

Bayesian Model Selection

for high-dimensional astrophysical generative models

How does data challenge our models?

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PostDoc

Information field theory group

Supervisor: **Torsten Enßlin**

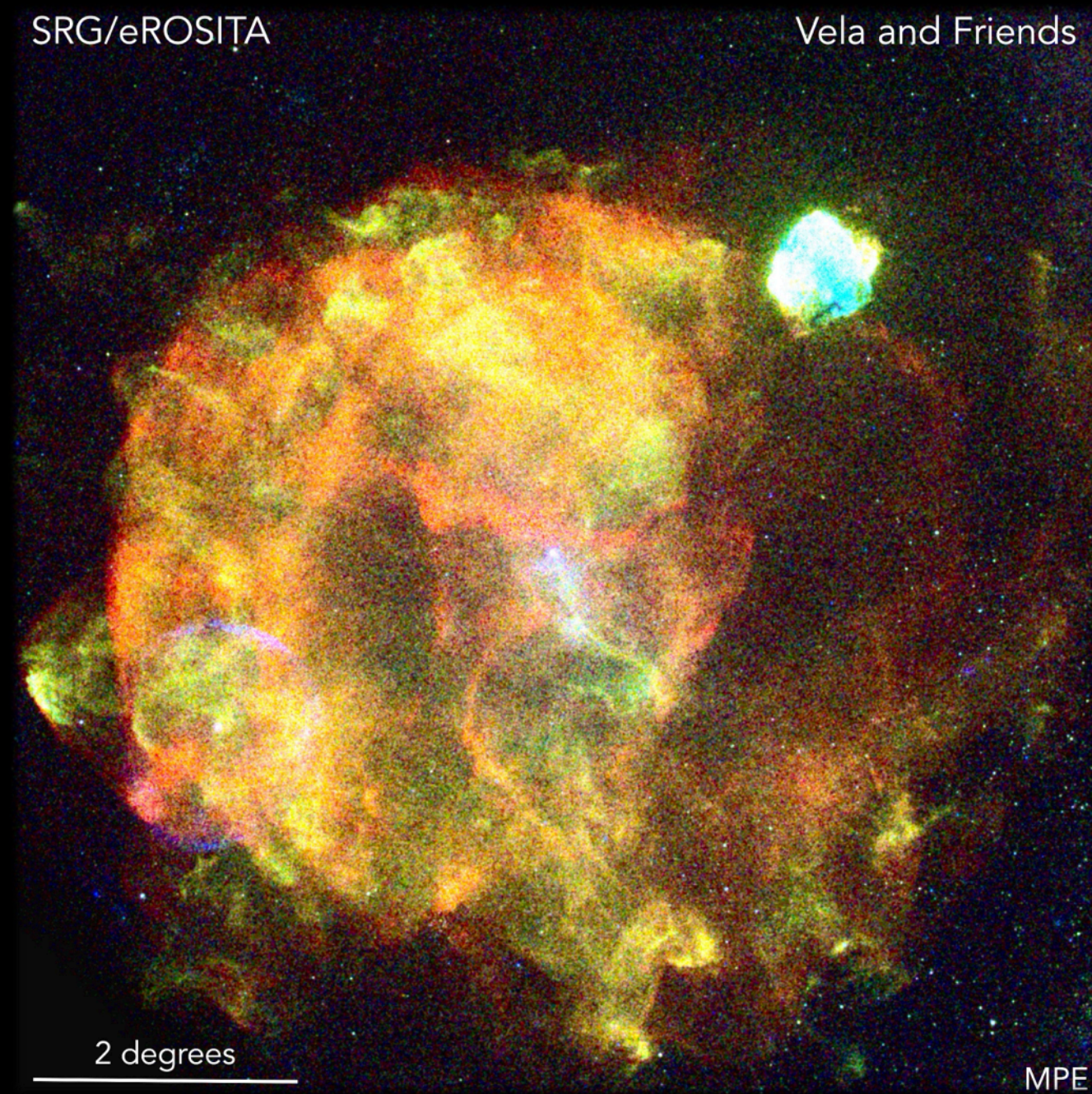
September 4th 2026
ML4ASTRO 3
University of Malta



