc		
d (AU)	20589921.6	20573645.4	20568881.5	20568152.5	20567292.3
p_i ($''$)	0.01001777	0.01002569	0.01002801	0.01002837	0.01002879
$p_\odot - p_i$ (μarcsec)		-7.93	-10.25	-10.60	-11.02
$\Delta\psi_i$ (μarcsec)		-7.99	-10.30	-10.60	-11.00
			distance $\sim$ 1000 pc		
d (AU)	202664764	201098829	200644596	200575248	200493473
p_i ($''$)	0.00101776	0.00102569	0.00102801	0.00102837	0.00102879
$p_\odot - p_i$ (μarcsec)		-7.93	-10.25	-10.60	-11.02
$\Delta\psi_i$ (μarcsec)		-7.99	-10.30	-10.60	-11.00
			distance $\sim$ 10000 pc		
d (AU)	1751489760	1641052370	1611285280	1609051510	1603803810
p_i ($''$)	0.00011777	0.00012569	0.00012801	0.00012819	0.00012861
$p_\odot - p_i$ (μarcsec)		-7.93	-10.25	-10.42	-10.84
$\Delta\psi_i$ (μarcsec)		-7.99	-10.30	-10.60	-11.00



observed direction w.r.t. the BCRS $s^i = \mathcal{L}^i + \mathcal{P}^i + \mathcal{M}^i + \mathcal{D}^i + \mathcal{V}_s^i$

Each components is a complicated function of a set of standard parameters up to the accuracy required

parallactic shift $\pi^i \equiv \frac{1}{R_0}[\vec{l}_0 \times (\vec{x} \times \vec{l}_0)]^i$

proper motion and independent parameter

$\mu^i \equiv R_0^{-1}[\vec{l}_0 \times (\vec{v}_{*0} \times \vec{l}_0)]^i$
$v_r \equiv (\vec{l}_0 \cdot \vec{v}_{*0})$
$\nu^i \equiv [\vec{l}_0 \times (\dot{\vec{v}}_{*0} \times \vec{l}_0)]^i$
$a_r \equiv (\vec{l}_0 \cdot \dot{\vec{v}}_{*0})$
$\zeta^i \equiv [\vec{l}_0 \times (\ddot{\vec{v}}_{*0} \times \vec{l}_0)]^i$
$z_r \equiv (\vec{l}_0 \cdot \ddot{\vec{v}}_{*0})$

tangential velocity $\vec{l} \times (\vec{v}_* \times \vec{l})] = R_0^{-1}\vec{\mu}$

relativistic astrometric parameters at 0.1μas in pM/pN (for a Barnard like-star)

- barycentric direction

$$
\begin{aligned}
\mathcal{L}^i \quad = \quad & -l_0^i \Big\{ 1 - \pi^2 + (\vec{\Pi} \cdot \vec{\mu})[\Delta t + c^{-1}(\Delta \vec{x} \cdot \vec{l}_0)] \\
& + \delta_{km} \dot{\Xi}^k \left[c^{-1}(l_0^m + \pi^m + \mu_{app}^m \Delta t) + c^{-2} \left(\dot{\Xi}^m - 2 l_0^m (\vec{l}_0 \cdot \vec{v}_s) + v_s^m \right) \right] \\
& + \delta_{km} v_s^k \Bigg[c^{-1} \left(l_0^m + \pi^m + \mu_{app}^m \Delta t + \frac{1}{2} \dot{\mu}_{app}^m \Delta t^2 \right) + c^{-2} \Bigg(\frac{3}{2} l_0^m + \frac{1}{2} v_s^m \\
& - l_0^m (\vec{l}_0 \cdot \vec{v}_s) + \frac{3}{2} \mu_{app}^m (\vec{l}_0 \cdot \vec{v}_s) \Delta t + \frac{1}{2} \mu_{app}^m (\vec{\mu}_{app} \cdot \vec{v}_s) \Delta t^2 \Bigg) + c^{-3} \Bigg(l_0^m (\vec{l}_0 \cdot \vec{v}_s) \\
& + 2 w(\mathbf{x}_s) l_0^m - \frac{1}{2} v_s^m (\vec{l}_0 \cdot \vec{v}_s) \Bigg) \Bigg] \Big\} + c^{-2} q F^{ij} l_0^j [1 - c^{-1}(\vec{l}_0 \cdot \vec{v}_s)] \\
& + O(\epsilon^3 \cdot 10^{-1})
\end{aligned}
$$

- parallactic parameter

$$
\begin{aligned}
\mathcal{P}^i \quad = \quad & \pi^i \Big\{ \delta_{km} l_0^k [\Pi^m - c^{-1}(\dot{\Xi}^m - v_s^m)] \\
& - [1 + v_r R_0^{-1}(1 + \beta_r) \Delta t] \Big\} + O(\epsilon^3 \cdot 10^{-1})
\end{aligned}
$$

- proper motion parameter

$$\mathcal{M}^i = \mu^i_{app}\Delta t\left\{1 - c^{-1}[(\vec{\mu}_{app}\cdot\vec{v}_s)\Delta t + \delta_{km}l_0^k(\dot{\Xi}^m + v_s^m) + \frac{1}{2}l_0^k c^{-1}v_s^m(\vec{l}_0\cdot\vec{v}_s)]\right\}$$
$$-\mu^i(\vec{\Pi}\cdot\vec{l}_0)[\Delta t + c^{-1}(\Delta\vec{x}\cdot\vec{l}_0)] - qF^{ij}\mu^j_{app}\Delta t$$
$$+\dot{\mu}^i\Delta t^2\left[\left(\frac{1}{2}+\beta_r\right) + \frac{1}{2}c^{-1}(\vec{l}_0\cdot\vec{v}_s)\right] + \frac{1}{6}\ddot{\mu}^i\Delta t^3$$
$$+O(\epsilon^3\cdot 10^{-1})$$

- deflection parameter

$$\mathcal{D}^i = c^{-1}\dot{\Xi}^i\left[1 + \delta_{km}l_0^k c^{-1}(\dot{\Xi}^m - v_s^m)\right]$$
$$+ \quad O(\epsilon^3\cdot 10^{-1})$$

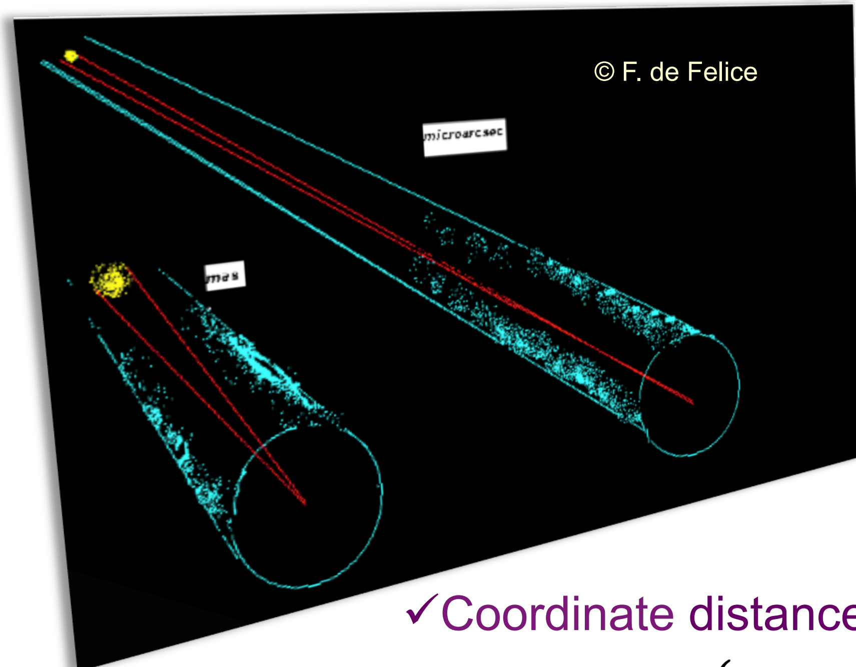
Relativistic effects in the solar systems depending on the method chosen in order to integrate the null geodetic

- aberrational parameter

$$\mathcal{V}_s^i = c^{-1}v_s^i\left\{1 + \delta_{km}v_s^k c^{-1}\left[-\frac{1}{2}l_0^m\left(1 - c^{-1}(\vec{l}_0\cdot\vec{v}_s)\right) + \frac{1}{2}\mu^m_{app}\Delta t + 2w(\mathbf{x}_s)\right]\right\}$$
$$+c^{-3}qF^{ij}v_s^j + O(\epsilon^3\cdot 10^{-1})$$

M. T. Crosta, *Methods of Relativistic Astrometry for the analysis of astrometric data in the Solar System gravitational field*, Ph.D. thesis, Università di Padova, Centro Interdipartimentale di Studi e Attività Spaziali (CISAS) "G. Colombo" (2003).

stellar distances..



