Transport of Solar Flare Energetic Electrons in the Heliosphere: Comparison of Hard X-Ray, Radio and in-Situ Observations With a Turbulent Scattering Transport Model

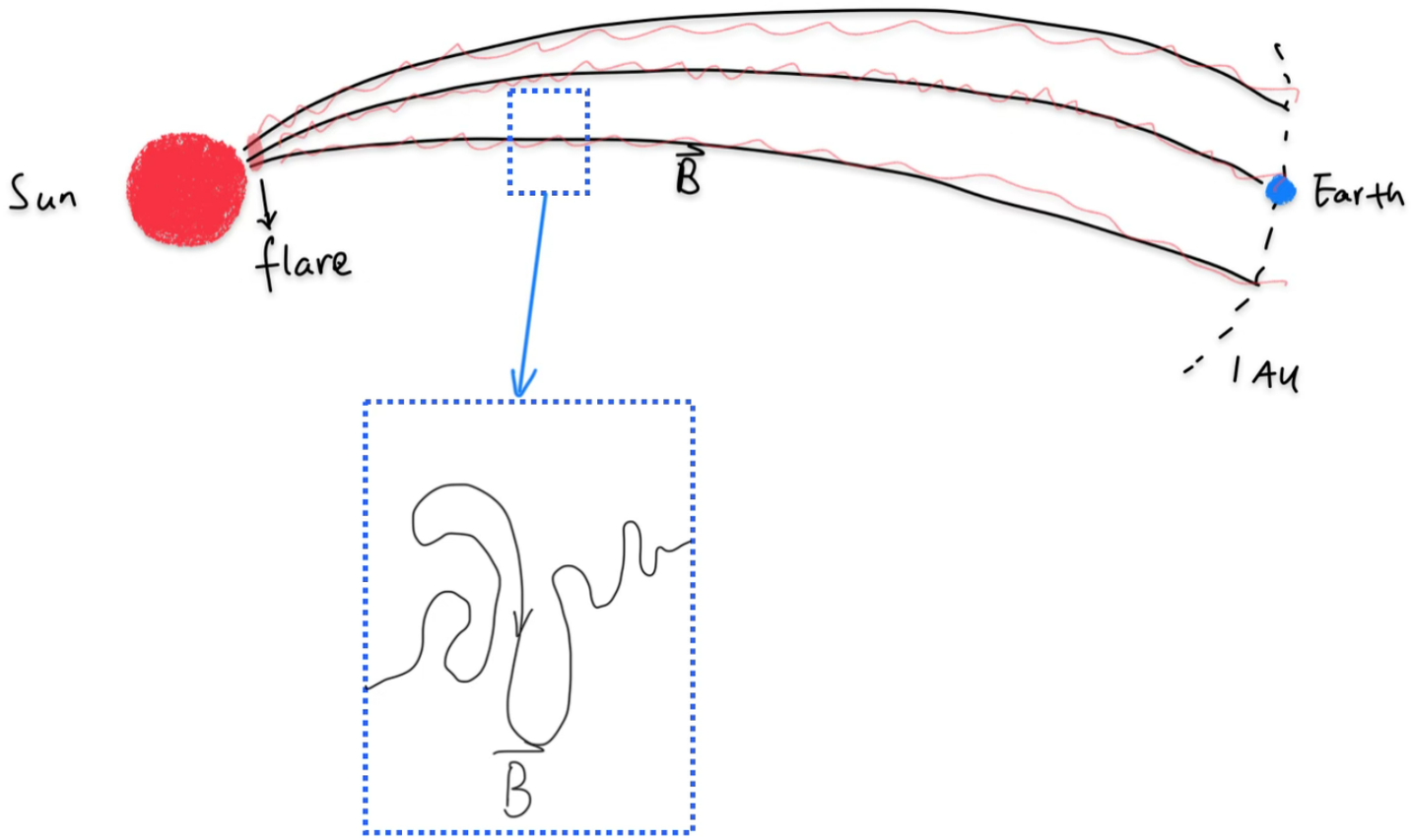
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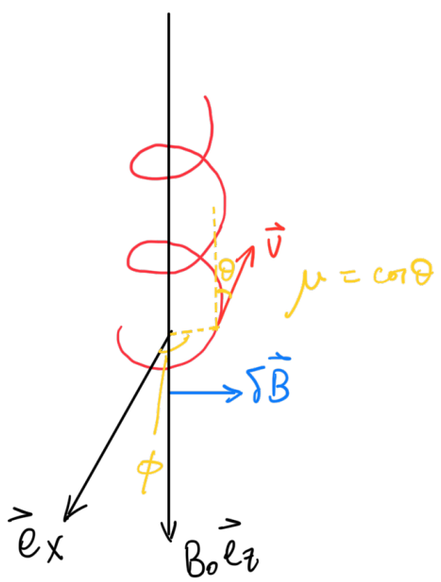
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Alliance Meeting 2026

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Turbulent scattering of electrons





$$\gamma m \dot{\mathbf{v}} = \frac{e}{c}\left[\mathbf{v} \times \left(B_0 \mathbf{z} + \delta \mathbf{B}\right)\right]$$

Randomly changes
Small quantity
Perpendicular to the mean field lines

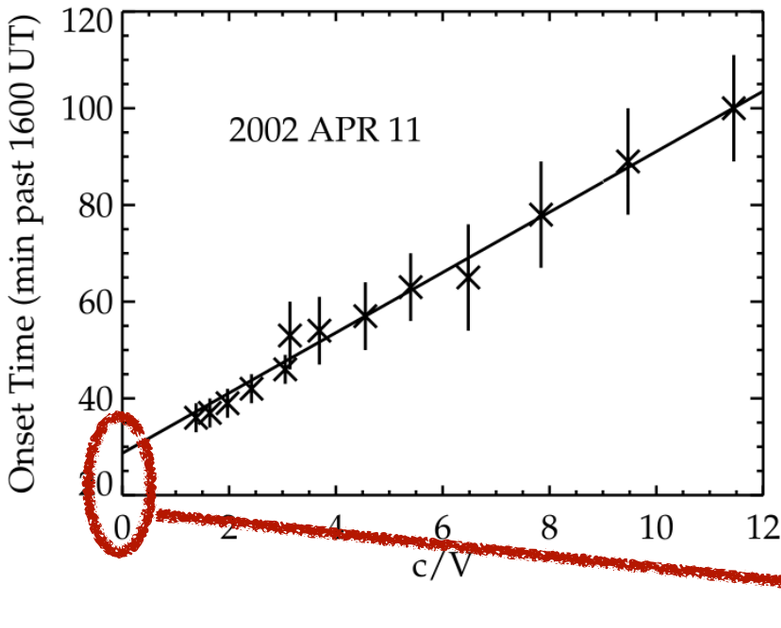
Turbulent scattering in the interplanetary space

