EXTREME PULSARS

21.07.2025

From 9 to 40 - Sexten

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SEXTEN CENTER FOR ASTROPHYSICS RICCARDO GIACCONI

NEUTRONENSTERN

OUTLINE

1. **Pulsar physics and gamma-ray emission**:
   - Rotation-powered pulsar **magnetosphere**
   - **Particle acceleration** scenarios and **gamma-ray** production mechanisms
2. **State of the art pulsar results:**
   - **Teasers** for what **ASTRI and CTAO** can do…
3. **Perspectives and conclusions**

Ideal magnetohydrodynamic plus axial symmetry:

$\vec{v} = \vec{\Omega} \times \vec{r} + \kappa_{(r,z)} \vec{B}$

PULSAR MAGNETOSPHERE

Plasma filled **magnetosphere**

Ohm's law: $\rho\vec{\mathbf{j}} = (\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$

Ideal plasma (infinite conductivity):

$$\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}} = 0 \qquad \vec{\mathbf{E}} \cdot \vec{\mathbf{B}} = 0$$

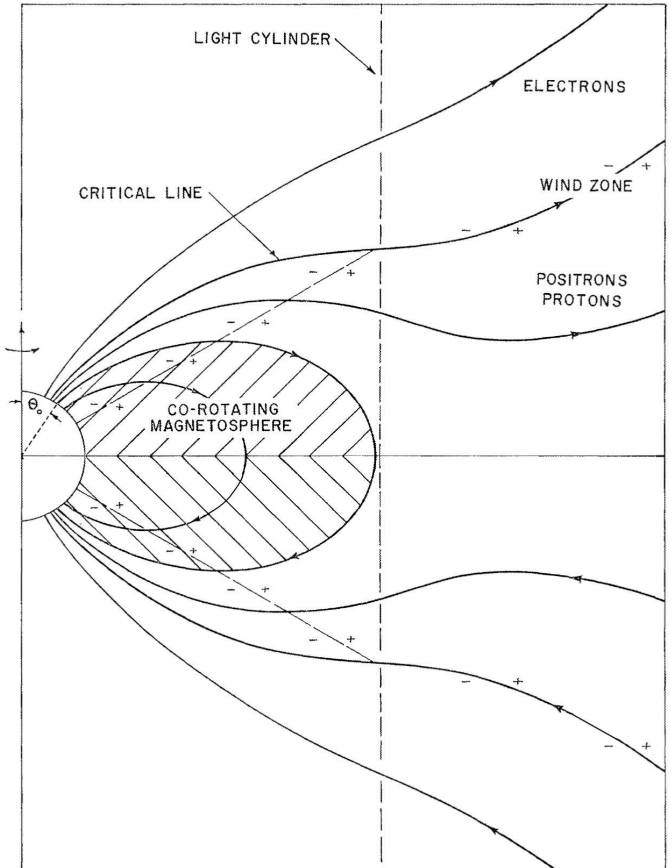
Magnetic field lines are **electric equipotential**

Particles **bound to the field lines** and in **co-rotation** with the neutron star

Co-rotation radius limited by the **speed of light**:
Light cylinder $R_L = {}^{c}/_{\Omega}$

Open magnetosphere (!!!)

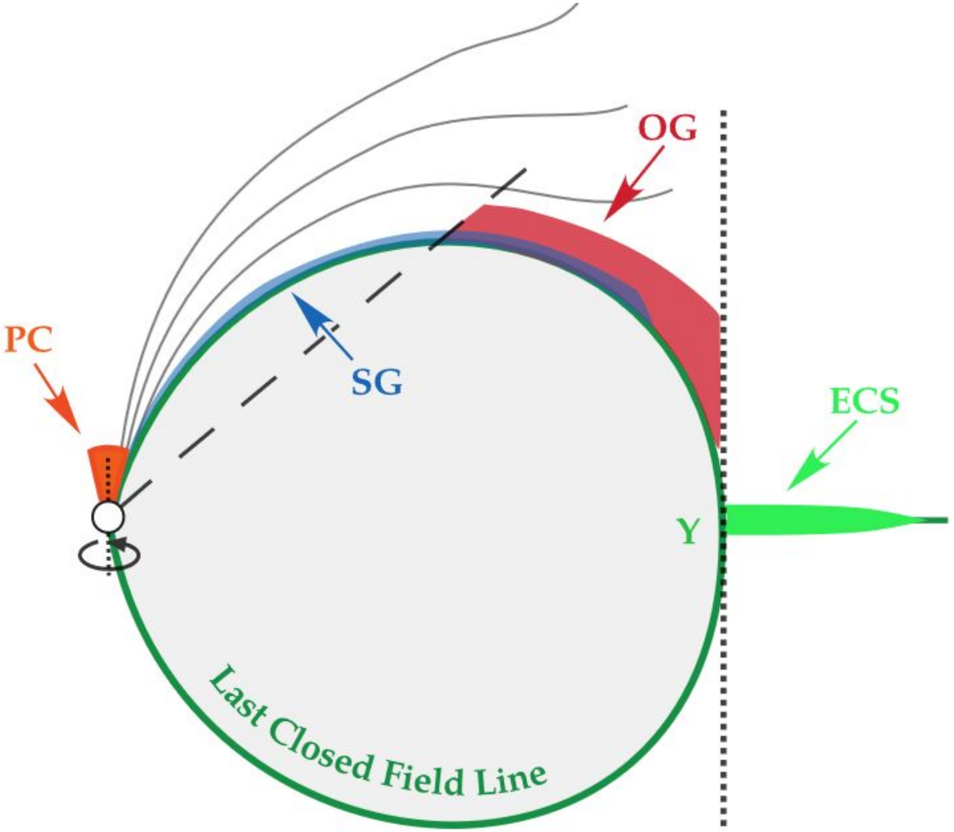
Particles stream away **leaving the star**
Loss of energy: **pulsar spins down**



Goldreich & Julian, 1969

PARTICLE ACCELERATION

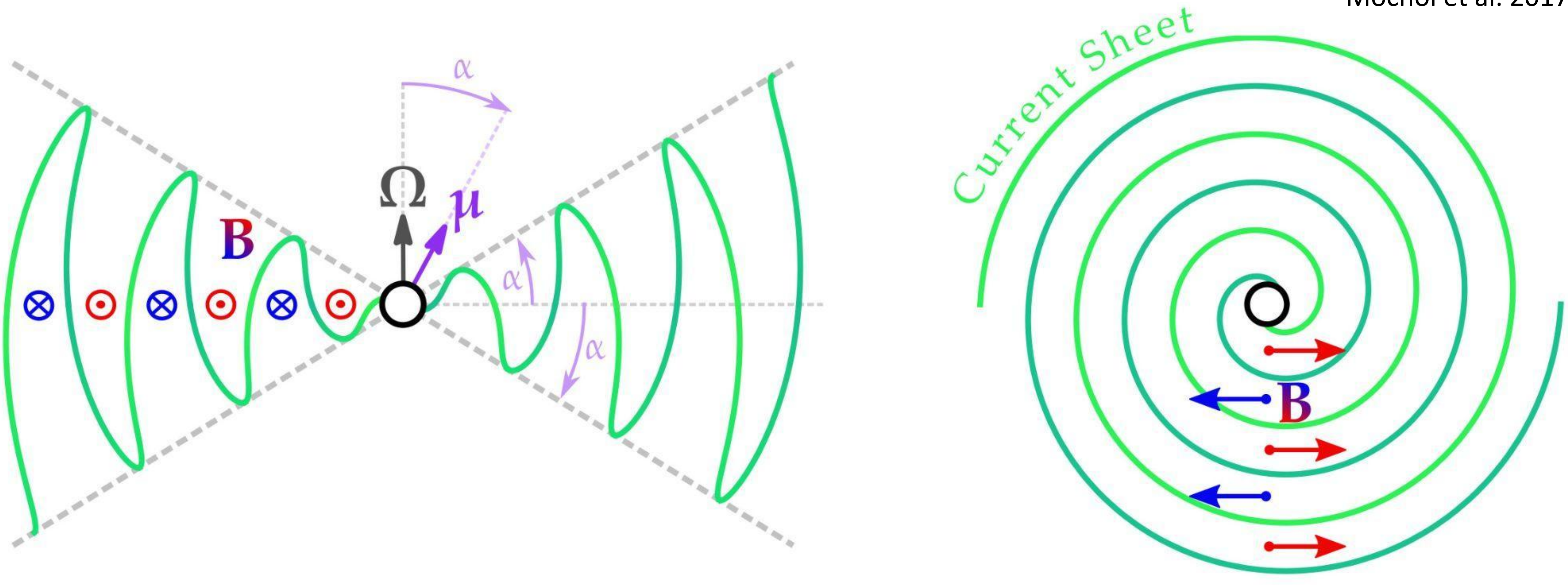
- **Particle acceleration** possible only in **defects** of the ideal **plasma:**
  - **Where** are they? **What** sustains them?
- **Gamma rays** via **synchro-curvature** radiation or **inverse-Compton** scattering:
  - **Competition** with $\gamma \rightarrow e^{+}e^{-}$ **absorption** in the **strong** $\vec{B}$
  - Spectral **cut-offs** due to maximum electron energy
- **No analytical** solution!
  - Theoretical modeling **limited** by **computational methods** and power



Modified from Hirotani, 2011

MAGNETOSPHERE CHARGES

Mochol et al. 2017

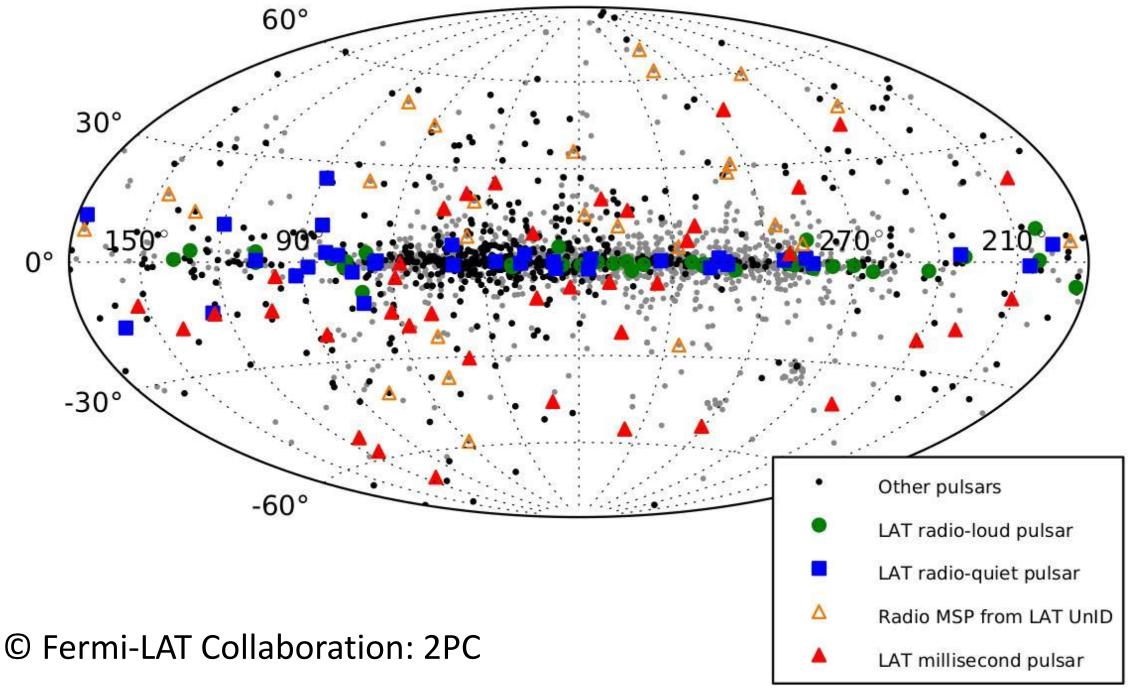


Meridian Plane

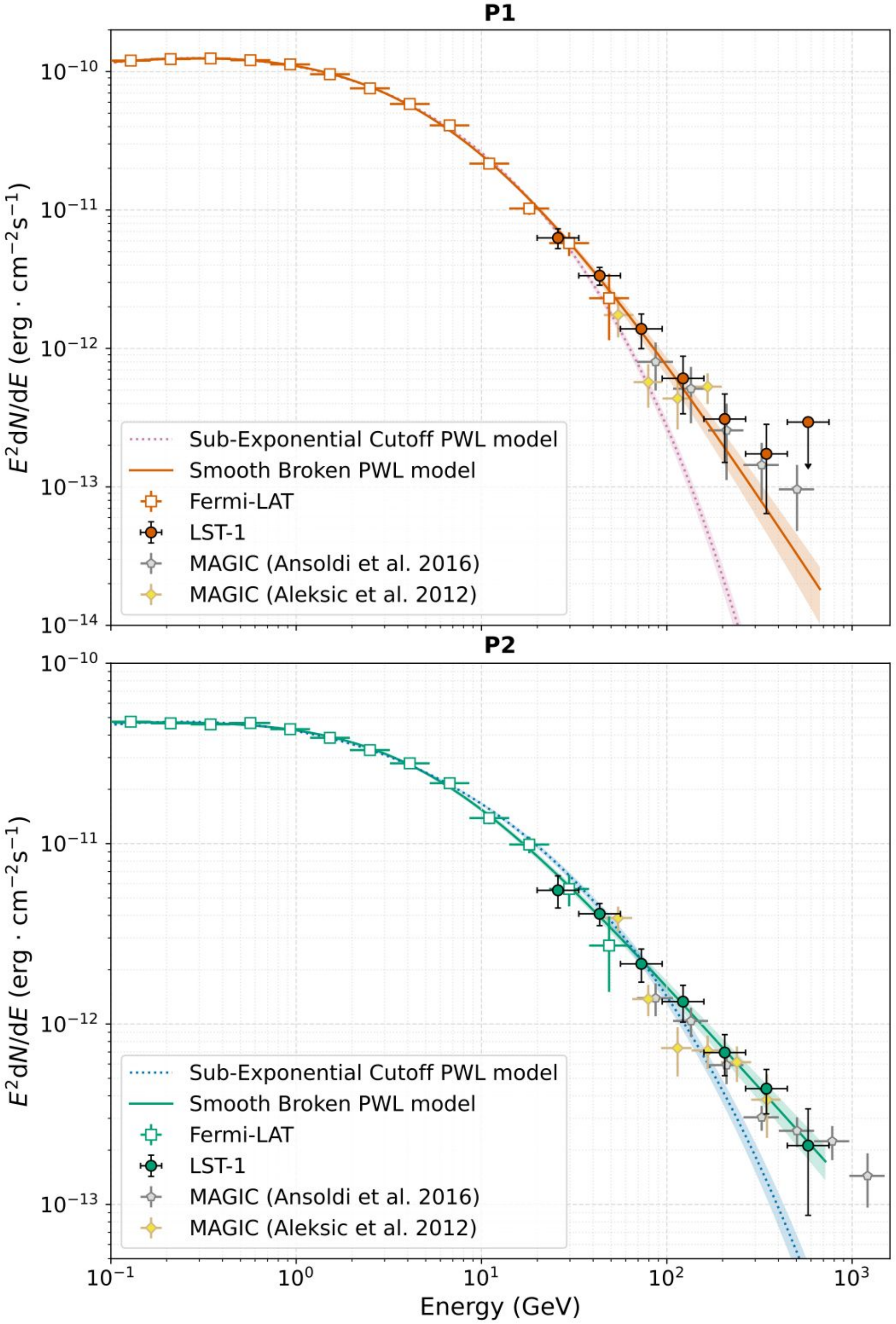
Equatorial Plane

PULSARS

- **Hundreds of gamma-ray pulsars** in the 100 MeV – 10 GeV band
- Only **5(=3+2) above 50 GeV**
- Only **2 above 1 TeV**
- Only **1 above 10 TeV**
- No consensus on the **emission** details: several **competing models**
- **Magnetosphere structure** and **dynamics** still poorly understood
- What allows some **pulsars** to **emit at VHE energies**?



© Fermi-LAT Collaboration: 2PC



LST-Collaboration, 2024

CRAB PULSAR

Power-law like tail reaching (and surpassing) **TeV energies**.

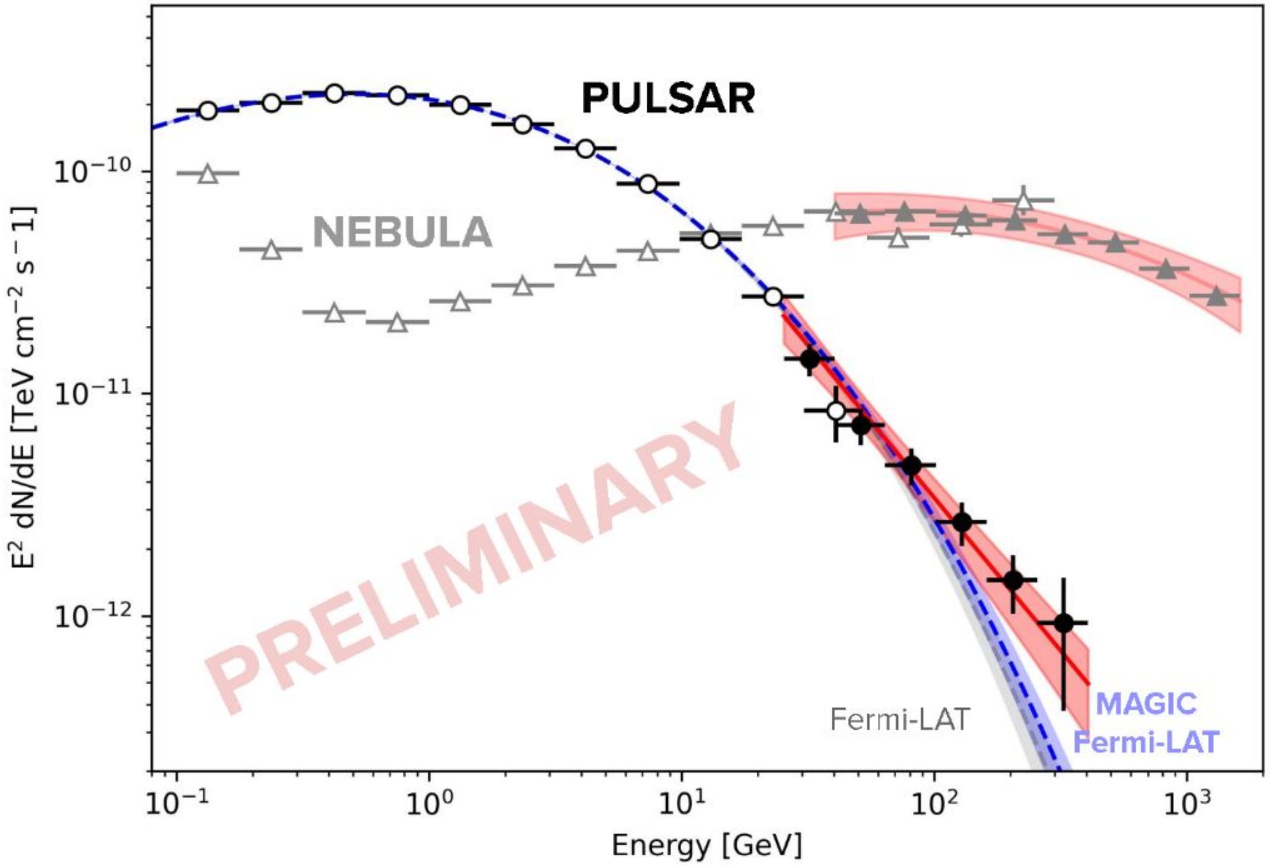
Single smooth spectrum connecting to CR peak at **GeV scale**.

Large **spectral variation** across different **phases**.

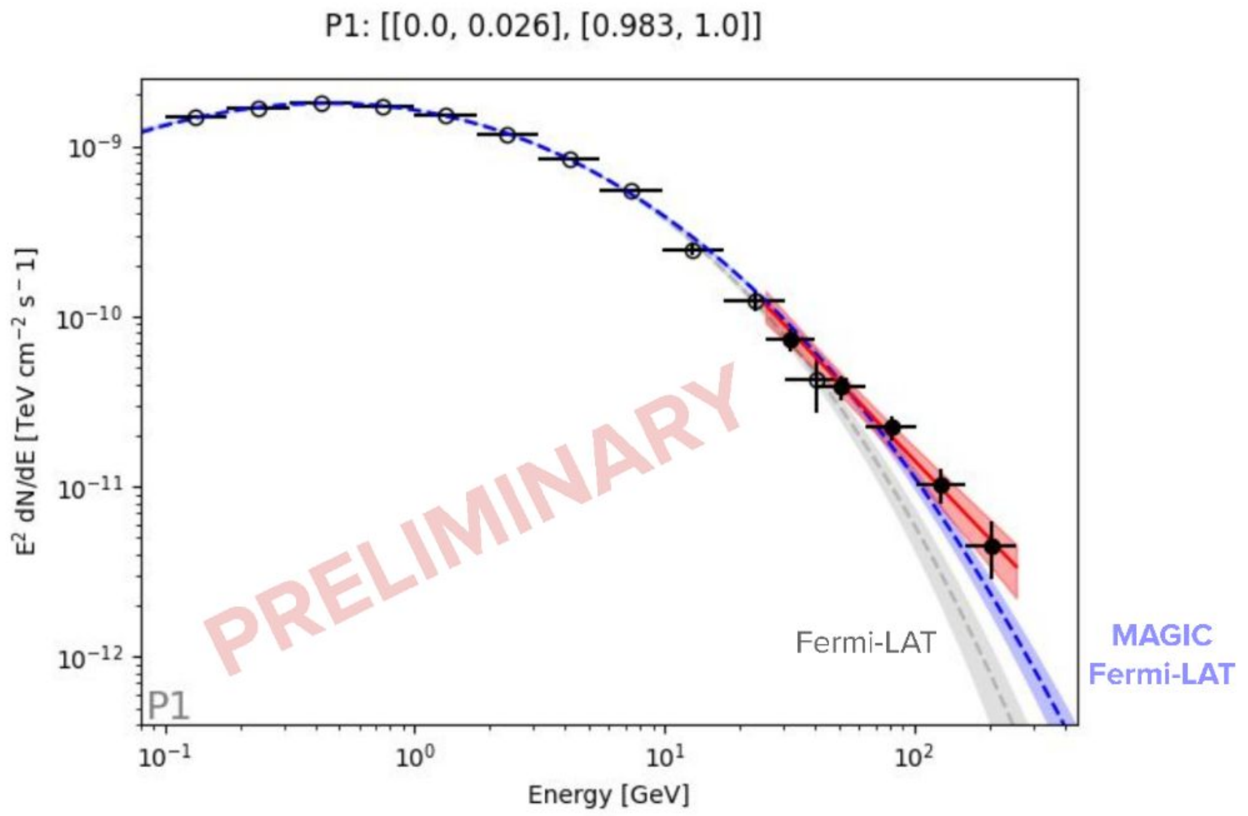
Challenges all classic **emission models**; presently **unexplainable**.