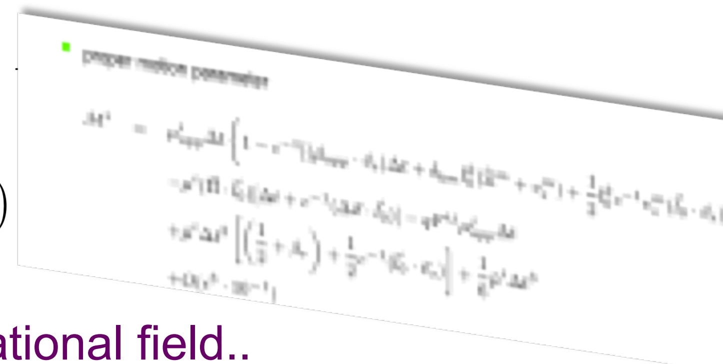
the local line-of-sight in RAMOD determines the stellar distances from within a local curved space-time

the distance to the stars takes into account the metric of the solar system space-time

$$m - M = 5 \log r_{pc} - 5$$

✓Coordinate distance (e.g.static solution of RAMOD)

$$x_*^k = x_0^k + \bar{\ell}_o^k \Delta\hat{\tau} + \frac{2G}{c^2} \sum_a \mathcal{M} \left\{ \frac{\bar{\ell}_o^k}{2} Log \left[\frac{(\vec{\bar{\ell}}_o \cdot \vec{r}_o) + r_o}{(\vec{\bar{\ell}}_* \cdot \vec{r}_*) + r_*} \right] \left[\frac{\bar{\ell}_o^k}{2r_o} + \frac{d_o^k (\bar{\ell}_o \cdot \vec{r}_o)}{r_o d_o^2} \right] \Delta\hat{\tau} + \frac{d_o^k}{d_o^2} (r_o - r_*) \right\} + O\left(h^2\right)$$



✓Physical distance measured at the local gravitational field..

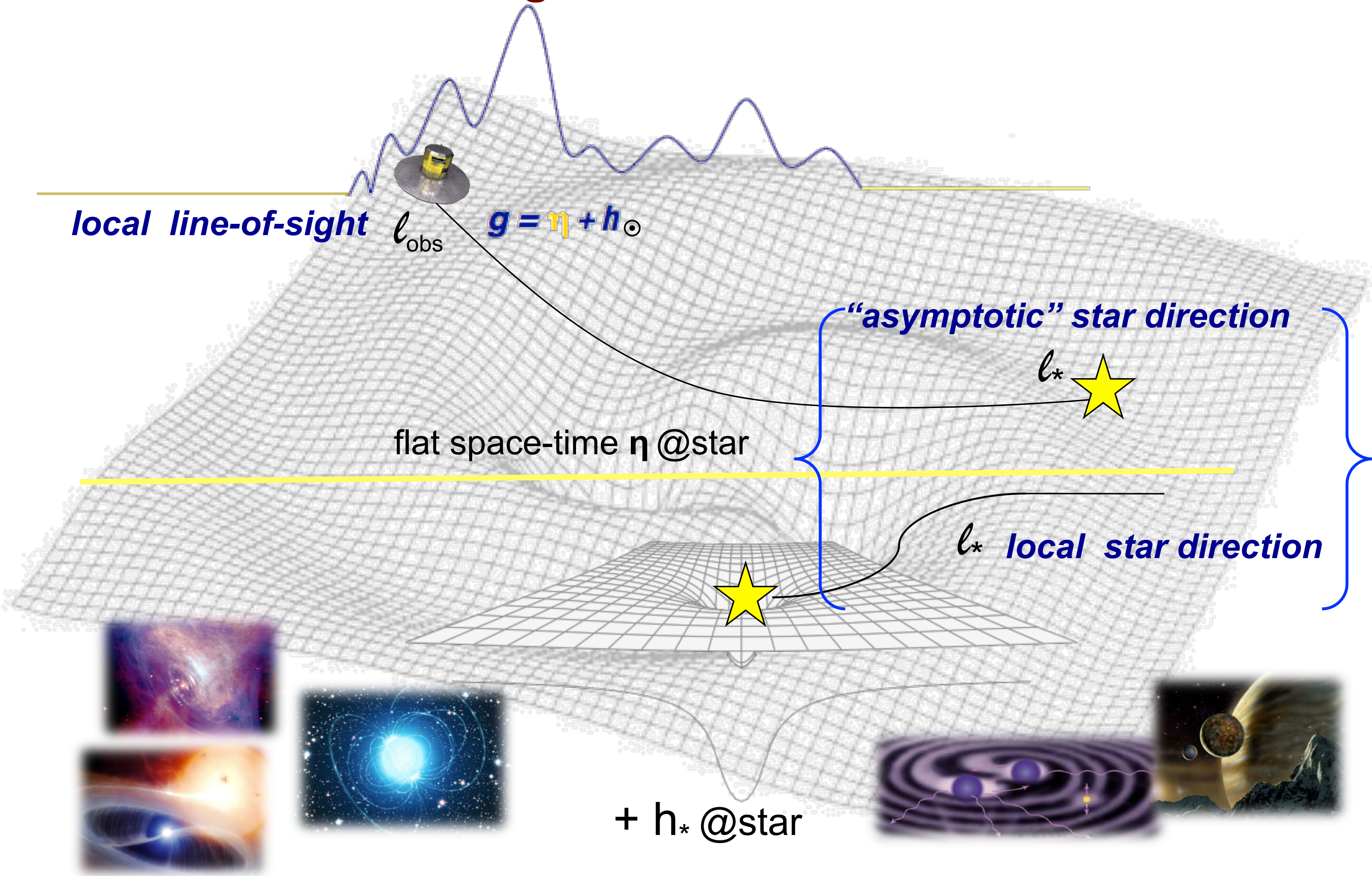
$$dL_u = \sqrt{P_{\alpha\beta}(u) dx^\alpha dx^\beta} \Longrightarrow$$

$\sigma_{m\text{-}M} \approx 2\ \sigma_d / d \approx 2\text{x}0.08 \approx 0.2$ **mag**!