MAX PLANCK INSTITUTE
FOR ASTROPHYSICS

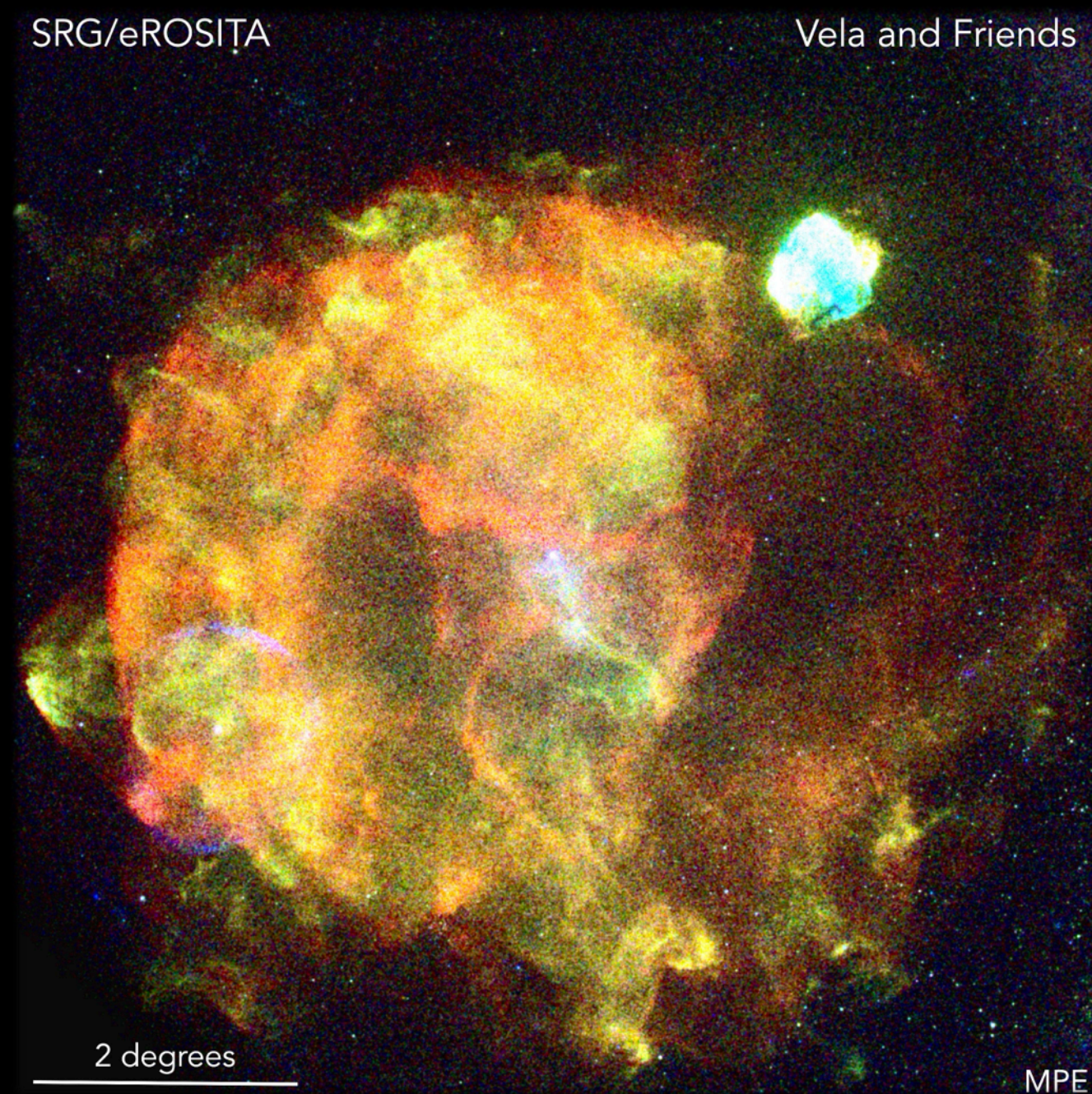


The problem



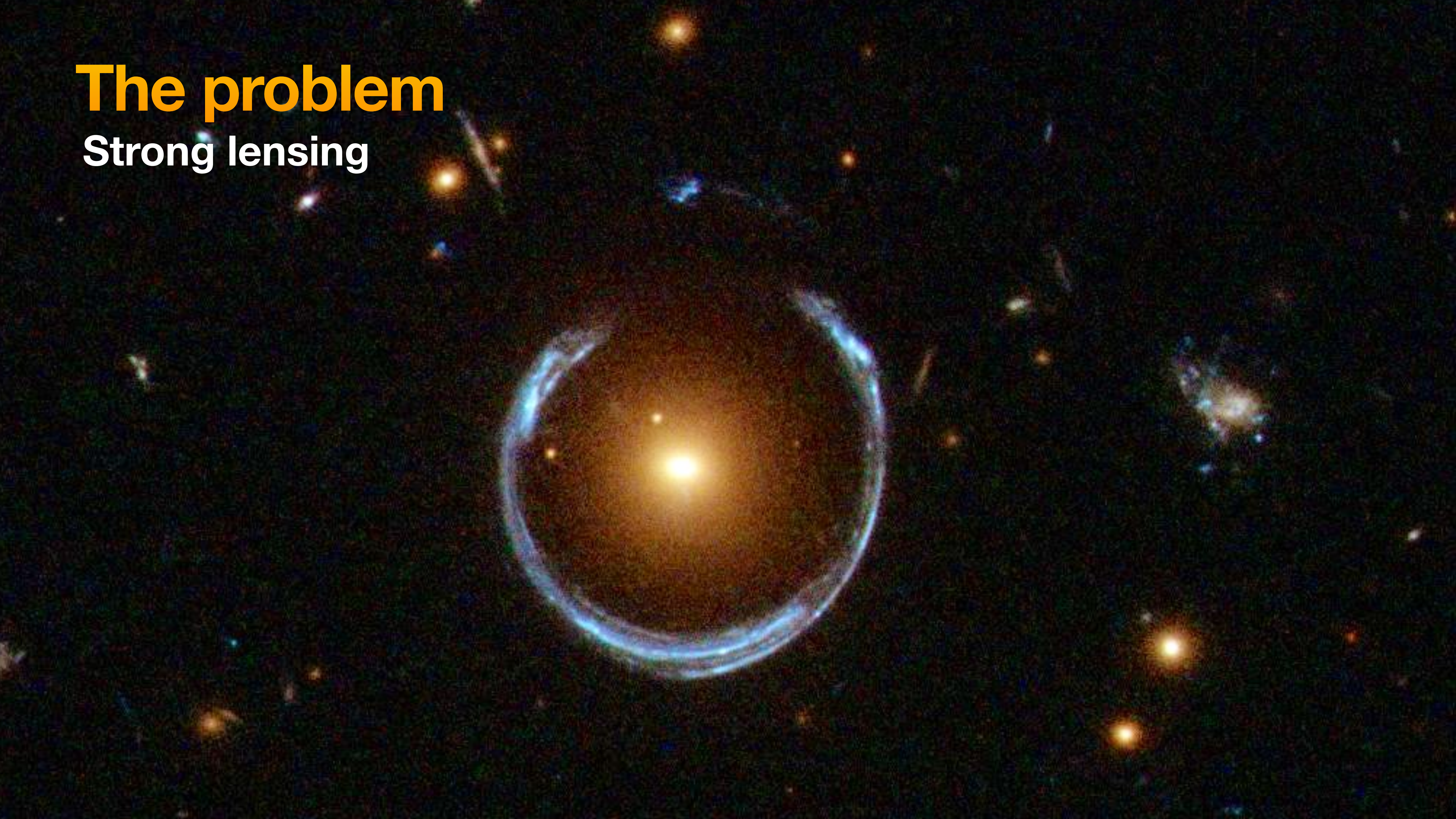
The problem

From great data come great responsibility



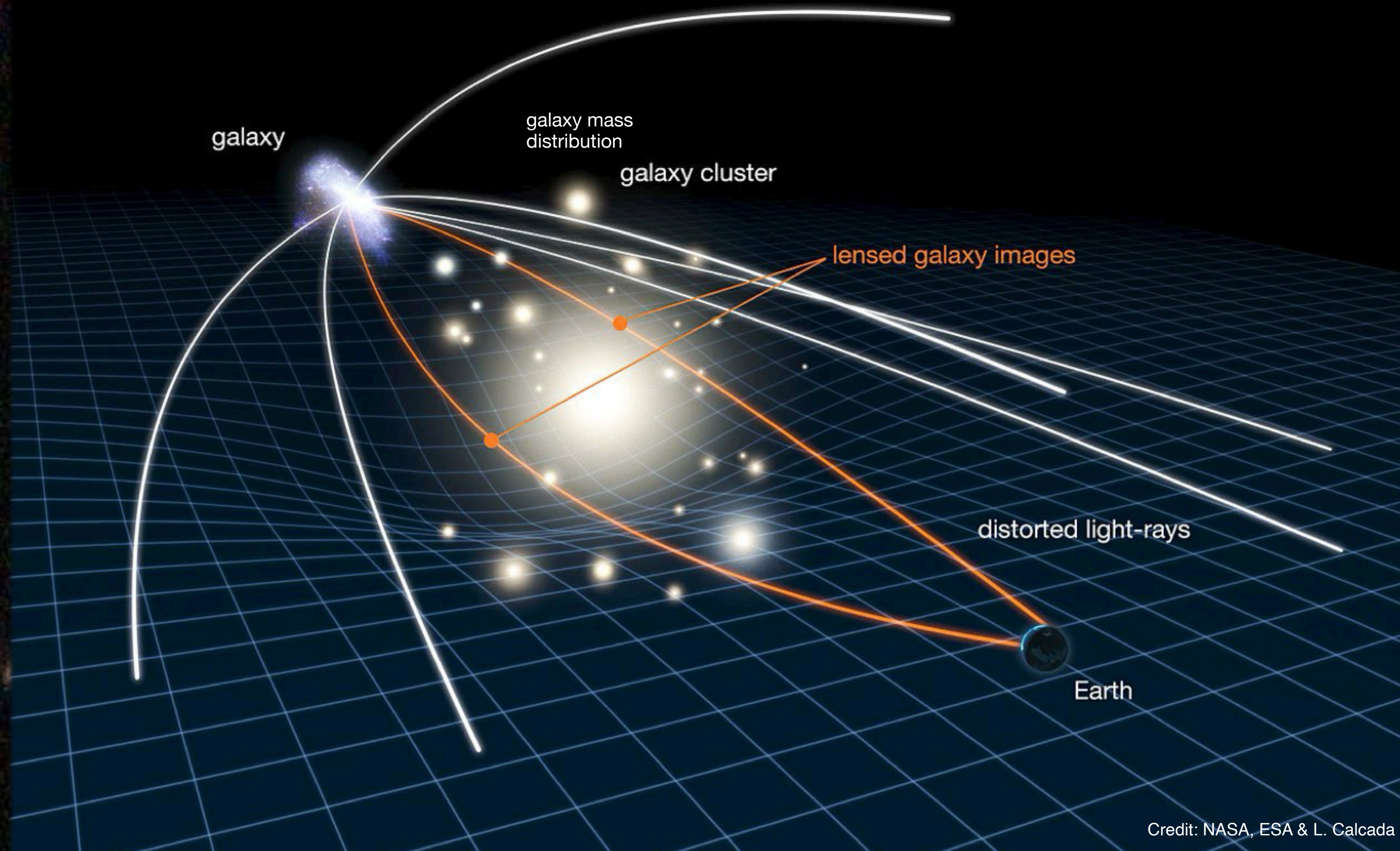
The problem

Strong lensing



The problem

Strong lensing



Modeling data with IFT

Information field theory

Bayes' Theorem

$$P(s | d) =$$