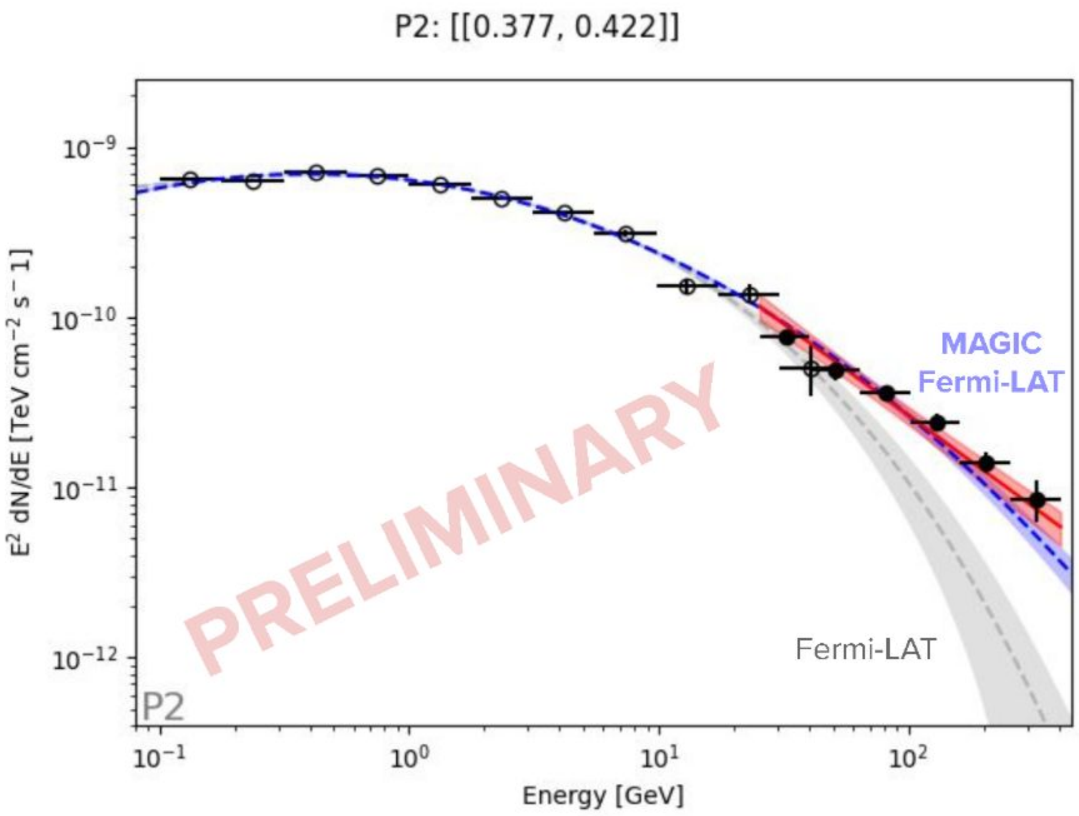
PHASE-AVERAGED SPECTRUM

- **Power-law** extension of the Fermi-LAT spectra:
  - Range: 25 GeV - 400 GeV
- **Nebula** gated in off phase interval (0.52,0.87)
- **Joint-fit** with LAT data:
  - MAGIC data "**drowned**" in LAT statistics
  - Compatible with cutoff from **curvature radiation**?



ADDING A DIMENSION: PULSED SPECTRA





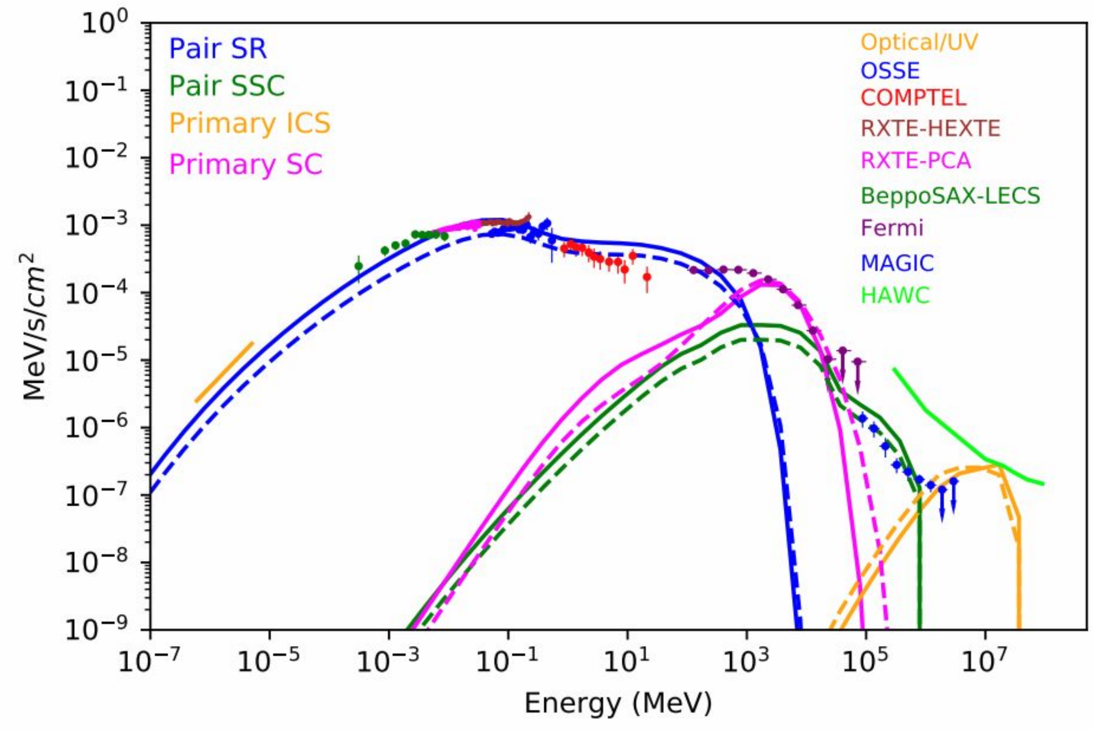
PWL Spectral Index (P1): **~3.5** (LST-1 ~3.4)
b(P1): 0.11±0.03

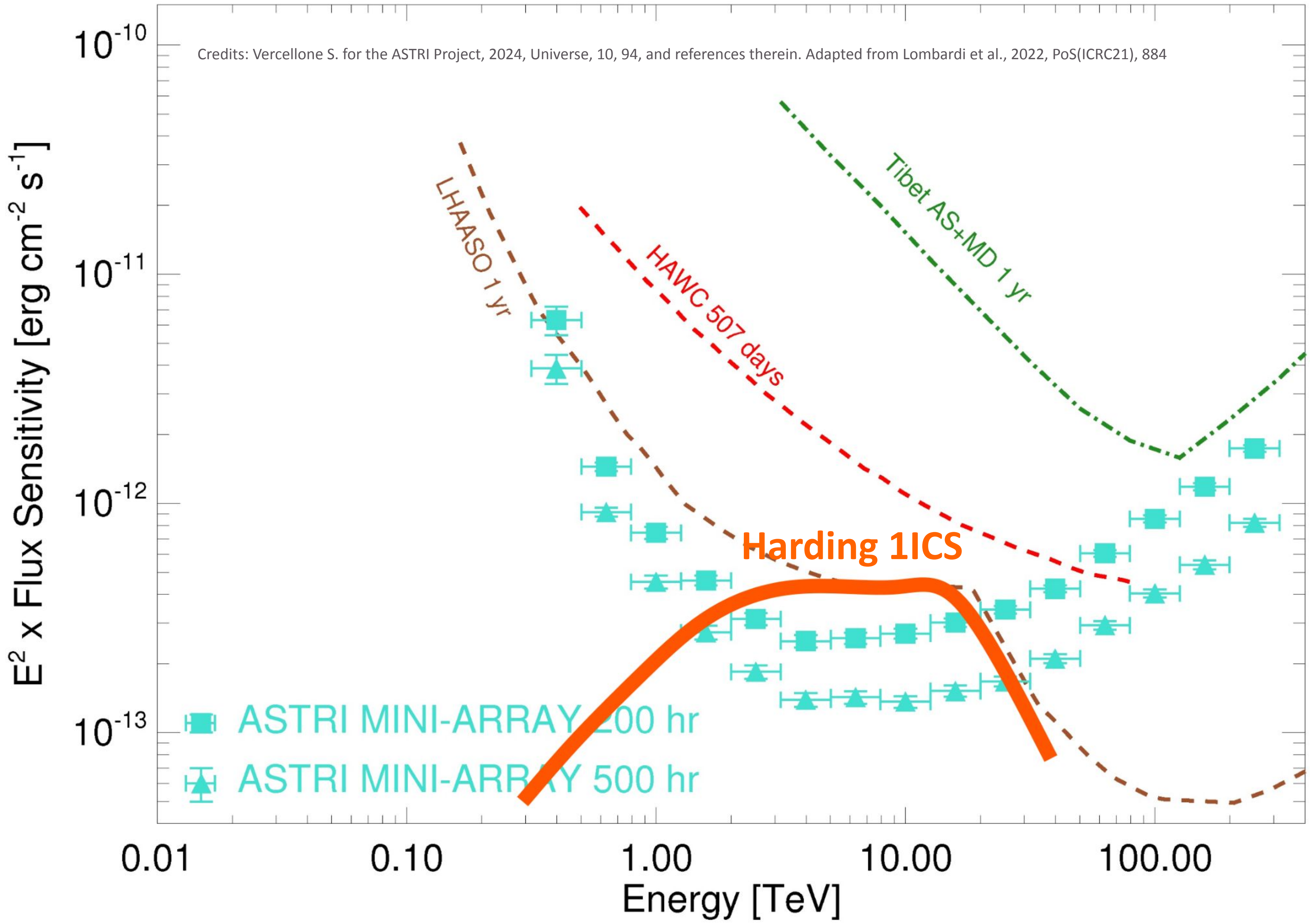
PWL Spectral Index (P2): **~3.1** (LST-1 ~3.0)
b(P2): 0.05±0.04

COMPETING MODELS

- Long march **towards** and beyond the **light cylinder**
- Two classes of models:
  - **Linear** parallel **E field** at the **Y point** (Harding et al., etc)
  - **Magnetic reconnection** in **striped wind** (Cerutti et al., etc)
- **Model degeneracy** impacted by **phase-dependent** spectra
- Need to **account phase dependency** in models

Harding et al., 2021





If **1ICS exists** then **wind** acceleration is in **trouble**.

If **1ICS** not there then **ECS acceleration** is in trouble.

Within **ASTRI**'s **capabilities** with **commissioning** data.

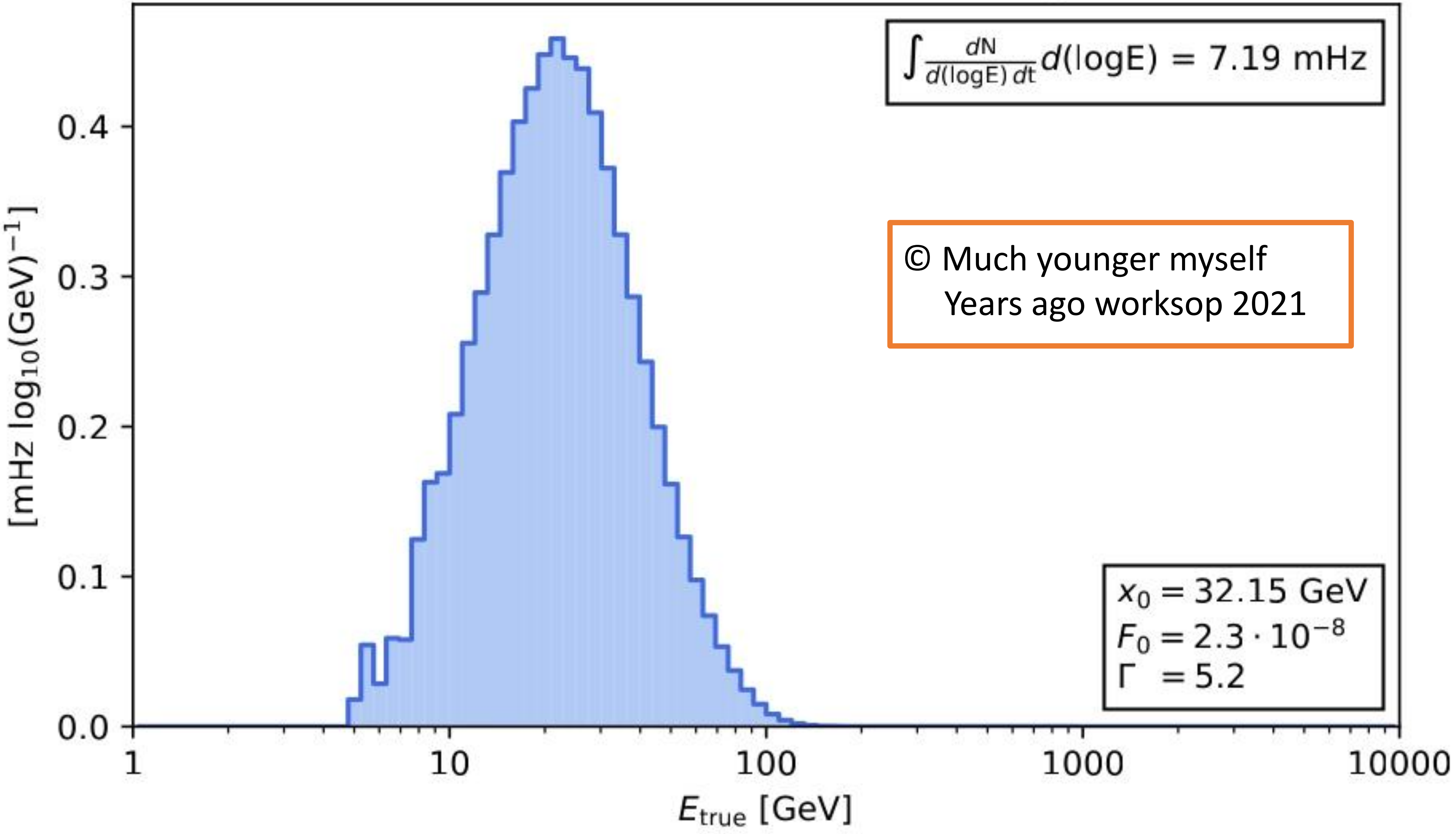
(Why hasn't LHAASO said anything yet?)

Sample usage: pulsar cookbook

- Multiply effective area with an assumed spectrum $\mathbf{dN_\gamma/dlog(E){\cdot}dA{\cdot}dt}$
  (with actual prefactor and correct units!)
- Differential excess event rate $\mathbf{dN_\gamma/dlog(E){\cdot}dt}$
  (Logarithmic threshold)
- Integrate over an energy range of interest: $\mathbf{R_{ex} = dN_\gamma/dt}$
  (Estimated excess event rate)
  (It's actually much cleaner to *incorporate the effective energy range of interest in the IRF itself*)
- Estimate a reasonable background rate from real data: $\mathbf{R_{off,0} \simeq N_h/t_{eff}}$
- *Important*: scale background rate to pulsar on data: $\mathbf{R_{off} = \alpha\ R_{off,0}}$
- Figure of merit for the pulsar: $\mathbf{s = R_{ex}\ /\ R_{off}^{0.5} \cdot 3600^{0.5}}$
  (in the extremely horrible and unsettling units of σ/hours$^{0.5}$)
- Significance vs time: $\mathbf{\Sigma = s{\cdot}t^{0.5}}$ Time to get a detection: $\mathbf{t_5 = 25/s^2}$

© Much younger myself
Years ago worksop 2021

Differential event rate: Geminga



LogAeff_zd10-30_Geminga_size20_eest25-100_t75h90fine.root

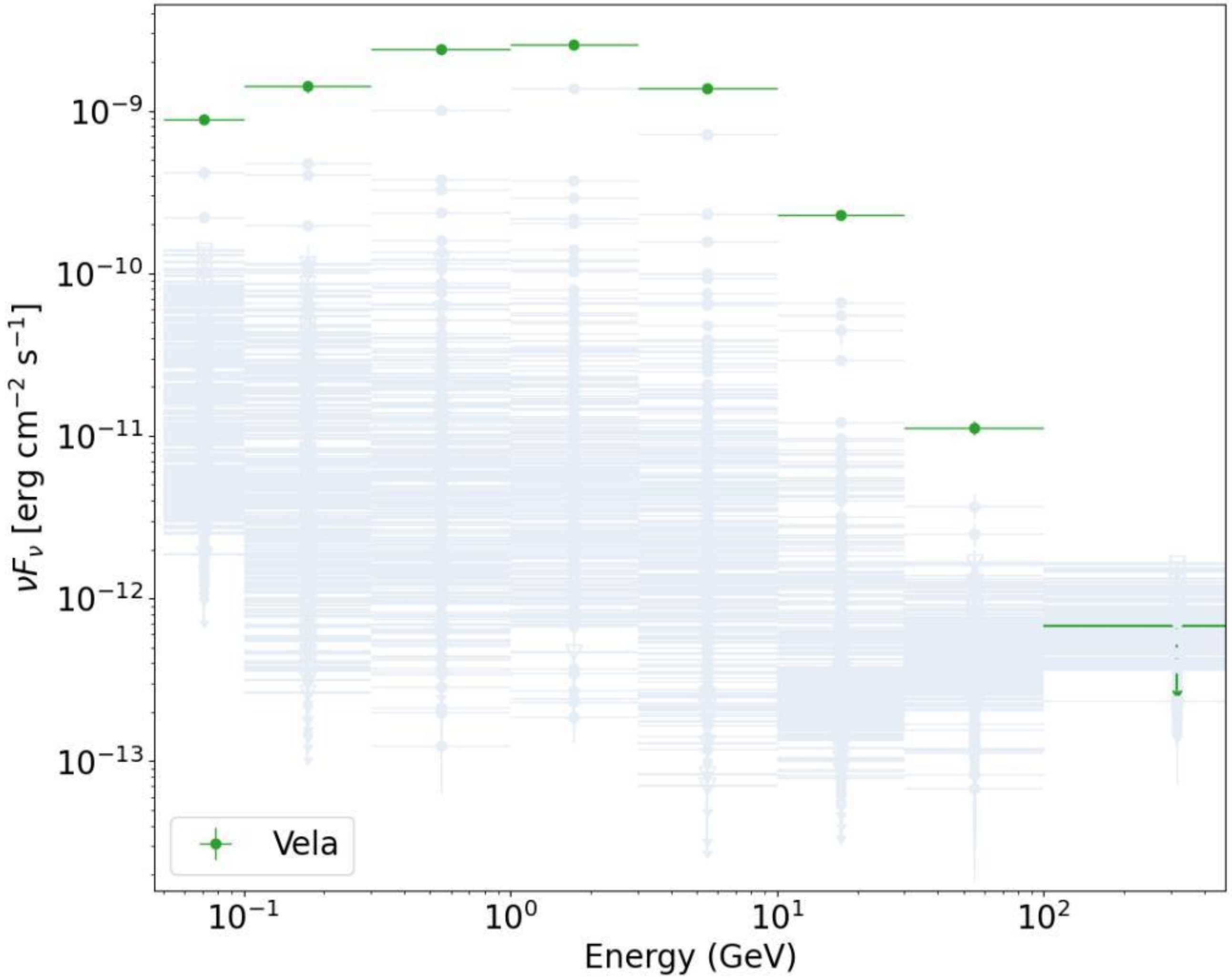
Flux units = $GeV^{-1}m^{-2}s^{-1}$

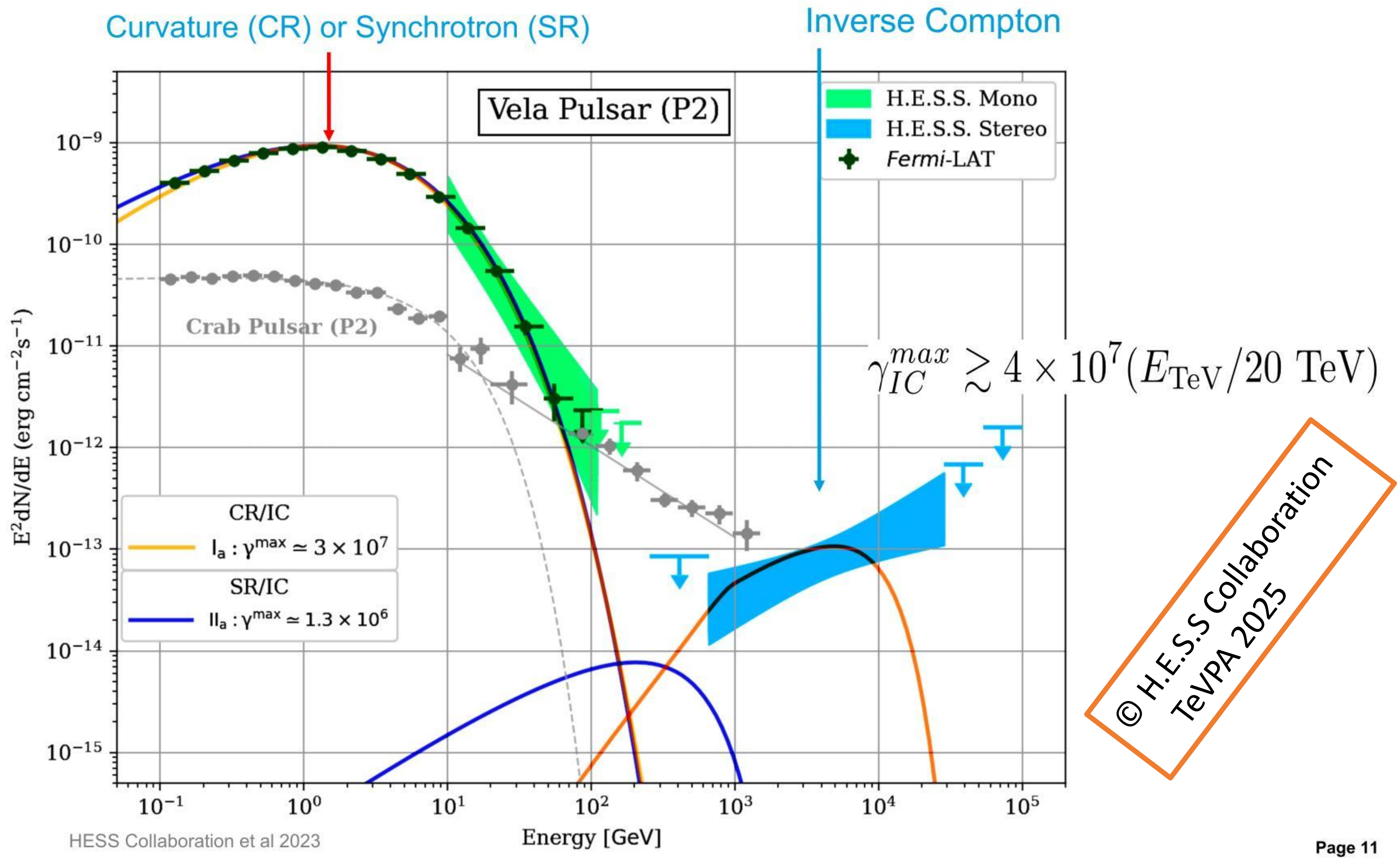
The Vela pulsar

Vela in the sub-100 GeV regime

Nearby d=287 pc
Young pulsar 11 kyr and period of 89 ms
Ė ~ 10^{37} erg/s

Spectra of LAT Pulsars in the 3PC





HESS Collaboration et al 2023

HUNTING FOR NEW PULSARS

MNRAS **000**, 1–17 (2016) Preprint 23 June 2017 Compiled using MNRAS LaTeX style file v3.0