Luminosità ed implicazioni con H_0

connessione implicita con altre aree tematiche

from the local line-of-sight to the local star direction



fully general-relativistic Doppler shift formula

Determination of the stellar velocity by taking advantage both of the spectroscopic and of the astrometric data supplied by the Gaia observations de Felice F Preti G Crosta M and Vecchiato A 2011 Astron. Astrophys. 528 A23+

- ω_* is the frequency of a photon as emitted by the star
- ω_{sat} is the corresponding reference frequency relative to the satellite rest-observer

$$\frac{\omega_*}{\omega_{\rm sat}} = \frac{1 - v_{*_{\rm rad}}}{\sqrt{1 - v_*^2}}$$

special-relativistic formula

$$\frac{\omega_*}{\omega_{\rm sat}} = \frac{1}{A(M_*, R_*)} \cdot \frac{\sqrt{(-g_{00})_{\rm sat}}}{\gamma_{\rm sat}(1 - v^\alpha_{\rm sat}\bar{\ell}_{(0)\alpha})}$$
$$\times \exp\left[-\frac{1}{2}\int_{\sigma_*}^{\sigma_0} (\bar{\ell}^i\bar{\ell}^j\partial_0 h_{ij} + \partial_0 h_{00})d\sigma\right] \cdot \frac{1 - v_{*_{\rm rad}}}{\sqrt{1 - v_*^2}}.$$

general-relativistic Doppler shift formula

$$\boxed{v_*^\alpha = v_{*_{\rm rad}}\bar{\ell}_*^\alpha + v_{*_\perp}^\alpha,}$$

c=1

$$v_*^\alpha = -\frac{P(u)^\alpha{}_\beta u_*^\beta}{u_*^\rho u_\rho};$$

$$A(M_*, R_*) = 1 + \frac{M_*}{R_*} + \mathcal{O}(4),$$
$$\gamma_{\rm sat}^{-1} = 1 - \frac{1}{2}v_{\rm sat}^2 + \mathcal{O}(4)$$

Radial velocities and distances are also crucial for
-understanding the connection between giant planets, brown dwarfs and extreme low-mass stars
-revealing relativistic effects and characterizing a sample of extra-solar planets

in 1.5 pN => (v/c)^3 = ε^3 ~ μ-as

Newtonian order

mas order

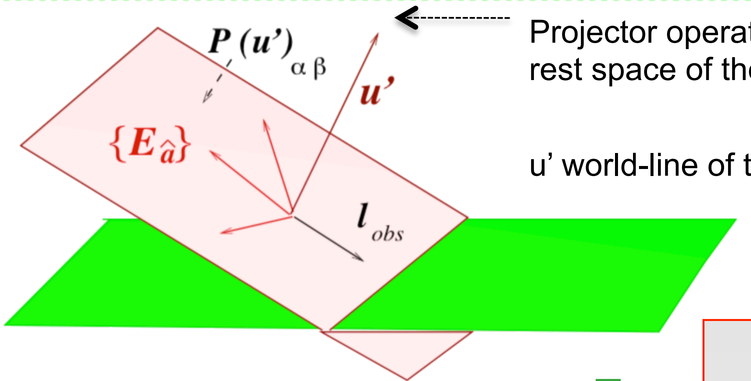
$$\begin{aligned}\frac{\tilde{\omega}_*}{\omega_{\rm sat}} - 1 &= \frac{1}{c}(v_* \cdot n - v_{\rm obs} \cdot n)\\ &+ \frac{1}{2c^2}\left[v_*^2 - 2(v_{\rm obs}\cdot n)(v_*\cdot n) - 2\Phi_{\rm obs} - v_{\rm obs}^2 + 2(v_{\rm obs}\cdot n)^2\right]\\ &+ \frac{1}{2c^3}\Big[(3(v_*\cdot n) - (v_{\rm obs}\cdot n))\, v_*^2 - \left(2\Phi_{\rm obs} + v_{\rm obs}^2 - 2v_{\rm obs}\cdot n)^2\right)(v_*\cdot n)\\ &\quad -6(v_{\rm obs}\cdot n)\Phi_{\rm obs} - (v_{\rm obs}\cdot n)v_{\rm obs}^2 - 2(v_{\rm obs}\cdot n)^3 + 2h_{0i}n^i\Big]\end{aligned}$$

μas order

In general, if (**x**,t) is computed at muas order (or 1.5 pN),

any F(**x**,t) =A (v/c) + B(v/c)^2 +C (v/c)^3 + O[(v/c)^4]

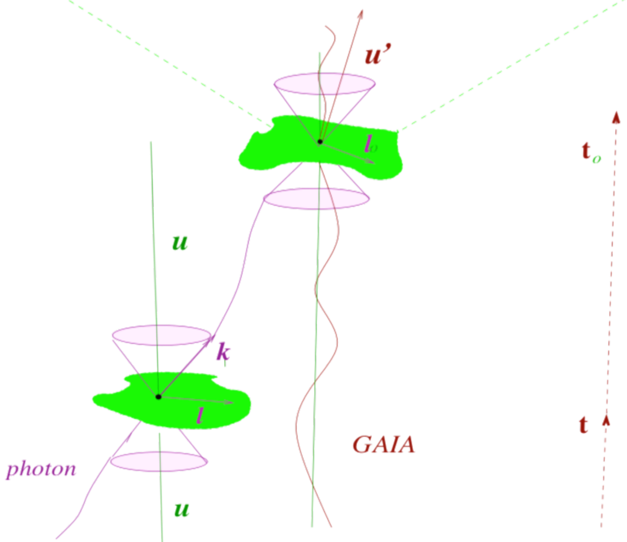
The astrometric observable



Projector operator onto the rest space of the satellite

u' world-line of the satellite

$E_\alpha^{\ \beta}$ "attitude tetrad"-> ESSENTIAL to define the boundary condition (Bini , Crosta, and de Felice, Class.Quantum Grav. 20, 4695, 2003



$$\cos\psi_{(E_{\hat{a}},\ell_{obs})} \equiv \mathbf{e}_{\hat{a}} = \frac{P(u')_{\alpha\beta}\,\ell^{\alpha}_{obs}\,\boldsymbol{E}^{\beta}_{\hat{a}}}{(P(u')_{\alpha\beta}k^{\alpha}k^{\beta})^{1/2}}$$

Observation equation

$$-\sin\phi\,\mathrm{d}\phi = \underbrace{\frac{\partial F}{\partial\alpha_*}\delta\alpha_* + \frac{\partial F}{\partial\delta_*}\delta\delta_* + \frac{\partial F}{\partial\varpi_*}\delta\varpi_* + \cdots}_{\text{Astrometric parameters}}$$

$$\sum_{ij}\frac{\partial F}{\partial\sigma_i^{(j)}}\delta\sigma_i^{(j)} + \sum_i\frac{\partial F}{\partial c_i}\delta c_i + \frac{\partial F}{\partial\gamma}\delta\gamma + \cdots$$

$$\boxed{\phi_{obs} - \phi_{calc}}$$

All derivatives are calculated at appropriate "catalog" values

The concept of the Global Sphere Reconstruction

$$\cos\phi \equiv F\left(\underbrace{\alpha_*,\delta_*,\varpi_*,\mu_{\alpha*},\mu_{\delta*}}_{\text{Astrometric parameters}},\underbrace{\sigma_1^{(1)},\sigma_2^{(1)},\sigma_3^{(1)},\sigma_1^{(3)},\sigma_2^{(3)},\sigma_3^{(3)}}_{\text{Attitude parameters}},\underbrace{c_1,c_2,\ldots,}_{\text{Instrument}}\underbrace{\gamma,\ldots}_{\text{Global}}\right)$$

Solving the linearized GSR sphere in the Least-Squares sense

Know
n

Unknown

Unknown

$$\sin\psi_i^{(1)}\,\Delta\psi_i^{(1)} = \frac{\partial f}{\partial\alpha_i}\Delta\alpha_i + \frac{\partial f}{\partial\delta_i}\Delta\delta_i + \frac{\partial f}{\partial\pi_i}\Delta\pi_i + \cdots + \frac{\partial f}{\partial\gamma}\Delta\gamma$$

$$\sin\psi_i^{(2)}\,\Delta\psi_i^{(2)} = \frac{\partial f}{\partial\alpha_i}\Delta\alpha_i + \frac{\partial f}{\partial\delta_i}\Delta\delta_i + \frac{\partial f}{\partial\pi_i}\Delta\pi_i + \cdots + \frac{\partial f}{\partial\gamma}\Delta\gamma$$

• • • • • •

$$\sin\psi_i^{(n)}\,\Delta\psi_i^{(n)} = \frac{\partial f}{\partial\alpha_i}\Delta\alpha_i + \frac{\partial f}{\partial\delta_i}\Delta\delta_i + \frac{\partial f}{\partial\pi_i}\Delta\pi_i + \cdots + \frac{\partial f}{\partial\gamma}\Delta\gamma$$

1 obs. $\Rightarrow$ 1 condition eq.