Information field theory

Bayes' Theorem

$$P(s | d) = \frac{P(s)}{P(d)}$$

Information field theory

Bayes' Theorem

$$P(s | d) = \frac{P(d | s) P(s)}{P(d)}$$

Information field theory

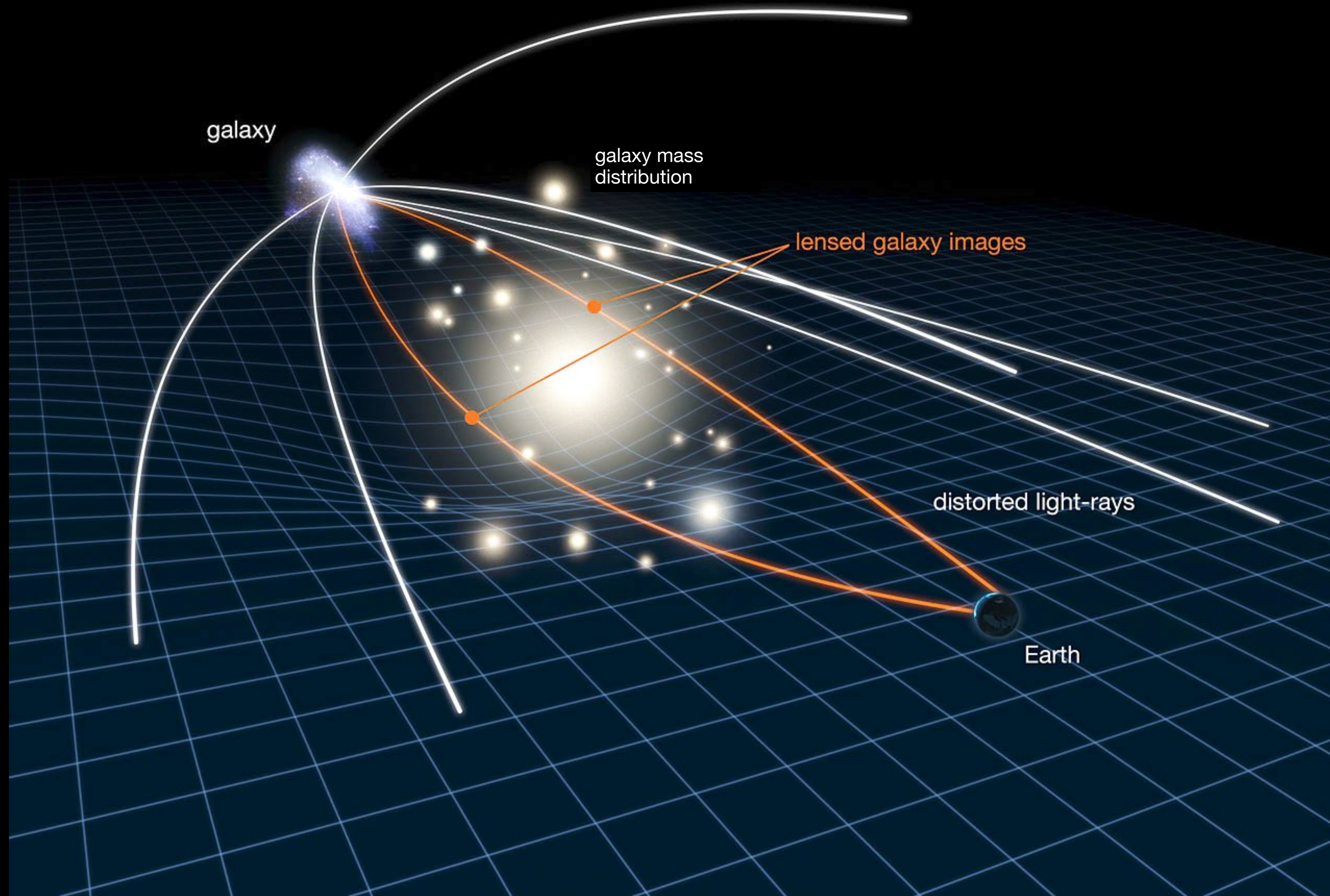
Bayes' Theorem

$$P(s | d) = \frac{P(d | s) P(s)}{P(d)}$$

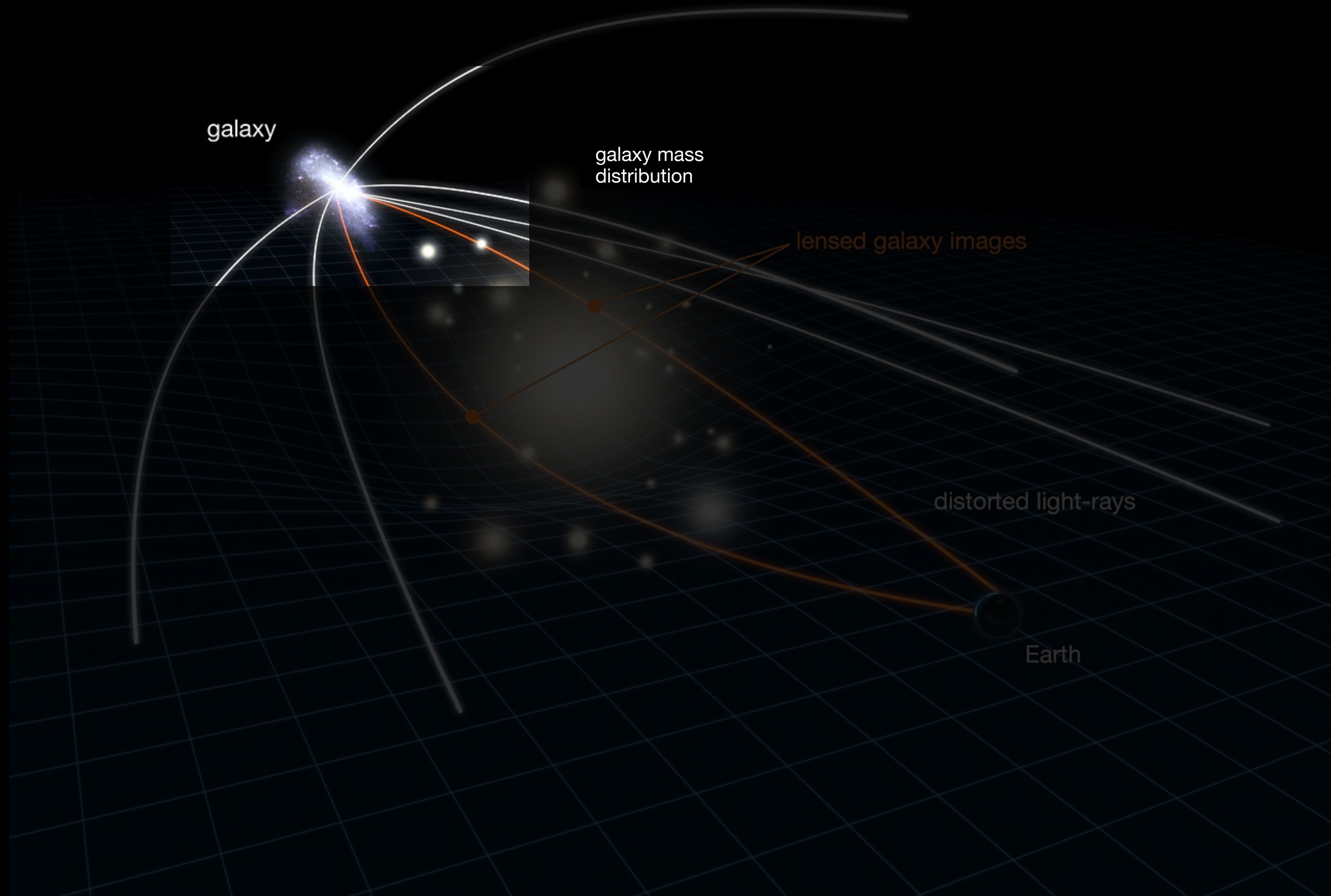
The prior

$$P(s | d) = \frac{P(d | s) P(s)}{P(d)}$$