Prospects for the detection of high-energy ($E>25$ GeV) *Fermi* pulsars with the Cherenkov Telescope Array

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23 June 2017

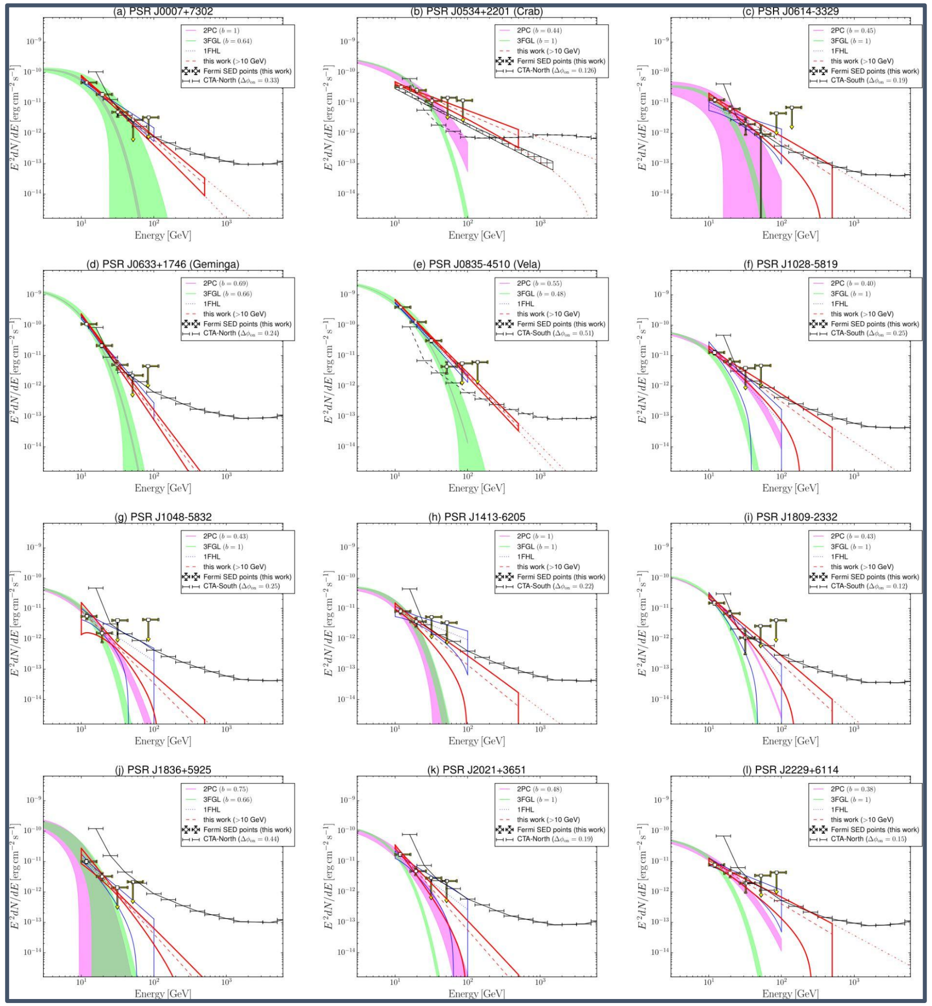
ABSTRACT
Around 160 gamma-ray pulsars were discovered by the *Fermi* Large Area Telescope (LAT) since 2008. The most energetic of them, 12 objects with emission above 25 GeV, are suitable candidates for the detection with the current and future Imaging Atmospheric Cherenkov Telescopes above few tens of GeV. We perform an analysis of the *Fermi*-LAT data of these high-energy pulsars in order to determine if such objects can be detected with the Cherenkov Telescope Array (CTA). Our goal is to forecast the significance of their point source detection with CTA. We analyze 5 years of the *Fermi*-LAT data fitting the spectra of each pulsar at energies $E>10$ GeV with a power-law function. Assuming no spectral cut-off, we extrapolate the resulting spectra to the very high energy range (VHE, $E>0.1$ TeV) and simulate CTA observations of all 12 pulsars with the `ctools` software package. Using different analysis tools, individual CTA sensitivity curves are independently calculated for each pulsar and cross-checked with the `ctools` results. Our simulations result in significant CTA detections of up to 8 pulsars in 50 h. Observations of the most energetic *Fermi* pulsars with CTA will shed light on the nature of the high-energy emission of pulsars, clarifying whether the VHE emission detected in the Crab pulsar spectrum is present also in other gamma-ray pulsars.

Key words: pulsars: individual: PSRs J0007+7303, J0534+2201, J0614−3329, J0633+1746, J0835−4510, J1028−5819, J1048−5832, J1413−6205, J1809−2332, J1836+5925, J2021+3651, J2229+6114 – gamma-rays: stars

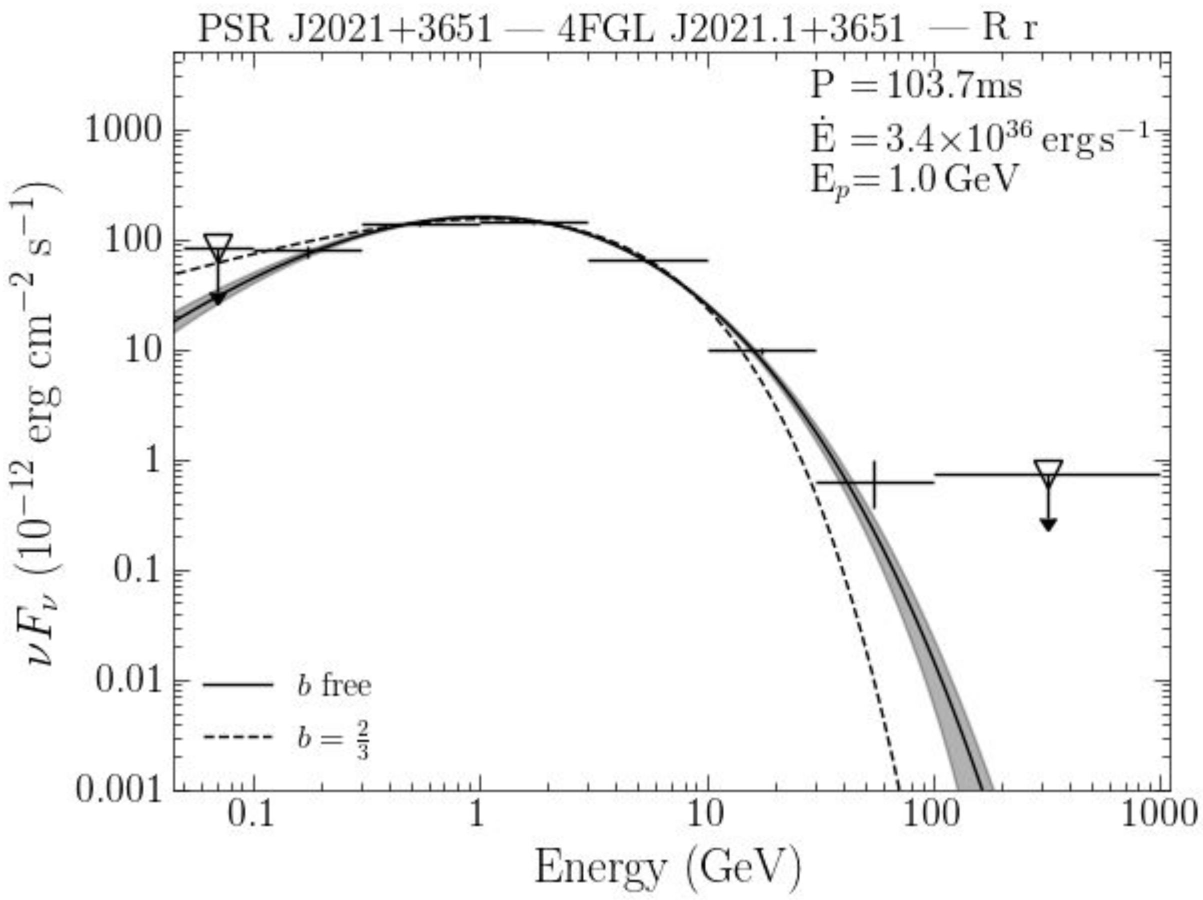
1 INTRODUCTION

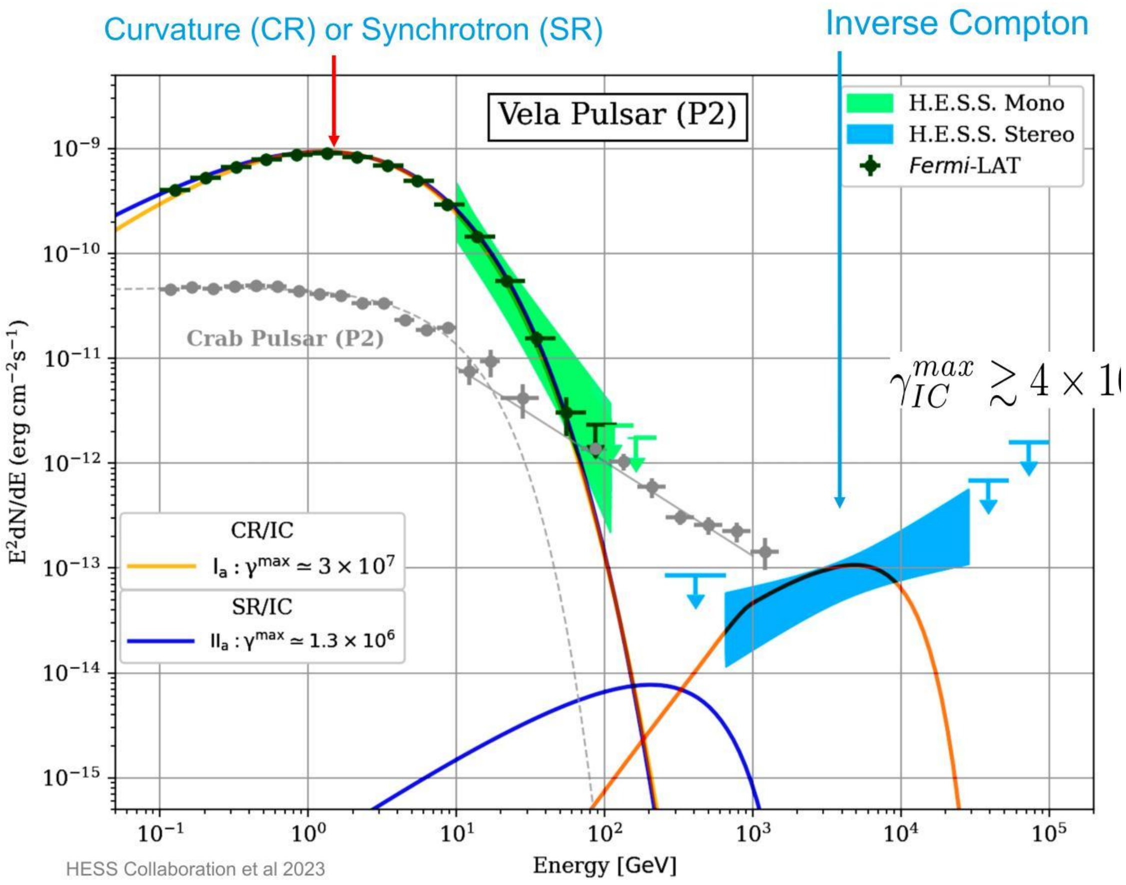
Pulsars are energetic neutron stars, which produce radiation in a broad energy range: from radio up to extremely energetic gamma-rays (see e.g. Becker & Truemper 1997; Manchester et al. 2005; Mignani 2011; Abdo et al. 2013; Ansoldi et al. 2016). These compact objects have strong magnetic fields of about $B = 10^9 - 10^{15}$ G for young isolated pulsars and $B < 10^9$ G for millisecond pulsars. The rotational energy of pulsars is lost mainly in the form of a Poynt-

have on- and off- peak intervals and, in some cases, a bridge interval in phase. The on-peak phase interval can be characterized by one (or a few) prominent and sharp emission peak(s) (also called "on-pulse emission"). The off-peak (OP) emission is usually dominated by the background from the surrounding nebula and/or the pulsar itself. The average level of the bridge emission (if present) between two peaks is higher than that of the off-peak phase interval. One of the most recent investigations of the gamma-ray pulsars light curves was presented by Pierbattista et al. (2015). Gamma-ray observations



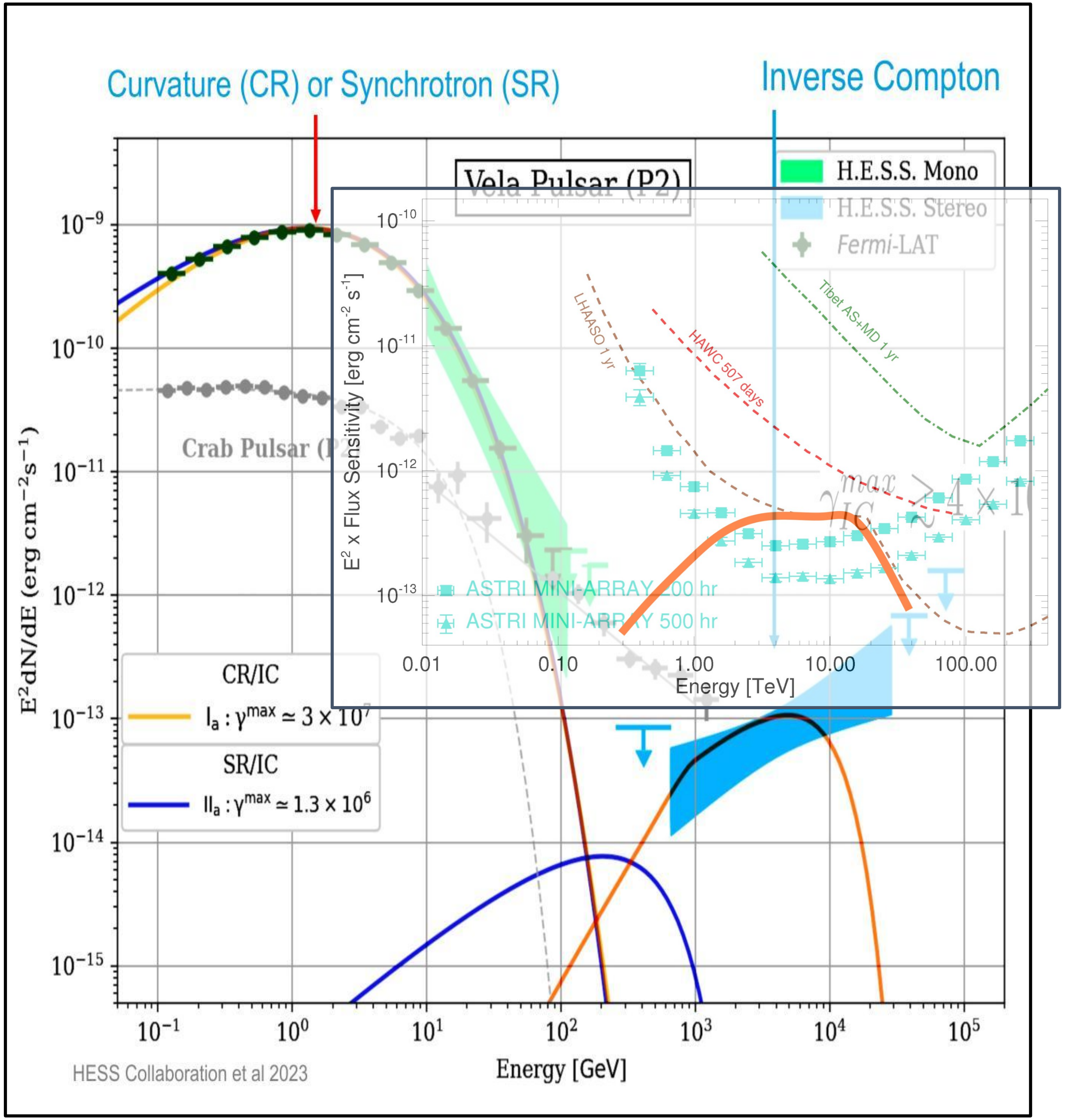
PSR J2021+3651: DRAGONFLY vs VELA

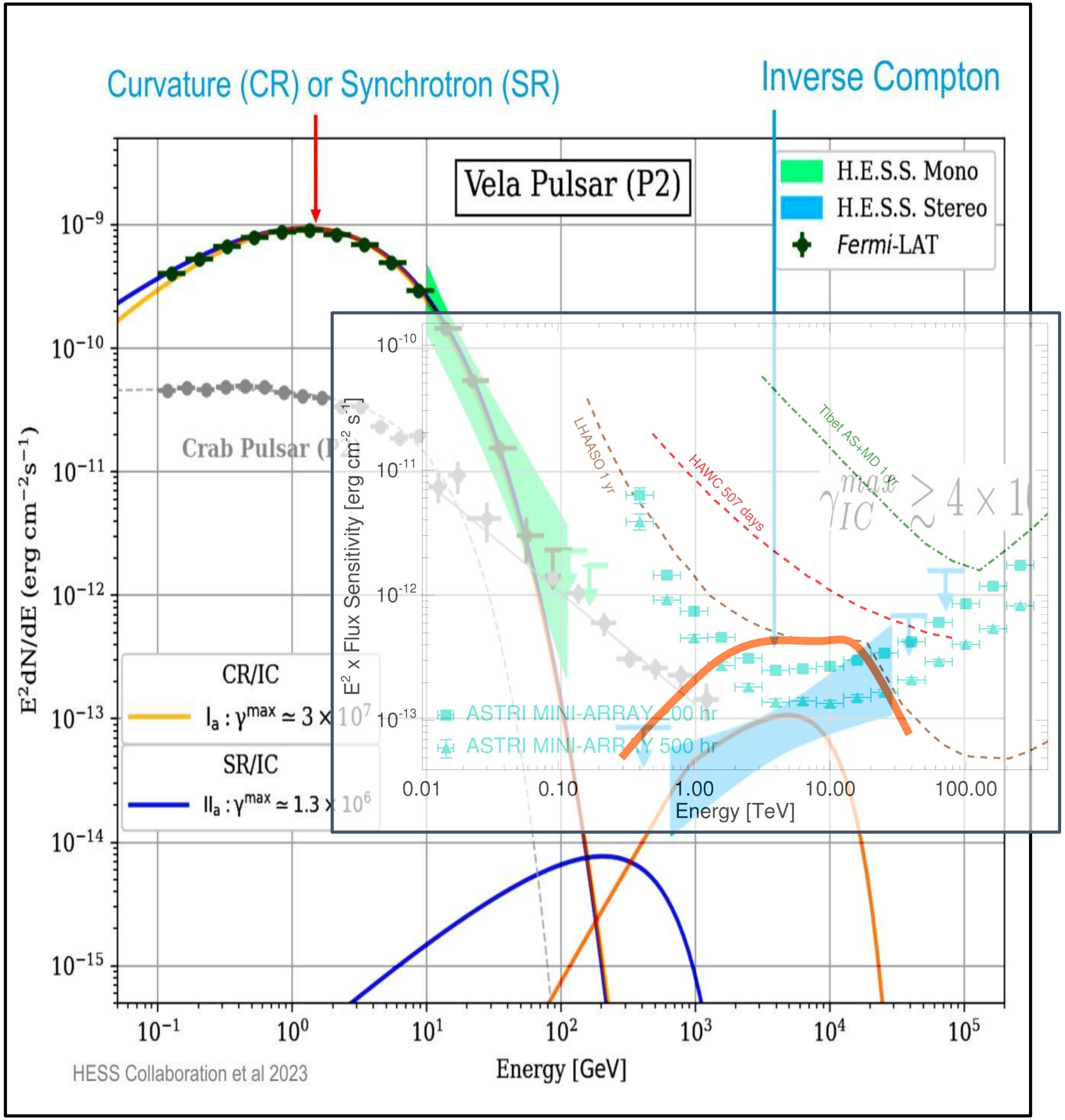




“Maybe by the time this is detected we have warp-drive technology to probe it directly…”

Reviewer Two





"Did reviewer two actually account for the background reduction due to phase selection?"

Author's Comment

PART III
PERSPECTIVES

SUMMARY

- **Pulsars** with **Cherenkov** telescopes are **extreme objects**:
  - Extreme **“natural particle accelerators”**
  - **Extreme feat** to detect them
- We have seen three objects:
  - **Crab pulsar**: young, energetic, emission **keeps on** up to **1 TeV**
  - **Vela pulsar**: - , **additional component** up to **20 TeV**
  - **Geminga**: middle-age, weaker, **seamlessly keeps up** to **100 GeV**
- Evolution of the **theoretical framework**

PERSPECTIVES

- Twenty years ago: only a **handful of gamma-ray pulsars**…
  - … **Fermi-LAT** arrives to town in **2008**…
  - More than **300 objects** now detected by **Fermi-LAT**.
  - **Particle acceleration** from the **neutron star magnetosphere** and **wind**, cutoff at **few GeV**.
  - Greatly advanced our **astrophysics understanding** of these exotic objects, but also enables **fundamental Physics** searches and links to **gravitational wave** (PTA)

PERSPECTIVES

- Now: only a **handful of extreme pulsars** at energies above **10 GeV** and **one** at the **10 TeV** scale**:**
  - … **CTAO** and **ASTRI** are coming to town…
  - **Inverse-Compton** components at the very-high energies? **Universal features** or target dependent?
  - **Multi-wavelength tomography** of a pulsar magnetosphere?
  - Improved fundamental Physics limits: **dark matter**, **exotic interactions**
  - **Evolution** of pulsars in time? **Population** study?

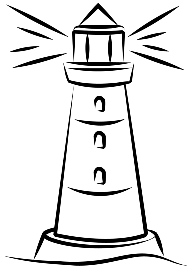
WHAT?!?
YOU WANT MORE??