(linearized) system of solution with dimensions ~$10^{10}\times10^{8}$

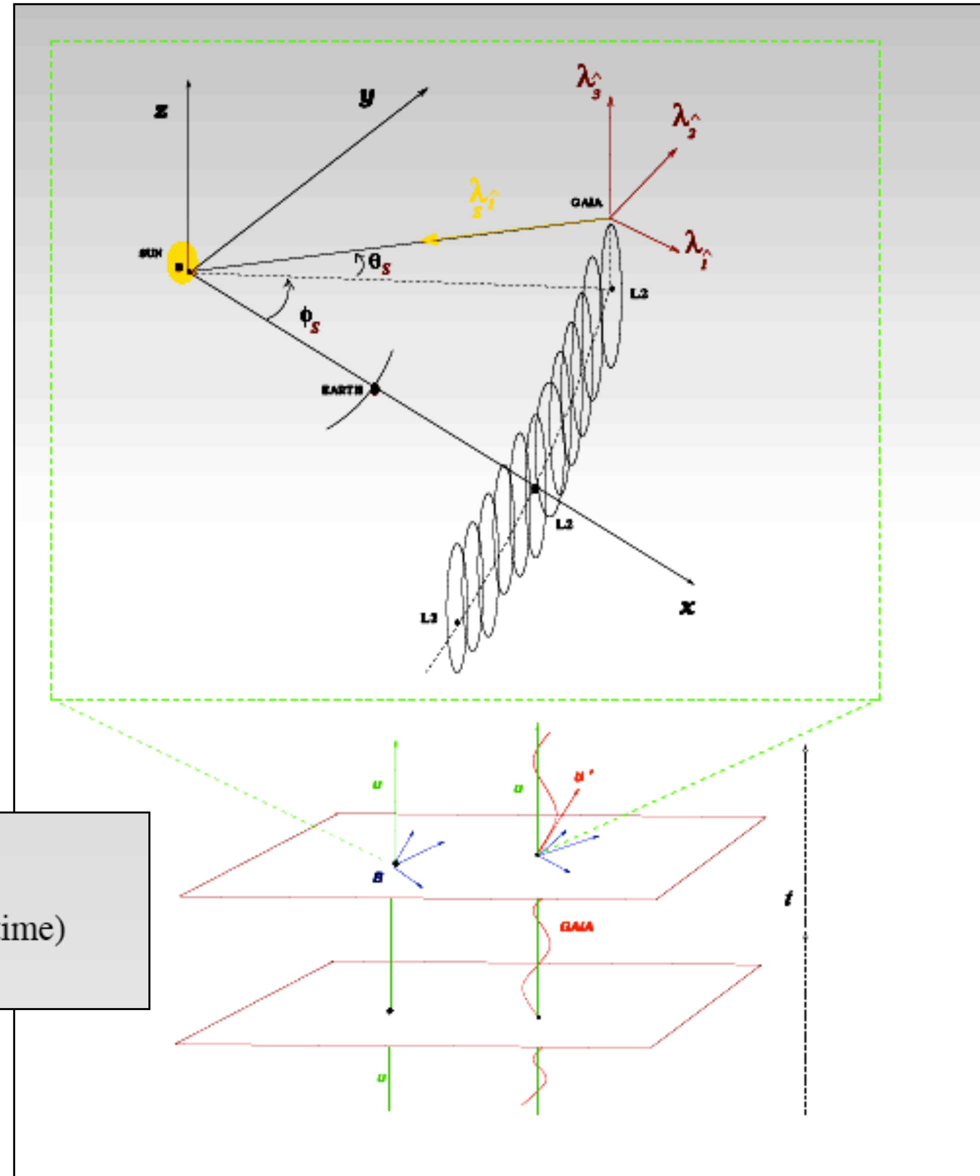
iterative method (LSQR, Paige, C. & Saunders, M. A. 1982, ACM Trans. Math. Software, 8, 43)

→ A real Galilean experiment in space: a massive repetition of the Eddington et al. astrometric test of GR with 21st century technology, <u>thank to the interface of analytical&numerical relativity methods</u>

The proper frame of the satellite

The **rest-frame** of an observer consists of a ***clock (satellite proper-time) + a space triad*** *of* orthonormal axes.

The **mathematical quantity** which defines a rest-frame of a given observer **is the *tetrad adapted to that observer, depending on the chosen metric.***



$$g_{\mu\nu}\lambda^{\mu}_{\hat{\alpha}}\lambda^{\nu}_{\hat{\beta}} = \eta_{\hat{\alpha}\hat{\beta}}$$

$\lambda_{\hat{0}} \equiv u'$ (space-time history of the observer in a given space-time)
$\lambda_{\hat{a}} \equiv$ spatial triad of space-like vectors

What is the actual attitude frame for Gaia in RAMOD?

3. The Gaia proper frame in RAMOD

*a. **Gaia attitude frame***

$$\boldsymbol{E}_{\hat{a}} = \mathcal{R}_1(\omega_r t)\mathcal{R}_2(\alpha)\mathcal{R}_1(\omega_p t)\underset{bs}{\boldsymbol{\lambda}}_{\hat{a}} \qquad \hat{a} = 1,\ 2,\ 3$$

The full relativistic analytical expression of E_a and λ_a are in Bini, Crosta, de Felice, 2003, Class. Quantum Grav., 20,2251/4695

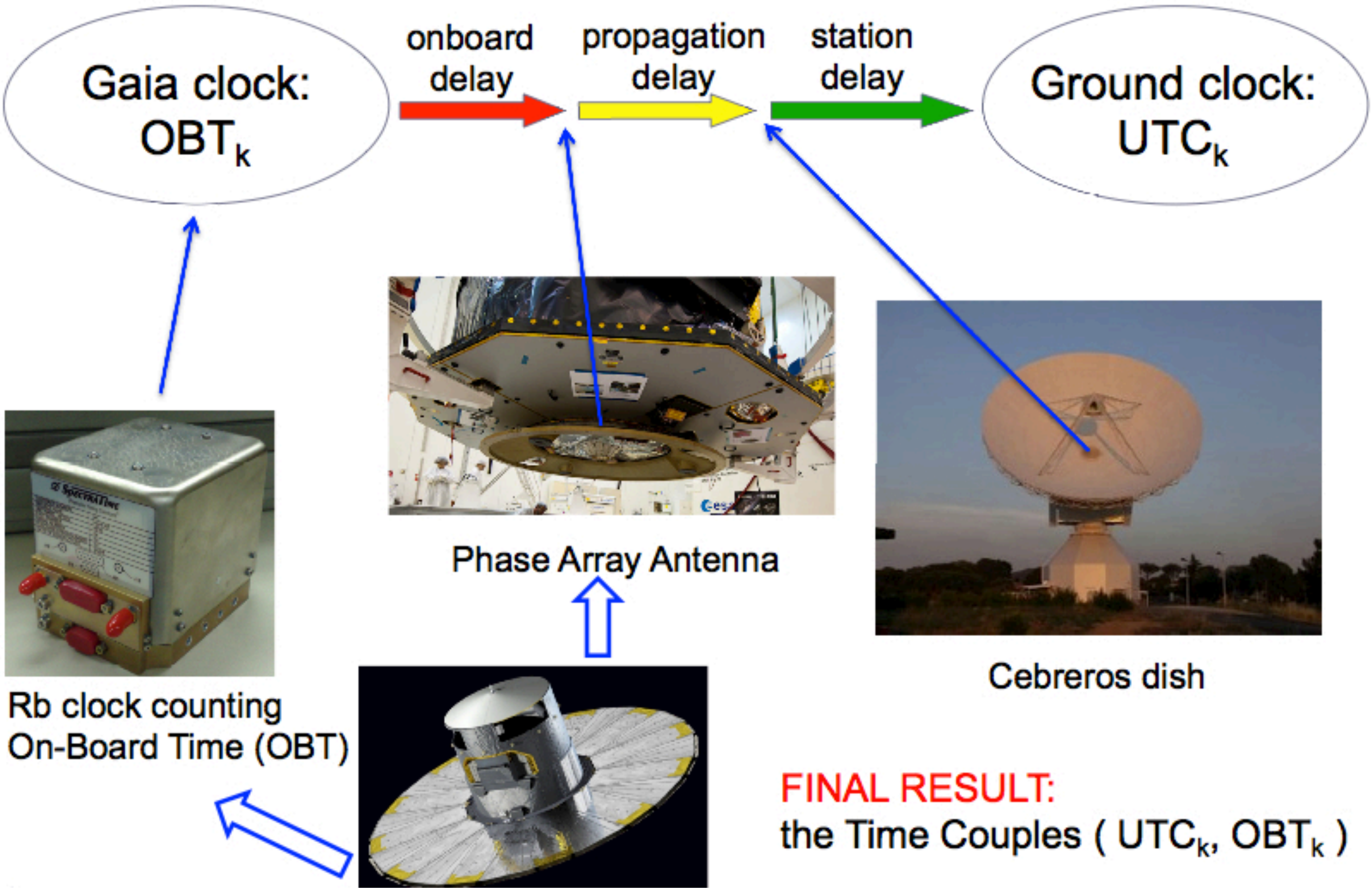
$$dT = -\frac{1}{c} g_{\alpha\beta} {u'}^{\alpha} dx^{\beta}$$