The prior

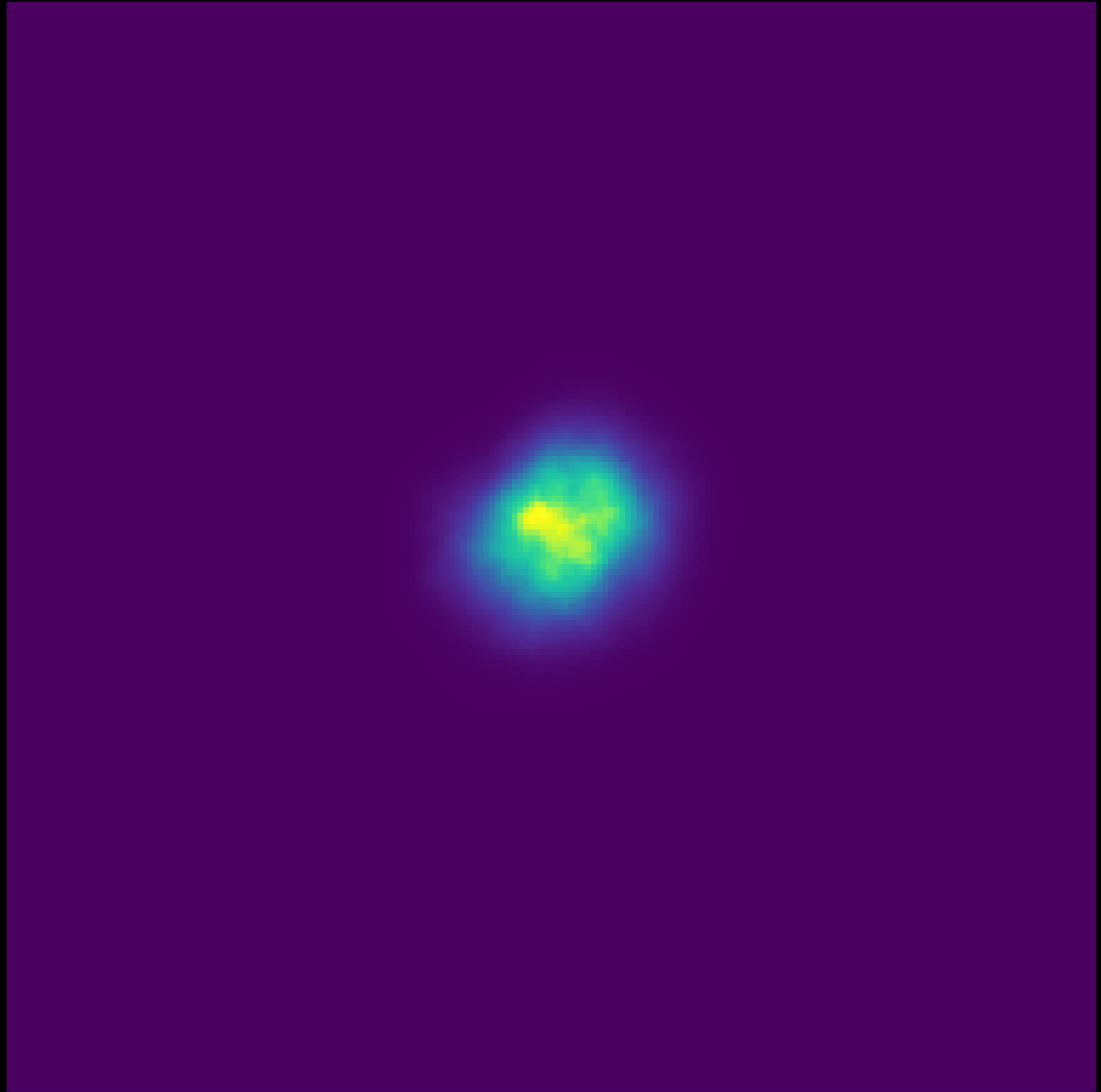


The source



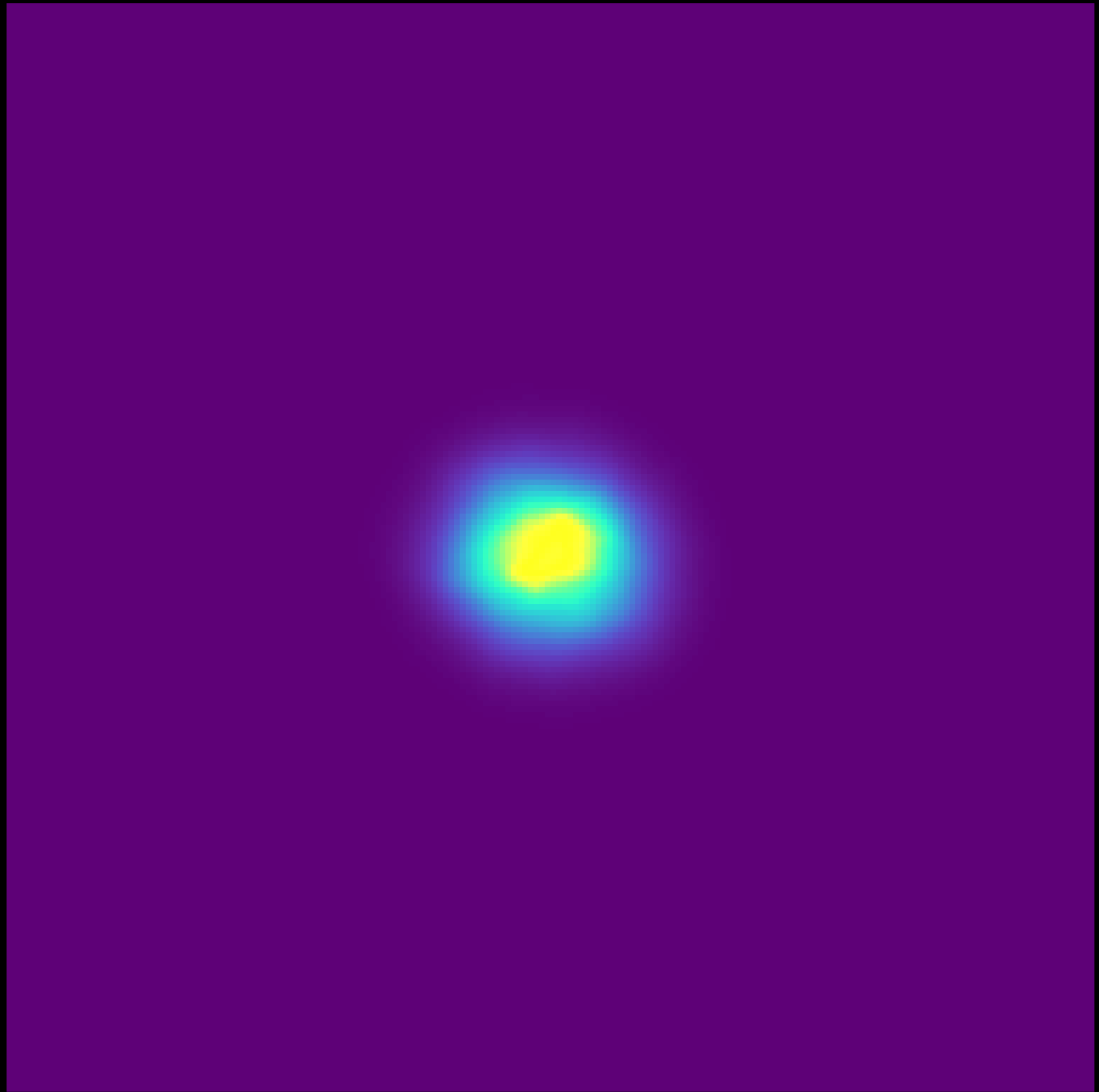
The prior galaxy

$$P(s)$$



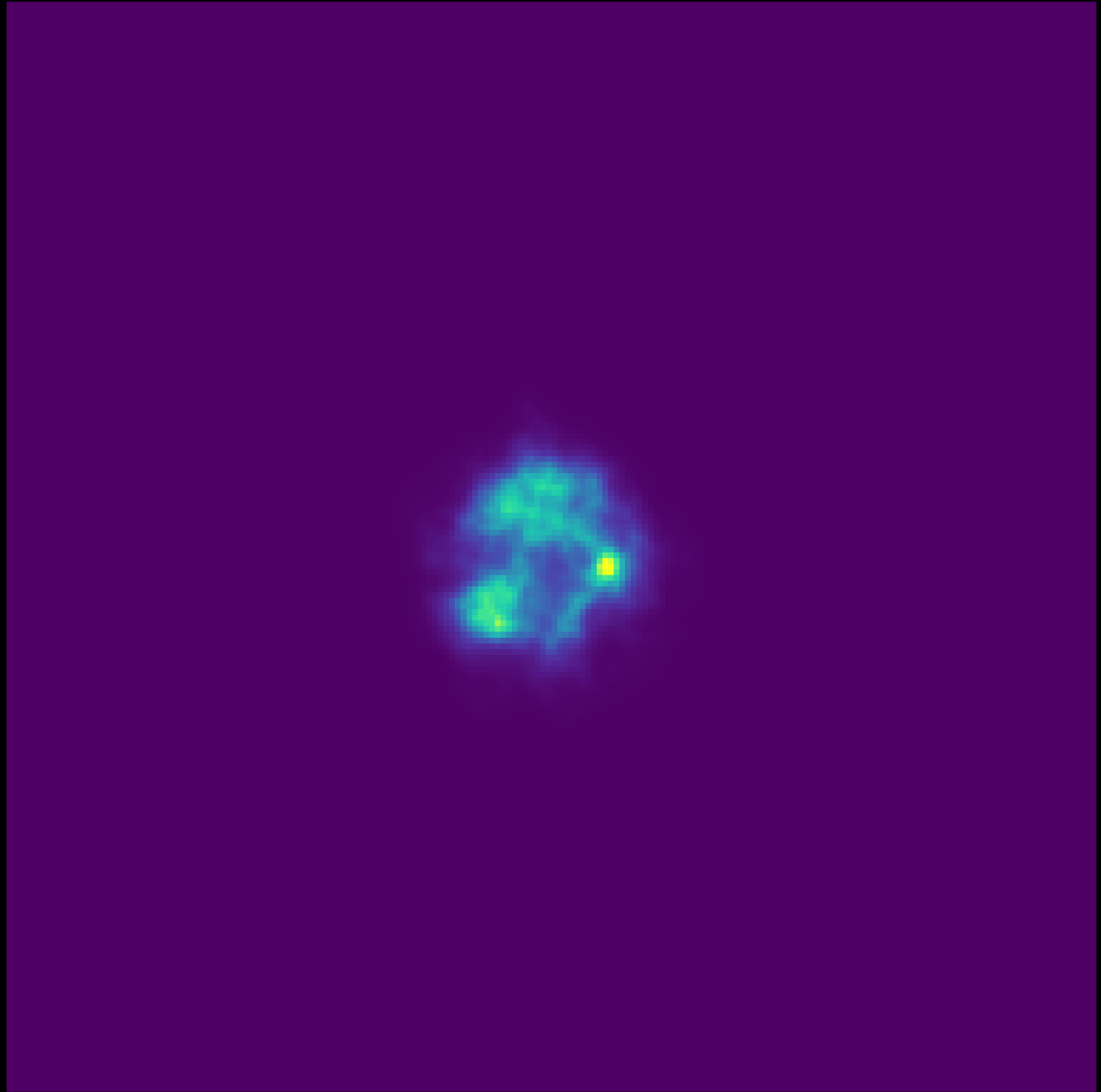
The prior galaxy

$$P(s)$$



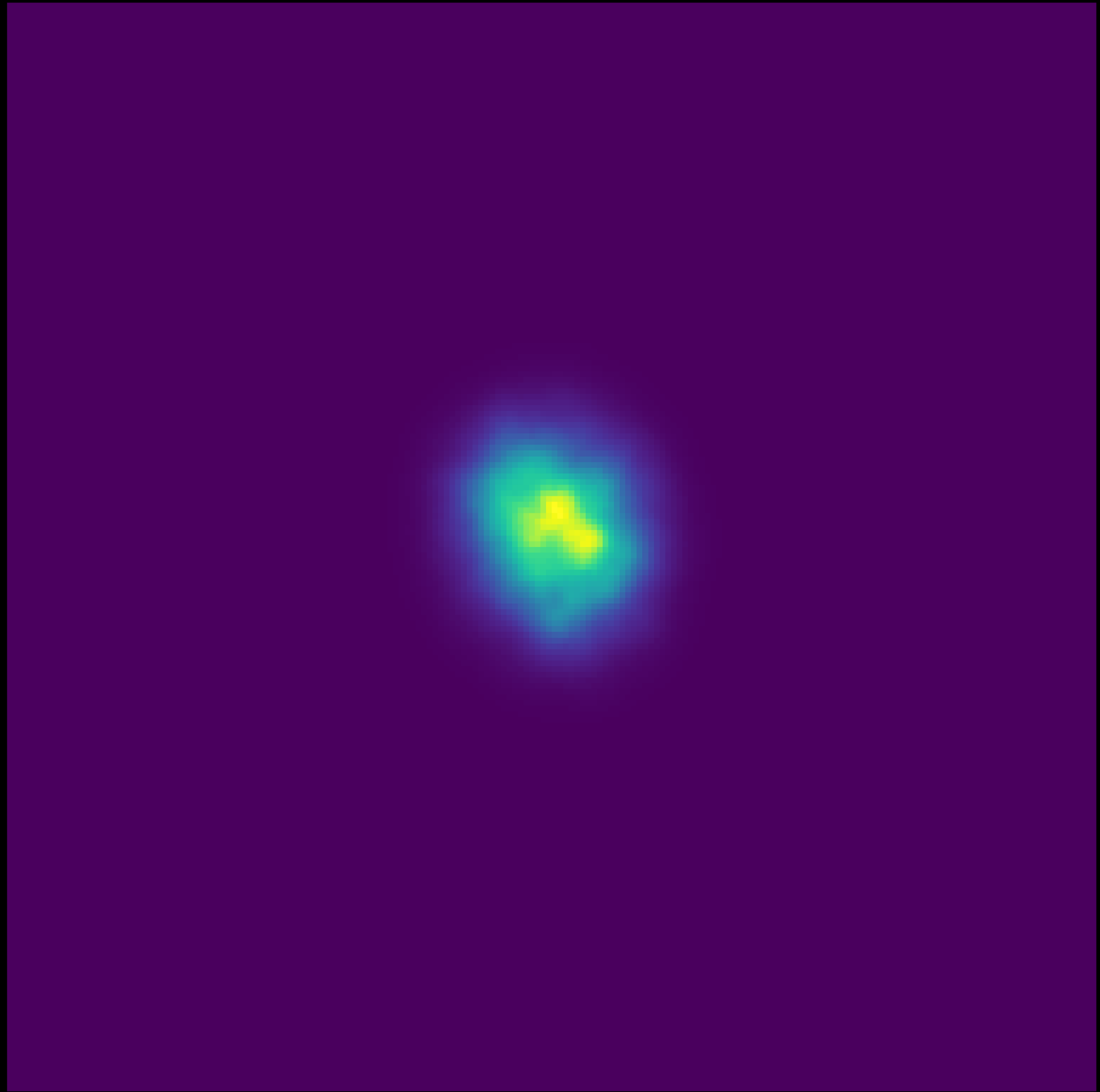
The prior galaxy

$$P(s)$$



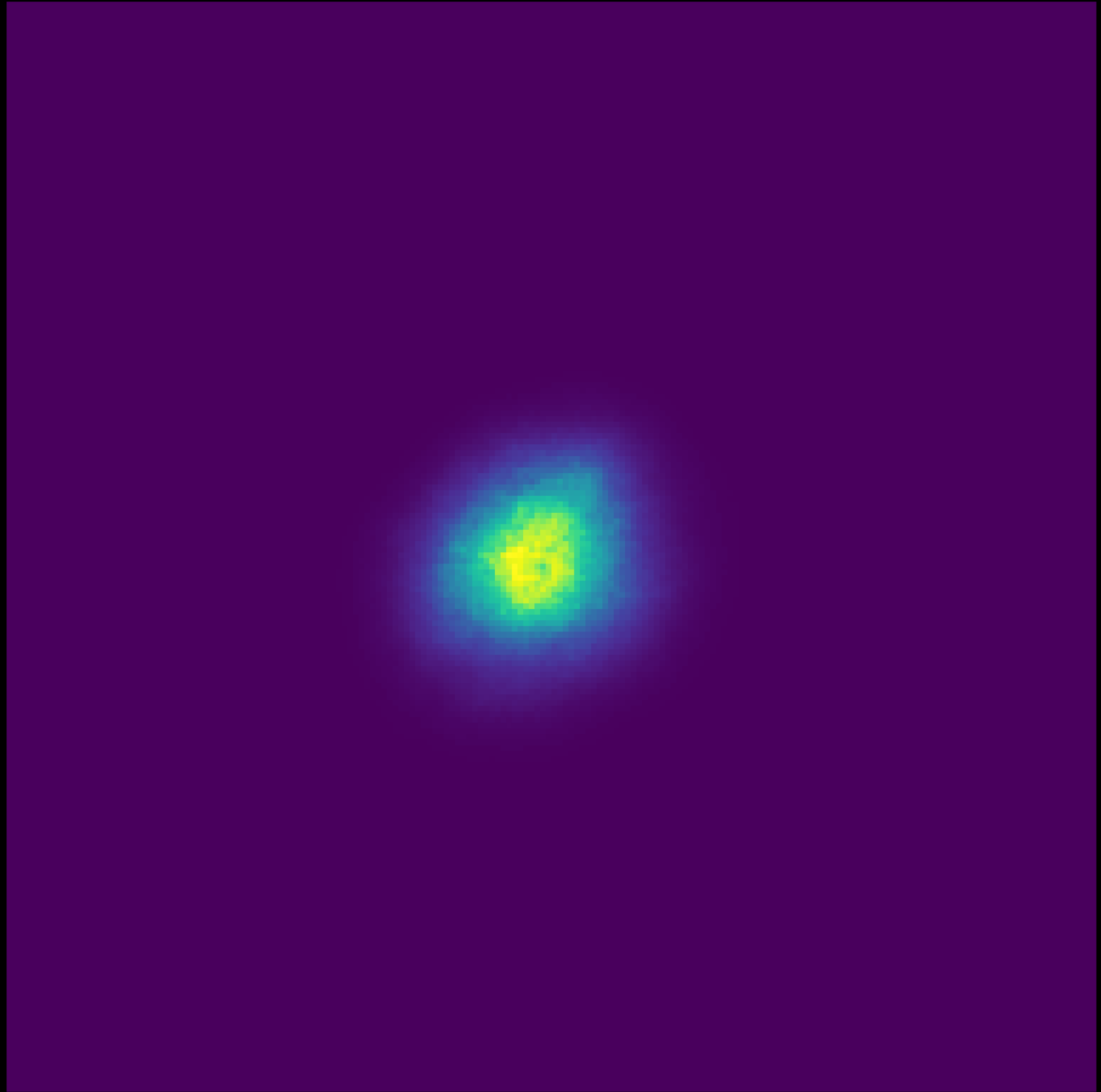
The prior galaxy

$$P(s)$$