PART I
PULSAR PHYSICS

PULSAR FACTS

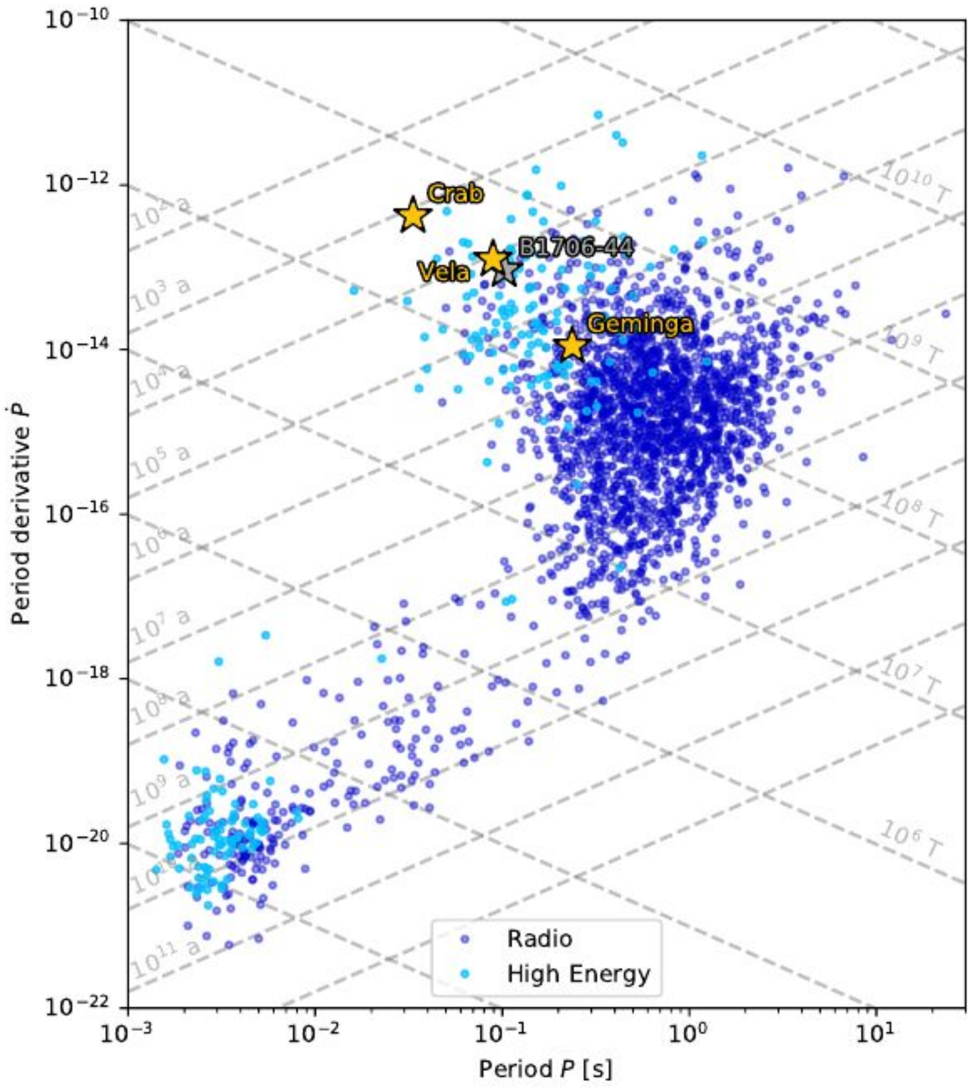
- More than **3000** known pulsars (300 gamma-ray ones).
- **Repeating, periodical signal** ("pulsating star" → pulsar). Due to the **rotation** of the neutron star.
- **Periods** between **1ms** and several seconds.
- Most of these are observed as **radio emitters** (radio-beacons).
- Several others emit at higher energies: optical, **X-rays**.
- By observing the emission: **measuring** of the **rotation period**.

Crab pulsar, Cambridge University, via Wikipedia



PULSAR P-PDOT DIAGRAM

- Classified with a diagram similar to the HR one for stars.
- Plotting the **period** of the pulsar versus the **period derivative**.
- Pulsar start fast (P<<1s) and **slow down** as they age.
- The **spin-down rate** depends on their **magnetic field**, which **decays** with time.
- **Different classes** can be identified in the diagram.



PULSAR MAGNETOSPHERE

Given the charge distribution in the interior of the neutron star, one can calculate the **electric field at the surface of the star** if it was in a vacuum. At its pole, its magnitude would be $E = \Omega R B$:

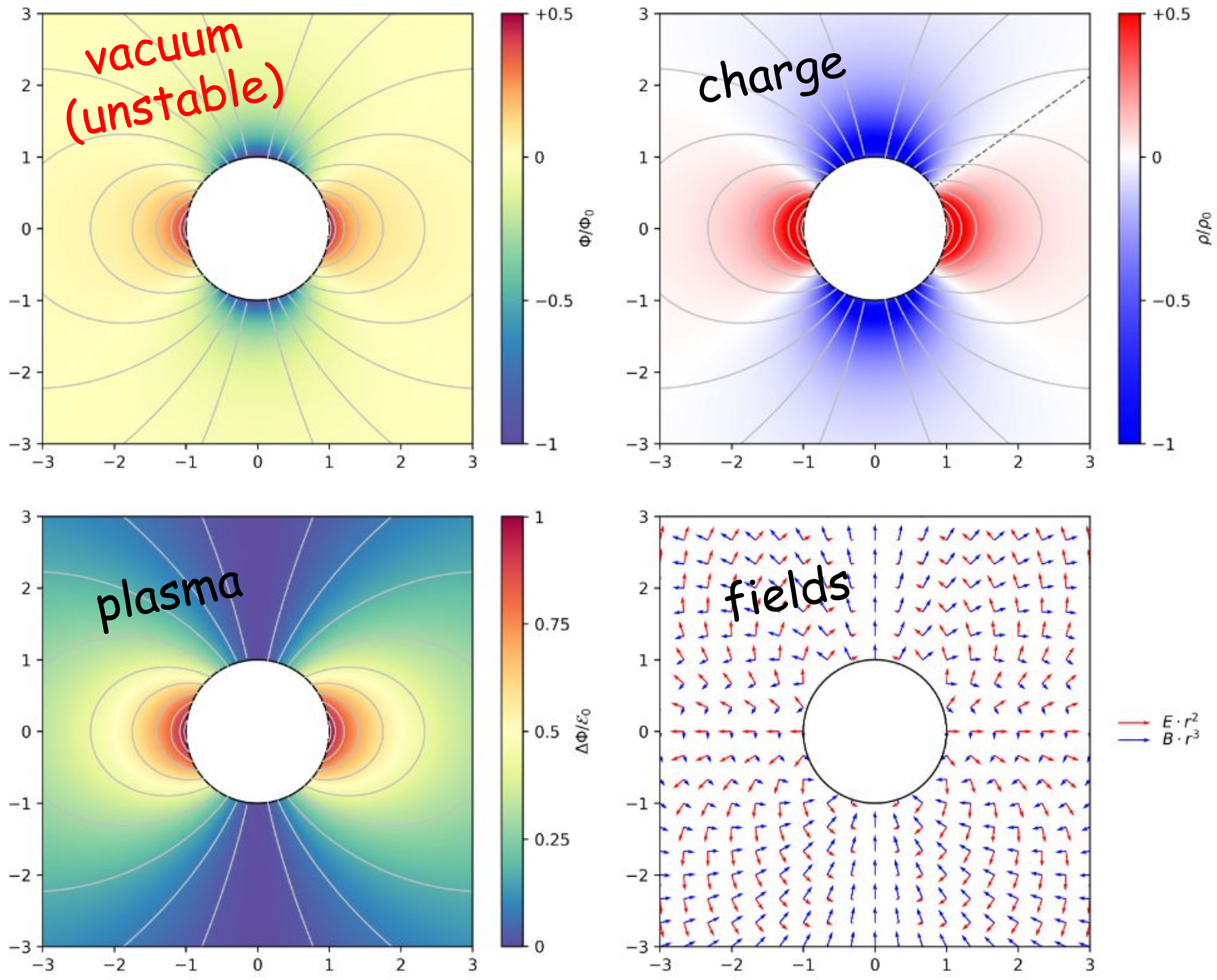
$$\mathrm{B} \sim 10^8\ \mathrm{T}, \quad \mathrm{R} \sim 10^4\ \mathrm{m}, \quad \Omega \sim 10\ \mathrm{rad/s}, \quad \rightarrow \quad \mathbf{E \sim 10^{13}\ V/m}$$

Let us compare the electric acceleration on an electron to the gravitational one:

$$a_{el} = \frac{|qE|}{m_e} \sim 10^{26}\ \mathrm{m/s^2} \qquad\qquad a_{gr} = G\frac{M}{R^2} \sim 10^{11}\ \mathrm{m/s^2}$$

The **electrostatic force dwarfs the gravity** of the neutron star! **Charged particles are ripped off the surface of the star!**

PULSAR MAGNETOSPHERE



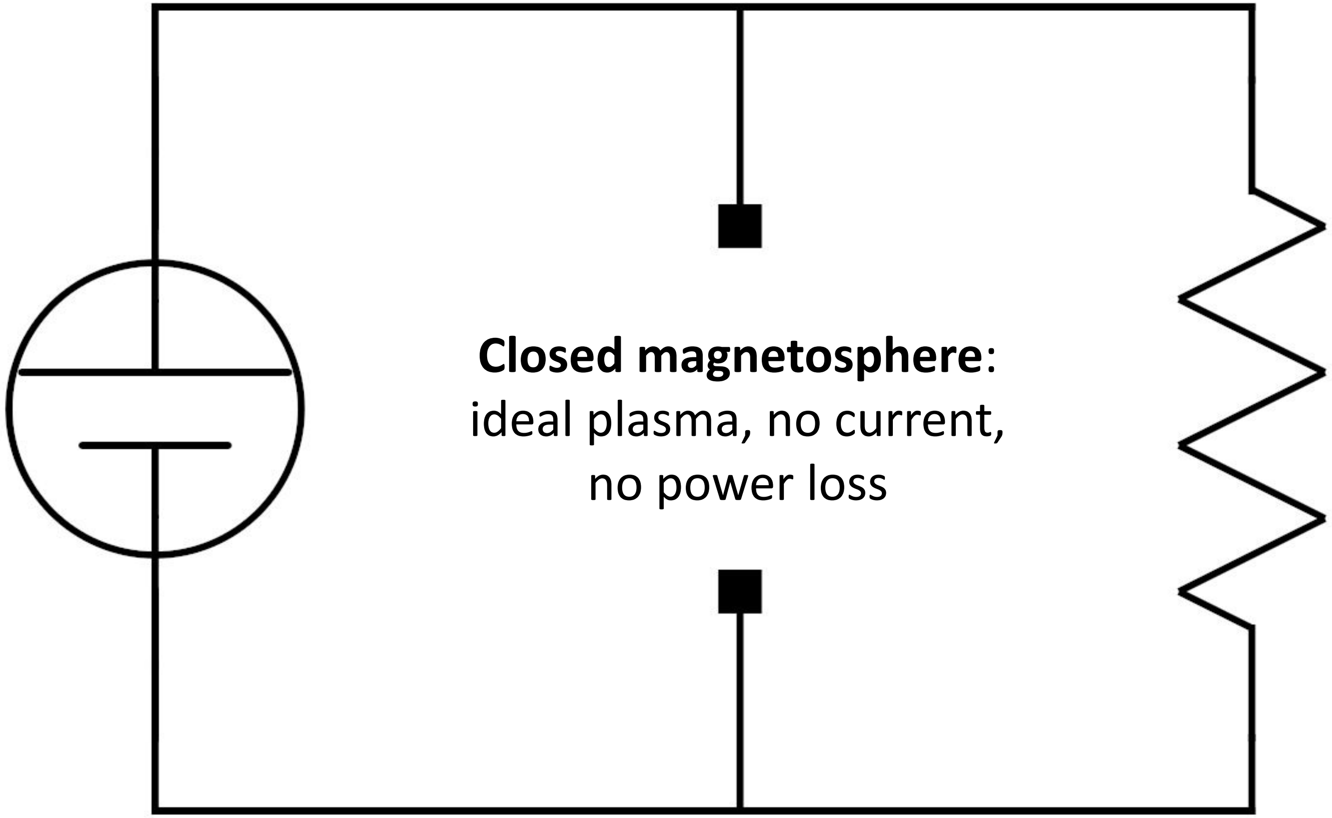
PULSAR MAGNETOSPHERE

- The **charges extracted from the surface** of the neutron star quickly become **ultra-relativistic**. They are **forced to spiral around and along the curved magnetic field lines** and, as a result, **radiate gamma-rays**. In the humongous magnetic field of the neutron star, these gamma-rays are **immediately re-absorbed** by the production of **electron-positron pairs**:

$$\boldsymbol{\gamma \rightarrow e^{+} e^{-}}$$

This **single-photon pair-production** process is **forbidden** in normal conditions (conservation of momentum requires that there are at least two photons), and is only possible in the **strong magnetic field** of the neutron star.

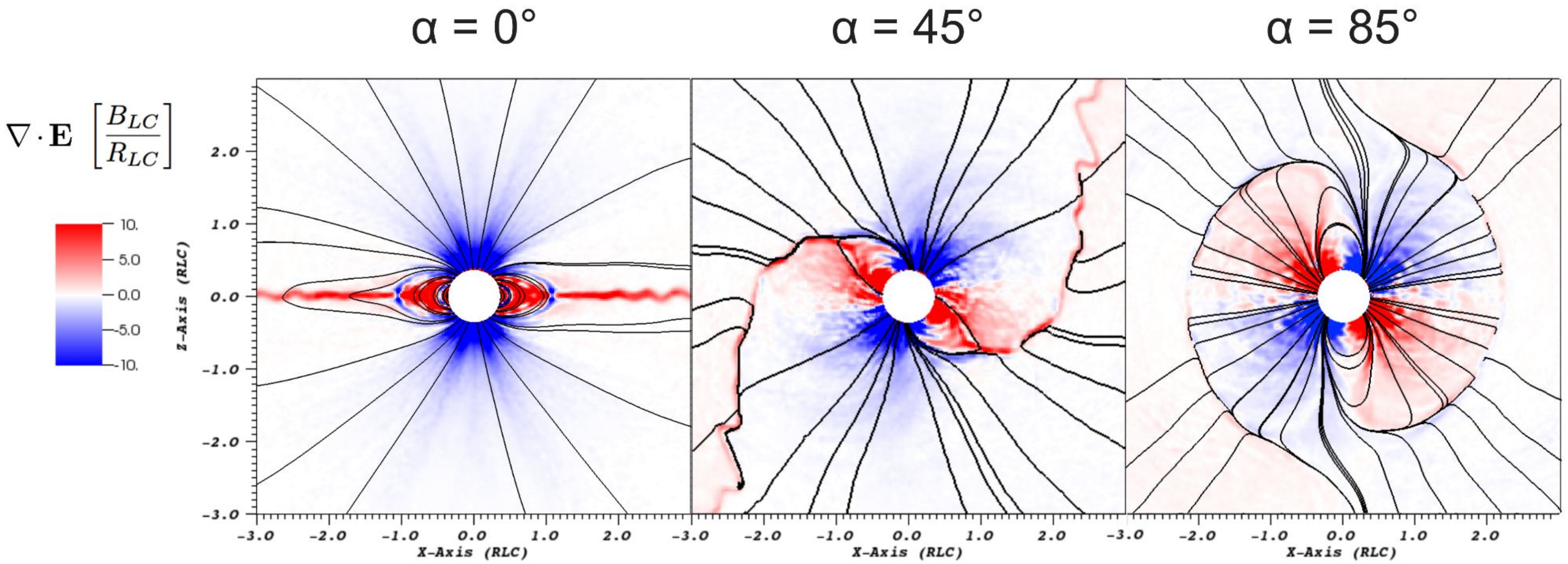
PULSAR MAGNETOSPHERE



If the pulsar's magnetic field is **tilted** with respect to the rotation axis, the **magnetic dipole radiation** will be **instantly absorbed by the plasm**a surrounding the star, and its energy passed on to the **pulsar wind**.

MAGNETOSPHERE CHARGES

Brambilla et al. 2018



GAMMA-RAY EMISSION MECHANISMS

Synchrotron

- Motion **around** the magnetic field lines.
- $E_c = \frac{3}{2}\hbar\frac{eB}{m_e}\gamma^2$

Curvature

- Motion **along** the magnetic field lines.
- $E_c = \frac{3}{2}\hbar\frac{c}{r_c}\gamma^3$ (r_c=curvature radius)

Electron spectrum $N_e(\gamma) = \gamma^{-\alpha}$:

Synchrotron:

$$N(E) = E^{-(\alpha+1)/2}$$
$$EN(E) = E^{-(\alpha-1)/2}$$

Curvature:

$$N(E) = E^{-(\alpha+2)/3}$$
$$EN(E) = E^{-(\alpha-1)/3}$$

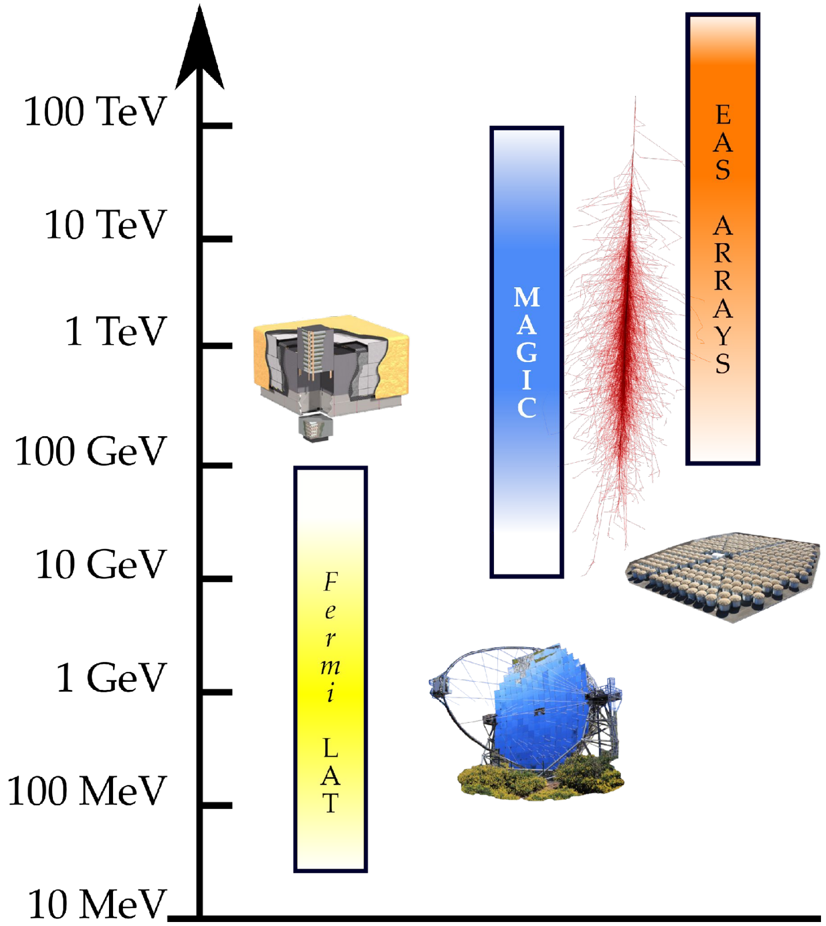
Inverse Compton: $N(E) = E^{-(\alpha+1)}, EN(E) = E^{-\alpha}$

Klein-Nishina!!

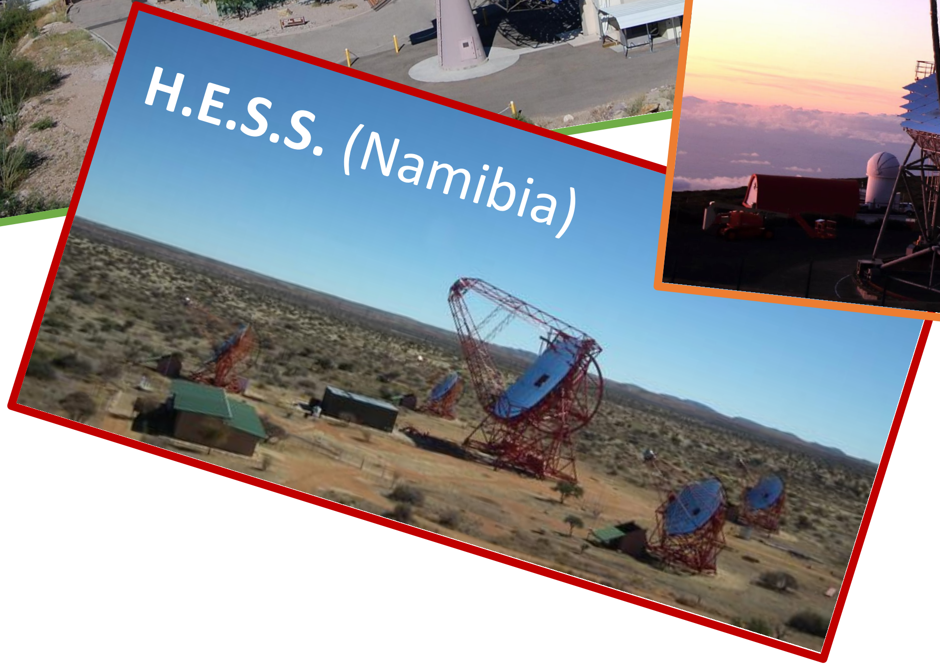
PART II
STATE OF THE ART

GAMMA-RAY ASTRONOMY

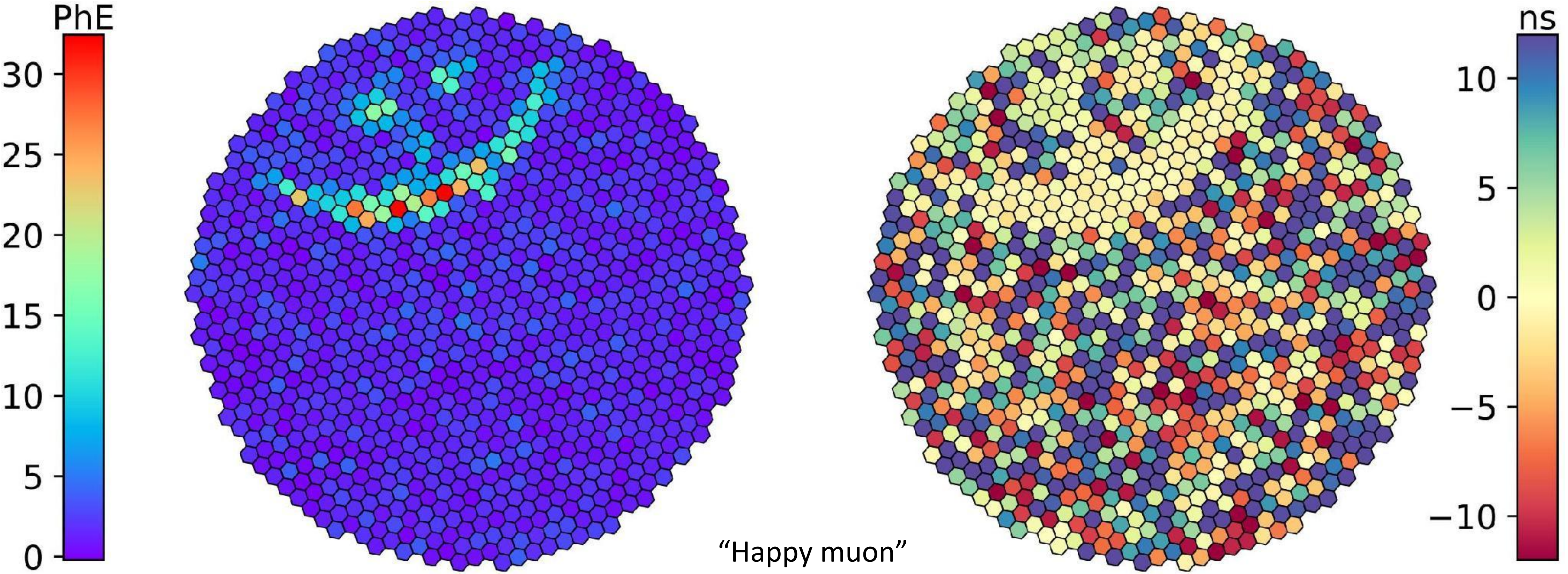
- ***Fermi*-LAT** (pair conversion) detection up to **tens of GeV**
- **Extensive air showers** at **VHE**:
  - Secondary particles
  - Cherenkov radiation
- **Sensitivity gap**: at **10 – 100 GeV**
- Hosts **interesting astrophysical phenomena**:
  - VHE **Pulsars**
  - Distant Active Galactic Nuclei
  - Gamma-Ray Bursts



IMAGING ATMOSPHERIC CHERENKOV TELESCOPES



IMAGING ATMOSPHERIC CHERENKOV TELESCOPES



“Happy muon”
Credit: G. Ceribella

CTAO: CHERENKOV TELESCOPE ARRAY

Next generation Cherenkov telescope observatory. **Multiple telescope** sizes to cover energies between **10 GeV** and **>100 TeV**. Open **observatory policy** surpassing the capabilities of **all previous** and existing instruments.