applying IAU resolutions

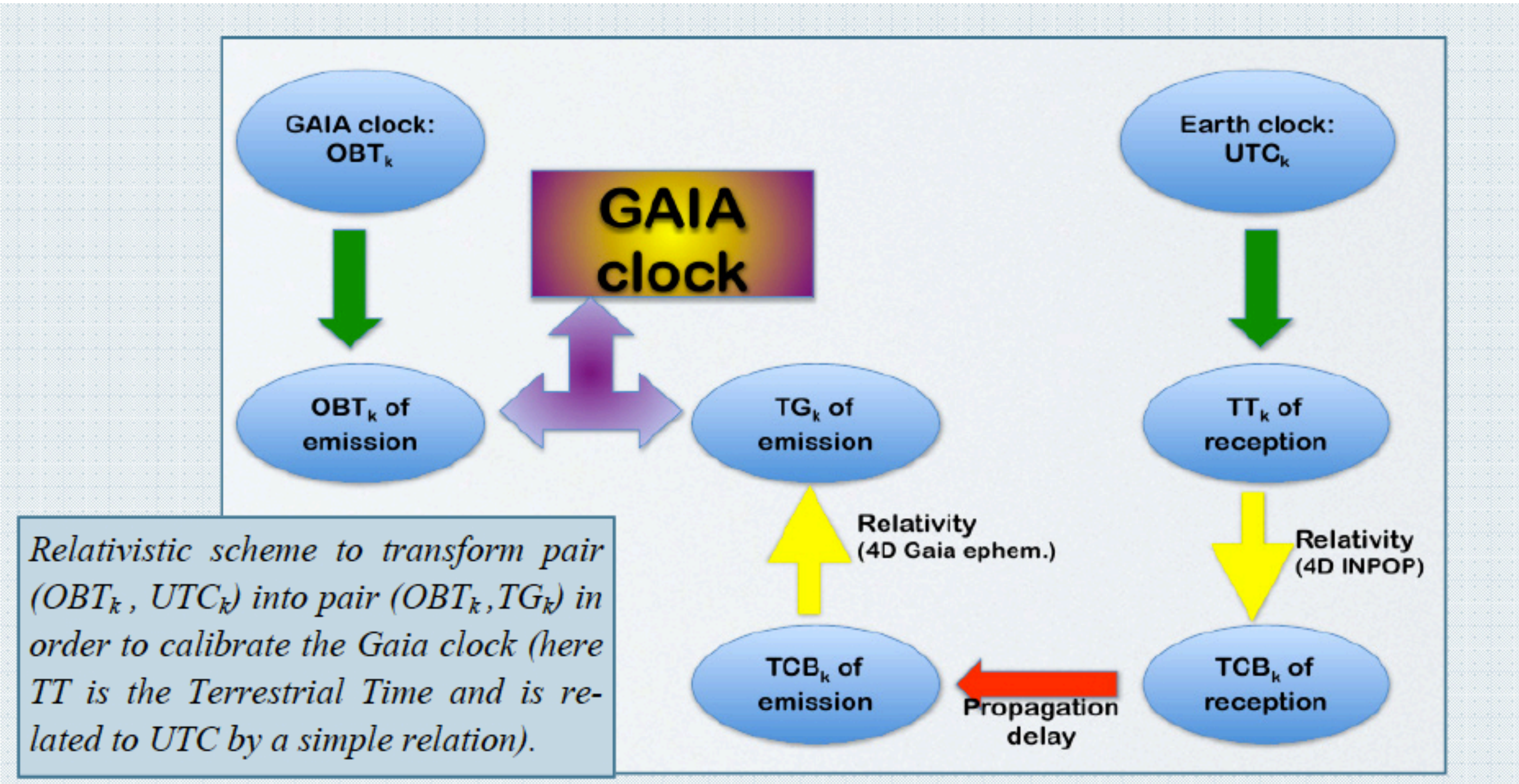
*b. **Gaia clock*** →

$$\begin{aligned} dT \quad &\approx \quad dt - c^{-2}\left[\left(\frac{v^2}{2} + w(\mathbf{x},t)\right) + v^i dr^i\right] \\ & +c^{-4}\left[\left(\frac{w^2(\mathbf{x},t)}{2} - \frac{v^4}{8} - \frac{3v^2 w(\mathbf{x},t)}{2} + 4w^i(\mathbf{x},t)v^i\right) dt \right. \\ & \left. +4w^i(\mathbf{x},t)dr^i - \left(3w(\mathbf{x},t) + \frac{v^2}{2}\right) v^i dr^i\right], \end{aligned}$$

(Crosta et al., *Proper frames and time scan for Gaia-like satellites, 2004, ESA livelink, tech.note, Crosta and Vecchiato 2010)*



http://www.gao.spb.ru/english/as/j2014/presentations/klioner.pdf



Relativistic scheme to transform pair (OBT_k , UTC_k) into pair (OBT_k ,TG_k) in order to calibrate the Gaia clock (here TT is the Terrestrial Time and is related to UTC by a simple relation).

by courtesy of C. Le Poncin-Lafitte (Observatoire de Paris, SYRTE)

Italian Data Processing Center

All Gaia operations activities (daily and cyclic) done in Italy are implemented at the DPCT, the Italian provided HW and SW operations system designed, built and run by ALTEC (To) and INAF-OATo for ASI.

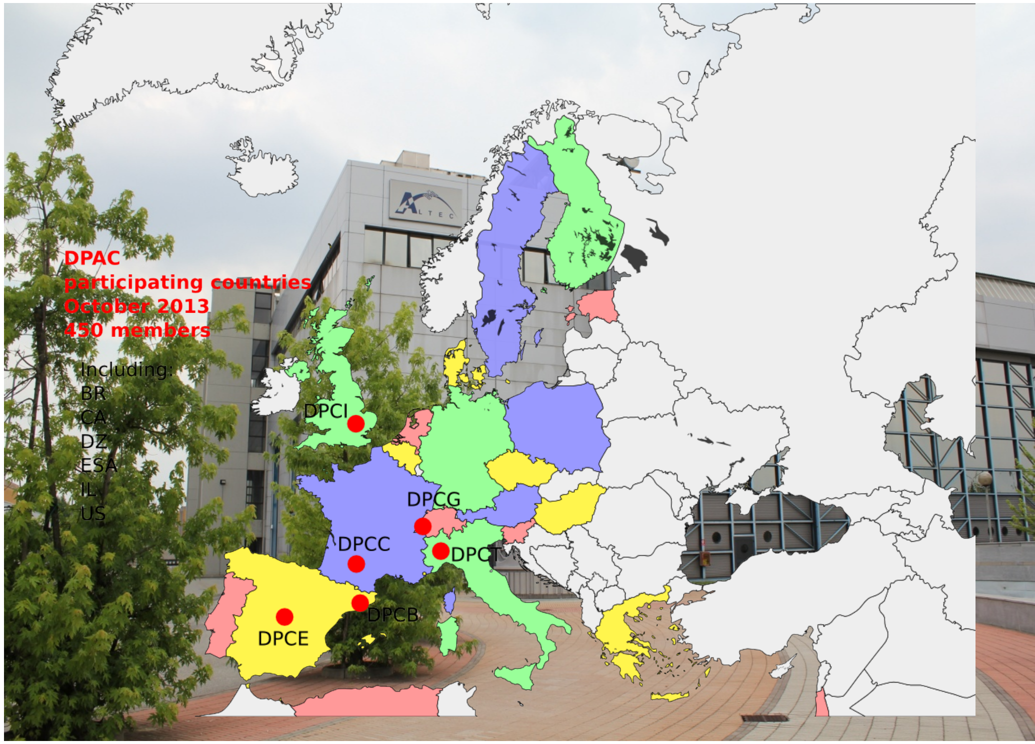
DPCT at full capacity. Accumulated other than 50 TB of data