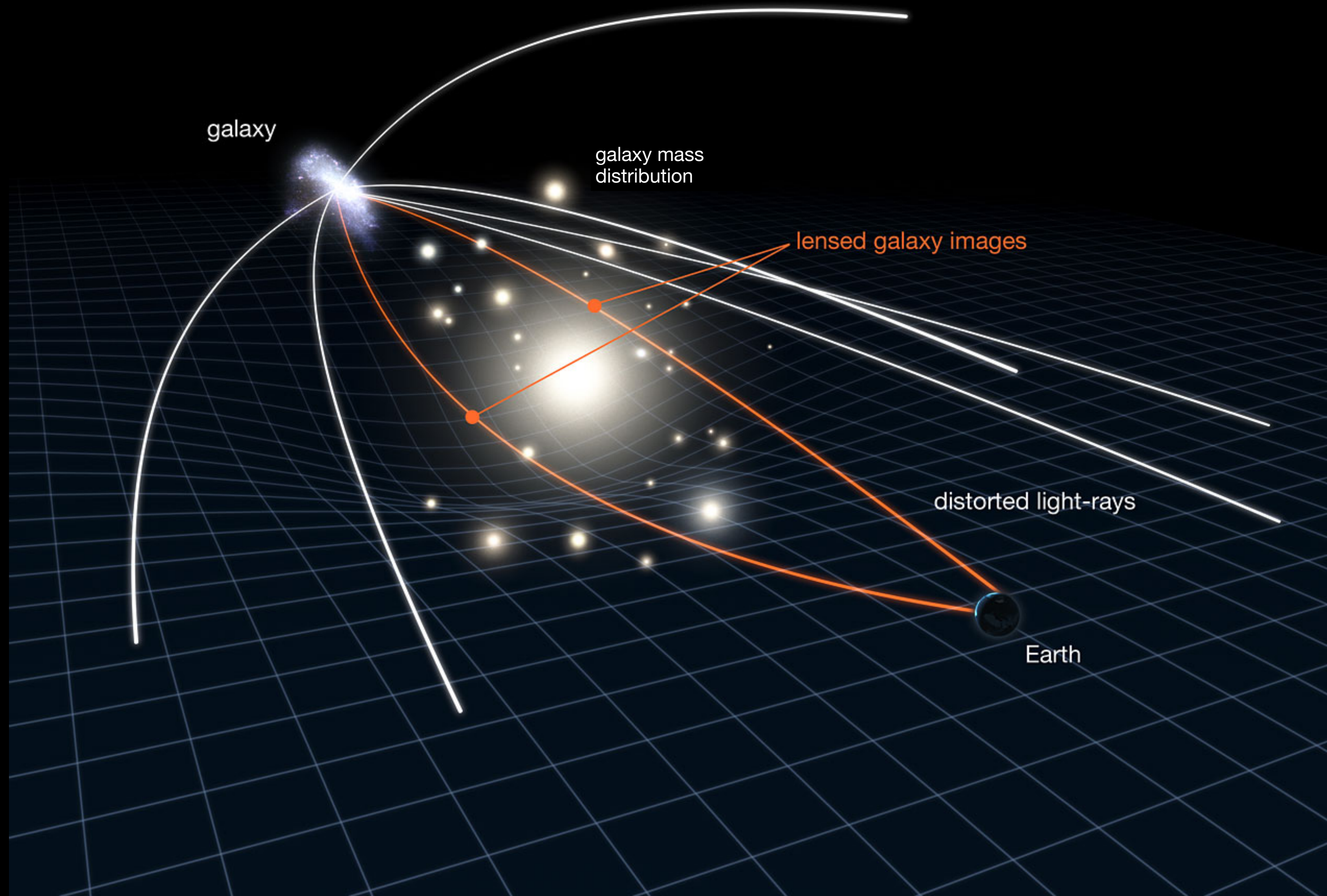


The prior galaxy

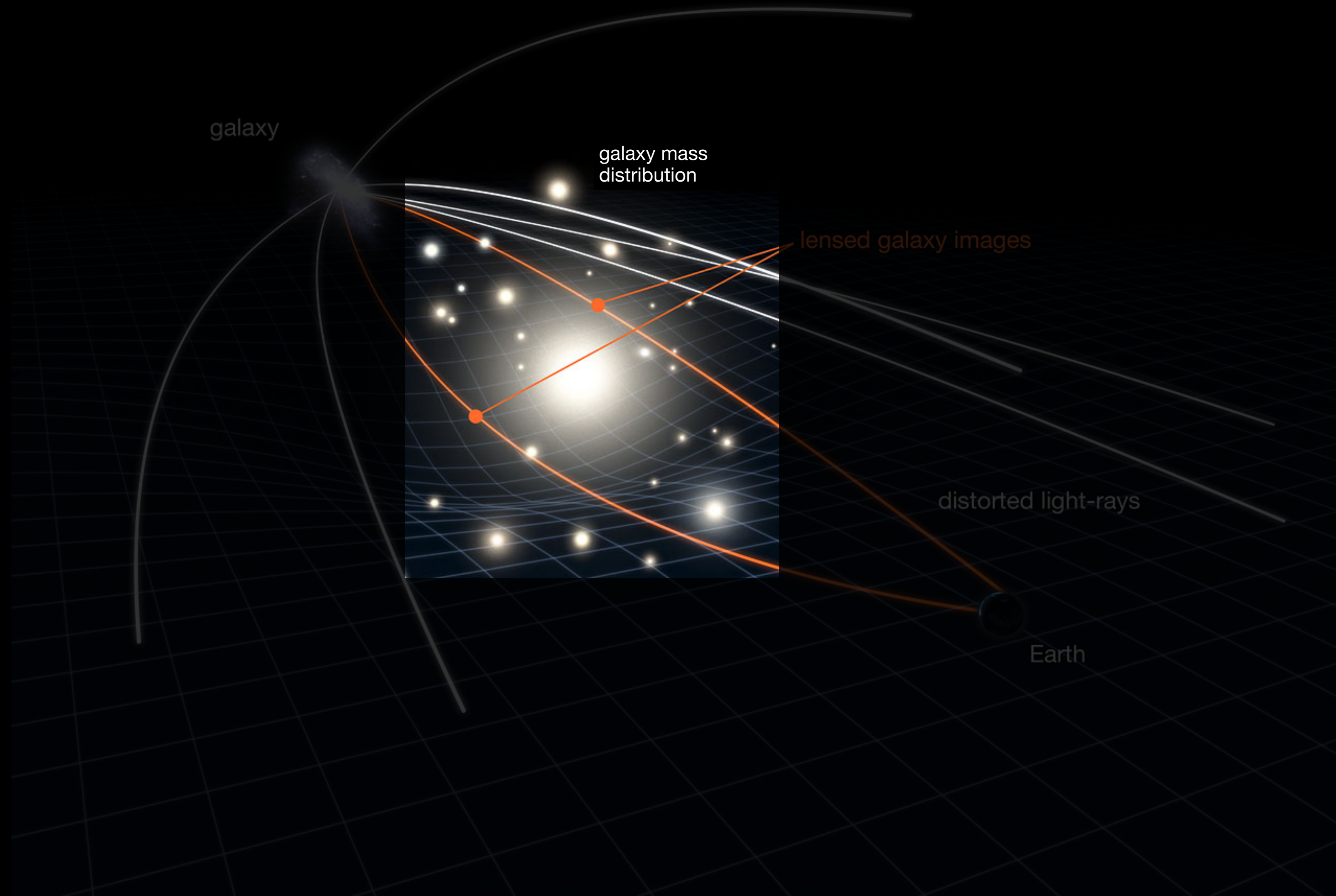
$$P(s)$$



The convergence



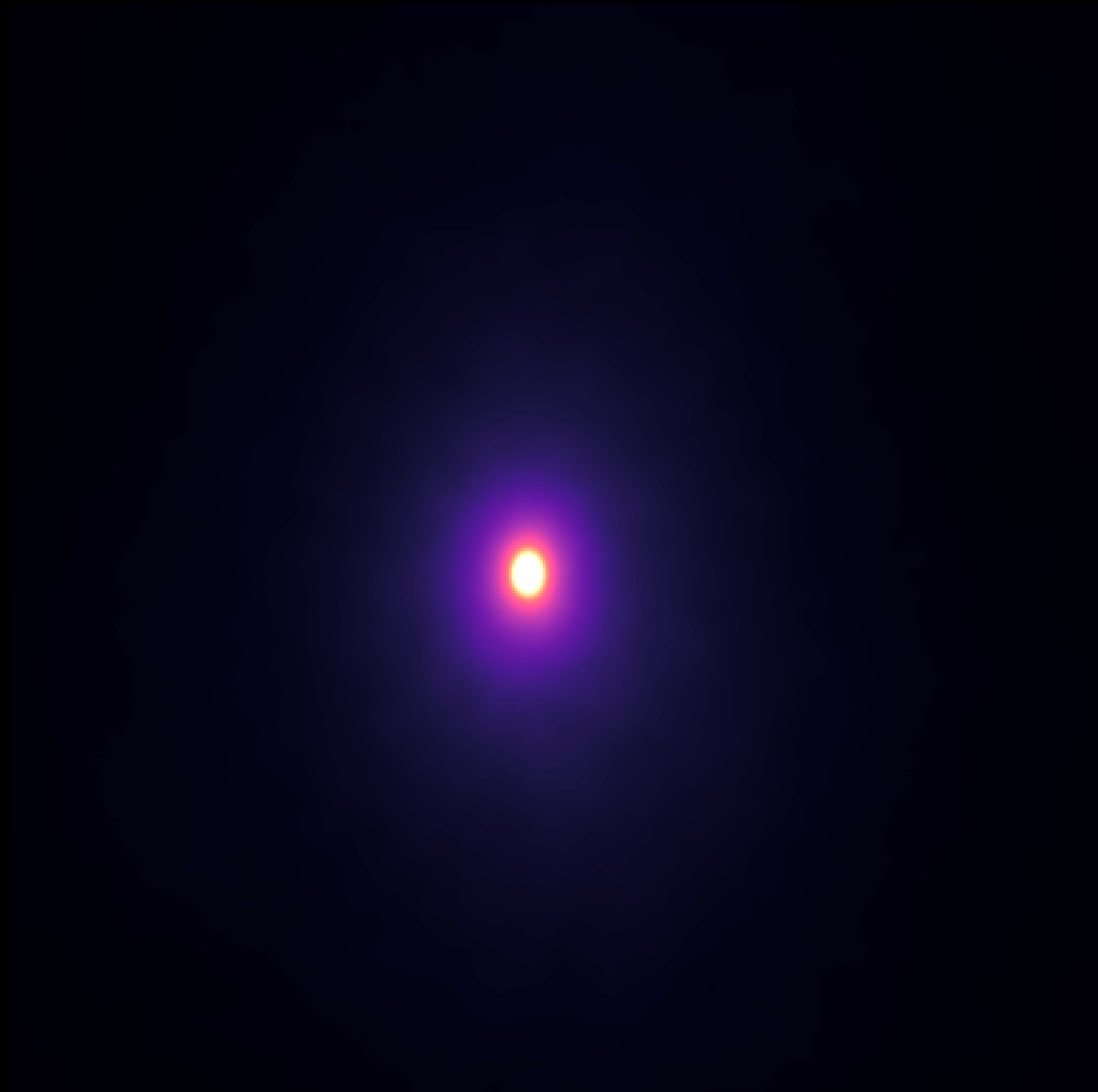
The convergence



The prior

Convergence

$$P(s)$$



The prior

Convergence

$$P(s)$$



The prior

Convergence

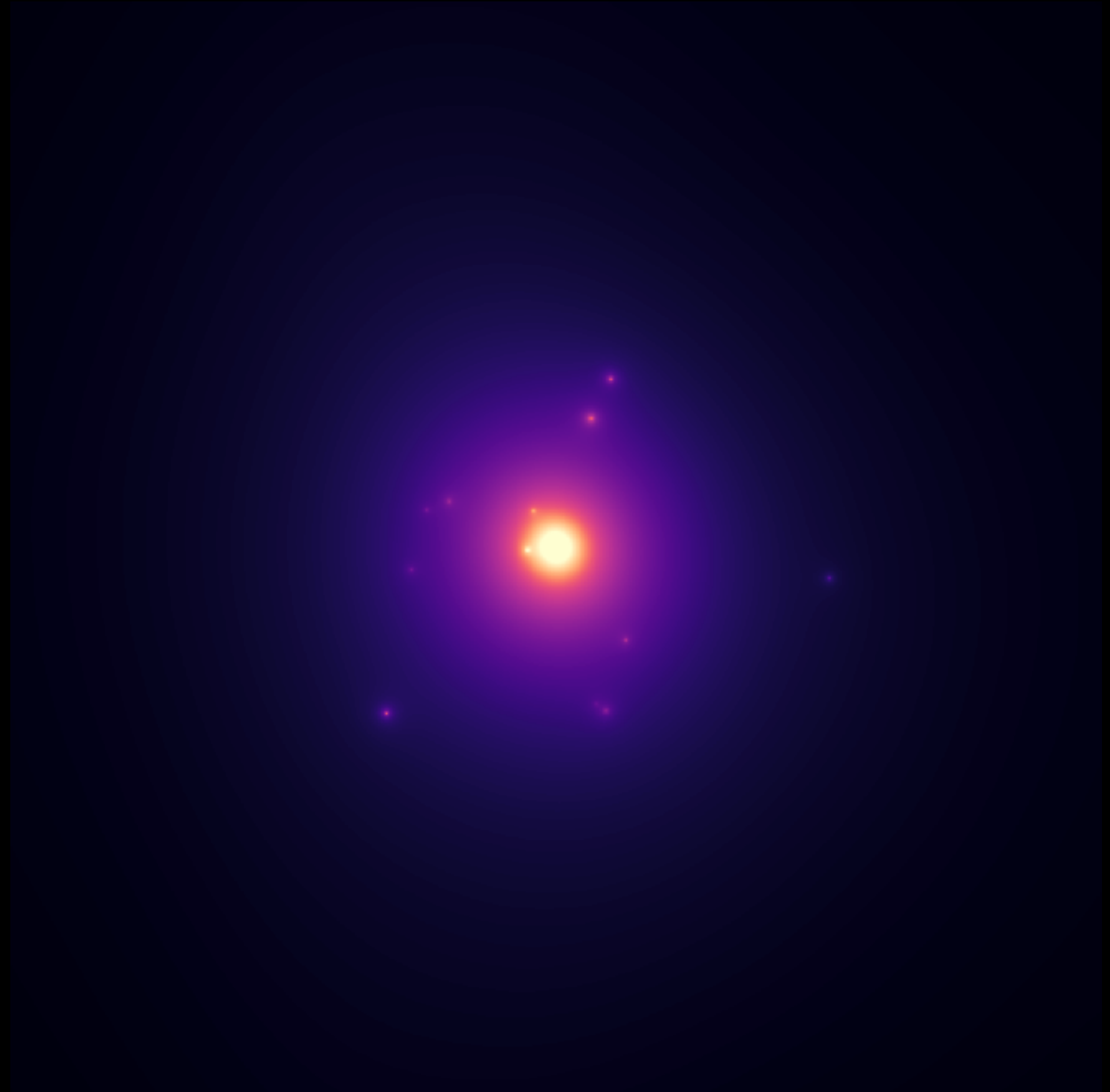
$$P(s)$$



The prior

Convergence