CTAO: CHERENKOV TELESCOPE ARRAY

LST-1 telescope (prototype) already **operating**/commissioning at the ORM.
LST-2,3,4 in construction/assembly. Expected **LST-4** first light: **end 2025**.

VHE PULSAR RESULTS

2008 Discovery of **pulsed VHE emission** of the Crab (>25GeV)

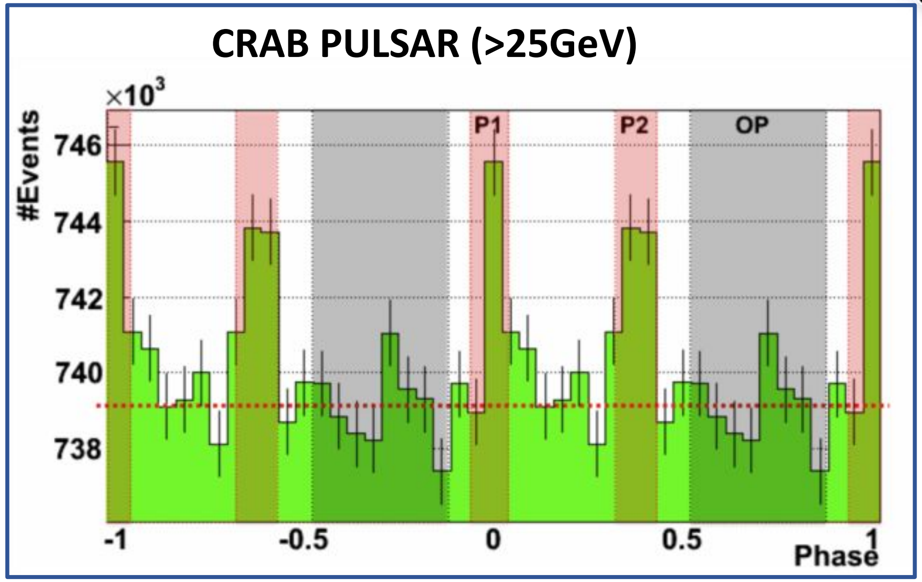
- Excludes polar-cap model

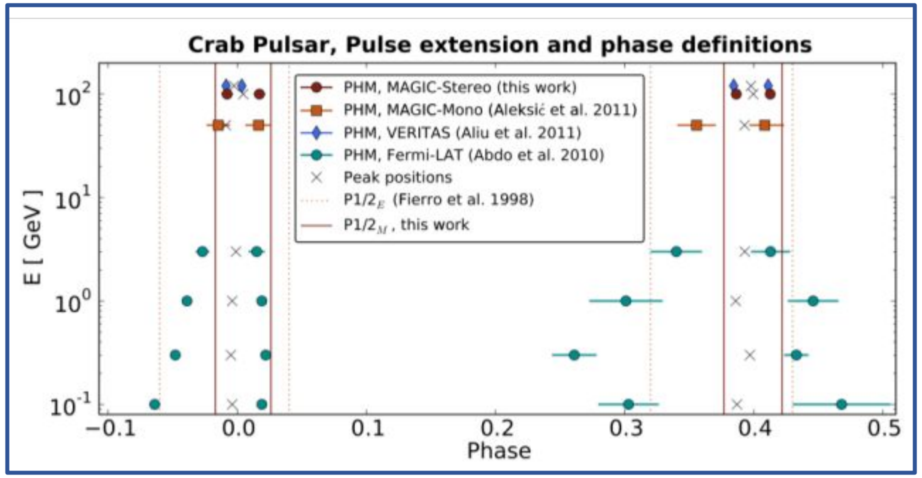
2009 **Fermi-LAT** kicks in

2011 **VERITAS**: Crab pulsar above **100 GeV**

2012 **Phase-resolved** spectrum up to **400GeV**

- Peak width decreases with energy





VHE PULSAR RESULTS

2014 Crab **bridge emission** above 50GeV

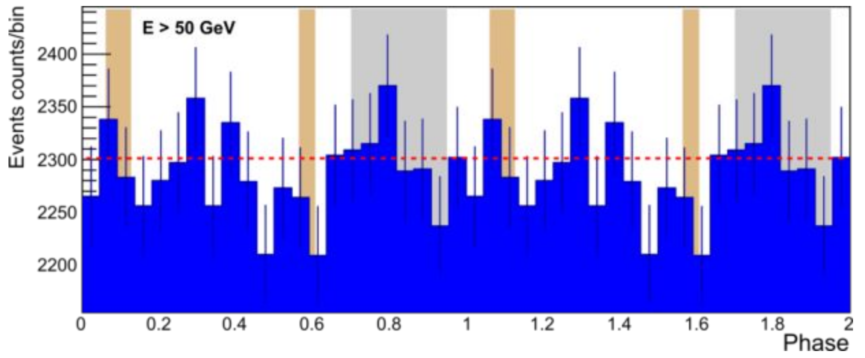
2016 Geminga searches with standard trigger above 50GeV

2016 **TeV pulsed** emission from the Crab!

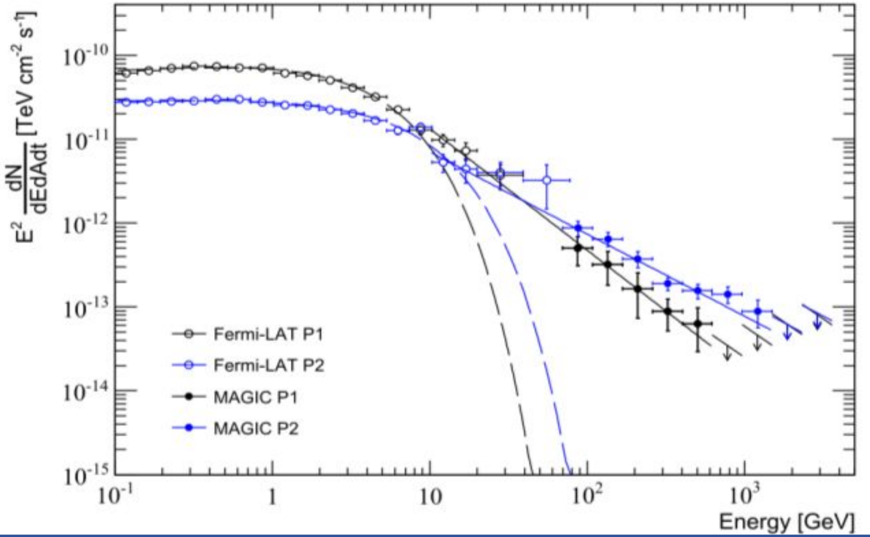
- Inverse Compton component

2017 Lorentz Invariance Violation studies with the Crab pulsar

GEMINGA PHASEOGRAM (>50GeV)



CRAB PULSAR SPECTRUM



CRAB PULSAR AND NEBULA

Young supernova remnant: **972 years**

Powerful pulsar: $4{\cdot}10^{31}$ W, or $10^5 \cdot$ Sun
Strong surface **magnetic field**: $4 \cdot 10^8$ T

Vast **Pulsar Wind Nebula**:

- Used as **standard candle** in high-energy astrophysics
- Sporadically **flares** (day timescale) in the **100 MeV – 1 GeV** band.

Pulsar detected by MAGIC above **1 TeV**:

- **Inverse-Compton** component

X-ray: NASA/CXC/SAO; Optical: NASA/STScI; Infrared: NASA-JPL-Caltech

GEMINGA PULSAR

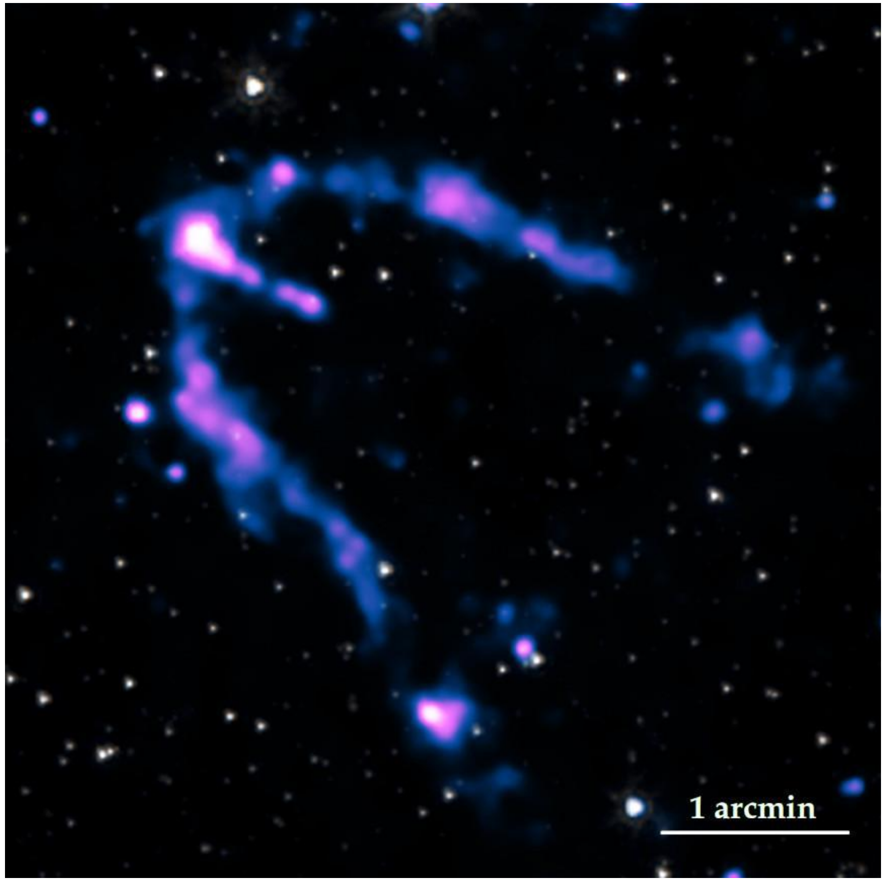
Middle aged pulsar: **$3 \cdot 10^5$ years**

Less powerful than Crab: 10 · Sun
Very close by: **250 pc** = **800 ly**

Unusual pulsar: **radio quiet,** second **brightest steady source** in the GeV sky

Embedded in a **vast TeV halo:**

- **Accelerated lepton** escape & diffusion
- **Contributor** to **positron spectrum** at the **Earth**



X-ray: NASA/CXC/PSU/B. Posselt et al; Infrared: NASA/JPL-Caltech

GEMINGA PULSAR

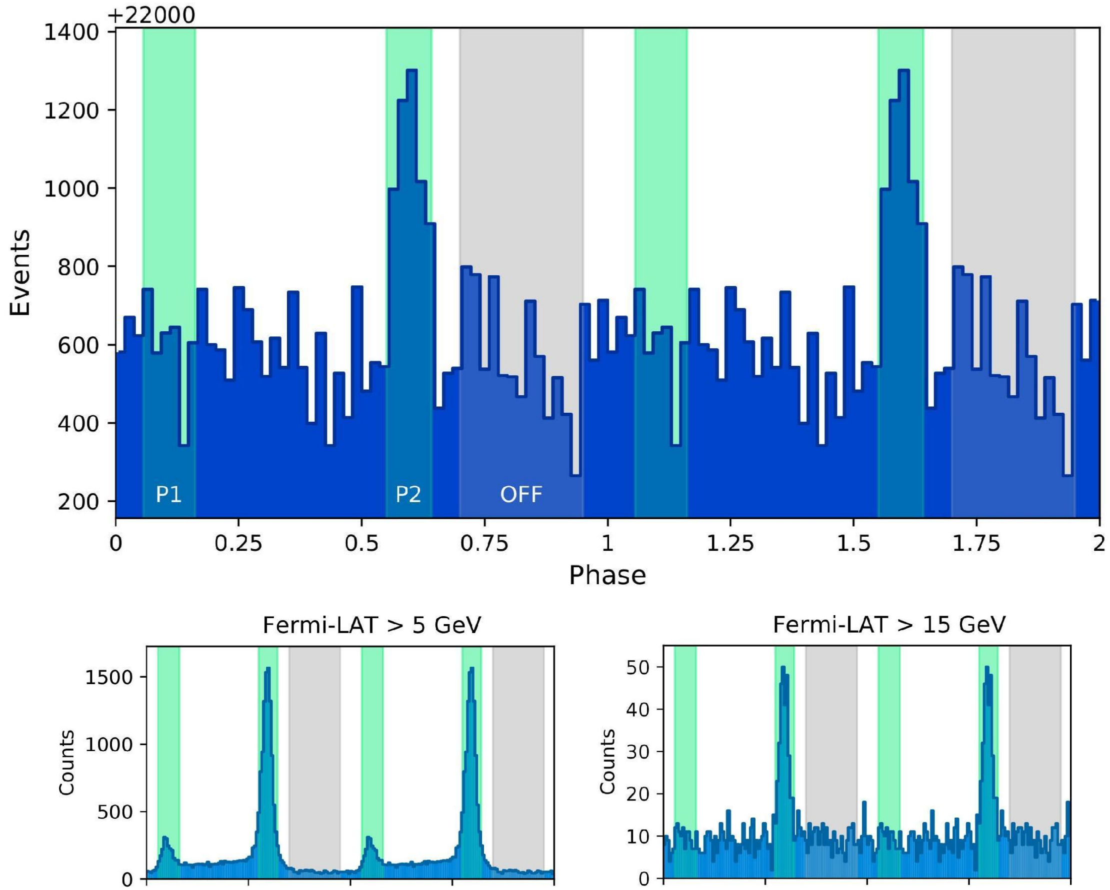
First ground-based detection of Geminga, 15 GeV – 75 GeV

Third very-high-energy pulsar, first **middle-aged** one

Detection significance: **6.3σ**

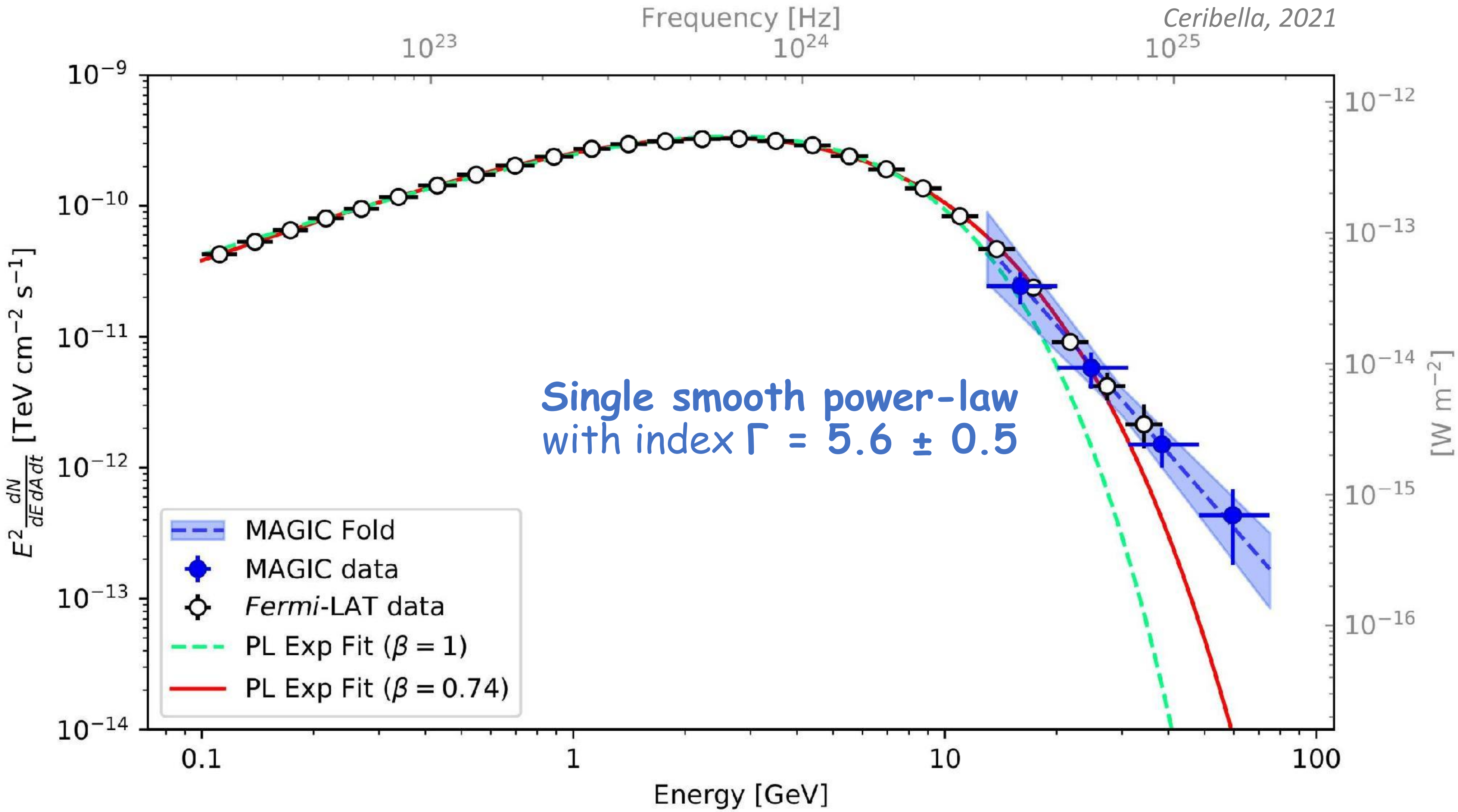
Reference article:

MAGIC Collaboration et al. (2020), *Detection of the Geminga pulsar with MAGIC hints at a power-law tail beyond 15 GeV*
10.1051/0004-6361/202039131



GEMINGA: MAGIC SPECTRUM

Ceribella, 2021



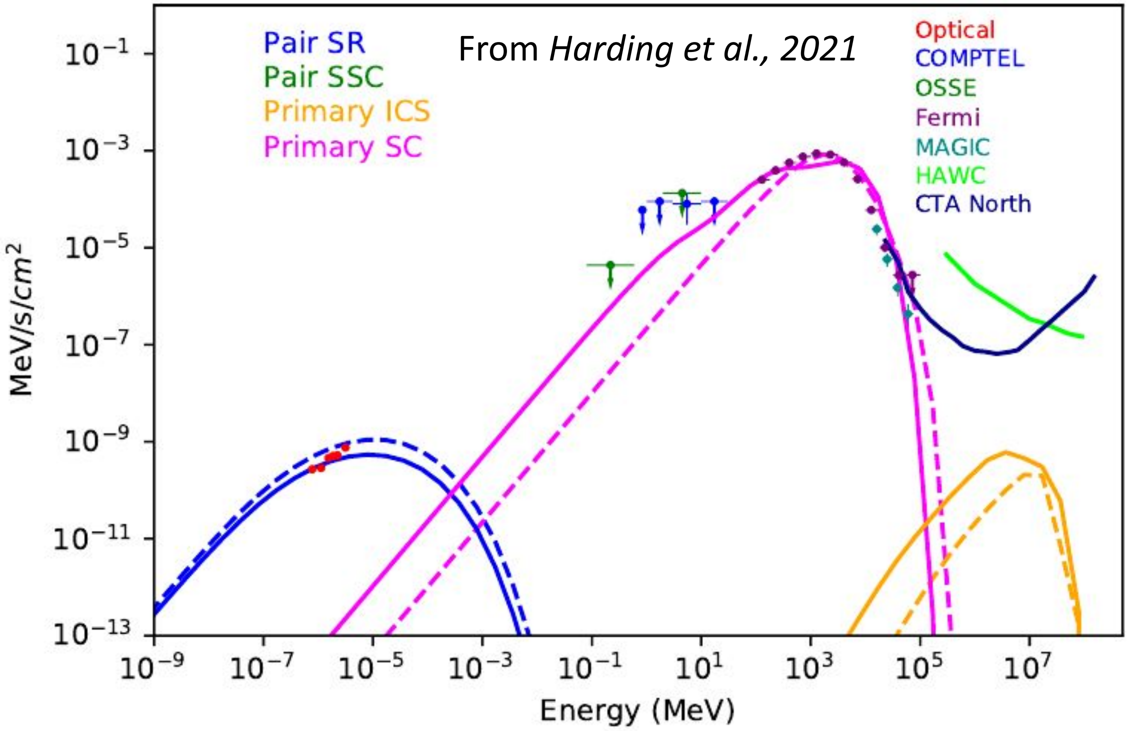
GEMINGA: CURRENT SHEET

Novel class of models supported by extensive **numerical simulations**.