Size at completion ~ 1.2 PB

The DPCT host the systems AVU:

- ❖ CCD-level precision and accuracy (Astrometric Instrument Monitoring - AIM)
- ❖ Accuracy at the Optical System level (Basic Angle Monitoring - BAM/AVU)
- ❖ Precision & accuracy on the celestial sphere (Global Sphere Reconstruction - GSR)

Essential components of Gaia's astrometric error budget

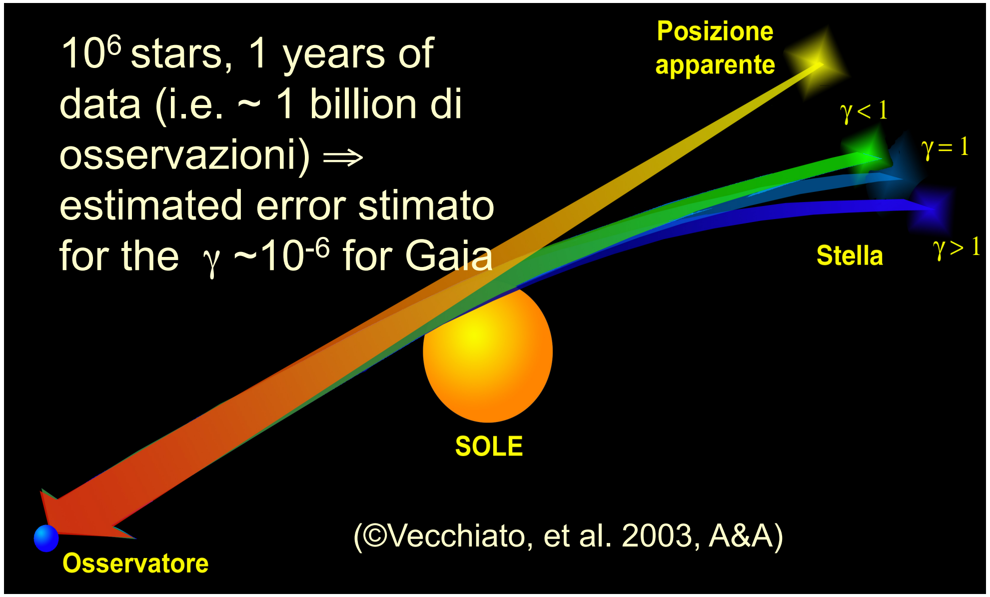


DPCT was established through a specific ASI contract via a partnership between INAF-OATo and ALTEC S.p.A.

- **M. Castronuovo (RC, MLA-SC repr.)**
- **B. Negri (EOS Head)**

This is the only Data Processing Center, within the network of 6 DPCs dedicated to Gaia, which specializes in the treatment of the satellite astrometric data

Global astrometric tests, PPN parameters



•In GR γ=1 -> measures the amount of space curvature generated by a unit mass

•Gravity theories alternative to GR require the existence of a scalar field coupled to gravity and predict it fades with time, so that its residue would manifest itself through very small deviations from Einstein's GR in the weak field regime

$$\left|\gamma - 1\right| \approx 10^{-5} - 10^{-7}$$

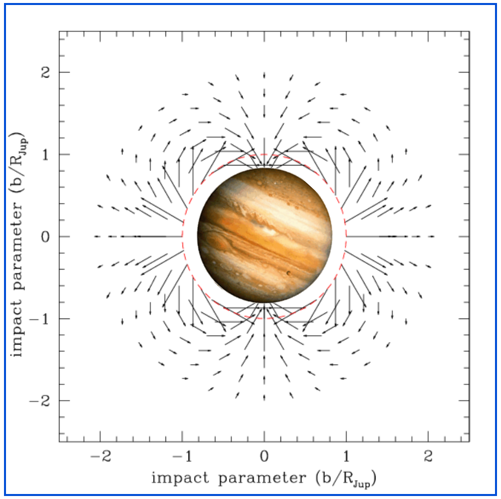
Future improvements of light deflection measurements in Solar System allow 10^{-8}!

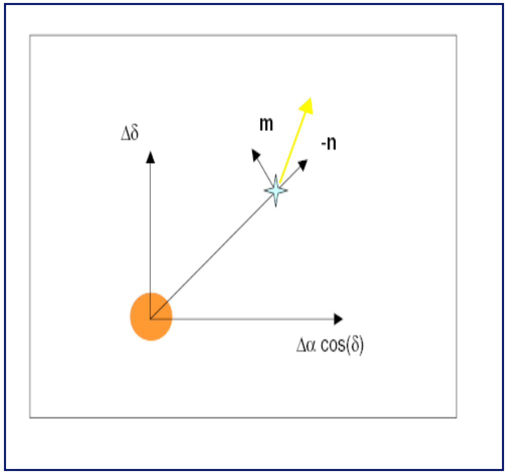
The evaluation of these deviations depends on the particular scalar-tensor theory adopted

- quantum theory of gravity, verification of inflationary models, violation of the principle of equivalence, constancy of the physical constants, low energy limit for string theory etc. ..
- f (R) gravity with no need of dark matter and dark energy, accelerated cosmological expansion, Galaxy cluster dynamics, Galaxy rotation curves and DM halos

Estimation of the β parameter through the integration of some thousands of NEOs orbits (Now: 10^{-4}, exp.: 10^{-4}) -> second-order term in the light defection

Quadrupole Light deflection by Jupiter





"q-effect"

3 to10σ level estimation

vector field of the stellar positions deflected around Jupiter, measurable by Gaia (Crosta & Mignard, QCG, 2006)

$$\begin{aligned}
\Delta\Phi &= \Delta\Phi_1 \mathbf{n} + \Delta\Phi_2 \mathbf{m}, \\
\Delta\Phi_1 &= \frac{2(1+\gamma)M}{b}\left[1 + \epsilon J_2 \frac{R^2}{b^2}(1 - 2(n\cdot z)^2 - (t\cdot z)^2)\right], \\
\Delta\Phi_2 &= \frac{4(1+\gamma)M J_2 R^2}{b^3}(m\cdot z)(n\cdot z)
\end{aligned}$$

•further confirmation of GR-> screening of some alternative theories

• evaluation of the quadrupole contribution in second order deflection effects

• Gravitomagntic and post-Newtonian effects of higher order related to the speed of Jupiter/Saturn

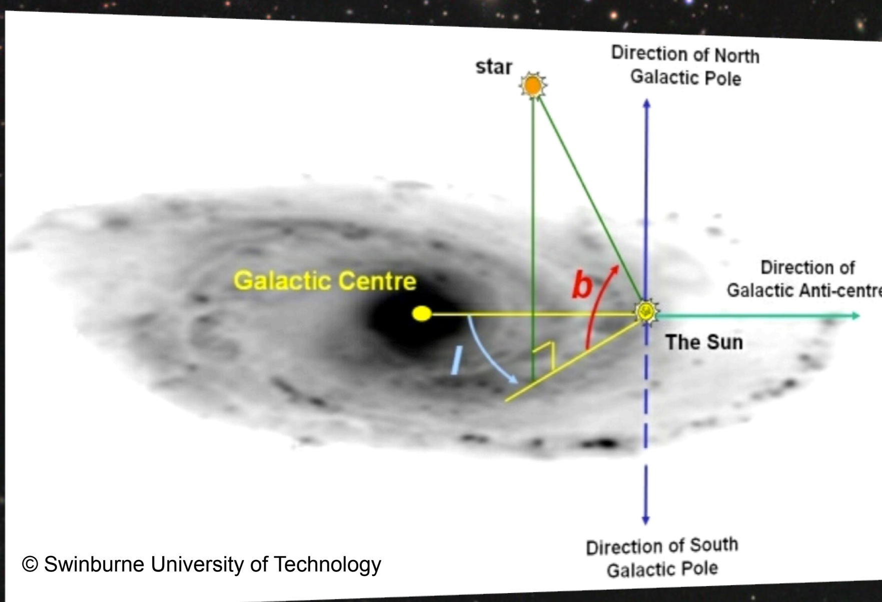
• Cluster of Galaxies -> evaluating the quadrupole contribution to the gravitational lensing, mass distribution