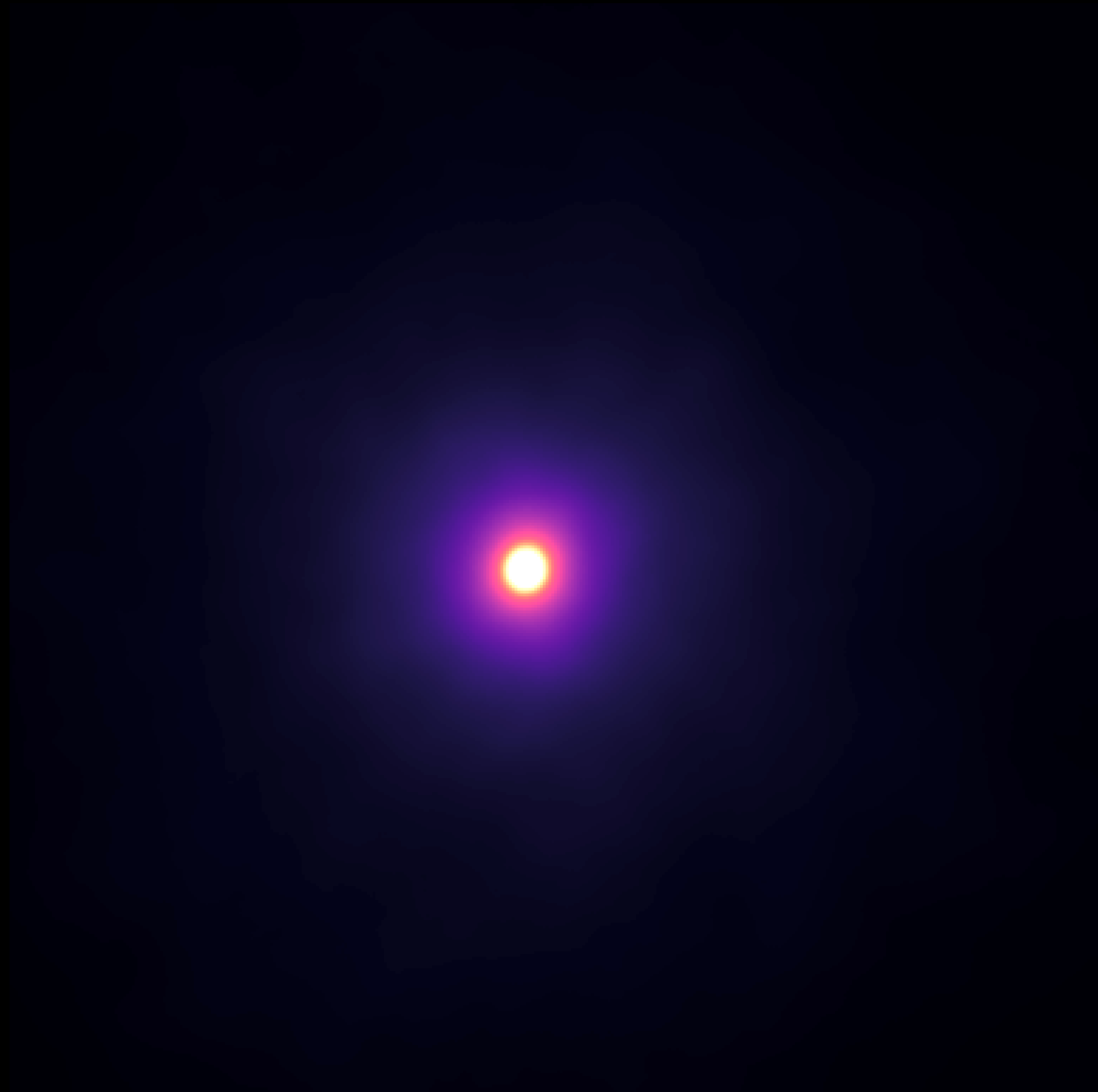
$$P(s)$$



The prior

Convergence

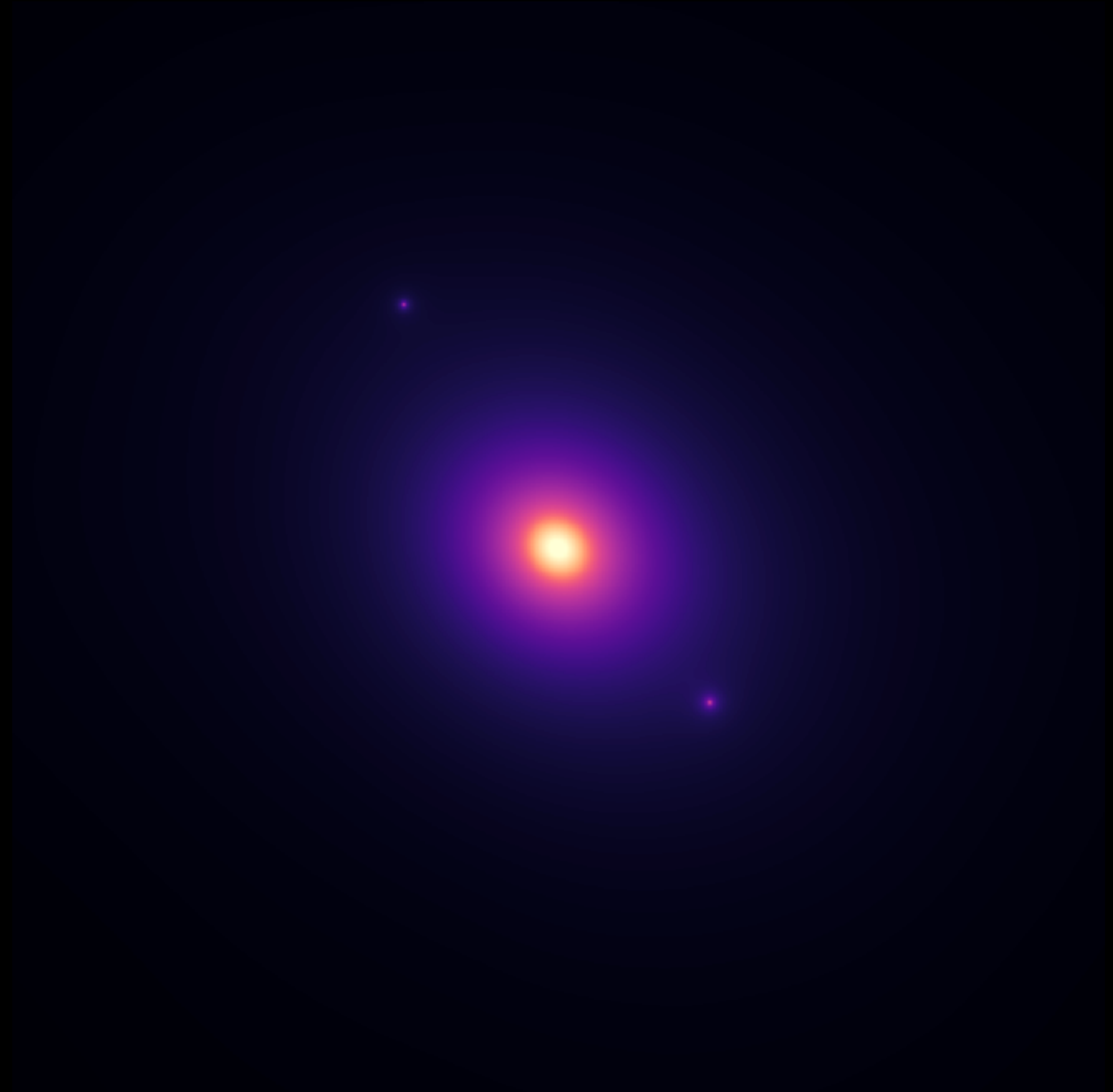
$$P(s)$$



The prior

Convergence

$$P(s)$$

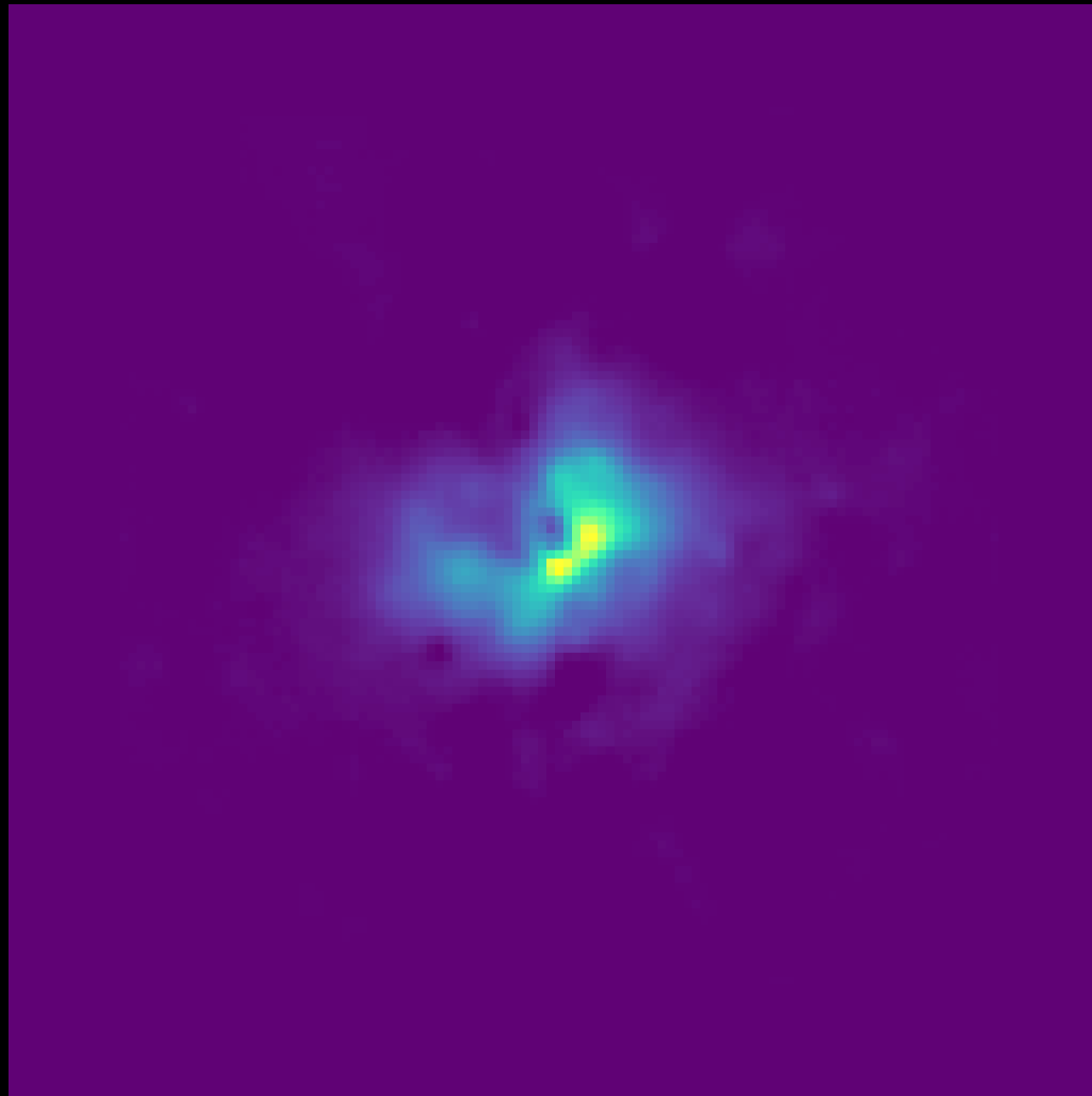


The likelihood

$$P(s | d) = \frac{P(d | s) P(s)}{P(d)}$$

The likelihood

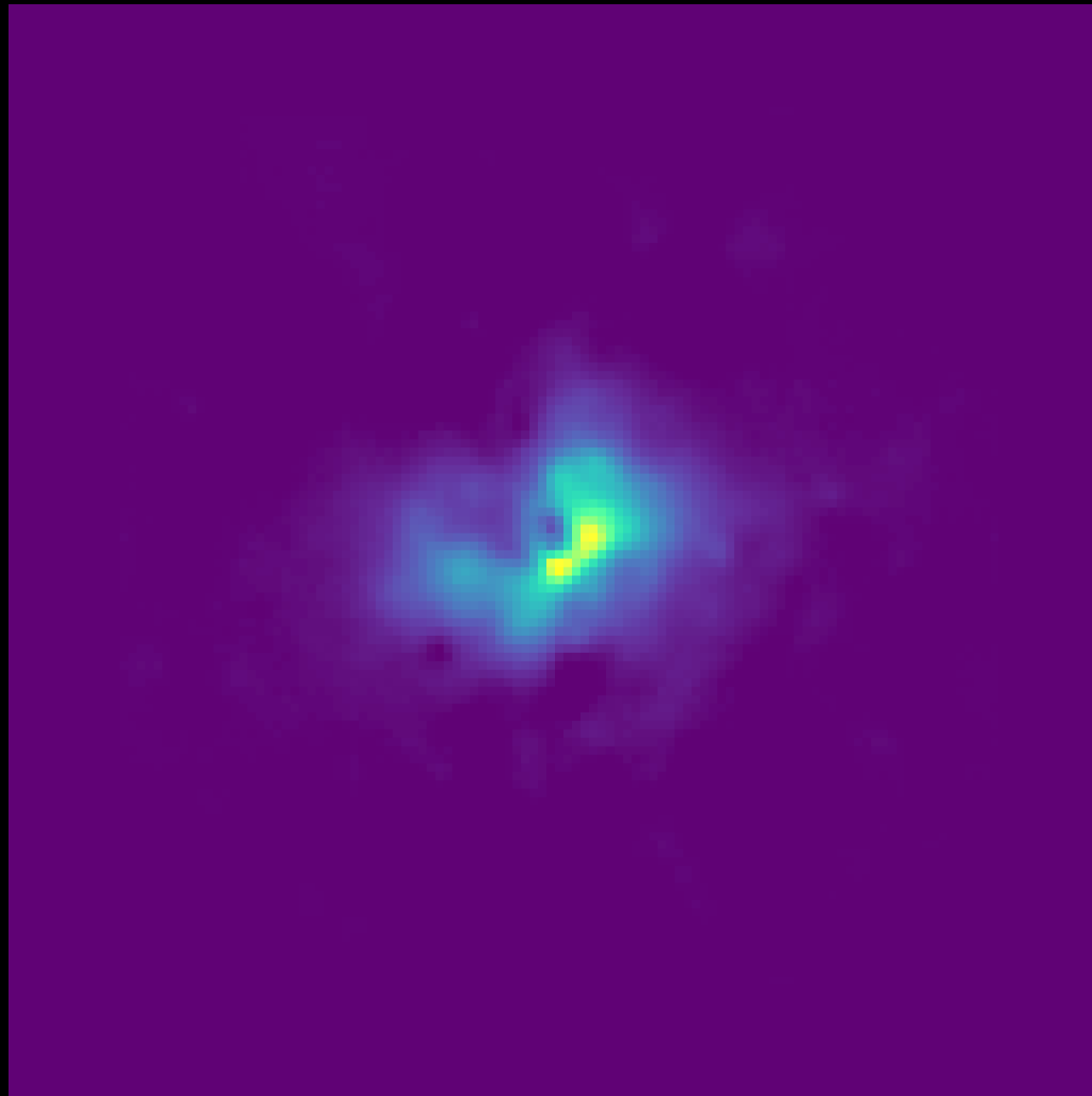
Strong lensing response



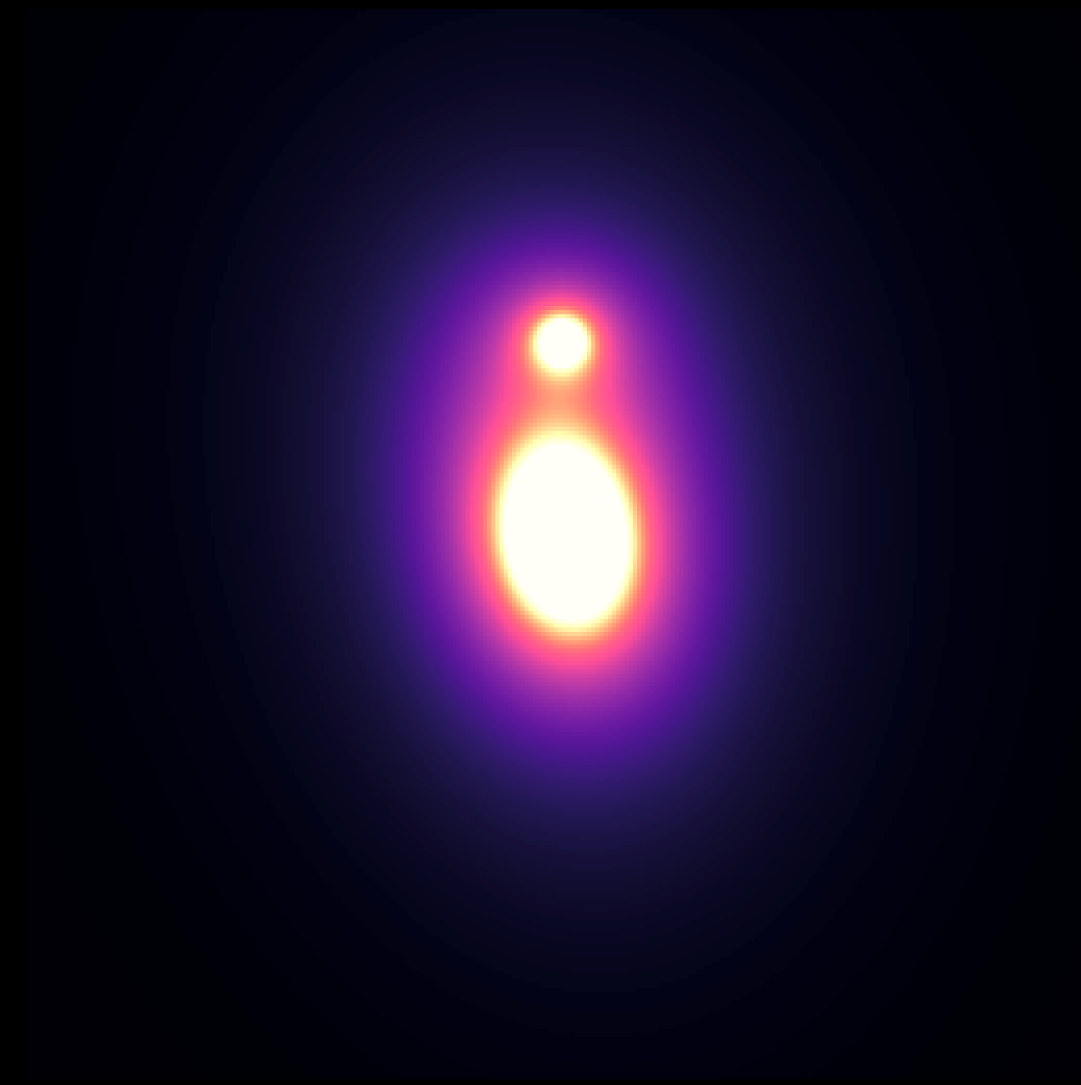