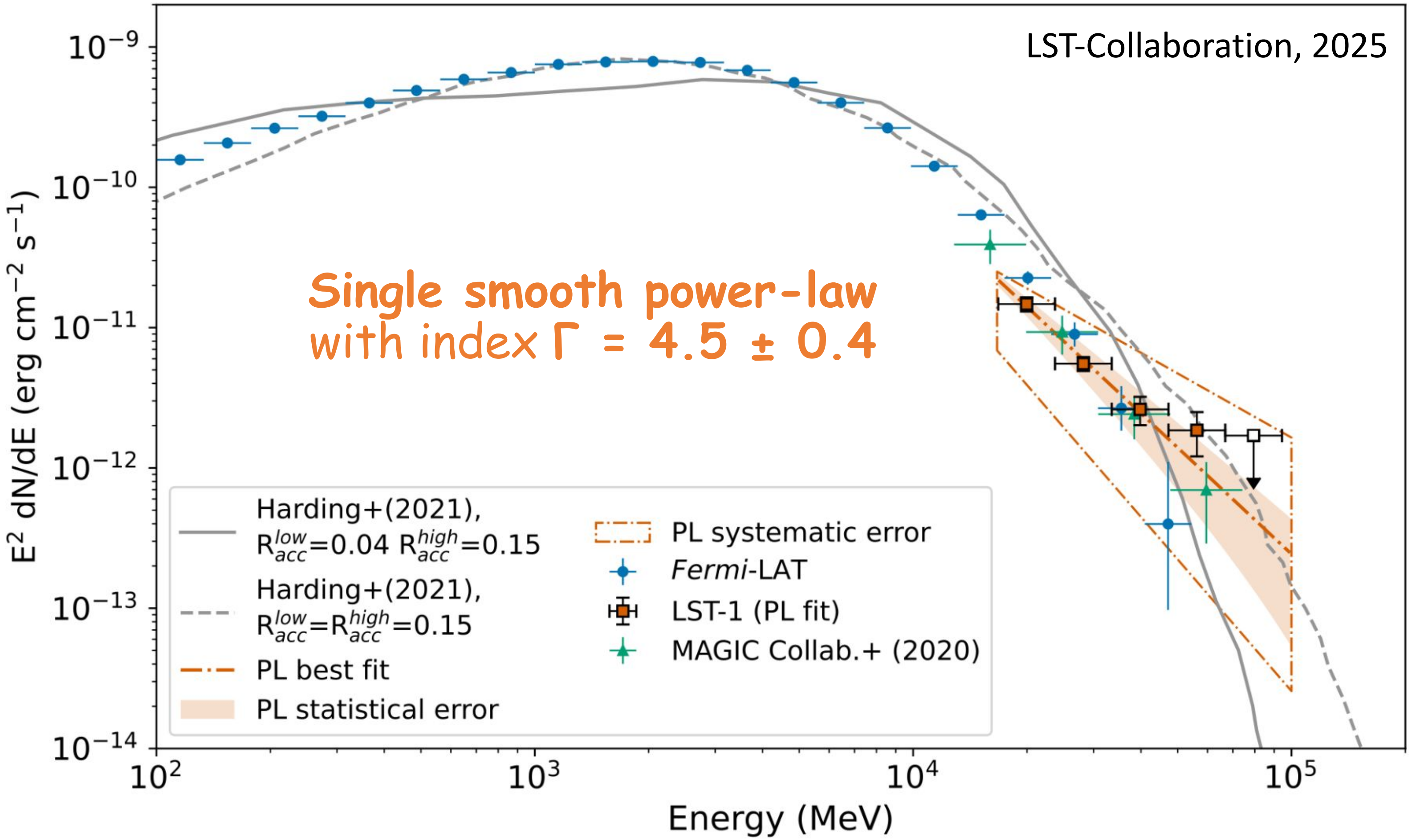
Acceleration just **beyond the light-cylinder**, at the Y point.

Geminga emission explained as primary **synchro-curvature**, with **no inverse-Compton** component.

Depending on assumptions on the (unobservable) **radio flux**, based on optical emission.



GEMINGA: LST-1 SPECTRUM



Power-law extension measured up to 60 GeV: **8σ in less time**!

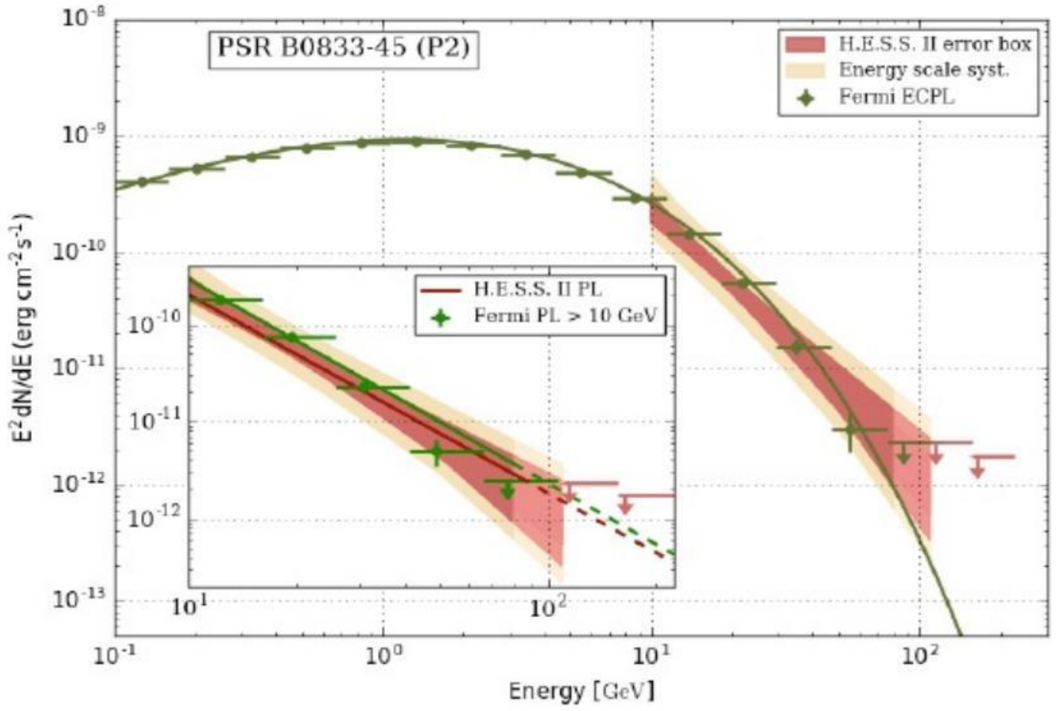
Confirms independently the MAGIC findings, but **prefers a harder spectrum**

Current sheet model (Harding 2021) consistent within systematics

Curvature radiation? Extra Inverse-Compton contribution?

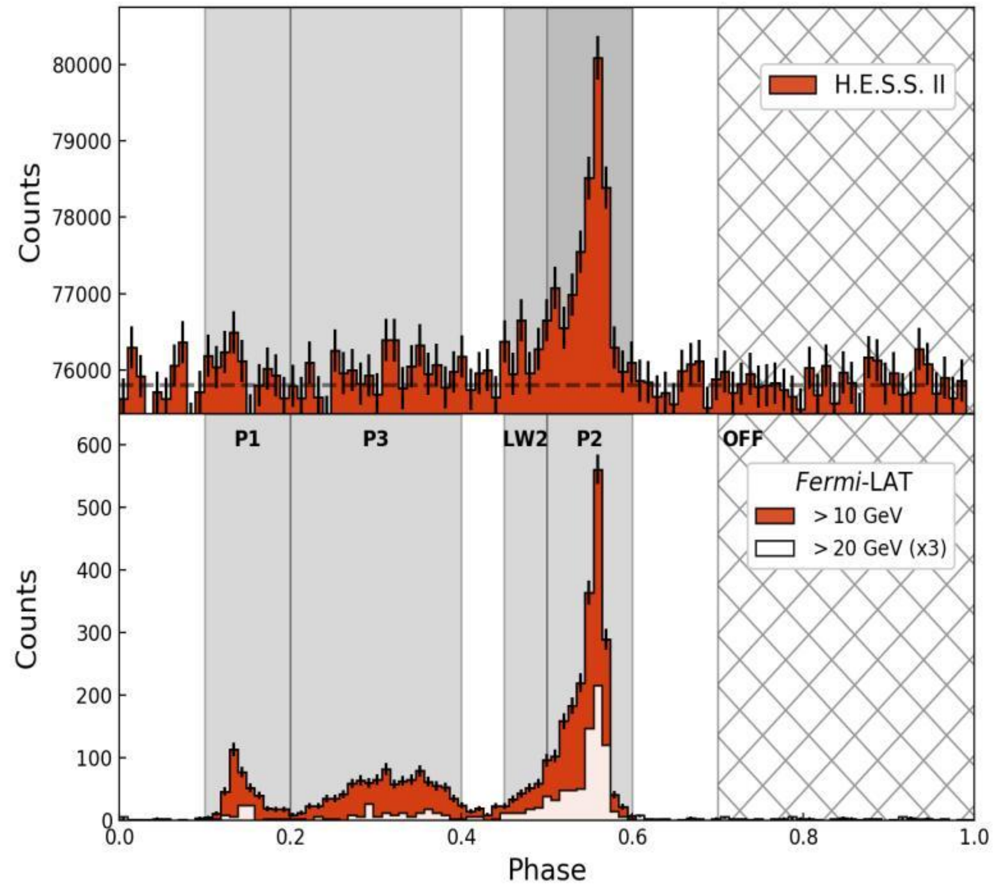
The Vela pulsar

Vela in the sub-100 GeV regime



Vela Pulsar, HESS Collaboration 2018

Peak evolves with energy



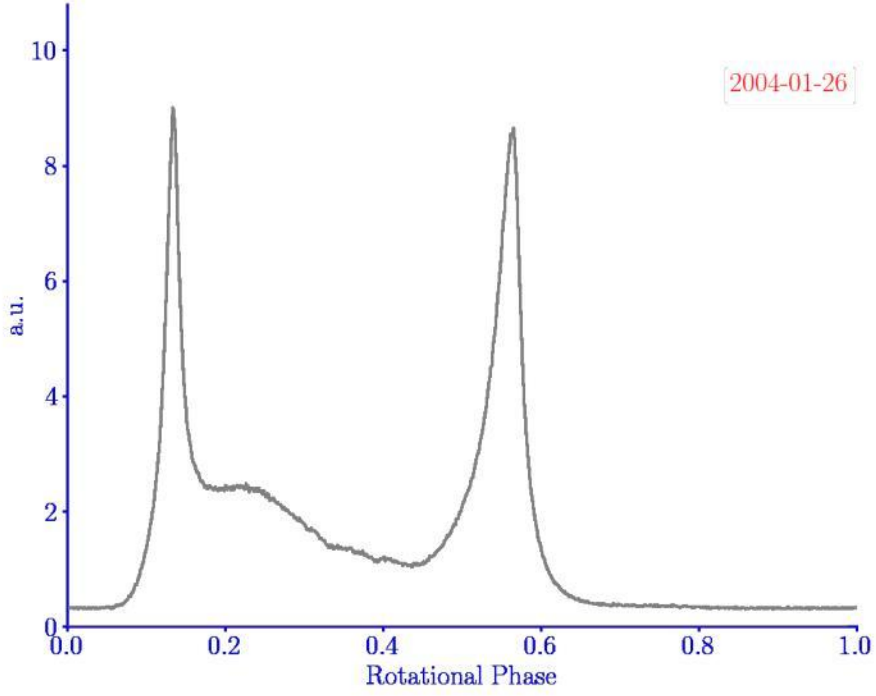
Detected in 40.3 h with HESS CT5

Light curve with 3 peaks, with P2 becoming prominent at high energies:

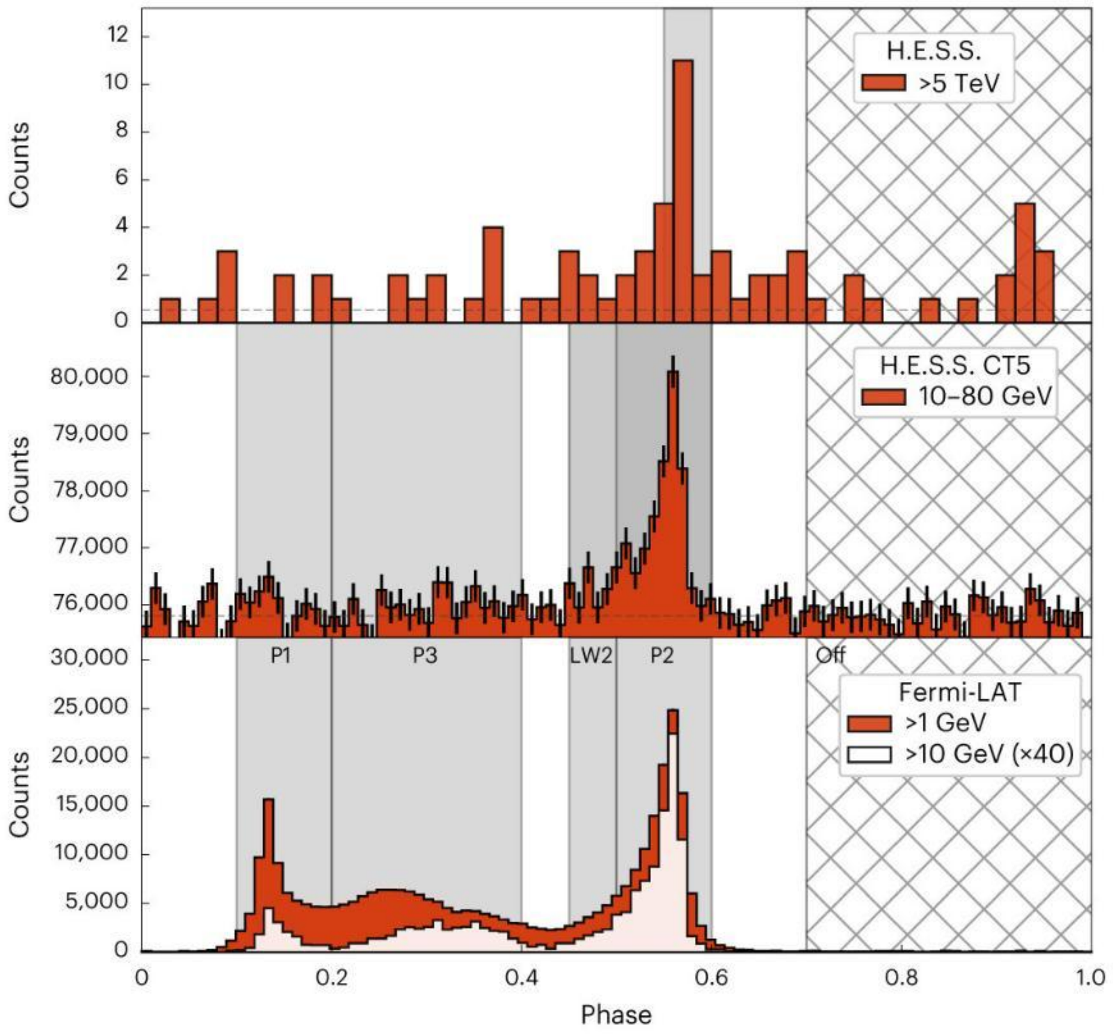
The Vela Pulsar in the TeV regime

Detection of Vela above 1 TeV

- **Data from 2004-2016 observations (80 h)**
- Phase-folding : ephemeris from LAT+ Parkes

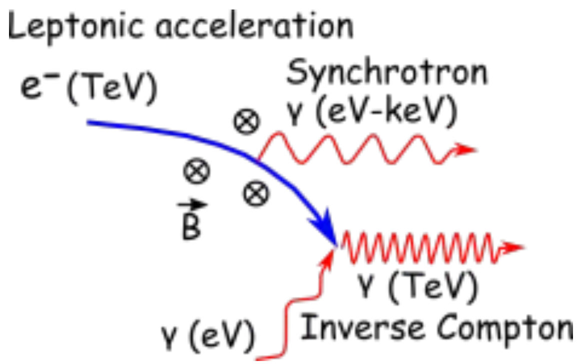


HESS Collaboration et al 2023



GAMMA-RAY EMISSION MECHANISMS

Once a **population of accelerated, ultra-relativistic particles** is produced, these can radiate **electromagnetic waves**.

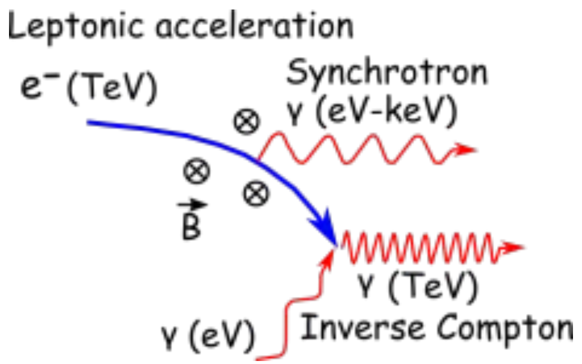


- **Synchrotron radiation**: produced by the **gyro-motion** of the charges around the magnetic field. If an ultra-relativistic electron has an energy $E = \gamma m_e c^2$, with $\gamma \gg 1$, its synchrotron emission will have a **peak followed by a sharp cut-off,** at:

$$E_c = \frac{3}{2}\hbar\frac{eB}{m_e}\gamma^2$$

GAMMA-RAY EMISSION MECHANISMS

Once a **population of accelerated, ultra-relativistic particles** is produced, these can radiate **electromagnetic waves**.

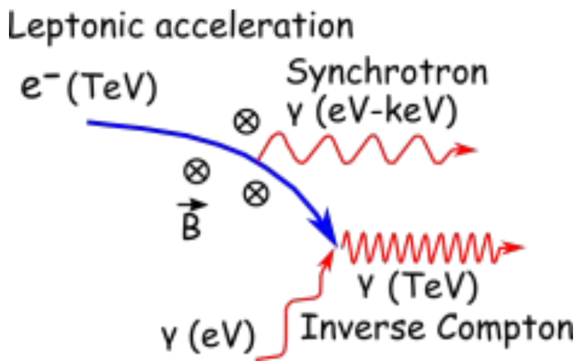


Curvature radiation: produced by the motion of the charges forced to curve **along the curved magnetic field** (this is often required by the ideal plasma magnetohydrodynamics). It is very similar to synchrotron, and often treated together as **synchro-curvature** radiation, but it has a stronger dependency on the Lorentz-factor of the electrons:

$$E_c = \frac{3}{2}\hbar\frac{c}{\ }\gamma^{3}$$

GAMMA-RAY EMISSION MECHANISMS

Once a **population of accelerated, ultra-relativistic particles** is produced, these can radiate **electromagnetic waves**.



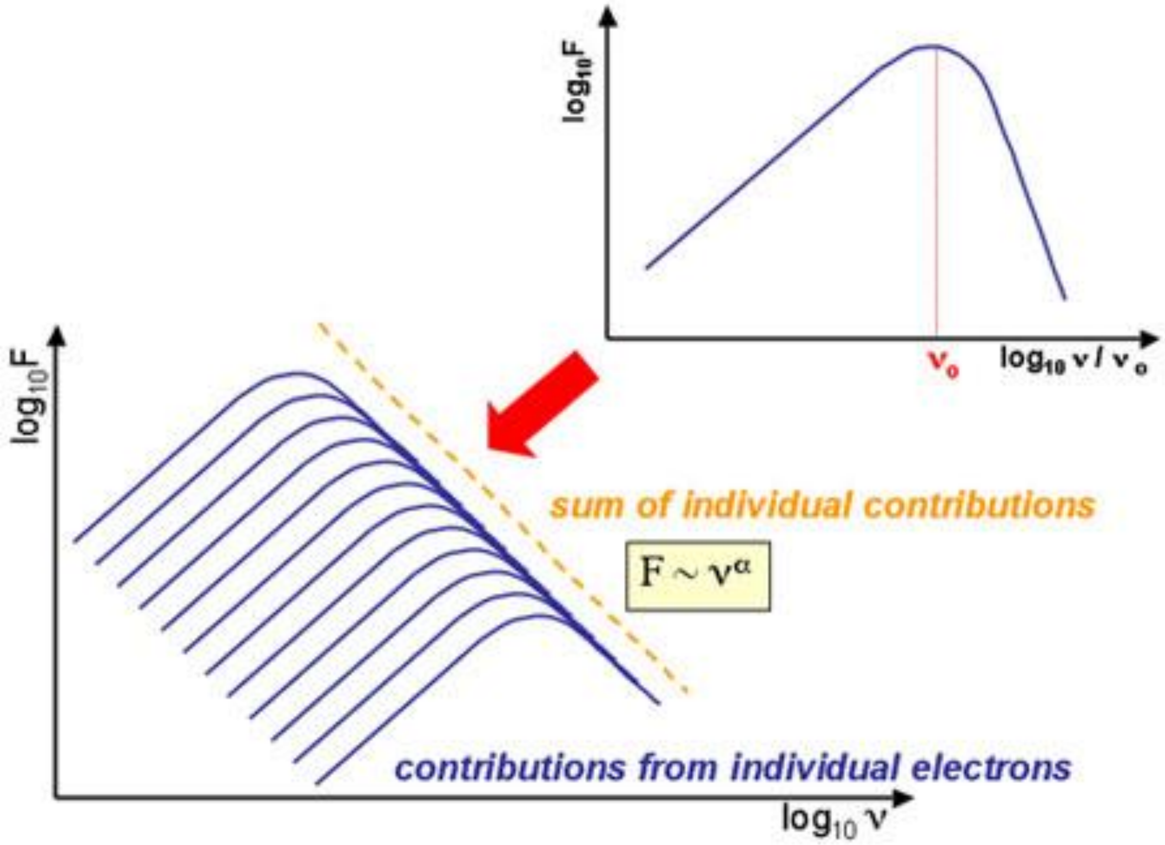
- Inverse-Compton scattering: produced when the **electrons "hit" a photon** with lower-energy, scattering it at a **much higher-energy one**. In the ultra-relativistic particle limit, when the energy of the electron E_e is much higher than the initial energy of the photon, the final photon energy E_γ is simply:

$$E_\gamma = \frac{1}{2} E_e = \frac{1}{2} \gamma m_e c^2$$

GAMMA-RAY EMISSION MECHANISMS

If the electrons themselves have a **distribution of energies**, the resulting **radiation spectrum** will be the composition of each contribution.

The **shape** of the **radiation spectrum** is **linked** to the shape of the **electron energy spectrum**. By measuring the radiation spectrum we can **infer the properties of the magnetospheric electrons**.



Credit: Swinburne University of Technology

EXTENSIVE AIR SHOWERS

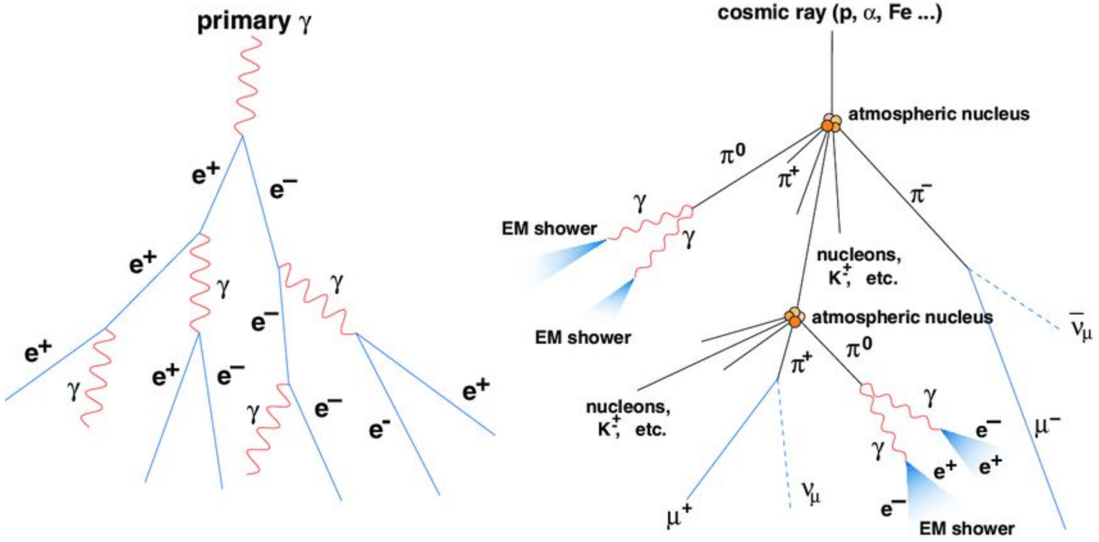
High-energy gamma rays travel to the Earth and enter the atmosphere.

