

# Multimessenger analysis with *xkn* & *bajes*

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Multimessenger Astronomy in the ET era  
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# *xkn*: a semi-analytic framework for KN modelling

## Goals:

- to provide a flexible, robust & computationally feasible KN model for
  - extensive parameter estimation analysis
  - multimessenger frameworks
- incorporate the relevant physics with a connection to the outcome of first principle simulations
  - heating rates
  - thermalization efficiency
  - atomic opacities
  - matter ejecta properties

*xkn* documented in Ricigliano et al 2024, available at <https://github.com/GiacomoRicigliano/xkn>

# *xkn*: hypothesis and method

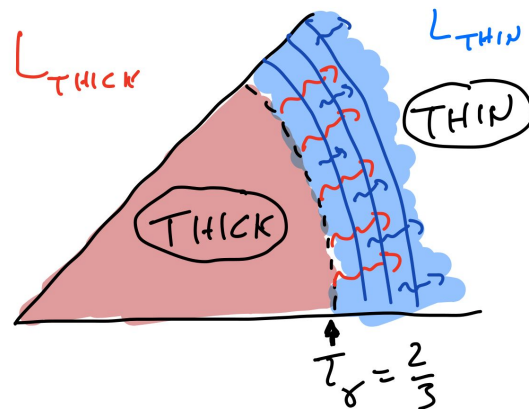
## Basic hypothesis:

- homologously expanding ejecta
- spherical symmetry
- photospheric conditions (within a few days post-merger, for GW170817-like KNe)
- gray, spatially homogeneous opacity

## Method:

- distinction between **optically thick** and **optically thin** conditions

$$\rho(t) = \rho_0 \left( \frac{t_0}{t} \right)^3 \quad \rho_0 = \frac{M_{\text{ej}}}{\frac{4}{3}\pi (v_{\text{max}} t_0)^3}$$



$$L(t) = L_{\text{thick}}(t) + L_{\text{thin}}(t)$$

# *xkn*: calculation of photospheric luminosity

## optically thick part

- radiation diffusion problem
- photospheric luminosity: solution of eigenvalue problem
- precise solution depends on temporal behavior of thermalization ( $f_{th}$ ), nuclear energy rate and gray opacity

$$\frac{DE}{Dt} - \frac{c}{3r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\chi} \frac{\partial E}{\partial r} \right) + \frac{4\dot{R}}{R} E = \dot{E}_{\text{heat}}$$

$$F = -\frac{c}{3\chi} \frac{\partial E}{\partial r}$$

$$\begin{aligned} L(t) &= 4\pi R^2(t) \left[ x^2 F(x, t) \right]_{x=1} \\ &= \frac{4\pi (v_{\text{max}} t_0)^3 \sqrt{2} E_0}{\tau} \sum_{n=1}^{\infty} (-1)^{n+1} n \pi \phi_n(t) \end{aligned}$$

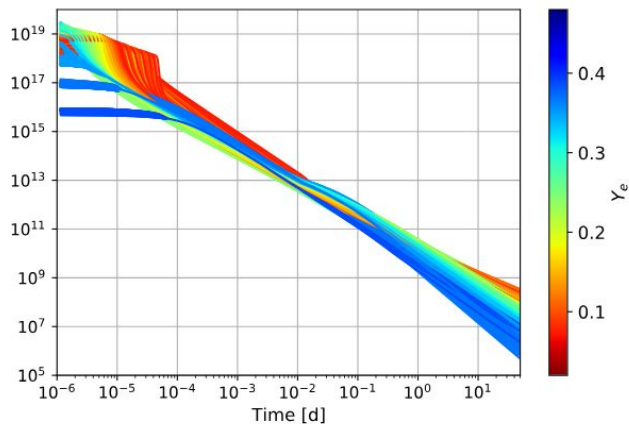
$$\phi'_n(t) + \left( \frac{t}{t_0 \tau} \right) (n^2 \pi^2) \phi_n(t) = \frac{(-1)^{n+1} \rho_0 \sqrt{2}}{n \pi E_0} \left( \frac{t}{t_0} \right) \dot{\epsilon} f_{th} \quad \dot{\epsilon} = \dot{\epsilon}_0 \left( \frac{t}{t_0} \right)^{-\alpha}, \quad f_{th} = f_{th,0} \left( \frac{t}{t_0} \right)^{-\beta}, \quad \kappa = \kappa_0 \left( \frac{t}{t_0} \right)^{-\gamma}$$

first proposed by Pinto & Eastman (2000) and later extended to KN by Wollaeger et al (2018)

# *xkn*: optically thin part and input physics

$$L_{\text{thin}}(t) = \sum_{i=1}^{N_{\text{thin}}} f_{\text{th},i}(t) \dot{\epsilon}(t) dM_i$$

Relevant input physics:



Wu et al 2022

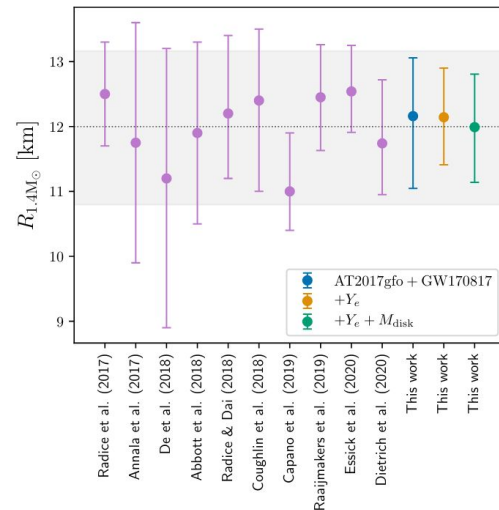
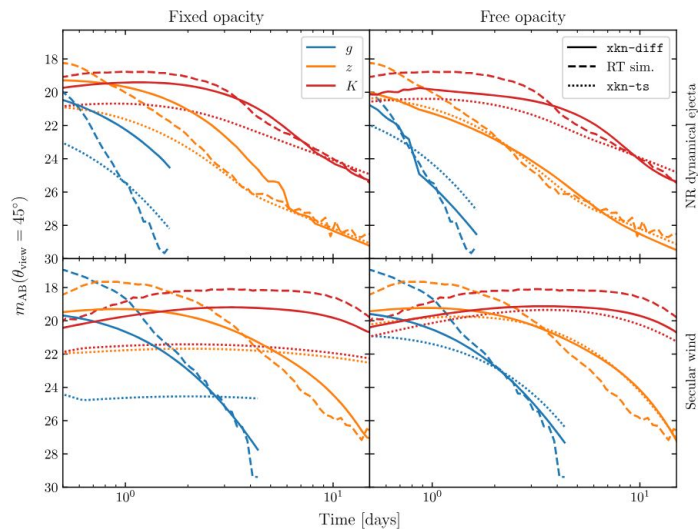
## optically thin part

- free streaming contribution from optically thin (Lagrangian) shells
- **heating rates**: numerical fit of the outcome of detailed nuclear network trajectories in  $(s, t_{\text{exp}}, Y_e)$  space
- **thermalization efficiency**: physically motivated fitting formula
- **gray opacity**: simple relation with elemental composition

see Chiesa et al 2024, Barnes et al 2016, Tanaka et al 2020

# *xkn*: from 1D luminosity to KN framework

- extension to 2D: ray-by-ray approach
- anisotropic density, opacity and speed profiles
- implemented alongside a simpler time scale KN models (Grossmann model)



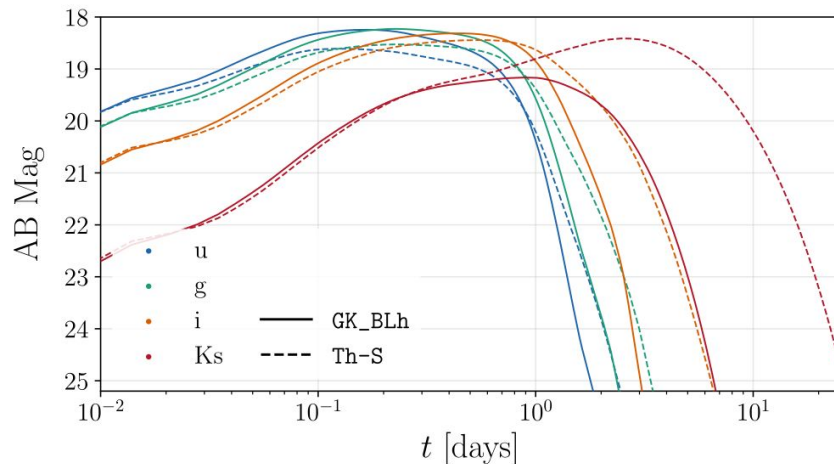
Breschi et al 2022

- photometry: broad band light curves
- black body emission, both from photosphere & optically thin shells
- validated against RT simulations

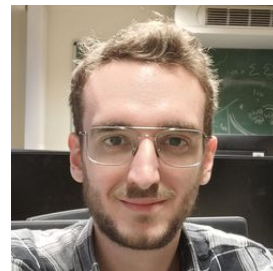
# Radiation hydrodynamics models with nuclear network

*SNEC*  $\Rightarrow$  *KNEC*+*NN*

- radiation hydrodynamics of the ejecta
- energy-dependent photon transport
- online nuclear network:
  - detailed spectral atomic opacities
  - nuclear energy
  - thermalization efficiency
- 2D or 3D ray-by-ray
- F. Magistrelli & S. Bernuzzi (Jena U), G. Ricigliano (TU Darmstat  $\rightarrow$  Tokyo U)