Source galaxy

The likelihood

Strong lensing response



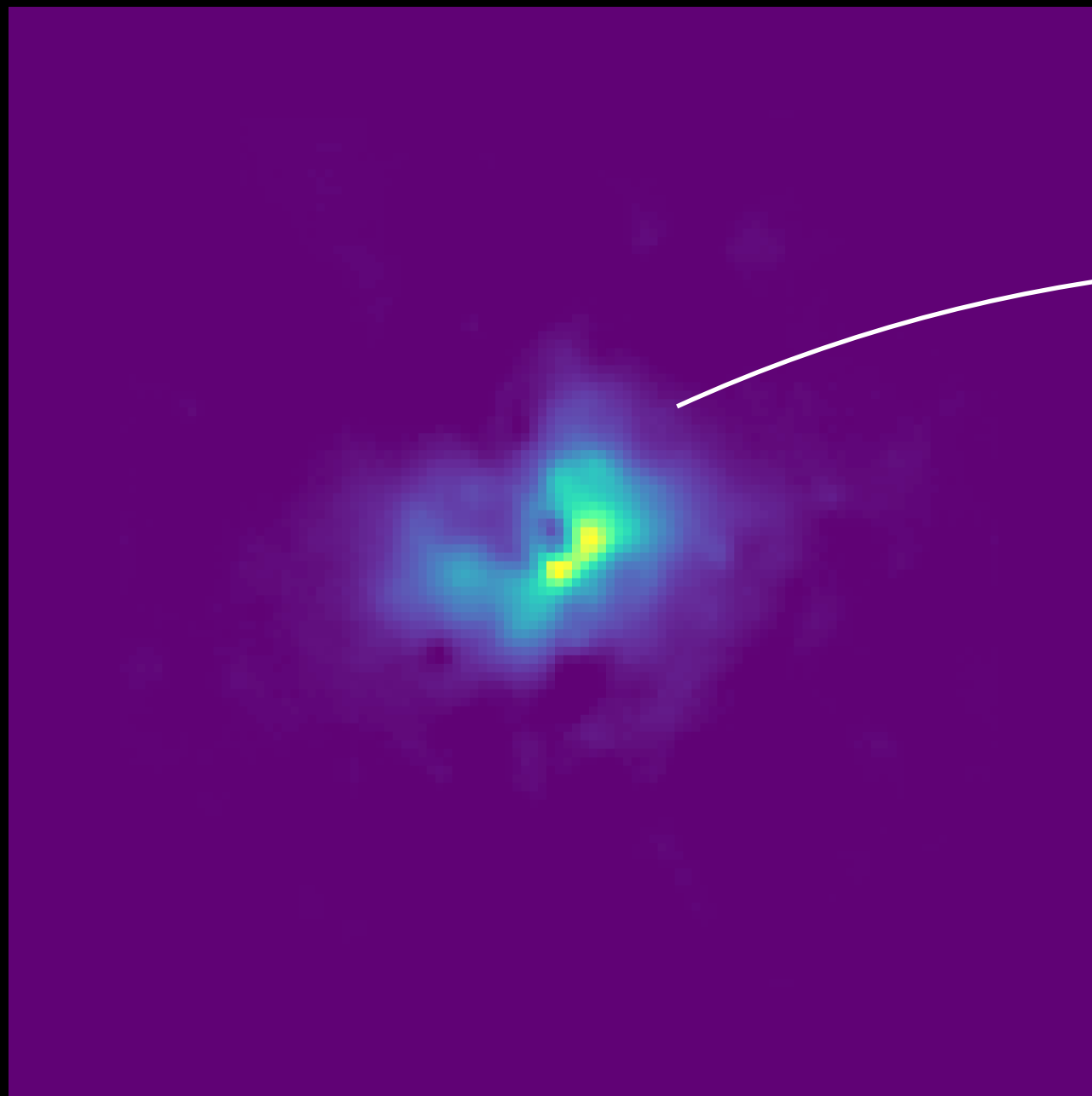
Source galaxy



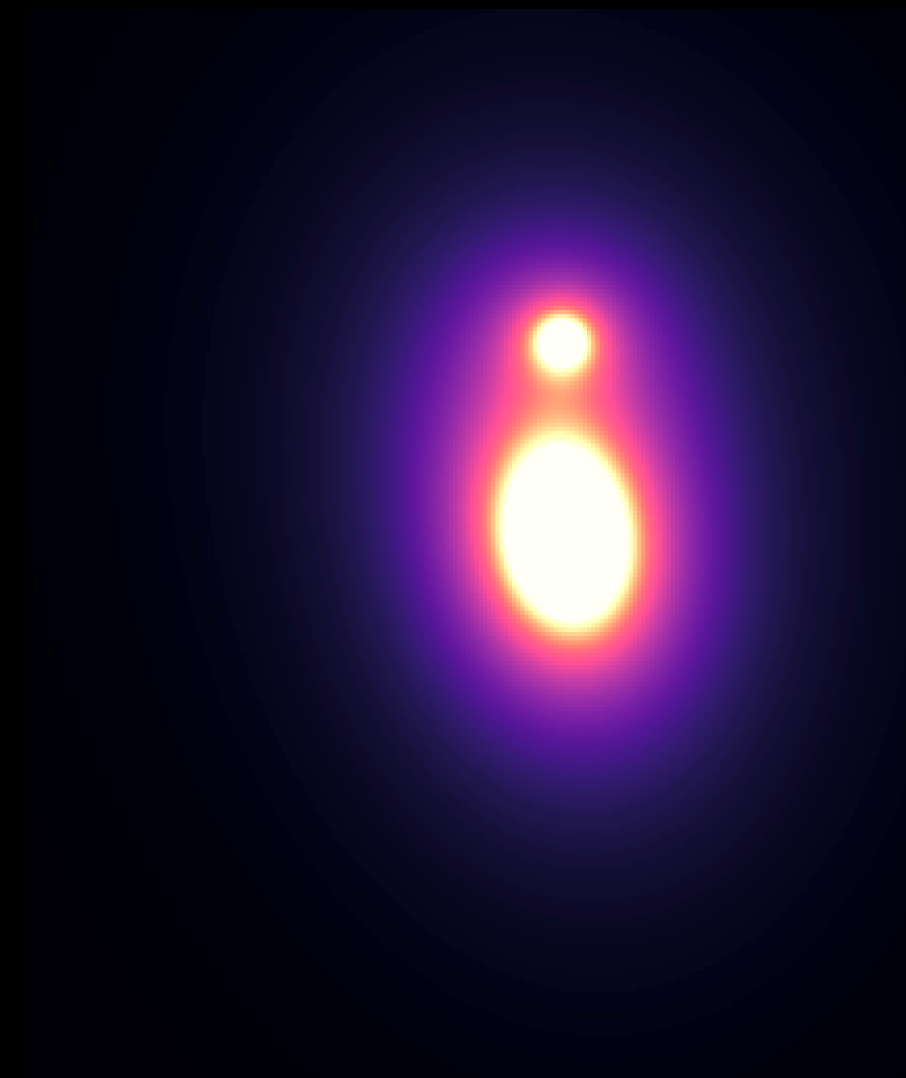
DM halo mass profile

The likelihood

Strong lensing response



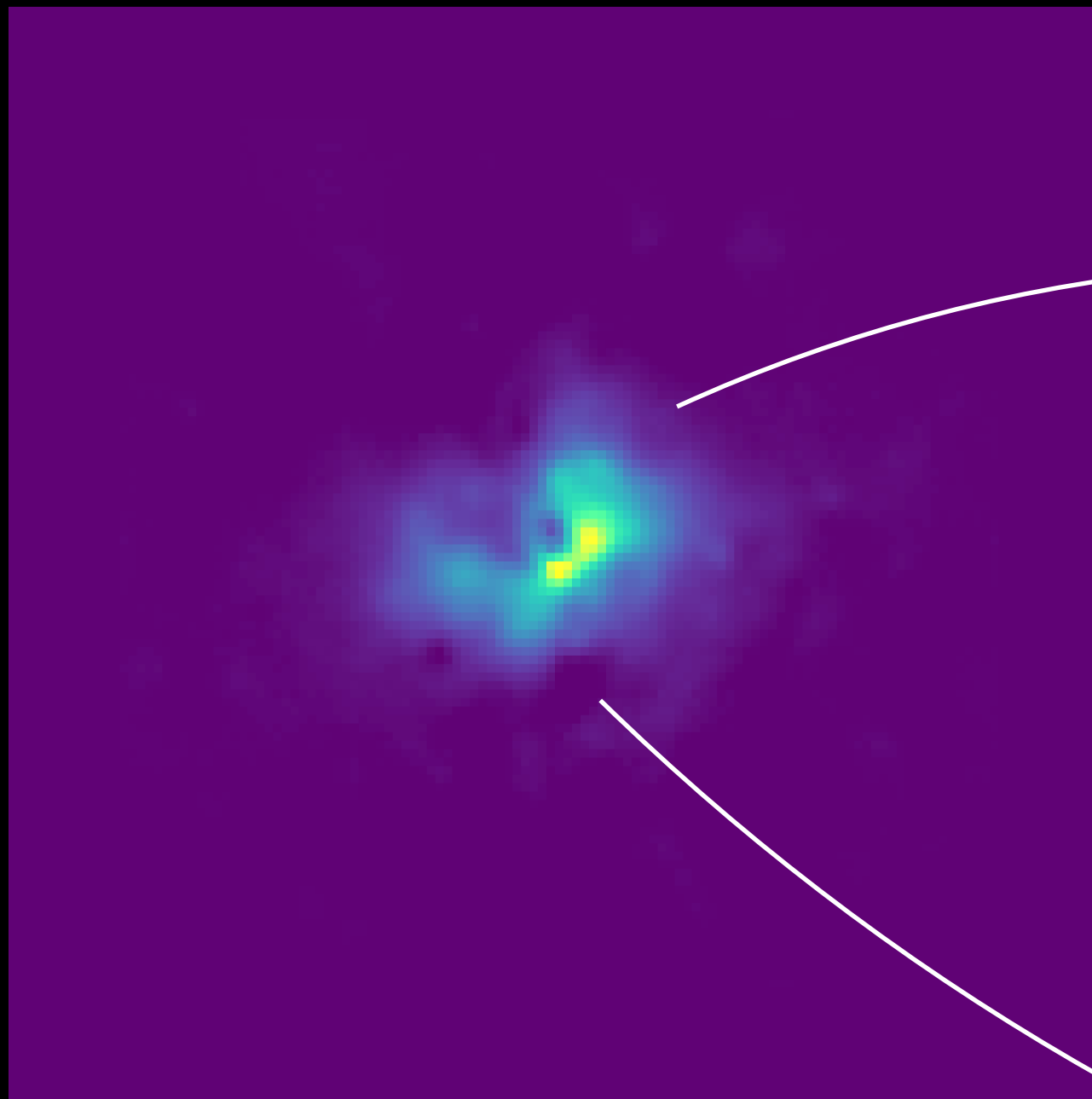
Source galaxy



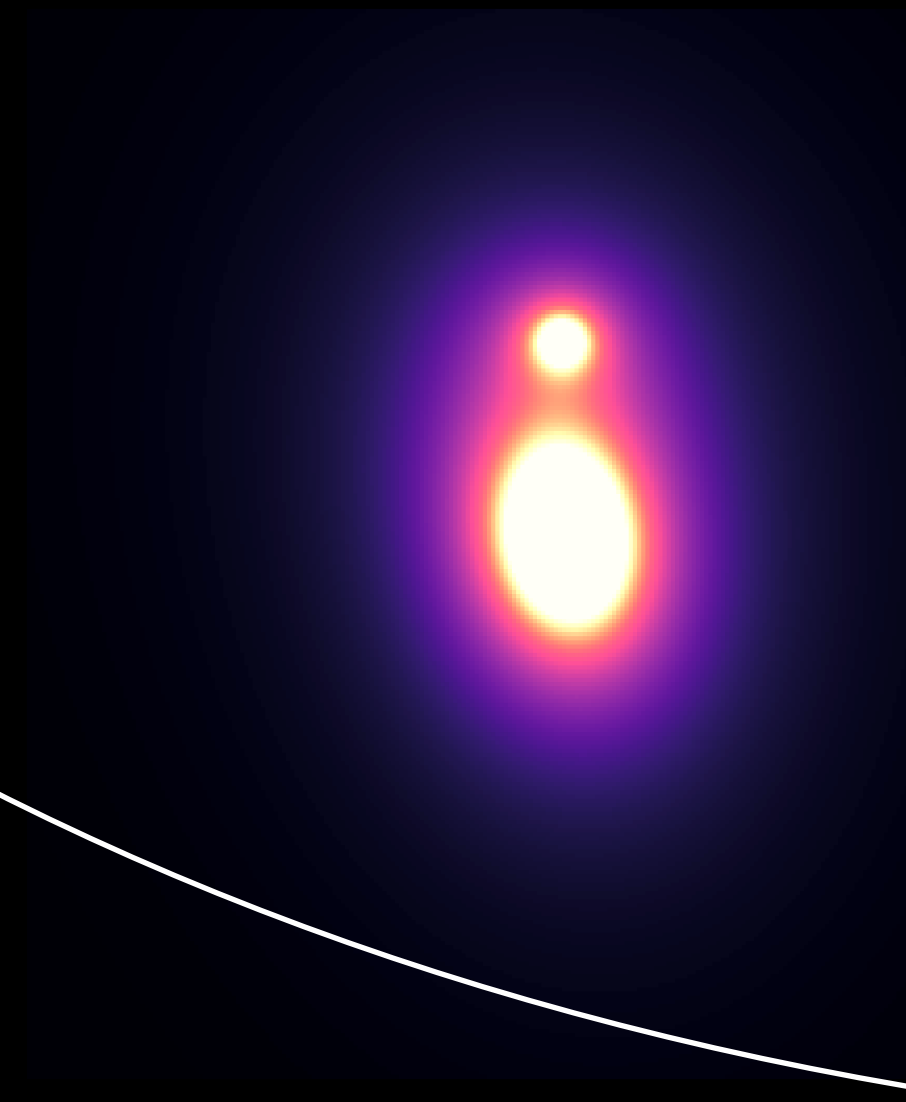
DM halo mass profile

The likelihood

Strong lensing response