They **interact** with the nuclei of air via electron-positron **pair production.**

Electrons emit **secondary** gamma rays via **Bremsstrahlung**, and the process repeats in cascade.

→ **Electromagnetic shower**

The cascade process stops when the **new generation** of electrons fall below an energy of **80 MeV** (ionization becomes dominant).



CHERENKOV EFFECT

The **limit speed** in nature is the **speed of light in vacuum**: c. However, in a continuous medium light travels at a speed v lower than c:

$$v_\gamma = \frac{c}{n}$$

where n is the **refractive index** of the medium. I stress again that the limit speed to ensure **causality** ("the speed of reality") is always c, also inside a material medium. It follows that it is **possible**, for a relativistic object, to **move faster than light in a medium** with a velocity v such that:

$$\frac{c}{n} < v < c$$

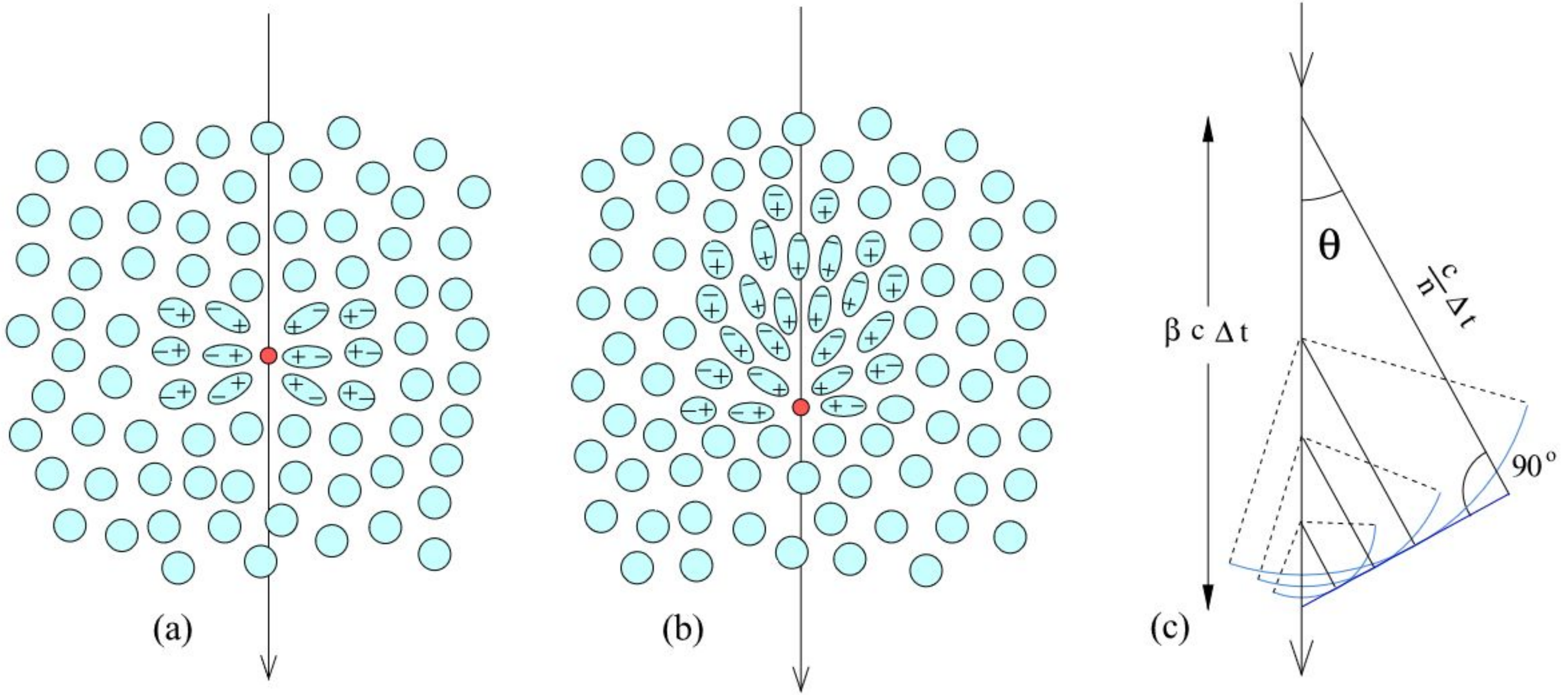
Please don't let Elon Musk know this!!

CHERENKOV EFFECT

In practice, any macroscopical object moving at a relativistic speed inside a material medium would immediately interact with it, release an enormous amount of energy, and disastrously explode together with the scientist attempting the experiment and the city hosting them (this means: **do not try this at home**).

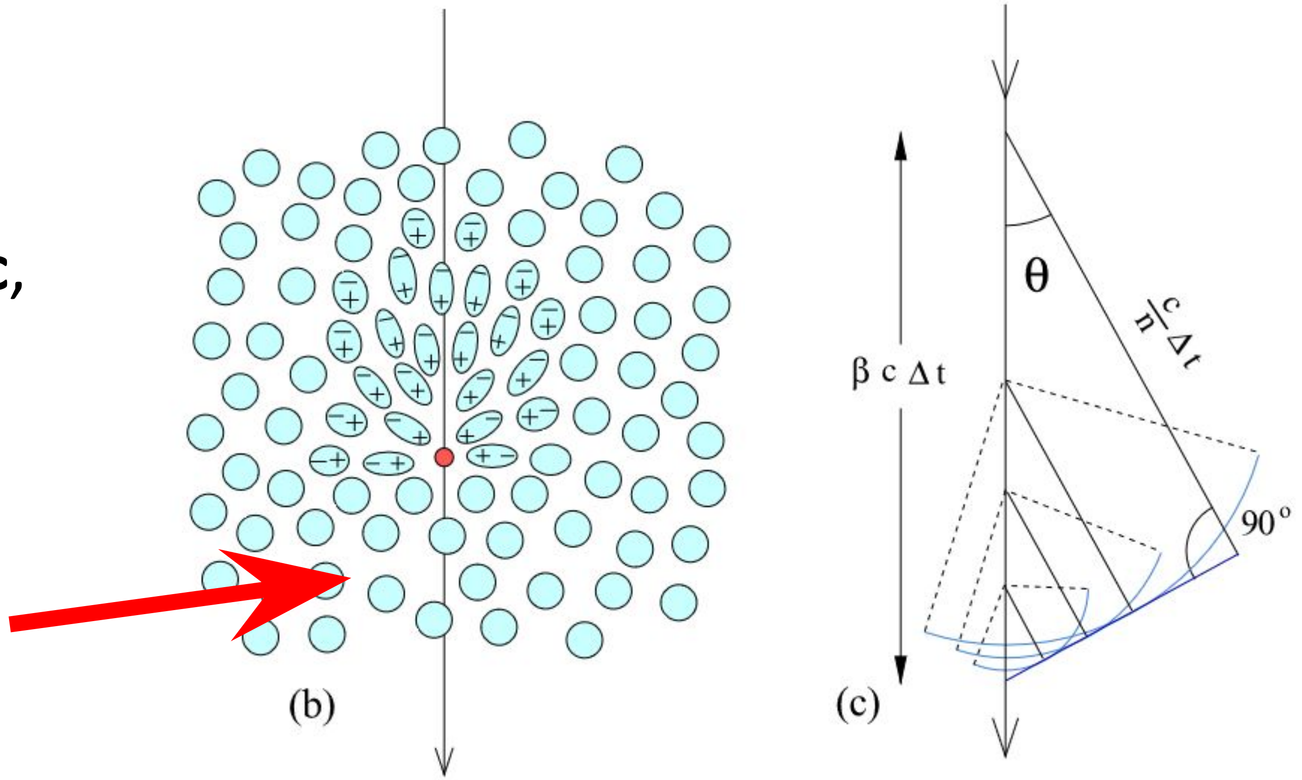
It is however possible (and safer) to **consider such scenario for particles** traversing a medium. In particular, **super-luminar motion of charged particles**, as charge is itself a source of electromagnetic energy, produces a “shock” of **instantaneous dipole depolarization** in the material it is travelling in, giving rise to the **emission of light**: **Cherenkov radiation**.

CHERENKOV EFFECT



CHERENKOV EFFECT

- Passing charges **polarize the medium** they are moving into.
- Normal **sub-luminar motion**: the polarization pattern is **symmetric**, and depolarization causes **no emission**.
- These particles are **completely unaware of the charge! Electromagnetic** interaction is mediated by **photons**, but the particle is **moving faster than light**!



CHERENKOV EFFECT

- The depolarization produces a **beamed, conical wave-front of light**, with an aperture angle ϑ given by:

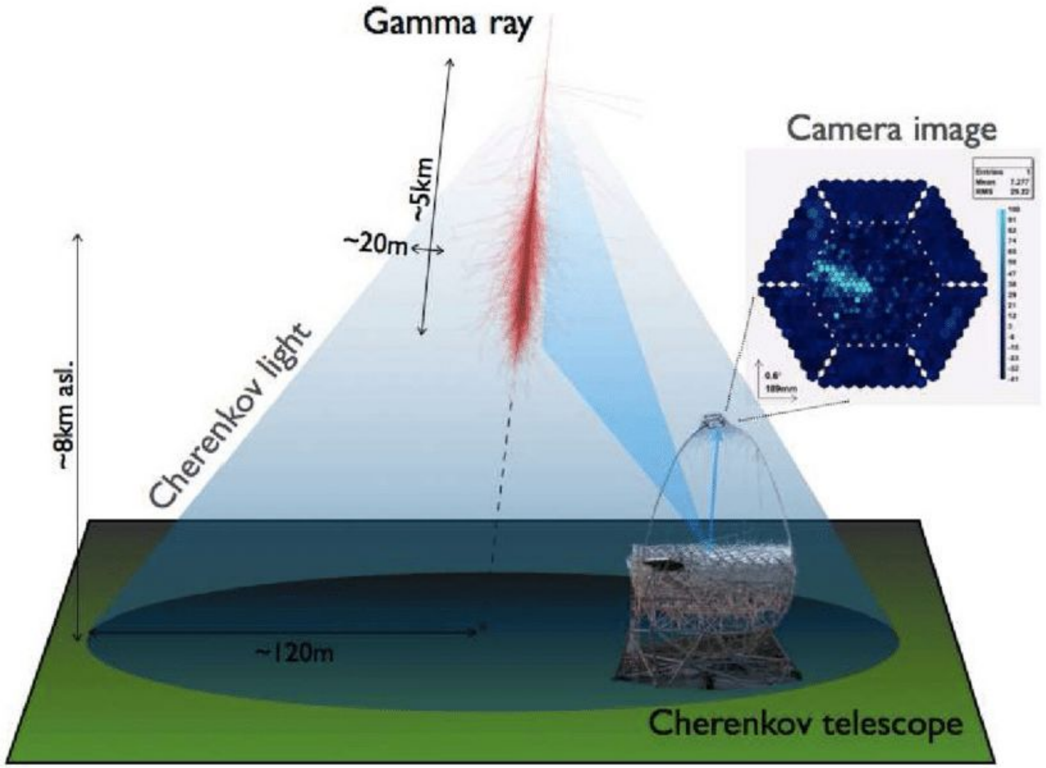
$$\cos\vartheta = \frac{1}{n\beta}$$

where n is the refraction index of the medium, and β is the speed of the particle divided by c. The aperture angle becomes zero when the particle speed equals the speed of light in the medium, that is:

$$\beta = \frac{1}{n}$$

For an electron in air, this corresponds to a threshold energy of around 20 MeV – 36 MeV. This means that **all electrons** generated **in an air shower emit Cherenkov radiation**.

IMAGING ATMOSPHERIC CHEREKOV TELESCOPES

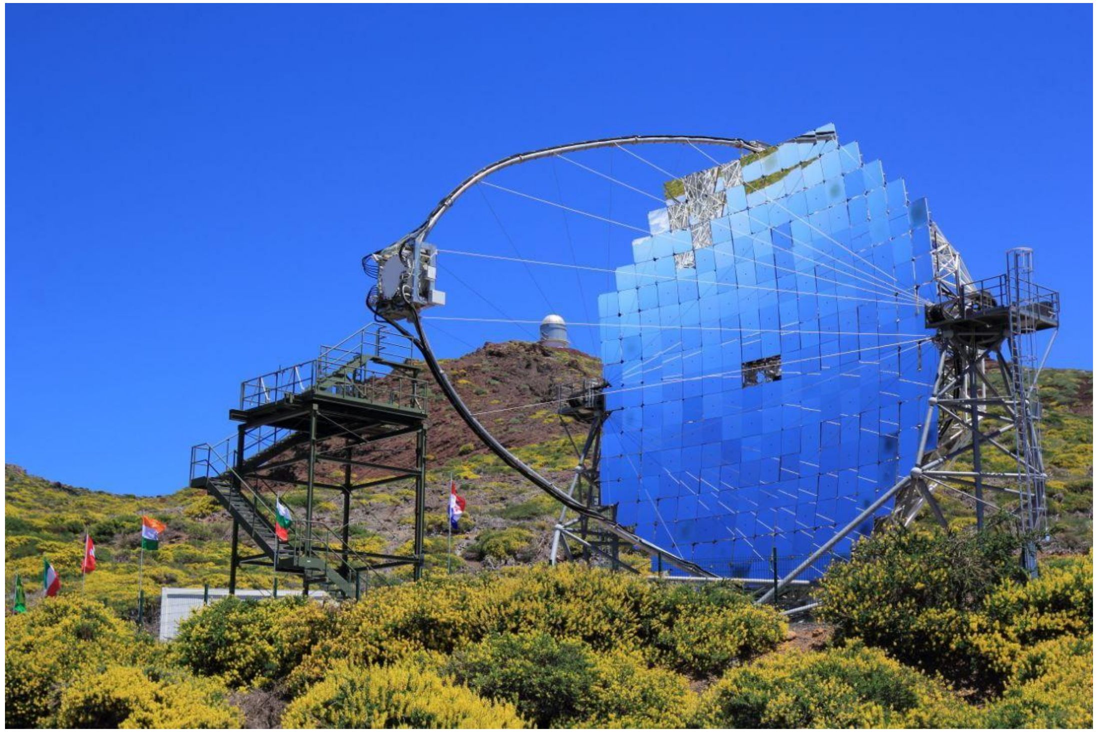


100 GeV → 10 ph./m^2 (approximately linear)

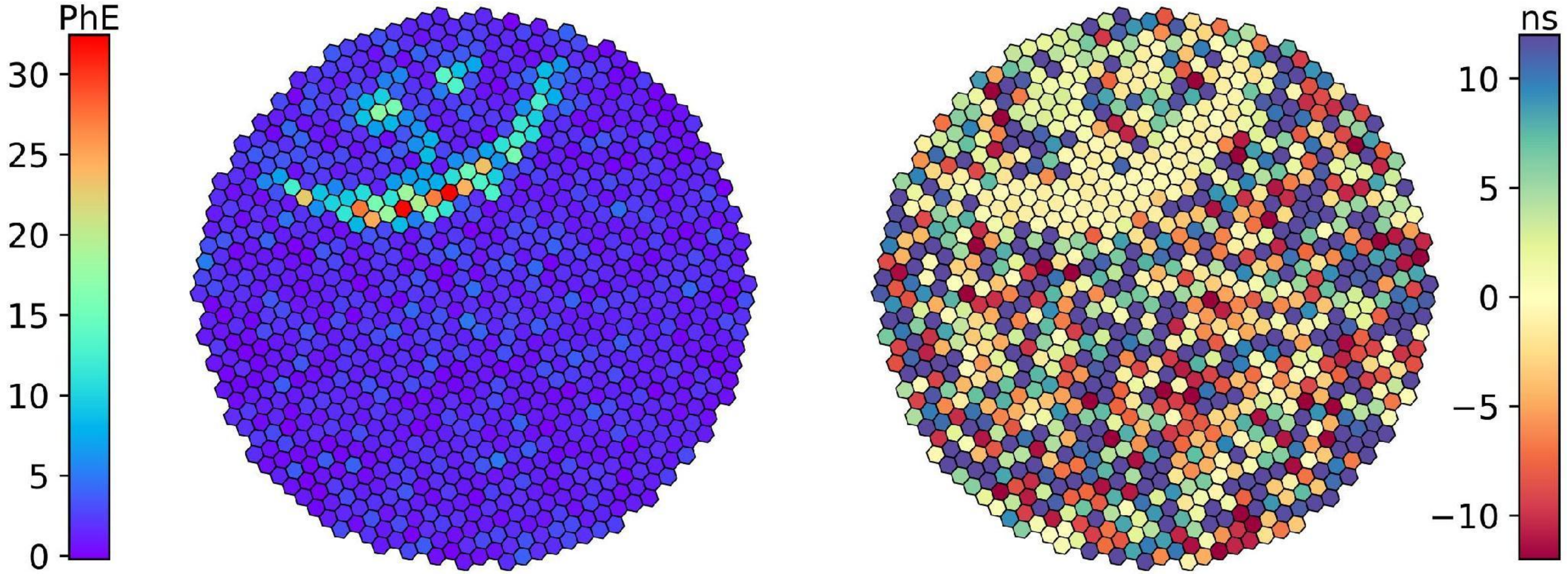
- Cherenkov cone of **1.4 deg**. Electron threshold energy of **21 MeV**.
- Each electron in the shower emit **Cherenkov radiation**.
- Overlapping wave-fronts **form a Cherenkov light pool** on the ground. Typical duration of **3 ns**.
- A telescope measures the **photon density** at the ground.
  Cherenkov photons ∝ Number of electrons ∝ Primary gamma-ray energy

MAGIC

- Two IACTs at the Canary Island of La Palma at **2200 m** above sea level.
- Diameter of **17m**, reflective surface of **240 m^2**. Asymptotical collection area of **10^5 m2** at 1 TeV.
- **Photo-multiplier tube cameras** with **sub-nanosecond** sampling (1.64 GHz).
- Total moving parts weight **60 tons**: “lightweight” telescope...
- Energy range between **20 GeV** and **100 TeV** (Very High Energy).



SHOWER IMAGES



- Each pixel: one **photo-multiplier tube** measuring the deposited **charge** (integrated in 3ns) and the photon **arrival time**.
- Position in the camera: different **incoming direction**.
- **Shape** and **intensity** of shower image → properties of the **primary particle**.

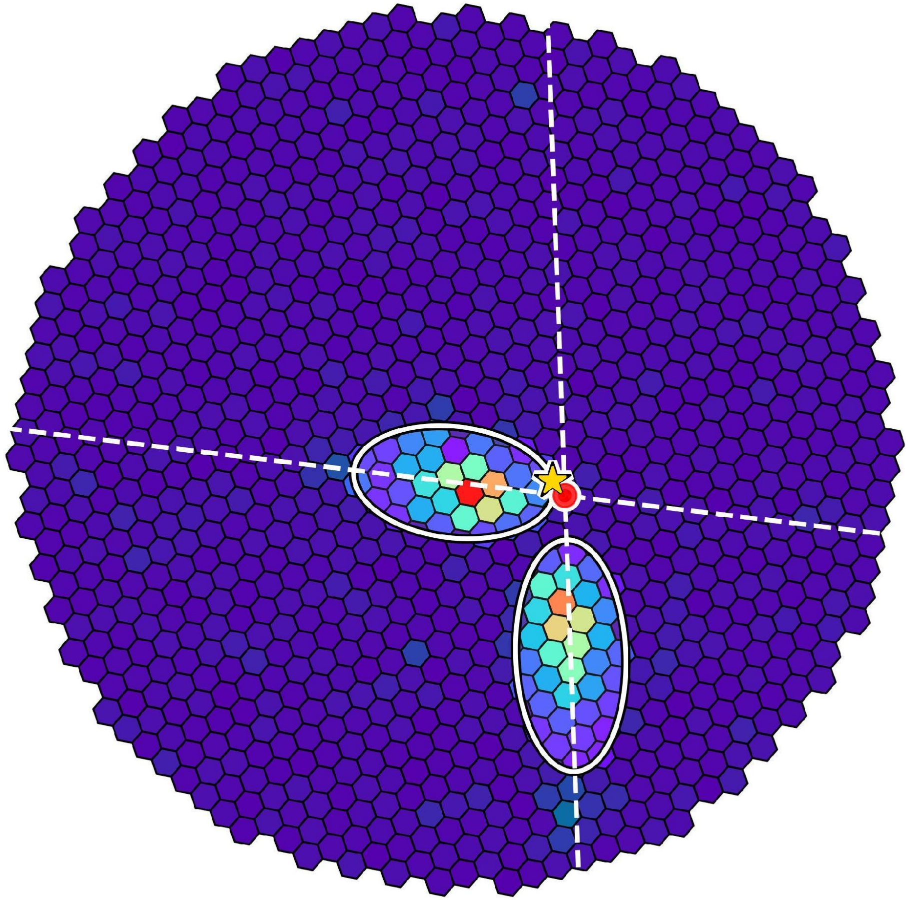
IMAGE PARAMETRIZATION

Total light → **Energy**
Image shape → Particle **type**
Timing → **Event** geometry

Stereoscopic system: **Incoming direction**, noise rejection

Reconstruction of the physical properties: **energy + direction** of **arrival** (momentum) and particle **type** (filtering cosmic-rays).

By **collecting many events** from the same object → **statistical determination of the spectrum**.



THE *HIDDEN* PART IV GOLDREICH-JULIAN THEORY

These slides are not part of your lecture. I added them here for those who may want to have more insights into the basic, simple theory of the pulsar magnetosphere according to (Goldreich-Julian). They require some familiarity with concepts of calculus and electromagnetism.

MAGNETOSPHERE ELECTRODYNAMICS

It is possible to understand **few key properties** of the **electrodynamics** of a **pulsar magnetosphere** without much previous knowledge about **plasma physics**. In the next we make some **very simplified** assumptions to understand the basic behavior (**Goldreich-Julian** model):

- Intrinsic magnetic field of the star aligned with the rotation axis.
- Axially-symmetric and stationary solution $\left(\frac{\partial}{\partial t} = 0\right)$
- Perfectly conducting electron-positron plasma with zero viscosity (ideal plasma).
- Neglecting inertia, gravity and other external forces.

A **fast-spinning**, **magnetized neutron star** acts as a **generator**, like the **dynamo** of a bicycle. The **magnetosphere** of the pulsar acts as a **circuit**. What we see as **pulsar emission** results from **losses** (“loads”) in the circuit.

GENERALIZED OHM'S LAW

- For a normal, linear conductive material one writes Ohm's law as:

$$\Delta V = R\, I,$$

relating the current I to the potential drop V through the resistance R. The resistance for a uniform material can be written as $R = \rho \frac{L}{A}$ with L the length of the material, A the cross area, and ρ the resistivity of the material. Inserting this into Ohm's law we can obtain:

$$\frac{\Delta V}{L} = \rho \frac{I}{A}$$

Taking the limit as $L \to d\mathrm{L}$ and $A \to dA$, we reach (the minus comes from the relative direction of L and dV):

$$-\frac{dV}{dL} = \rho \frac{dI_A}{dA}$$

GENERALIZED OHM'S LAW (II)

- Taking the limit as $L \rightarrow d\mathrm{L}$ and $A \rightarrow dA$, we reach (the minus comes from the relative direction of L and dV):

$$-\frac{dV}{dL} = \rho \frac{dI_A}{dA}$$

But the derivatives are nothing but the electric field E and the current density j:

$$E = \rho j$$

Or, in a more general vector form:

$$\vec{\mathbf{E}} = \rho \vec{\mathbf{j}}$$

This is sometimes known as the generalized Ohm's law.