- magnetic field is turbulent
- diffusing the population over multiple interactions on the electron velocity by the magnetic field fluctuations

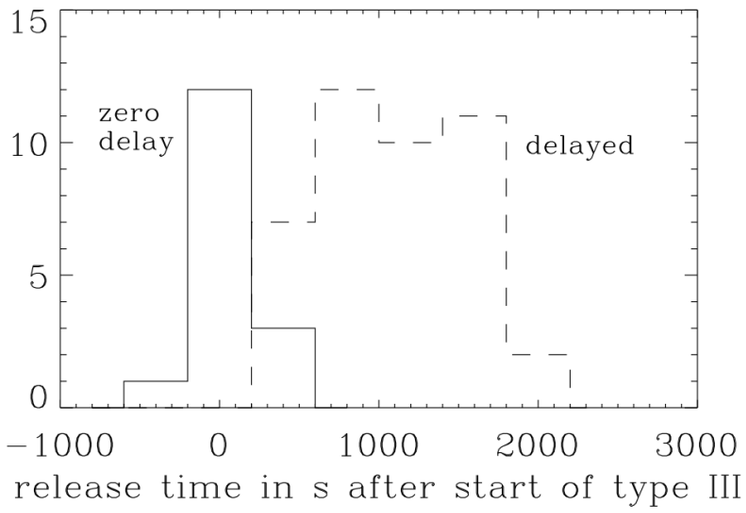
Electrons scattering by turbulence

Evidences

Delay of injection time



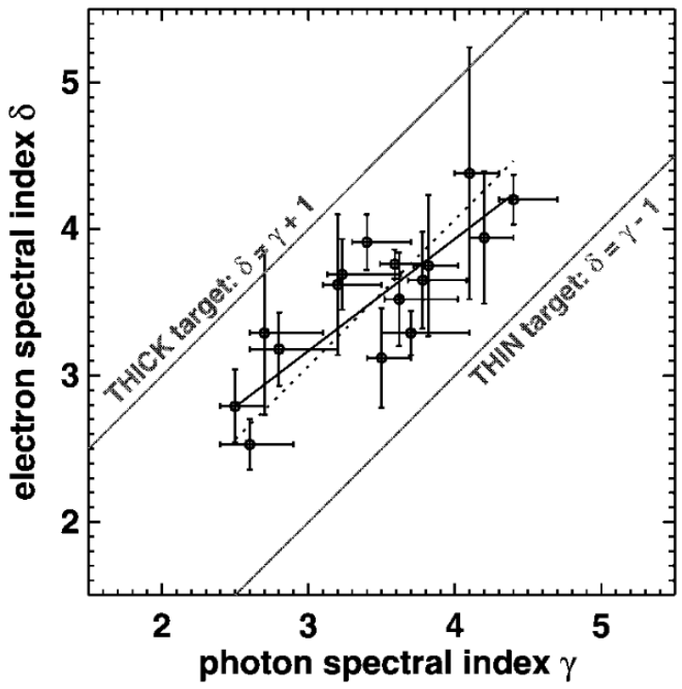
Kahler and Ragot (2006)



Krucker et al. (1999)

Delay of the inferred release time by tens of minutes from the HXR/radio determined release time

Spectra differ after propagation

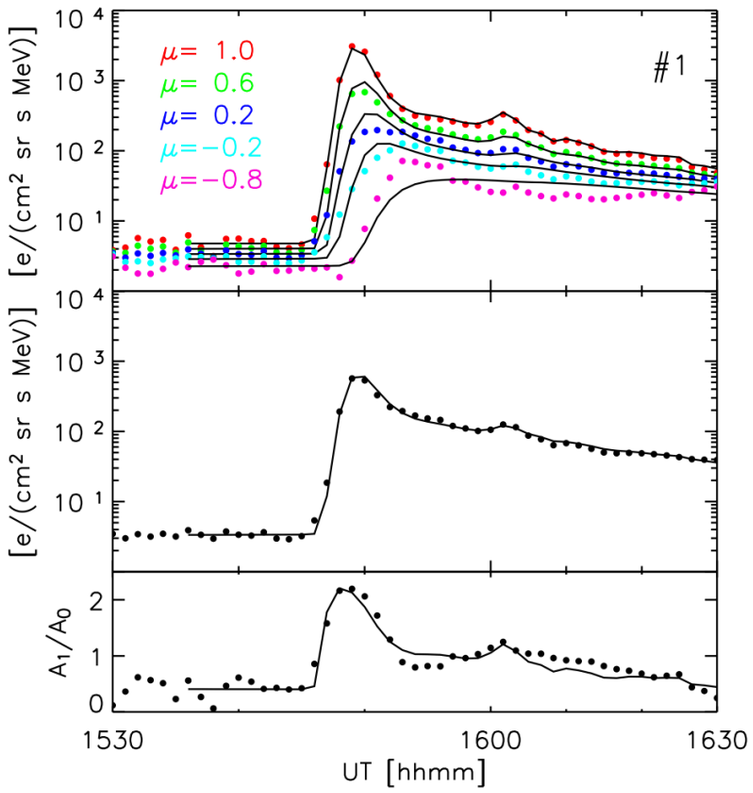


Krucker et al. (2007)