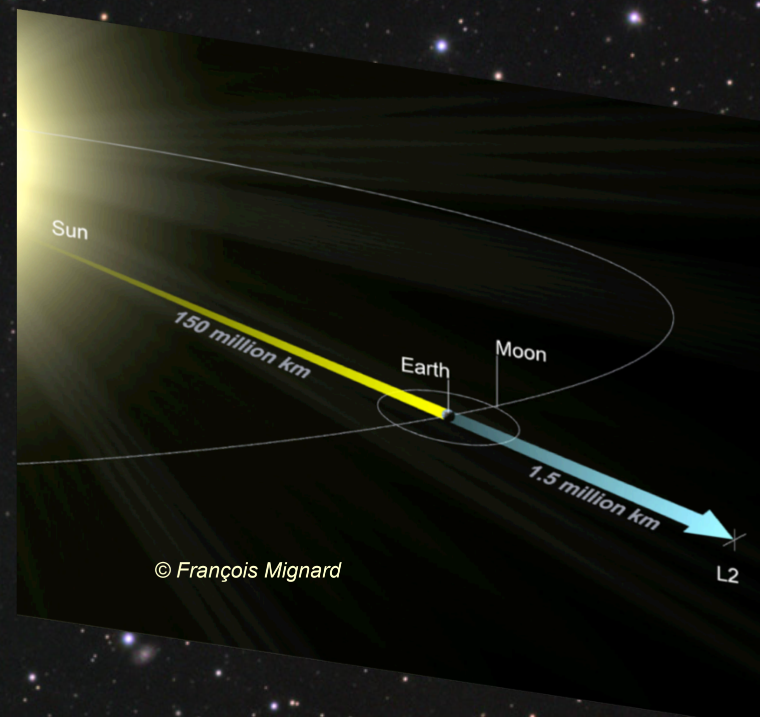
• Relativistic effects with the Quasars (Kopeikin et al.) and links between reference systems and dynamic ICRF (J.Souchay, A. Andrei et al.)

•first application of exact solution type Erez-Rosen (Bini, Crosta et al. 2013 Class. Quantum Grav. 045 009 30) for galaxy clusters

With Gaia 500,000 quasars,

Provide a new catalog of radio sources suitable for accurate radio-optical link between the Gaia Celestial Reference Frame (GCRF) and the International Celestial Reference Frame (ICRF).





Solar motion.. Reloaded

$$g_{00} = -1 + \frac{2}{c^2} w(t, \boldsymbol{x}) - \frac{2}{c^4} w^2(t, \boldsymbol{x}) \,,$$

$$g_{0i} = -\frac{4}{c^3} w^i(t, \boldsymbol{x}) \,,$$

$$g_{ij} = \delta_{ij} \left(1 + \frac{2}{c^2} w(t, \boldsymbol{x}) \right) .$$

BCRS metric at muas level acceleration of the Sun?

+ new terms according to the accuracy level achievable

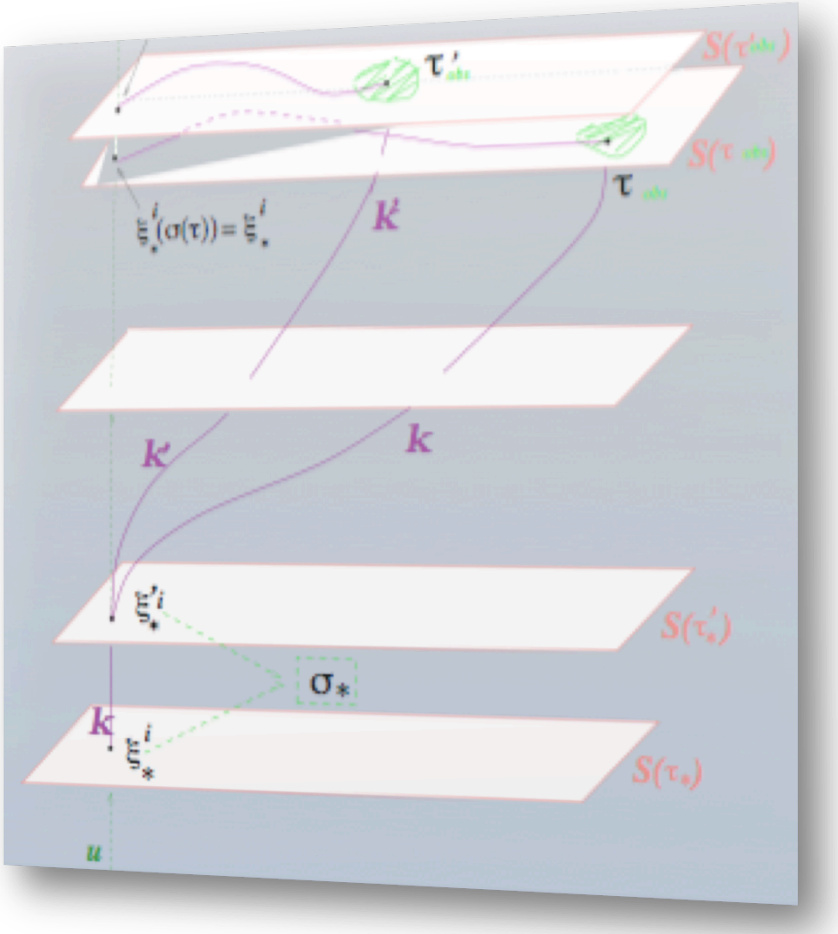
✓Repeated observations of the local line-of-sight of the same stars