OHM'S LAW AND MOVING CONDUCTOR (I)

This is sometimes known as the generalized Ohm's law:

$$\vec{\mathbf{E}} = \rho\vec{\mathbf{j}}$$

So far we have implicitly assumed that the conductor was at rest. If the conductor is moving at speed $\vec{\mathrm{u}}$ and there is a magnetic field $\vec{\mathrm{B}}$, the charges in the conductor will experience a Lorentz force. At equilibrium, the charges in the conductor will rearrange to create an electric field that balances the Lorentz force everywhere:

$$\cancel{q}\vec{\mathbf{E}}_{lor} = -\cancel{q}(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

The total electric field is the sum of the one due to the current and the "reaction" to the Lorentz force.

OHM'S LAW AND MOVING CONDUCTOR (II)

- The total electric field is the sum of the one due to the current and the "reaction" to the Lorentz force:

$$\vec{\mathbf{E}} = \rho\vec{\mathbf{j}} - \vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Which we rearrange to obtain:

$$\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}} = \rho\vec{\mathbf{j}}$$

In this equation $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are the electric and magnetic field, $\vec{\mathbf{j}}$ is the current density, $\vec{\mathbf{v}}$ is the velocity of the conductor, and ρ is the resistivity. If the conductor is ideal, the resistivity $\rho = 0$ and:

$$\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}} = 0$$

OHM'S LAW IN THE IDEAL PLASMA

- $$\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}} = 0$$

This is formally the same equation that we encountered during the Goldreich-Julian argument. However, there the velocity $\vec{\mathbf{v}}$ was the given rotation velocity of the (solid) neutron star. Here the velocity $\vec{\mathbf{v}}$ is an unknown, and since the conductor we are interested in is a **continuous** plasma, $\vec{\mathbf{v}}$ is a continuous **velocity field**.

In an **ideal plasma without external forces**, this equation relates the electric, magnetic, and velocity fields and must be satisfied in every point of space.

EQUIPOTENTIAL MAGNETIC FIELD LINES

- $$\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}} = 0$$

There are some interesting consequences from the ideal Ohm’s law.

$$\vec{\mathbf{B}} \cdot \left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right) = 0$$
$$\vec{\mathbf{B}} \cdot \vec{\mathbf{E}} + \cancel{\vec{\mathbf{B}} \cdot \left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)} = 0$$
$$\vec{\mathbf{B}} \cdot \vec{\mathbf{E}} = 0$$
$$\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}} V = 0$$

The **magnetic field lines** are also electric **equipotential lines**. This means that (as long as the plasma is ideal!) **no electric acceleration is possible along the lines of the magnetic field**.

PARALLEL VELOCITY FREEDOM

- $$\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}} = 0$$

Similarly, $\vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = 0$, the velocity is always perpendicular to the electric field. Let us break down the velocity field $\vec{\mathbf{v}}$ in a component $\vec{\mathbf{v}}_{\parallel}$ parallel to $\vec{\mathbf{B}}$ and a component $\vec{\mathbf{v}}_{\perp}$ perpendicular to it.

$$\vec{\mathbf{E}} \cancel{+ \vec{\mathbf{v}}_{\parallel} \times \vec{\mathbf{B}}} + \vec{\mathbf{v}}_{\perp} \times \vec{\mathbf{B}} = 0$$

$$\vec{\mathbf{E}} + \vec{\mathbf{v}}_{\perp} \times \vec{\mathbf{B}} = 0$$

The resulting equation does not constrain at all the velocity parallel to the magnetic field. The particles of the plasma are **free to slide along the magnetic field lines**.

POLOIDAL-TOROIDAL DECOMPOSITION

In the following we will use cylindrical coordinates (r,φ,z) and decompose the generic vector $\vec{\mathbf{a}}$ into a "toroidal" component $\vec{\mathbf{a}}_t = a_t\hat{\boldsymbol{\varphi}}$, and a "poloidal" component $\vec{\mathbf{a}}_p = a_r\hat{\mathbf{r}} + a_z\hat{\mathbf{z}}$.

We immediately see that the electric field $\vec{\mathbf{E}}$ can not have a toroidal component. If this was the case, it could either imply that:

- × The flux of the magnetic field across a circle enclosing the star varies with time. But this contradicts $\partial/\partial t = 0$!
- × There is a charge density that creates a toroidal electric field to balance a Lorentz force in that direction. But no axially-symmetric charge density can do that!

It follows that $\vec{\mathbf{E}}_t = 0$ and $\vec{\mathbf{E}} = \vec{\mathbf{E}}_p$, a purely poloidal field

POLOIDAL-TOROIDAL DECOMPOSITION (II)

- Since the electric field is purely poloidal, using the previous result on the equipotentiality we can also write:

$$\vec{\mathbf{E}} \cdot \vec{\mathbf{B}} = 0$$
$$\vec{\mathbf{E}} \cdot (\vec{\mathbf{B}}_t + \vec{\mathbf{B}}_p) = 0$$
$$\vec{\mathbf{E}} \cdot \vec{\mathbf{B}}_p = 0$$

This means that the electric field and the poloidal magnetic field are always perpendicular. As a result, we can write the electric field as:

$$\vec{\mathbf{E}} = (-u\hat{\boldsymbol{\varphi}}) \times \vec{\mathbf{B}}_p$$

Where $u = u(r, z)$ is an arbitrary (for now) function of the position. The reason for the minus sign will become clear in a moment.

POLOIDAL-TOROIDAL DECOMPOSITION (III)

- We decompose now the velocity and magnetic field in $\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}} = 0$ in poloidal and toroidal components:

$$\underset{\text{pol}}{\vec{\mathbf{E}}} + \underset{\text{tor}}{\vec{\mathbf{v}}_p \times \vec{\mathbf{B}}_p} + \underset{\text{pol}}{\vec{\mathbf{v}}_p \times \vec{\mathbf{B}}_t} + \underset{\text{pol}}{\vec{\mathbf{v}}_t \times \vec{\mathbf{B}}_p} \underset{\text{parallel}}{\cancel{+ \vec{\mathbf{v}}_t \times \vec{\mathbf{B}}_t}} = 0$$

We can identify and separate the poloidal and toroidal components in the equation:

$$\vec{\mathbf{E}} + \vec{\mathbf{v}}_p \times \vec{\mathbf{B}}_t + \vec{\mathbf{v}}_t \times \vec{\mathbf{B}}_p = 0$$

$$\vec{\mathbf{v}}_p \times \vec{\mathbf{B}}_p = 0$$

POLOIDAL-TOROIDAL DECOMPOSITION (IV)

- The second equation implies that the poloidal velocity and the poloidal magnetic field are parallel:

$$\vec{\mathbf{v}}_p \parallel \vec{\mathbf{B}}_p \;\rightarrow\; \vec{\mathbf{v}}_p = \kappa(r,z)\vec{\mathbf{B}}_p$$

The function $\kappa(r,z)$ is an arbitrary function of position. Similarly, the toroidal velocity is also, evidently, parallel to the toroidal magnetic field:

$$\vec{\mathbf{v}}_t \parallel \vec{\mathbf{B}}_t \;\rightarrow\; \vec{\mathbf{v}}_t = \lambda(r,z)\vec{\mathbf{B}}_t$$

Where $\lambda(r,z)$ is another arbitrary function. The **poloidal motion of the particles is “anchored” to the lines of the poloidal magnetic field**. We now insert these results back into the first equation.

POLOIDAL-TOROIDAL DECOMPOSITION (V)

- $$\vec{\mathbf{E}} + \vec{\mathbf{v}}_p \times \vec{\mathbf{B}}_t + \vec{\mathbf{v}}_t \times \vec{\mathbf{B}}_p = 0$$

Where:

$$\vec{\mathbf{E}} = (-u(r,z)\hat{\boldsymbol{\varphi}}) \times \vec{\mathbf{B}}_p = \left(-u(r,z)/\mathrm{B}_t\vec{\mathbf{B}}_t\right) \times \vec{\mathbf{B}}_p$$

$$\vec{\mathbf{v}}_p = \kappa(r,z)\vec{\mathbf{B}}_p \quad \text{and} \quad \kappa(r,z) = \mathrm{v}_p/\mathrm{B}_p$$

$$\vec{\mathbf{v}}_t = \lambda(r,z)\vec{\mathbf{B}}_t \quad \text{and} \quad \lambda(r,z) = \mathrm{v}_t/\mathrm{B}_t$$

By substituting and rearranging, the equation becomes scalar:

$$(-u/\mathrm{B}_t)\vec{\mathbf{B}}_t \times \vec{\mathbf{B}}_p + \kappa\vec{\mathbf{B}}_p \times \vec{\mathbf{B}}_t + \lambda\vec{\mathbf{B}}_t \times \vec{\mathbf{B}}_p = 0$$

$$-u/\mathrm{B}_t - \kappa + \lambda = 0$$

POLOIDAL-TOROIDAL DECOMPOSITION (VI)

After some more algebraic manipulation we obtain:

$$\mathrm{v}_t - u = \kappa \mathrm{B}_t = \frac{\mathrm{v}_p}{\mathrm{B}_p} \mathrm{B}_t$$

Let us now inspect the vector quantity $\vec{\mathbf{v}} - u\hat{\boldsymbol{\varphi}}$:

$$\vec{\mathbf{v}} - u\hat{\boldsymbol{\varphi}} = \begin{pmatrix} \mathrm{v}_p \\ \mathrm{v}_t \end{pmatrix} - \begin{pmatrix} 0 \\ u \end{pmatrix} = \begin{pmatrix} \mathrm{v}_p \\ \mathrm{v}_t - u \end{pmatrix} = \begin{pmatrix} \mathrm{v}_p \\ \frac{\mathrm{v}_p}{\mathrm{B}_p} \mathrm{B}_t \end{pmatrix} = \frac{\mathrm{v}_p}{\mathrm{B}_p} \begin{pmatrix} \mathrm{B}_p \\ \mathrm{B}_t \end{pmatrix} = \frac{\mathrm{v}_p}{\mathrm{B}_p} \vec{\mathbf{B}}$$

The vector is parallel to the (total) magnetic field!

POLOIDAL-TOROIDAL DECOMPOSITION (VII)

- $$\vec{\mathbf{v}} - u\hat{\boldsymbol{\varphi}} = \frac{\mathrm{v}_p}{\mathrm{B}_p}\vec{\mathbf{B}}$$

The vector is parallel to the (total) magnetic field! This means that **the velocity field** can everywhere be written as a **composition of** $u(r,z)\hat{\boldsymbol{\varphi}}$ and a **component parallel to the magnetic field**:

$$\vec{\mathbf{v}} = u(r,z)\hat{\boldsymbol{\varphi}} + \kappa(r,z)\vec{\mathbf{B}}$$

So far we have no idea of what $u(r,z)$ is. We will now prove that $u(r,z)\hat{\boldsymbol{\varphi}} = \vec{\boldsymbol{\Omega}} \times \vec{\boldsymbol{r}}$, where Ω is the angular frequency of the neutron star. We will plug $\vec{\mathbf{v}}$ back into Ohm's law, and note that the term parallel to $\vec{\mathbf{B}}$ disappears in the cross product.

IDEAL MAGNETOHYDRODYNAMIC

By applying the **curl** operator $\left(\vec{\boldsymbol{\nabla}}\times\right)$ to the ideal Ohm's law, we obtain the **ideal magnetohydrodynamic** equation:

$$\vec{\boldsymbol{\nabla}}\times\left(\vec{\mathbf{E}}+\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)=0$$

$$\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{E}}+\vec{\boldsymbol{\nabla}}\times\left(\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)=0$$

$$-\frac{\partial\vec{\mathbf{B}}}{\partial t}+\vec{\boldsymbol{\nabla}}\times\left(\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)=0$$

Since we are interested in a steady state solution, and $\frac{\partial}{\partial t}=0$:

$$\vec{\boldsymbol{\nabla}}\times\left(\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)=0$$

We now substitute for $\vec{\mathbf{v}}$ with the previous result

IDEAL MAGNETOHYDRODYNAMIC (II)

- $$\vec{\nabla} \times \left(\vec{v} \times \vec{B}\right) = 0$$
$$\vec{v} = u(r,z)\hat{\boldsymbol{\varphi}} + \kappa(r,z)\vec{B}$$

We note that the term $\kappa(r,z)\vec{B}$ does not contribute to the cross product, so that the equation reduces to:

$$\vec{\nabla} \times \left[u(r,z)\hat{\boldsymbol{\varphi}} \times \vec{B}\right] = 0$$

We first expand this with a vector calculus identity:

$$\vec{\nabla} \times \left(\vec{u} \times \vec{B}\right) = \vec{u}\left(\vec{\nabla} \cdot \vec{B}\right) - \vec{B}\left(\vec{\nabla} \cdot \vec{u}\right) + \left(\vec{B} \cdot \vec{\nabla}\right)\vec{u} - \left(\vec{u} \cdot \vec{\nabla}\right)\vec{B} = 0$$

Divergence of B is zero (Maxwell) ☺

Divergence of toroidal field is zero (Helmholtz) ☺

This is horrible!! ☹☹☹

IDEAL MAGNETOHYDRODYNAMIC (III)

- $$(\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{u}} - (\vec{\mathbf{u}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}} = 0$$

With some algebra and a little patience, the above equation can be reduced to this equation relating the components of $\vec{\mathbf{B}}$ with the derivatives of the velocity (homework for you!):

$$\mathrm{B}_r \frac{\partial u}{\partial r} + \mathrm{B}_z \frac{\partial u}{\partial z} - \frac{u\mathrm{B}_r}{r} = 0$$

And if we write $u(r, z) = r\omega(r, z)$, with ω being the angular velocity associated to u at the (r, z) position, we finally reach:

$$\mathrm{B}_r \frac{\partial \omega}{\partial r} + \mathrm{B}_z \frac{\partial \omega}{\partial z} = 0$$

COROTATION VELOCITY

- We finally reach:

$$B_r \frac{\partial \omega}{\partial r} + B_z \frac{\partial \omega}{\partial z} = 0$$

Since there is no variation of ω(r,z) along the azimuthal direction φ (axial symmetry), this is the same as saying that:

$$\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}} \omega = 0$$

This is also known as Ferraro's isorotation law.

The **angular velocity $\boldsymbol{\omega}$ does not change along the magnetic field lines**. And since the magnetic field lines are attached to the pulsar surface, that is rigidly moving at $\boldsymbol{\Omega}$, this means that $\boldsymbol{\omega}(\cancel{r,z}) = \boldsymbol{\Omega}$ is constant and the same as the pulsar frequency everywhere!

SUMMARY (SO FAR)

- We have shown that in an **axisymmetric**, **steady** magnetosphere around a **rigidly rotating body**, filled with an **ideal plasma**, the velocity of the plasma is:

$$\vec{\mathbf{v}} = \overrightarrow{\boldsymbol{\Omega}} \times \vec{\boldsymbol{r}} + \kappa(r, z)\overrightarrow{\mathbf{B}}$$

That means: the motion of the plasma is a composition of a **rigid rotation with the star**, and a motion **along the magnetic field**. We will see immediately that this fact produces a stunning phenomenon in the case of a rapidly spinning neutron star: the existence of a limit surface, the **light cylinder** of the pulsar, beyond which the magnetic field must become **toroidal**, and the plasma is lost into space as a **pulsar wind**.

THE LIGHT CYLINDER

As the co-rotation velocity is always $u = r\Omega$, **at a distance** $\boldsymbol{R_l}$ **sufficiently far from the star** the co-rotation should **happen at a speed greater than the speed of light**:

$$R_l = c/\Omega$$

The surface at $r = R_l$ is called the **light cylinder** of the pulsar. The electrons in the plasma must obviously move **slower than the speed of light**, but at the same time:

$$\vec{\mathbf{v}} = \vec{\boldsymbol{\Omega}} \times \vec{\boldsymbol{r}} + \kappa(r,z)\vec{\mathbf{B}}$$

The only way to ensure that $|\vec{\mathbf{v}}| < c$ is that $\vec{\mathbf{B}}$ becomes toroidal after the light cylinder so that the co-rotation velocity is “neutralized”.

THE LIGHT CYLINDER

The only way to ensure that $|\vec{\mathbf{v}}| < c$ is that $\vec{\mathbf{B}}$ becomes toroidal after the light cylinder so that the co-rotation velocity is “neutralized”.

$$\vec{\mathbf{v}} = \left[\vec{\boldsymbol{\Omega}} \times \vec{\boldsymbol{r}} + \kappa(r,z)\vec{\mathbf{B}}_t\right] + \kappa(r,z)\vec{\mathbf{B}}_p$$

The terms in the square brackets must (at least partially) cancel out beyond the light cylinder:

$$\vec{\boldsymbol{\Omega}} \times \vec{\boldsymbol{r}} + \kappa(r,z)\vec{\mathbf{B}}_t = 0 \quad \text{and} \quad \kappa(r,z) = \mathrm{v}_p/\mathrm{B}_p$$

$$|\mathrm{B}_t| = |r\Omega/\mathrm{v}_p||\mathrm{B}_p| > |c/\mathrm{v}_p||\mathrm{B}_p| > |\mathrm{B}_p|$$

The **toroidal magnetic field is dominant beyond the light cylinder.**

PULSAR WIND

The **toroidal magnetic field is dominant beyond the light cylinder.** When viewed from one of the poles, the field lines look more and more like one of these drawings as they approach the cylinder:

N S

The pulsar is spinning counterclockwise for the symbol on the left (north pole view) and clockwise for the one on the right (south pole view). Magnetic field lines are **“swept back”** as they approach the light cylinder.

PULSAR WIND

- Magnetic field lines are **“swept back”** as they approach the light cylinder.

Consider now a **magnetic field line that crosses the light cylinder**. Were it to **close itself** somewhere outside the cylinder and **come back to the star**, in at least a point along the path $B_t = 0$.* By the previous arguments, this would imply particles on that field line would need to **co-rotate faster than the speed of light**. Hence such a line **cannot come back to the star**, and instead **opens up** streaming to infinity.

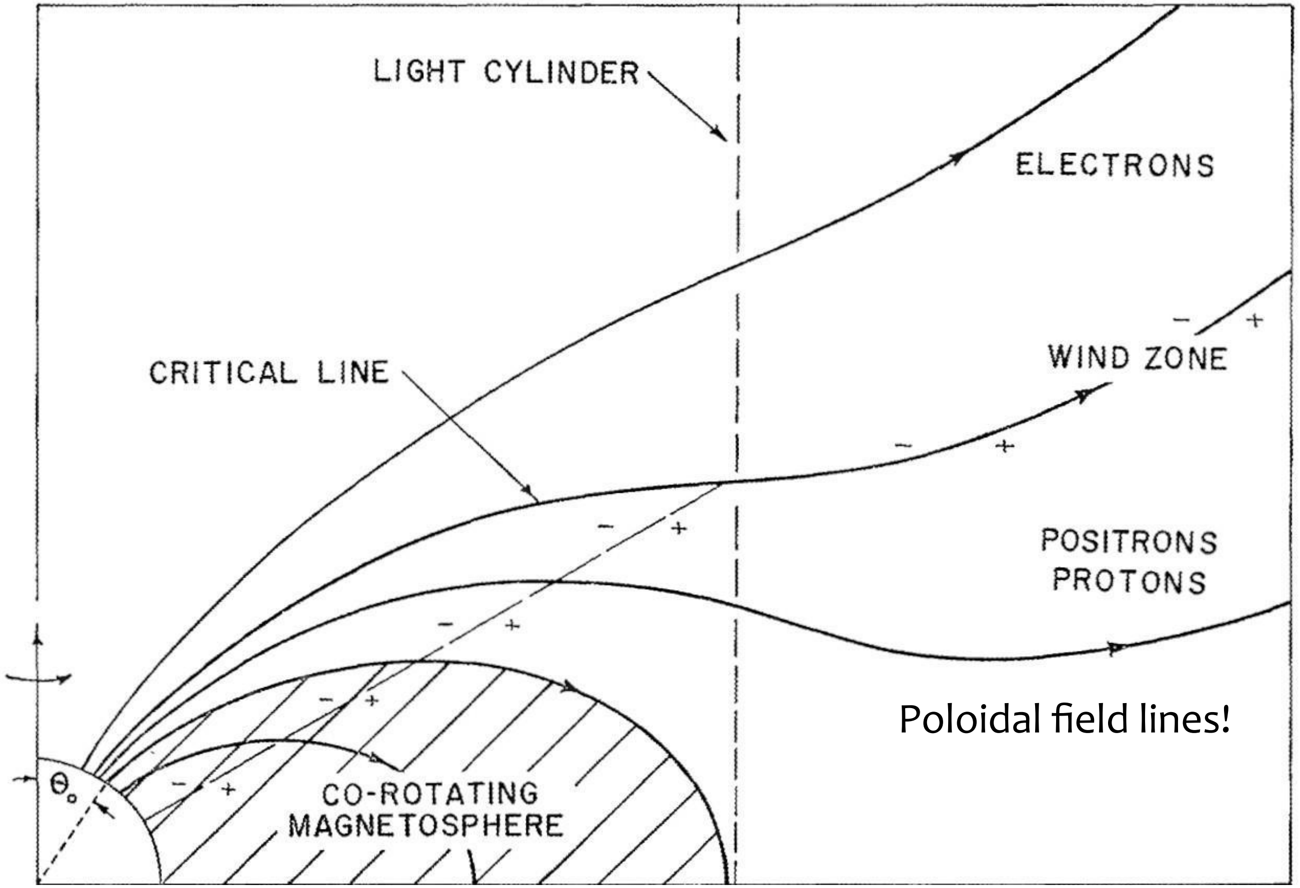
* Note: this argument neglects the case in which the pulsar magnetosphere has an internal twist, meaning it is axially-symmetric, but not reflection-symmetric along the equatorial plane. In such case, the field lines look like the same drawing (one of □ or □) both when viewed from the north or south pole. These twisted configurations are not dynamically stable, but are of interest in the case of magnetars.

PULSAR WIND

We have shown that **beyond the light cylinder**, magnetic **field lines** can **not cross again the cylinder**, and instead **stream to infinity**. These magnetic field lines are **open** (they merge in the turbulent intergalactic magnetic field and close there) and **do not come back** to the pulsar. The **particles** that stream along these lines **ultimately leave the pulsar forever** and become the **pulsar wind**.

With the above argument we have shown that even an **aligned pulsar** possesses a mechanism to **slow down**. Its gargantuan **electromagnetic energy** is ultimately **converted** in kinetic energy of the (leptonic) **pulsar wind**, that carries it away forever and **disperses** it in the surrounding space. This is the process powering **pulsar wind nebulae** (Crab).

MAGNETOSPHERE STRUCTURE



Poloidal field lines!