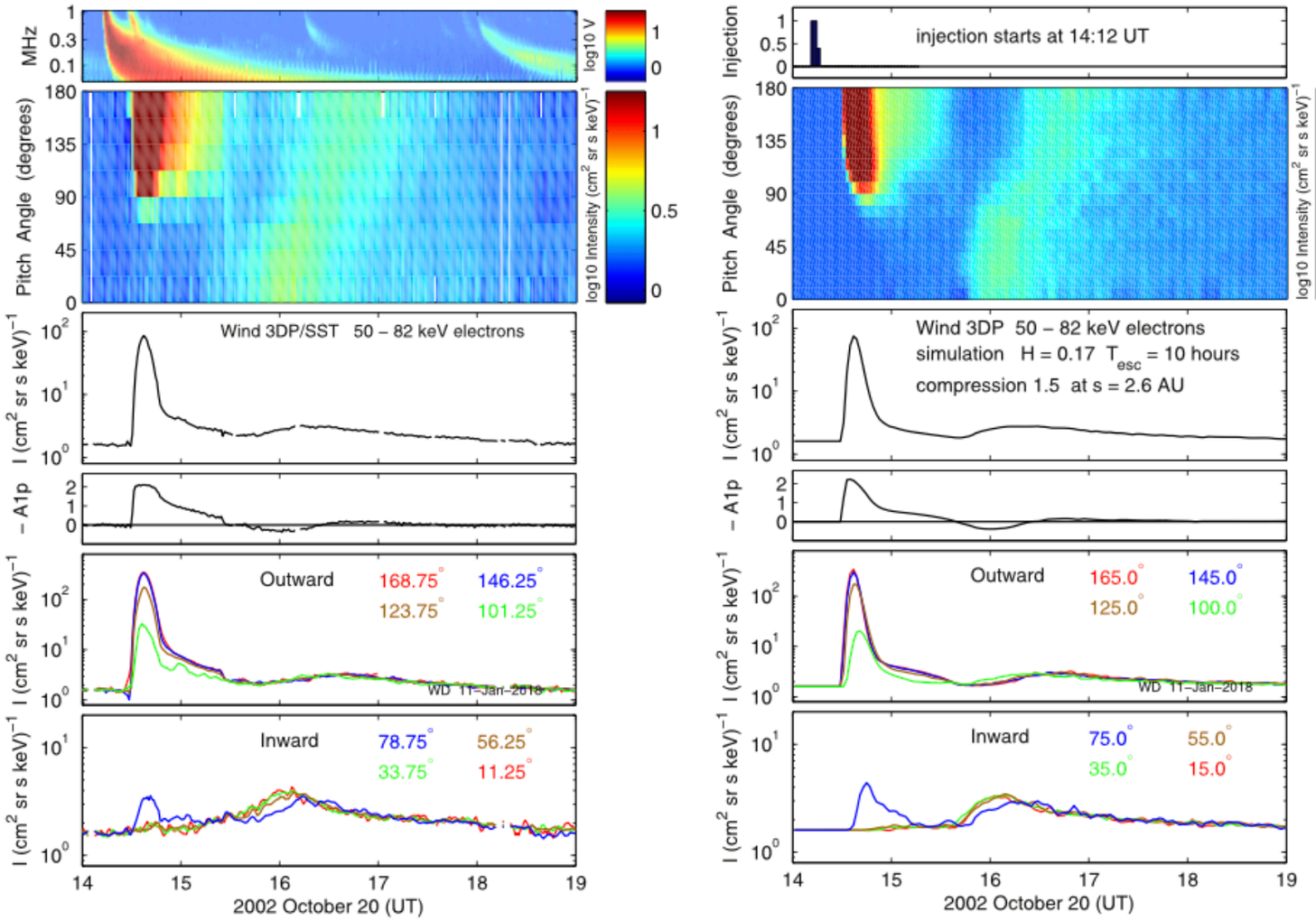
In-situ electron spectral index differs from the HXR derived electron spectral index by both thin-target or thick-target models

Previous works

Agueda & Lario (2016)



Dröge et al. (2018)



Successfully reproduced the observed in-situ electron flux and pitch-angle distributions from simulations, but they

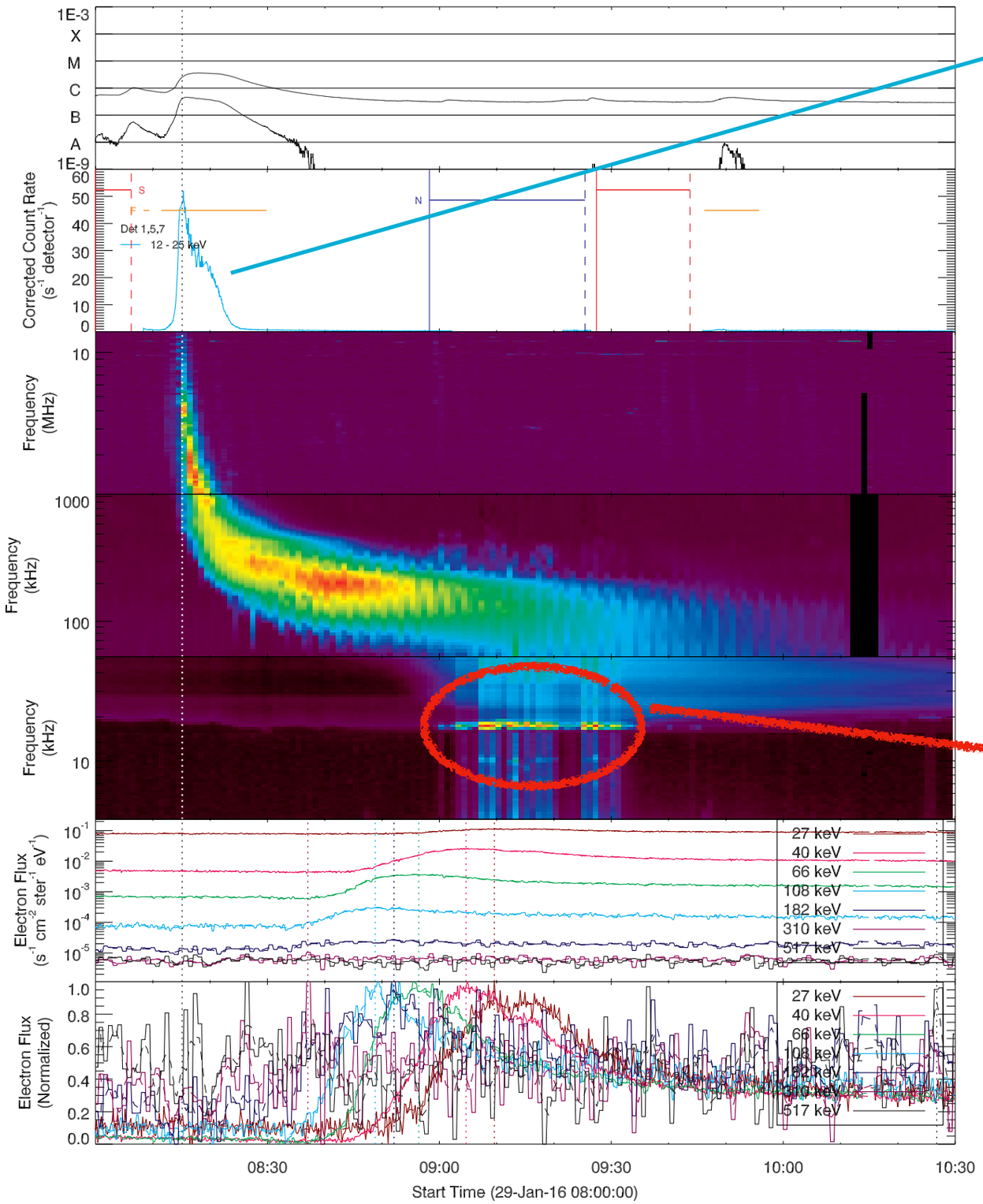
- **invoke flexibility in determining the injection history and time delay of the arrival**
  - derived from model fitting / back-tracing
- **mostly on mono-event and mono-energy**