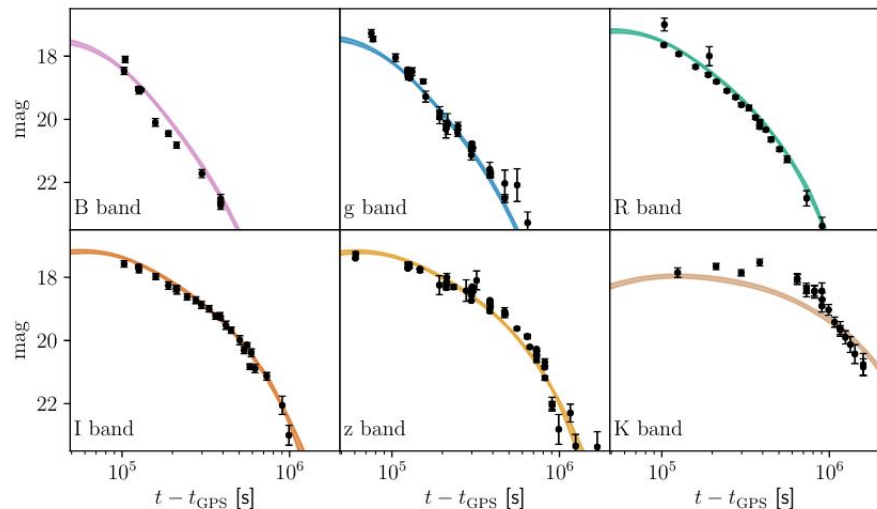


Magistrelli et al, arxiv



# *bajes*: an open source nested sampling algorithm

- parallel & lightweight framework for Bayesian inference of MM transients
- nested sampling → **model selection**
- Python modular package
  - minimal dependencies on external libraries
  - adaptable to the majority of the Bayesian models
  - various sampling methods
- developed and tested in the context of GW data analysis: 1 full analysis in 1 day on 128 CPUs

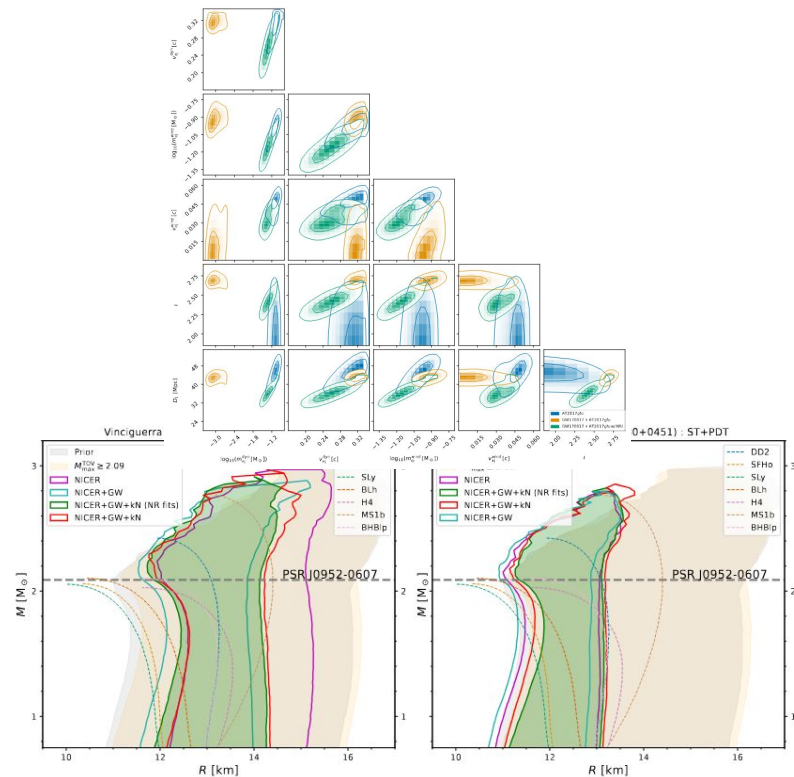


LC bands from GW+KN inference, from  
Breschi et al 2024



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EOS constraints from GW+KN inference, from Breschi et al 2024

# *bajes*: status of MM (GW+EM) analysis capabilities

- Nester sampling:
  - *bajes*
- GW models
  - *TEOBResumSPA*
  - *IMPRhenomPv2\_NRTidal*
- KN models
  - *Grossmann*
  - *xkn*
- GRB afterglow model:
  - *afterglowpy*
- additional key features:
  - *joint likelihood* for multiple datasets
  - *NR-informed relations* for ejecta properties
- First tests and calibration procedure
  - M. Breschi & S. Bernuzzi (Jena U)
  - V. Fusaro (MSc student in Trento)
  - G. Huez (PhD student in Jena)
  - collaboration with G. Stratta and A. Rossi (INAF-OAS Bologna)



# Pros & Cons of *bajes+xkn*

## Pros

- flexible & transparent
- easily extendable
- no need of interpolation inside large parameter space
- direct connection with first principle calculations concerning
  - ejecta properties
  - nucleosynthesis yields

## Cons

- accuracy bounded to performance
- limited accuracy in opacity & RT
  - no energy dependence
  - limited time evolution behavior
- lack of lateral motion & lateral fluxes
- systematic uncertainties in *xkn* (difficult to quantify)
- actual meaning and usage of NR fitting formulae