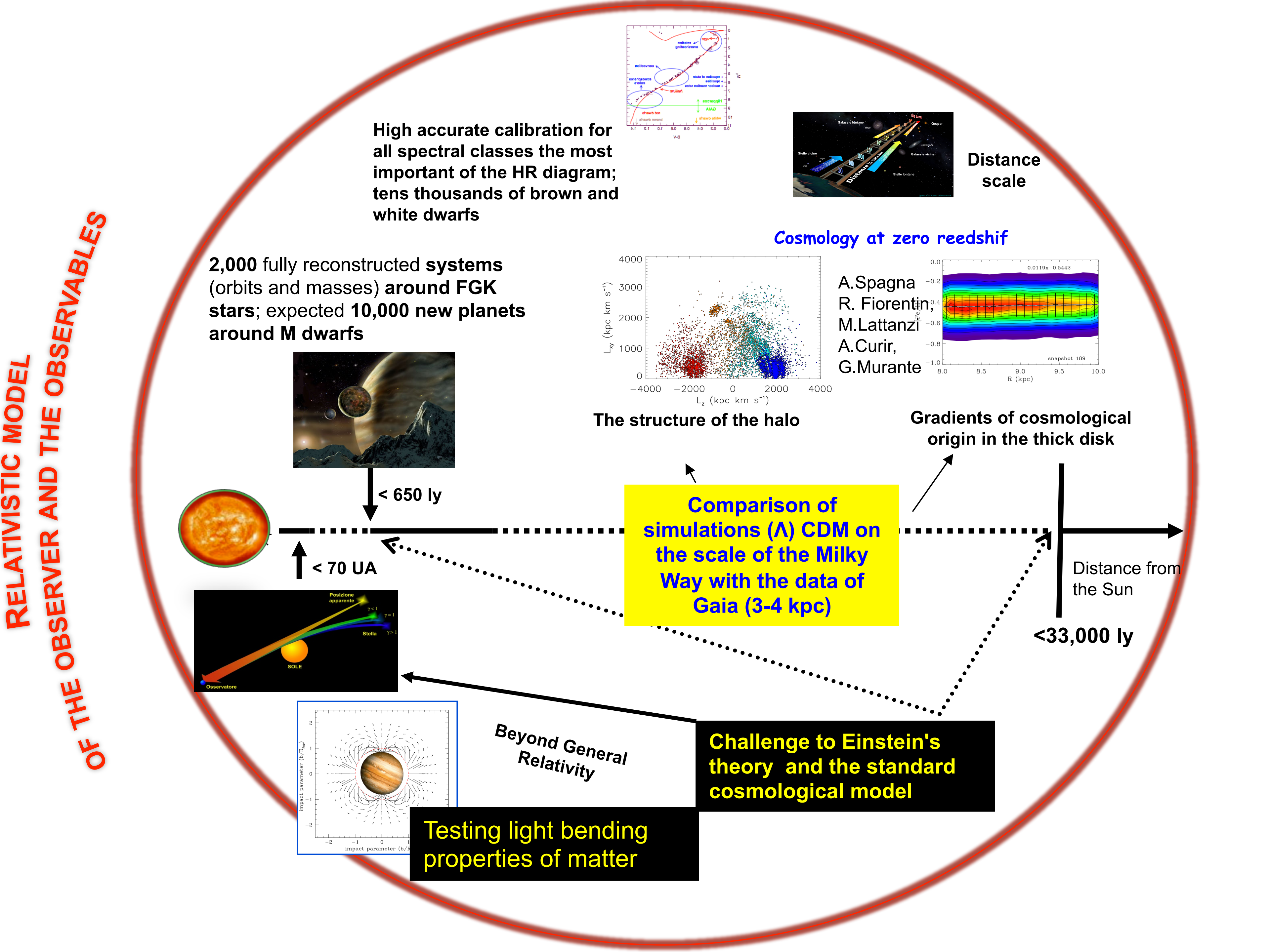
✓"inverse parameter problem" approach in order to statistically determine the metric (maybe outside the Solar System?)

$$g_{\alpha\beta} \text{ to } g_{\alpha\beta} + \delta g_{\alpha\beta}$$

"The measurement act is not simply an 'observed value' (of a set of 'observed values') but an 'state of information' acquired on some observable parameter"

Inverse Problem Theory and Methods for Model Parameter Estimation by Albert Tarantola, SIAM 2005




RELATIVISTIC MODEL
OF THE OBSERVER AND THE OBSERVABLES
High accurate calibration for all spectral classes the most important of the HR diagram; tens thousands of brown and white dwarfs
Distance scale
Cosmology at zero reedshif
2,000 fully reconstructed systems (orbits and masses) around FGK stars; expected 10,000 new planets around M dwarfs
A.Spagna
R. Fiorentin,
M.Lattanzi
A.Curir,
G.Murante
The structure of the halo
Gradients of cosmological origin in the thick disk
< 650 ly
< 70 UA
Comparison of simulations (Λ) CDM on the scale of the Milky Way with the data of Gaia (3-4 kpc)
Distance from the Sun
<33,000 ly
Posizione apparente
Stella
SOLE
Osservatore
Beyond General Relativity
Challenge to Einstein's theory and the standard cosmological model
Testing light bending properties of matter

Ricordiamo: La tecnica delle misure fondamentali ha raggiunto i 10 μas e può raggiungere 1 μas ed oltre; siamo nella situazione in cui se nuovi modelli non vengono approntati i dati (le osservazioni) non potrebbero essere correttamente sfruttati!

PROPOSTA DI TEMATICA FONDAMENTALE…(Big Question)(?)*:*
NATURA E QUANTITÀ DELLA DM (GRAVITA') NELLA VIA LATTEA

- Ovvero: se stiamo usando la corretta gravità → la stima della quantità di materia gravitazionale è più corretta (e indirettamete si contribuisce alla 'natura' della DM) → confronto locale (MW scale) con modelli ΛCDM che utilizzerebbero la corretta gravità per predire la composizione dello spazio delle fasi con l'astrometria e la spettroscopia di Gaia (comprese le Gaia complementary surveys)

quindi

- Qual è la corretta gravità? *Testing GR*!
- In ogni caso modelli ΛCDM correnti usano le equazioni 'sbagliate': F=ma!

- Siamo nella MA4, MA1? *NO*! Ma serve coordinamento e sinergie

- Nuovi paradigmi culturali: esempio, geodetica dei fotoni deve essere parte del bagaglio come le equazioni degli interni stellari

- Costruire *curricula*, piano di studi, fortemente multidisciplinari ed all'uso della relatività generale. Serve colmare il **gap culturale- filosofico** che si crea quando si porta avanti l'innovazione senza riguardare da dove si è partiti

- Eventuali sedi INAF specializzate e piano di reclutamento: necessità di medio e lungo termine

- Coordinameto a livello nazionale anche tra i vari enti (INFN, CNR,INRiM, Università, etc)

- Contributo alla preparazione del documento di Macroarea che includa un piano/ orizzonte di sviluppo e motivazioni scientifiche di medio e lungo termine (Gaia, EUCLID!!, Theia,…; tests di relatività generale, gravità per MW modeling,….)

One century after General Relativity we must rethink the Mach's principle: how much the local universe can affect on our knowledge of the global universe?

after Gaia Astrometry (i.e. α,δ,μ,π, v_{rad} etc..) becomes part of the fundamental physics and, in particular, in that of gravitation

✓now, with Gaia

The realization of the **relativistic celestial sphere** is a scientific validation of the absolute parallax and proper motions in Gaia

But reaching 10-20 μas accuracy on individual parallax and annual proper motions for bright stars (V<16) is also the key **possibly to perform**

- **the largest GR experiment ever attempted from space:** *given the number of celestial objects (a real Galilean method applied on the sky!) and directions involved (the whole celestial sphere!), the largest experiment in General Relativity ever made with astrometric methods (since 1919)t*
- **new test of GR predictions**
- to fully probe the MW (outer) halo (mass content and distribution) and compare the prediction of Lambda-CDM models

✓what new, with Gaia

- But all the goals of Gaia will not be achieved without the correct characterization and exploitation of the ``relativistic'' astrometric data.

  Any Gaia-like observer is positioned inside the Solar System, a <u>weak gravitational regime which turns out to be "strong" when one has to perform high accurate measurements</u>

- Any discrepancy between the relativistic models, if it can not be attributed to errors of different nature, will mean either a limit in the modeling/ interpretation - that a correct application of GR should fix - and therefore a validation of GR, or, maybe, a clue that we need to refine our approach to GR

✓what needed, after Gaia

➢**Beyond the micro-arcsecond?** **Gaia represents ONLY the 0-step**… increasing the level of the measurement precision requires to refine consistently the metric of the solar system, the solution for the null geodesic and so on..

➢**Once a relativistic model for the data reduction has been implemented, any subsequent scientific exploitation should be consistent with the precepts of the theory underlying such a model**

➢Then, the astronomers need to be ready to exploit all of the scientific potential of the local measurements entangled to the varying gravitational fields from within the Solar System and to maximize its impact.

One century after General Relativity we must rethink the Mach's principle: how much the local universe can affect on our knowledge of the global universe?

after Gaia Astrometry (i.e. α,δ,μ,π, etc..) becomes part of the fundamental physics and, in particular, in that of gravitation

Piano di sviluppo

- ✓ Serve FORMAZIONE per creare expertise utili ad affrontare non solo tecnologie sempre più avanzate ma anche i problemi teorici connessi alle osservabili in gioco, allo loro interpretazione e conseguente sfruttamento scientifico

- ✓ Serve colmare il **gap culturale- filosofico** che si crea quando si porta avanti l’innovazione senza riguardare da dove si è partiti

- ✓ Serve cercare sinergie anche tra macroaree diverse er essere competitivi