Motivations

- **Investigate on how turbulent scattering influences the transport of electrons**

- collect many events to give **statistical results**
  - can be **traced back to a solar flare origin**
    - **the injection history** are more convincingly determined from HXR observations
- do modeling and simulations to reproduce the statistical behavior
  - reveal the distance- and energy-dependence of the turbulent scattering

Event selection

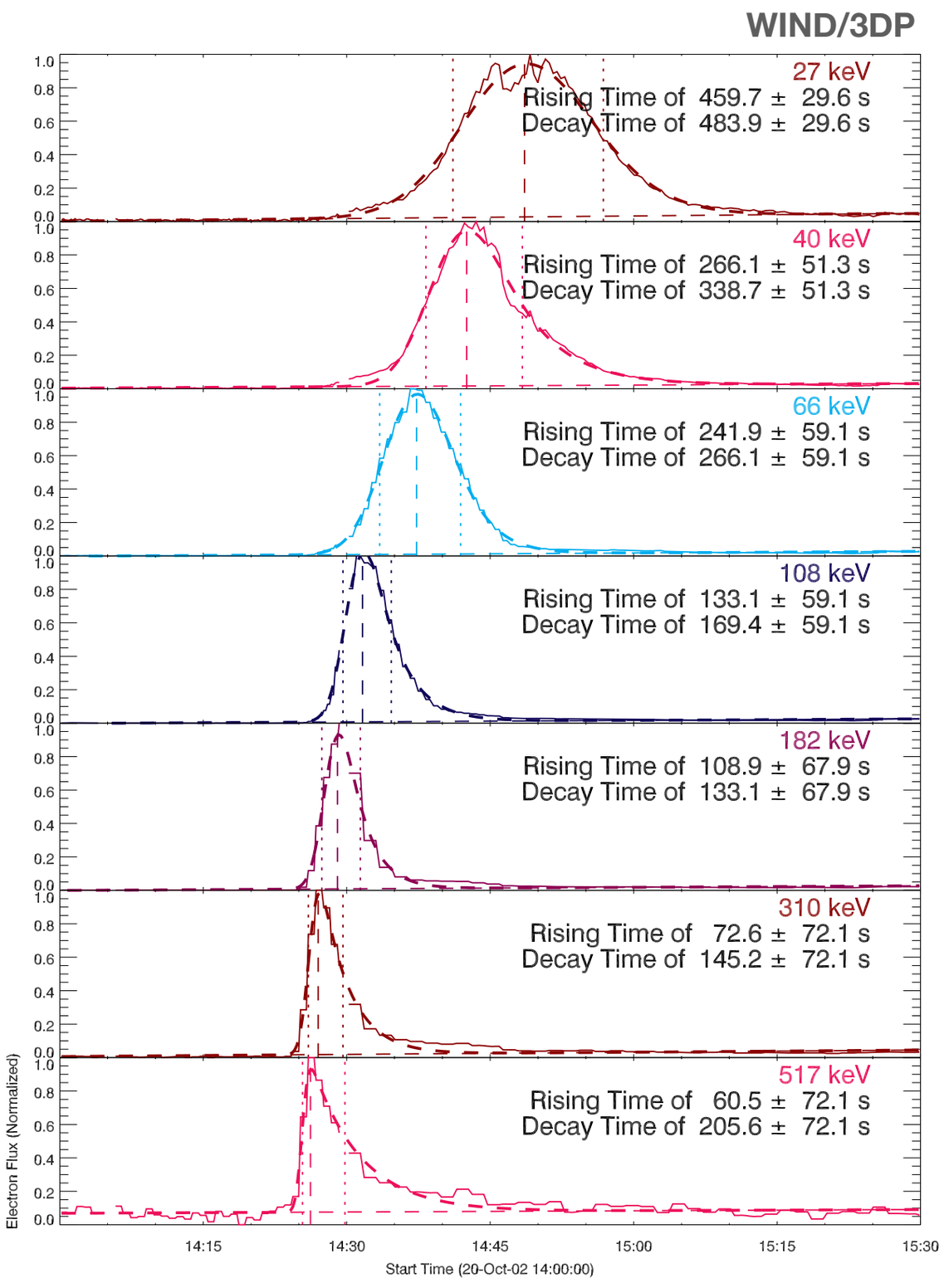


Single Injection

Collect electron events that can be traced back to a solar flare origin

- **single HXR peak — the injection time of electrons is certainly decided**
  - simultaneous upwards/downwards injection within the error range
- **type III burst** in time consistence with the HXR peak, and terminates near 20 kHz
- with the Langmuir waves near 20 kHz — the injected electrons are propagating along the open flux tubes and reach near-Earth
- clear **in-situ electron flux** enhancements conjugate with the Langmuir wave evidence — the electrons are with the same origin with HXR/radio

Time parameters extraction

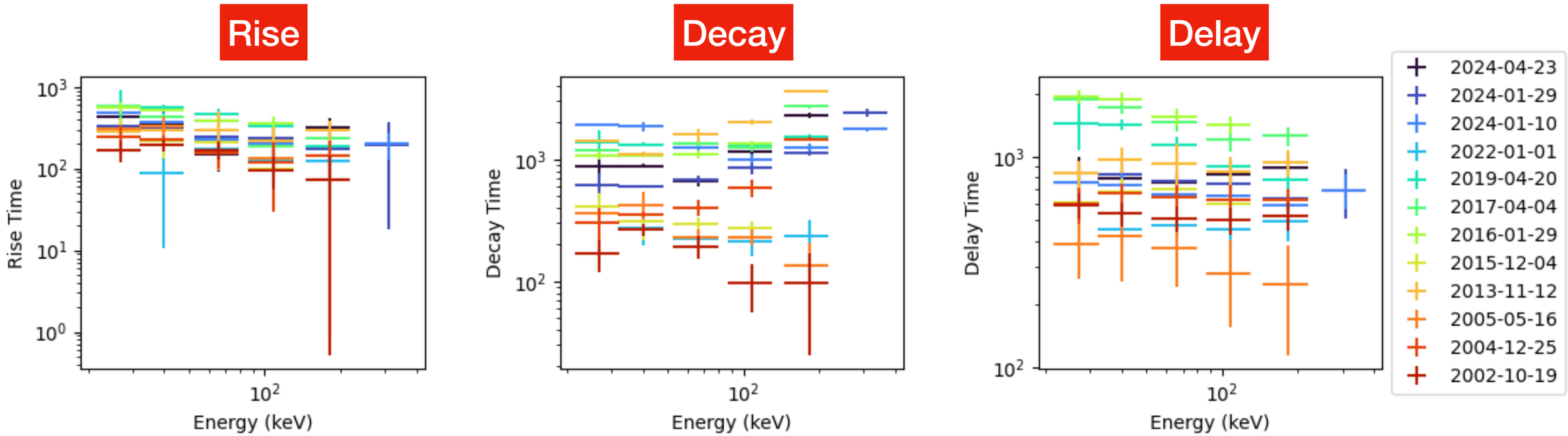


- With the electron flux profiles, we can obtain some parameters for evaluating the effect of scattering:

- **rise time** and **decay time:** from half magnitude of the normalized peak

- peak (HXR) —> peak (SEPs) timing: **delay time** of the arrival of electrons

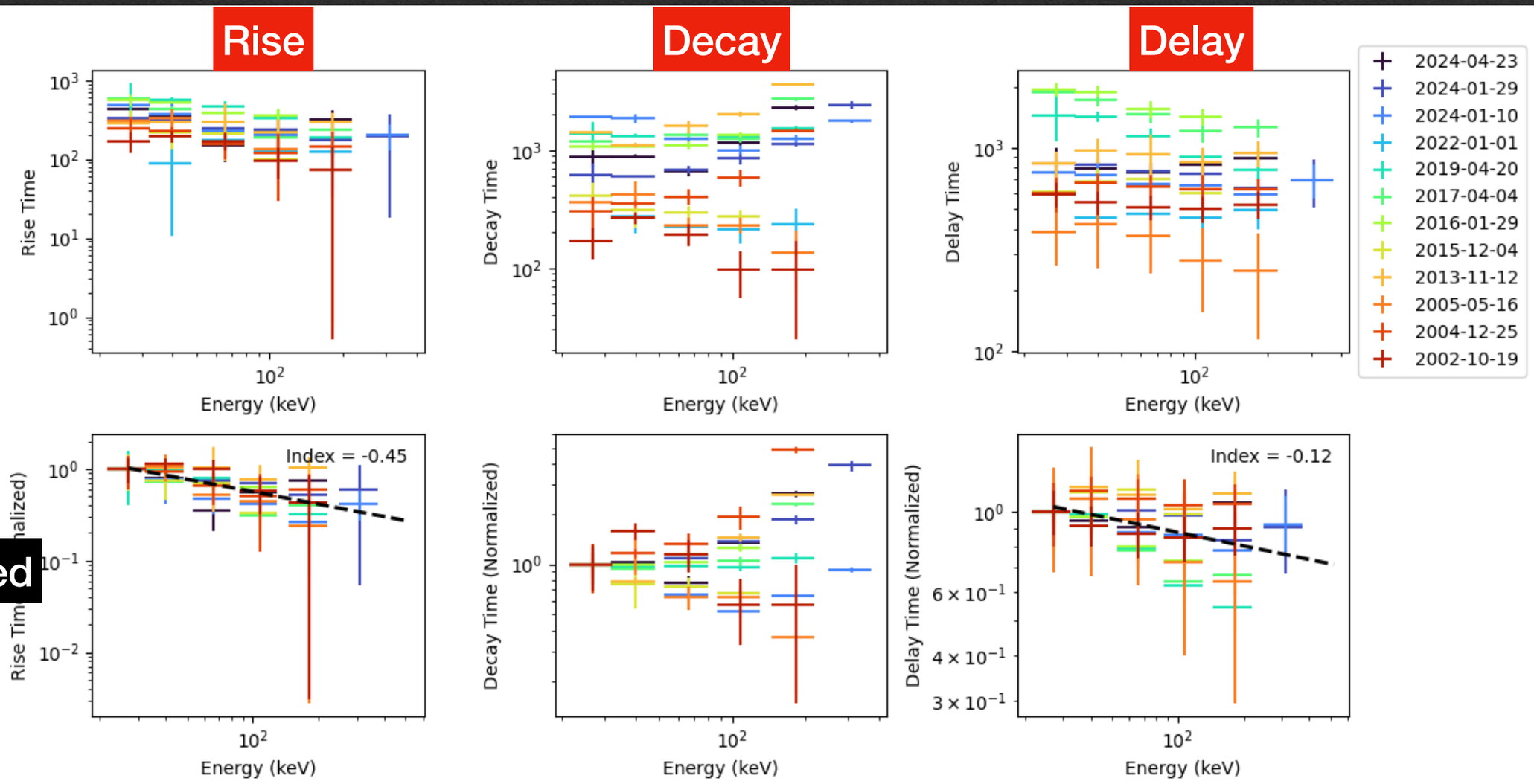
Statistical results from observations



Found a number of events meeting the criteria

- Typical values for the three time parameters
  - Rise time: 100 – 600 s
  - Decay time: 100 — 3000 s
  - Delay time: 200 — 2000 s

Statistical results from observations



Found a number of events meeting the criteria

- For **rise time** and **delay time:** decreasing trend with electron energy (power-law)
- For the **decay time:** not a single trend
  - an increasing trend for some events with relatively high decay times, and a decreasing trend for those with relatively low decay times

Transport model

Fokker-Planck transport equation

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial z} + \frac{v(1-\mu^2)}{2L_z}\frac{\partial f}{\partial \mu} - \frac{\partial}{\partial \mu}\left[D_{\mu\mu}\frac{\partial f}{\partial \mu}\right] = q(z,\mu,t)$$

convection term — magnetic focusing — pitch-angle scattering — injection of electrons

$$\mu = \cos\theta = \frac{v_{//}}{v_{\perp}}$$

$D_{\mu\mu}$ — diffusion coefficient

characterize the changing rate of electrons' pitch angle μ by

$$D_{\mu\mu} \sim \frac{(\Delta\mu)^2}{\tau} \qquad \frac{\partial D_{\mu\mu}}{\partial \mu} = -\frac{\partial \mu}{\partial t}$$

- adopt a form of $D_{\mu\mu} = \frac{v}{2\lambda}(1-\mu^2)(|\mu| + h)$
- λ is set to depend on the distance z along the field line and the electron energy E
  - $\lambda = \lambda_{1\,\mathrm{AU}}(\frac{z}{1.2\,\mathrm{AU}})^{\kappa}(\frac{E}{27\,\mathrm{keV}})^{\alpha}$

Simulation setup

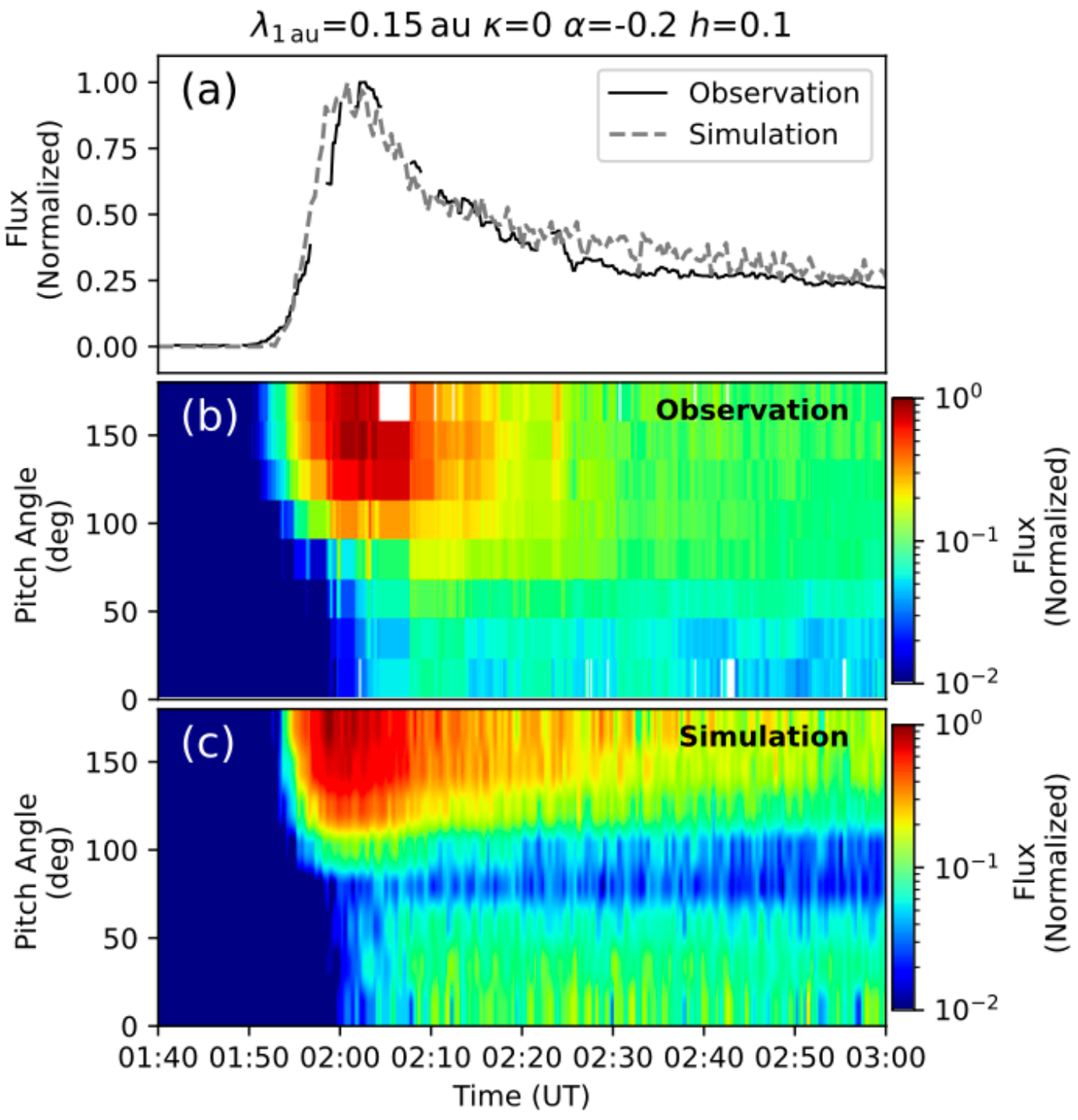
Transport model $$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial z} + \frac{v(1-\mu^2)}{2L_z}\frac{\partial f}{\partial \mu} - \frac{\partial}{\partial \mu}\left[D_{\mu\mu}\frac{\partial f}{\partial \mu}\right] = q(z,\mu,t)$$

Transformed into the equivalent stochastic differential equations (SDEs):

$$dz = \mu v dt$$

$$d\mu = \left(\frac{\partial D_{\mu\mu}}{\partial \mu} + \frac{v(1-\mu^2)}{2L_z}\right)dt + \sqrt{2D_{\mu\mu}}dW_\mu(t)$$

Simulation products



Obtained simulated in-situ **electron flux profiles** and **pitch-angle distributions**

- extract the rise, decay, and delay time same as for observations

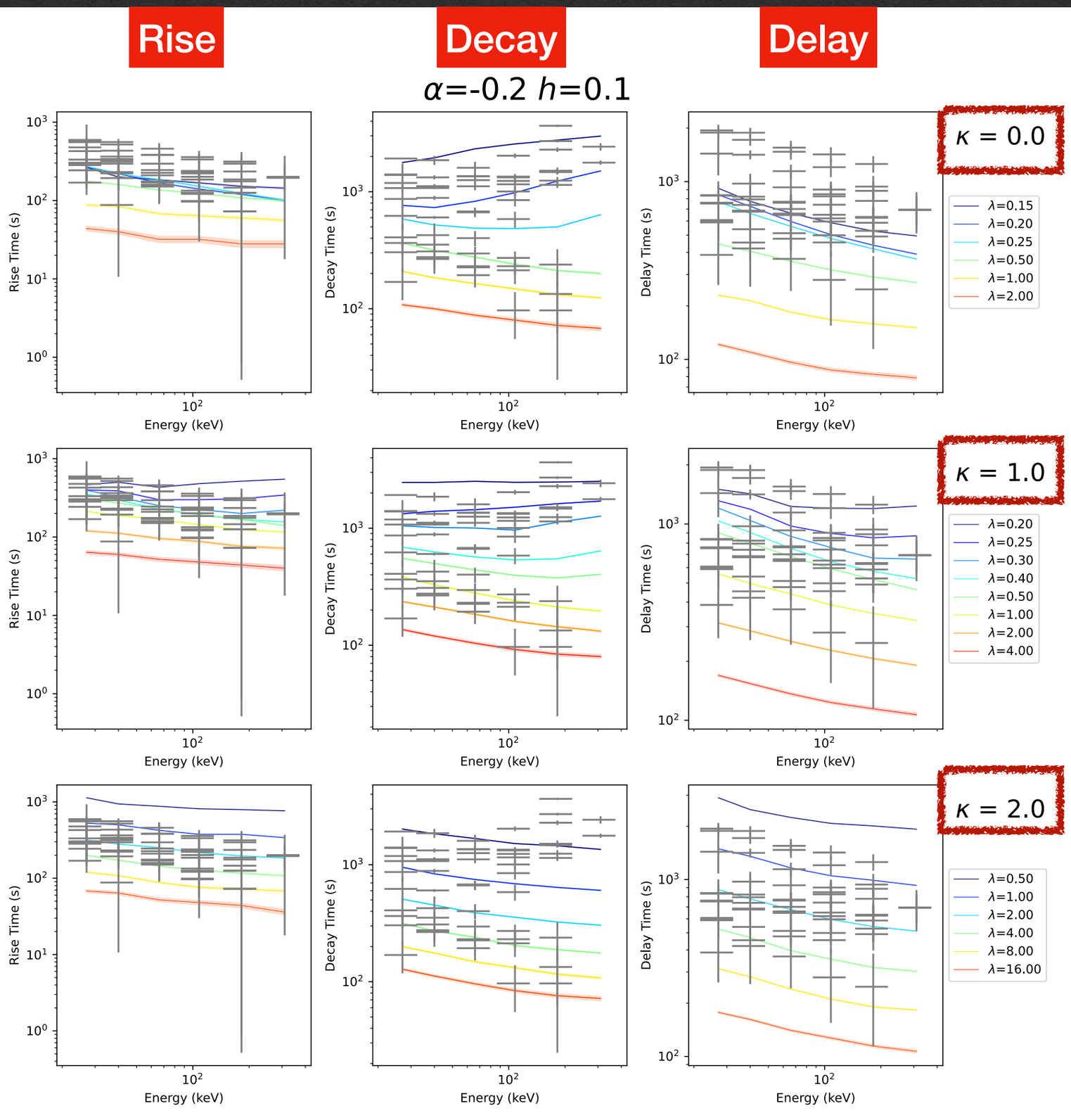
Simulation experiments over a lot of parameter combinations

- Scaling parameter $\lambda_{1\,\mathrm{AU}}$ — 0.1 to several AU
- κ — 0, 1, 2
- α — 0, -0.2, -0.4

$$D_{\mu\mu} = \frac{v}{2\lambda}(1-\mu^2)(|\mu|+h)$$

$$\lambda = \lambda_{1\,\mathrm{AU}}\left(\frac{z}{1.2\,\mathrm{AU}}\right)^{\kappa}\left(\frac{E}{27\,\mathrm{keV}}\right)^{\alpha}$$

Distance dependence

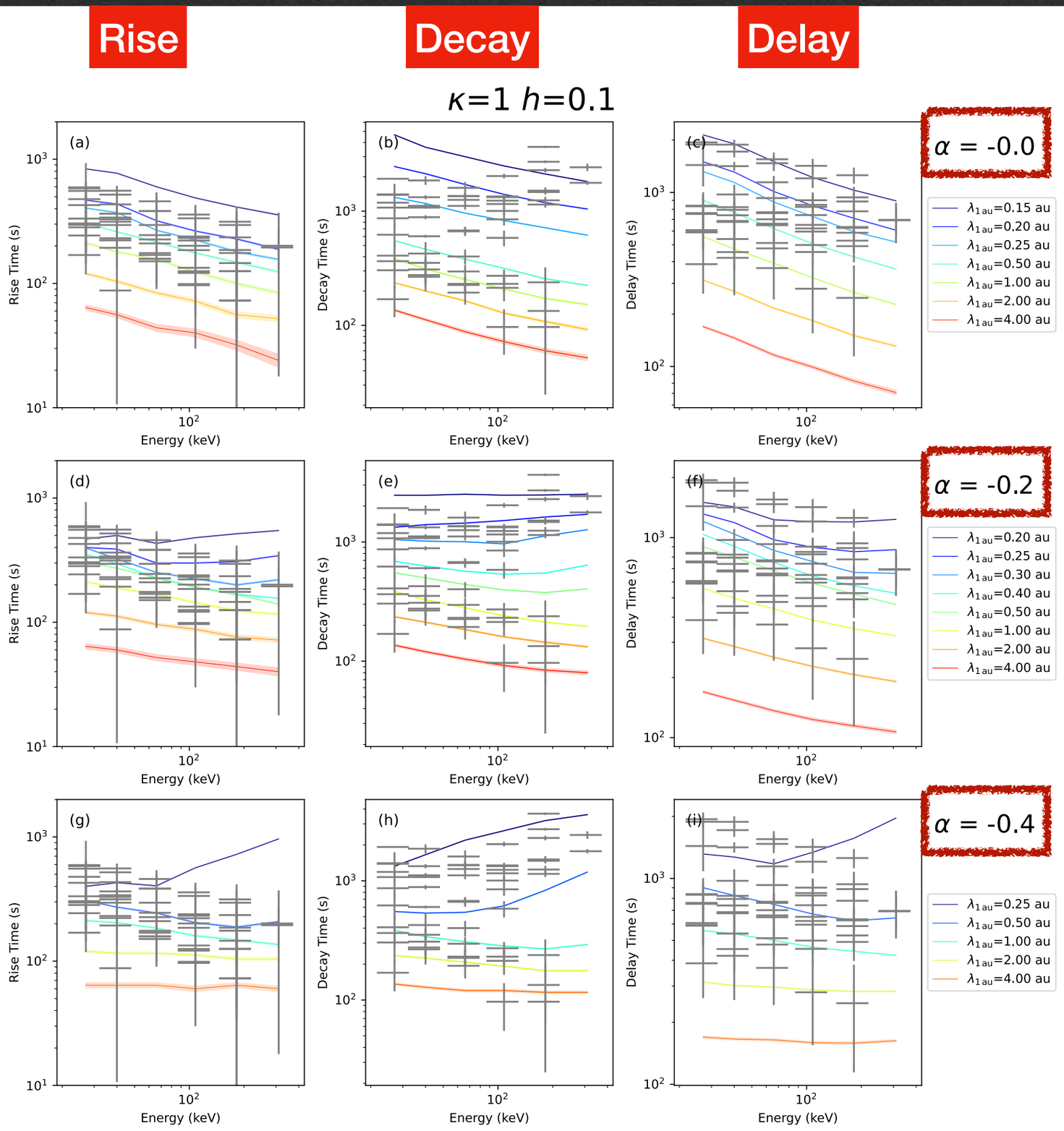


$$D_{\mu\mu} = \frac{v}{2\lambda}(1-\mu^2)(|\mu|+h)$$

$$\lambda = \lambda_{1\,\mathrm{AU}}\left(\frac{z}{1.2\,\mathrm{AU}}\right)^{\kappa}\left(\frac{E}{27\,\mathrm{keV}}\right)^{\alpha}$$

- Distance-independent scattering *($\kappa = 0$; top row)* show a wide range of acceptable results for the decay time
- but the rise time and the delay are not well matched
- $\kappa = 1$ **produces the best coverage of all the observed quantities**: rise time, decay time, and delay

Energy dependence



$$D_{\mu\mu} = \frac{v}{2\lambda}(1-\mu^2)(|\mu| + h)$$

$$\lambda = \lambda_{1\,\mathrm{AU}}\left(\frac{z}{1.2\,\mathrm{AU}}\right)^{\kappa}\left(\frac{E}{27\,\mathrm{keV}}\right)^{\alpha}$$

- $\alpha = 0$ (energy-independent) only produces decreasing trends in the various times with electron energy
- $\alpha < 0$ needed for stronger scattering at higher energy
  - flatter time vs. energy behavior
  - $\alpha = -0.2$ **reproduces time trends the best**

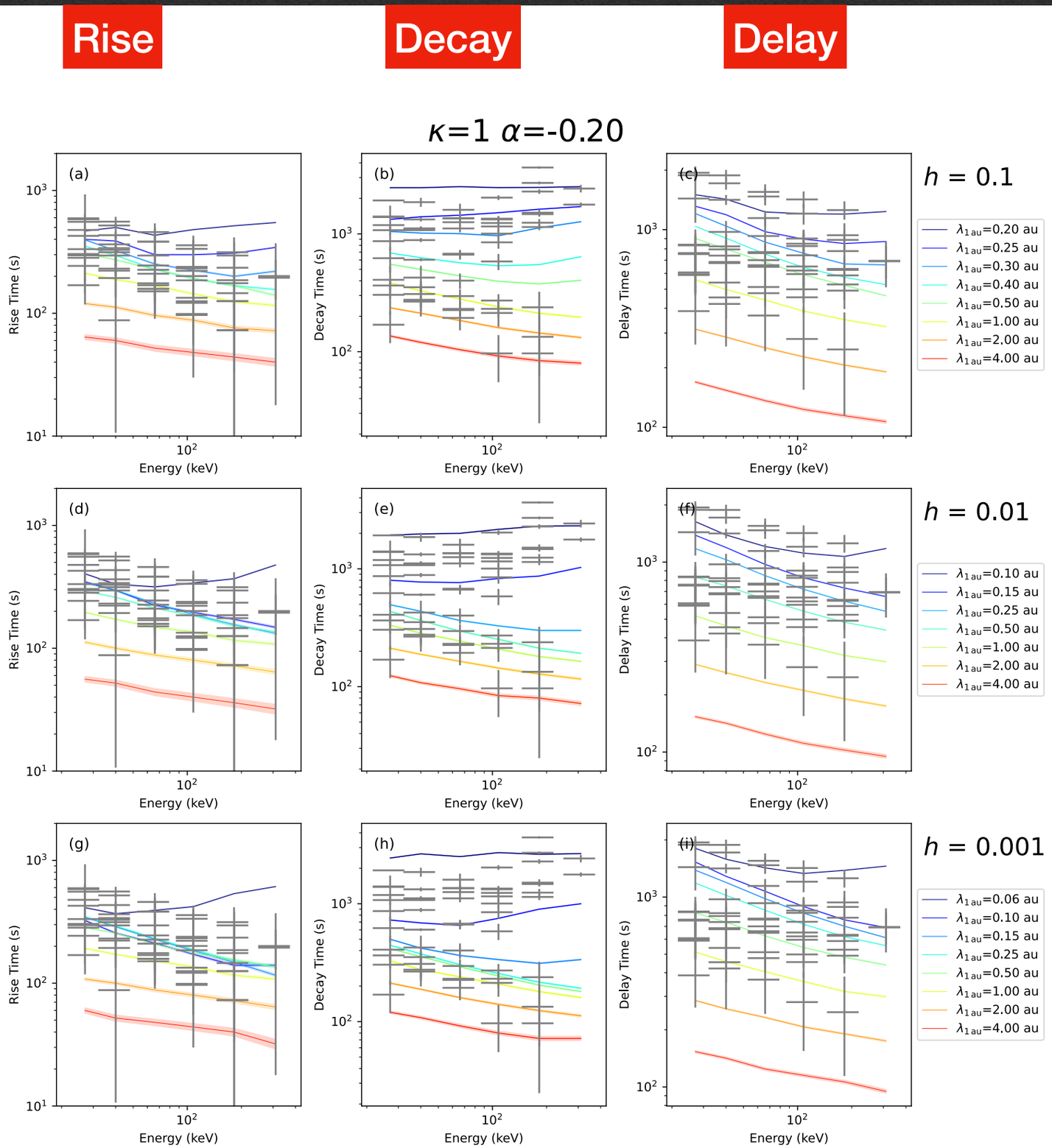
Summary

We find that

- The typical ranges of the observed rise time, decay time and delay time of SEPs are 100-600 s, 100-3000 s, and 200-2000 s, respectively.
- An electron transport model with **a distance- and energy-dependent scattering rate (or scattering mean free path)** that best reproduces the observed time vs. energy relationships across all events as
  - $D_{\mu\mu} \sim z^{-\kappa} E^{0.4-\alpha} = z^{-1} E^{0.6}$
  - $\lambda_{\parallel} \sim \lambda_{\parallel,1\,\mathrm{AU}}\, z\, E^{-0.2}$

$$\lambda_{\parallel} = \frac{3v}{8}\int_{-1}^{1}\frac{(1-\mu^2)^2}{D_{\mu\mu}}\mathrm{d}\mu = \frac{3\lambda}{2}\left[(1-h^2)\ln\left(1+\frac{1}{h}\right)+h-\frac{1}{2}\right]$$

Parameter h



$$D_{\mu\mu} = \frac{v}{2\lambda}(1-\mu^2)(|\mu| + h)$$

$$\lambda = \lambda_{1\,\mathrm{AU}}\left(\frac{z}{1.2\,\mathrm{AU}}\right)^{\kappa}\left(\frac{E}{27\,\mathrm{keV}}\right)^{\alpha}$$

$$\lambda_{\parallel} = \frac{3v}{8}\int_{-1}^{1}\frac{(1-\mu^2)^2}{D_{\mu\mu}}\mathrm{d}\mu = \frac{3\lambda}{2}\left[(1-h^2)\ln\left(1+\frac{1}{h}\right) + h - \frac{1}{2}\right]$$

- h fills the scattering gap around $\mu \sim 0$
- For weaker scattering, $\mu \sim 1$, the effect from h is negligible
- For stronger scattering, h and λ both contributes on the mean free path and the scattering strength