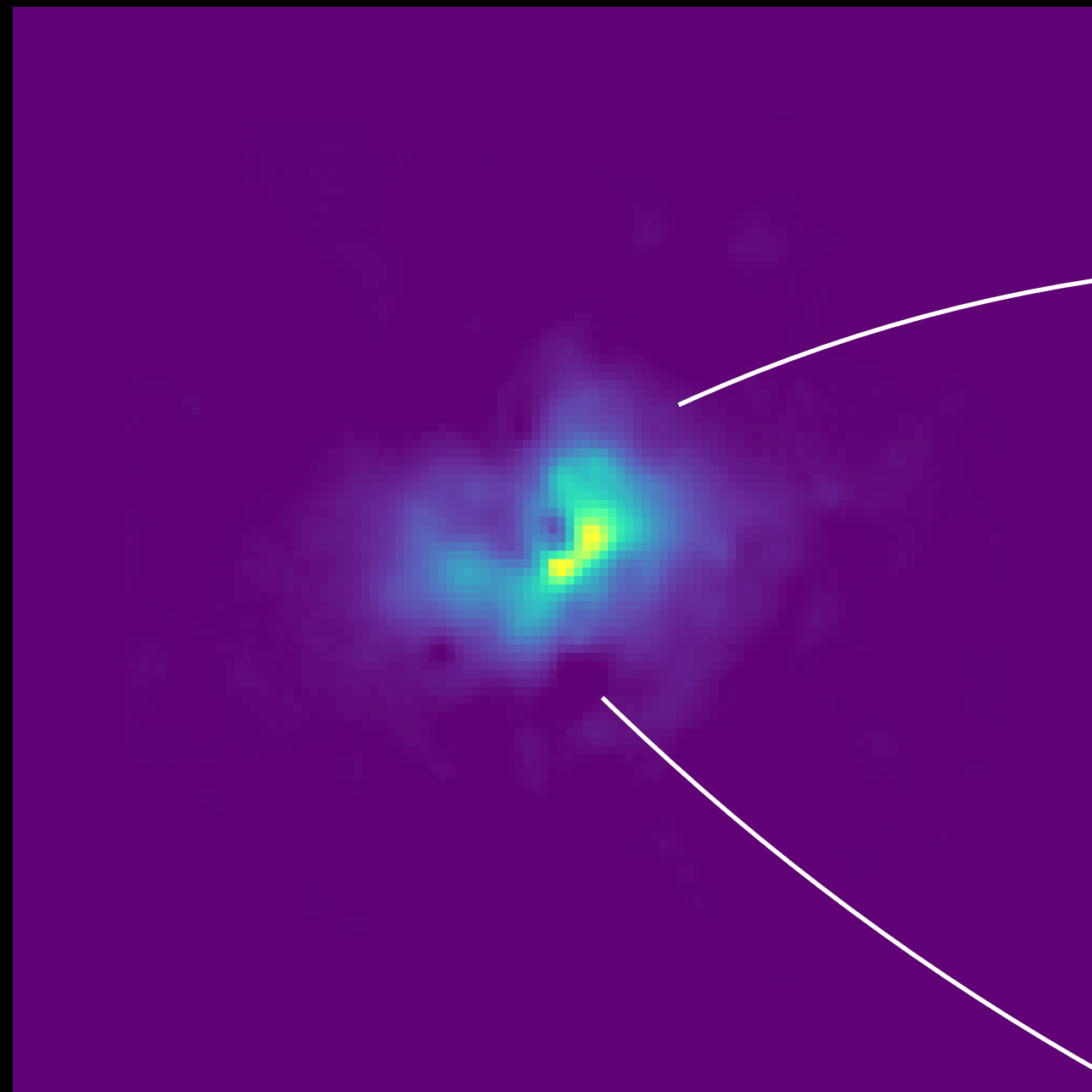
Source galaxy



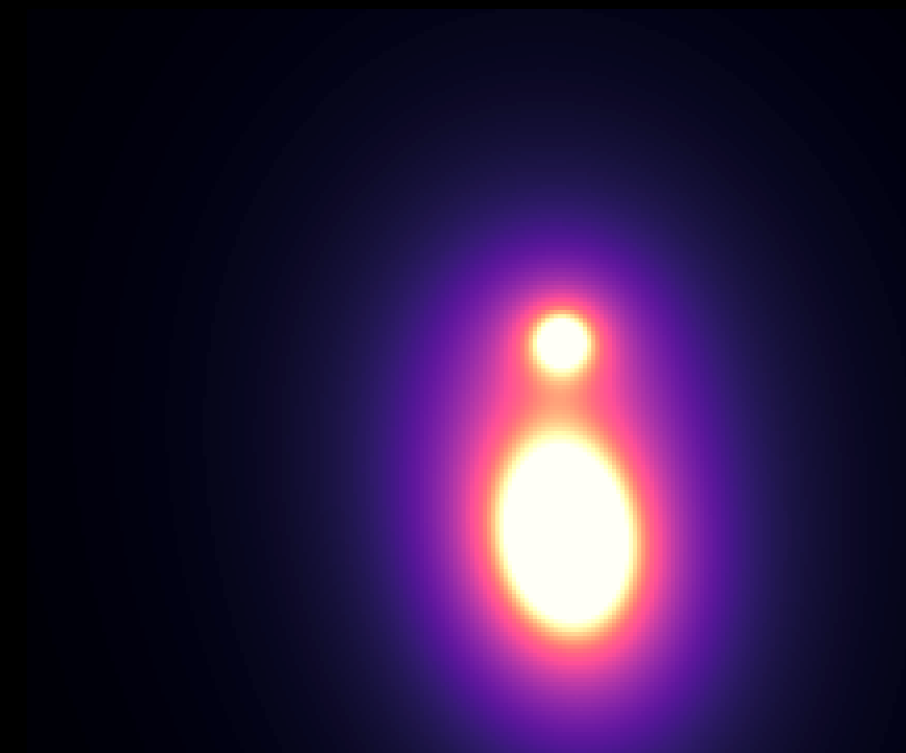
DM halo mass profile

The likelihood

Strong lensing response



Source galaxy

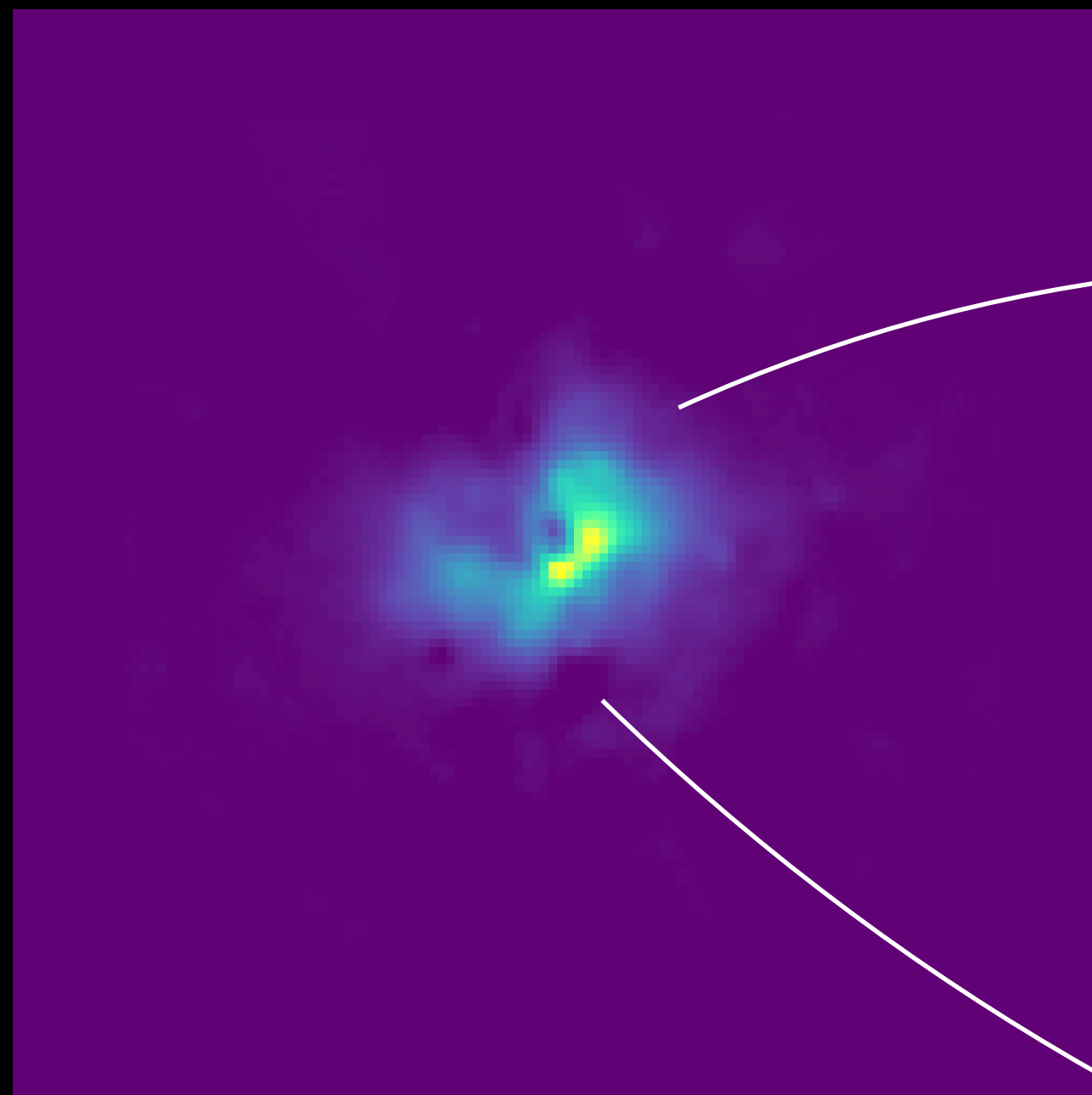


DM halo mass profile

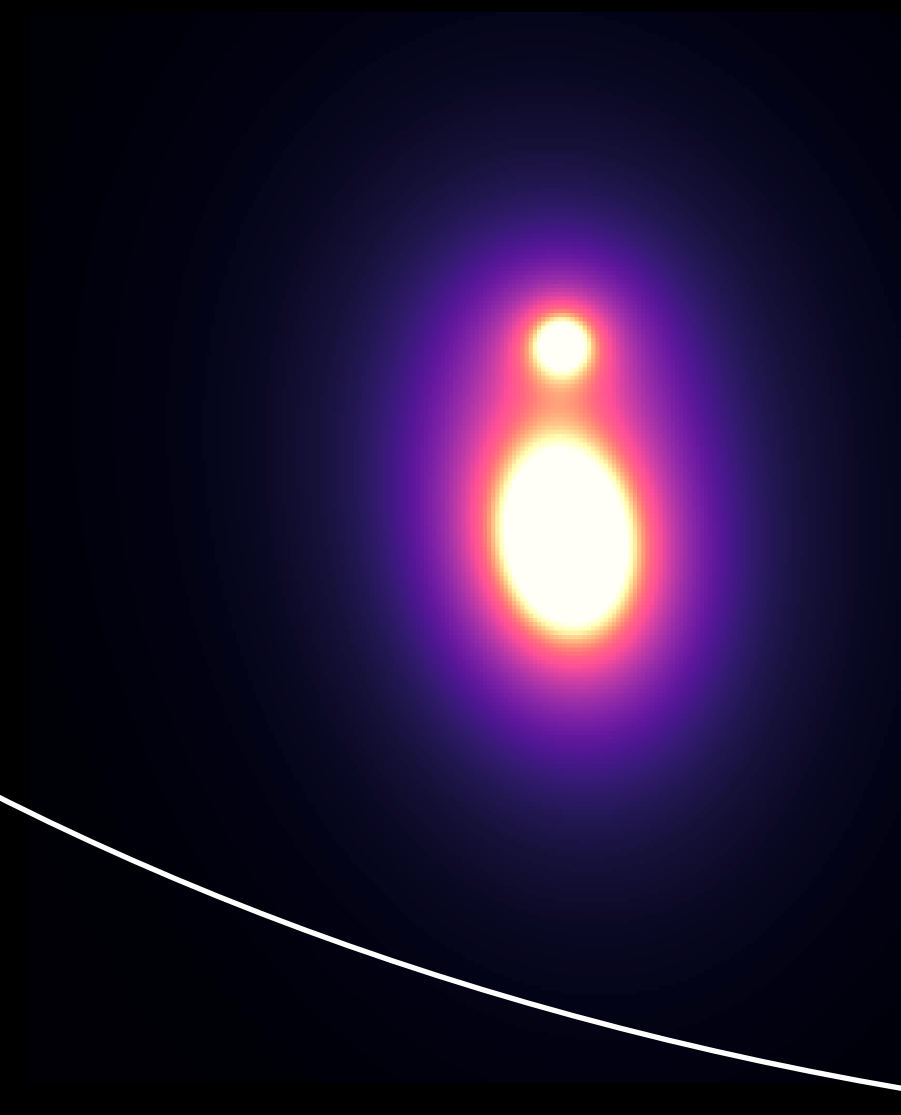
$$\nabla \cdot \alpha = 2\kappa$$

The likelihood

Strong lensing response

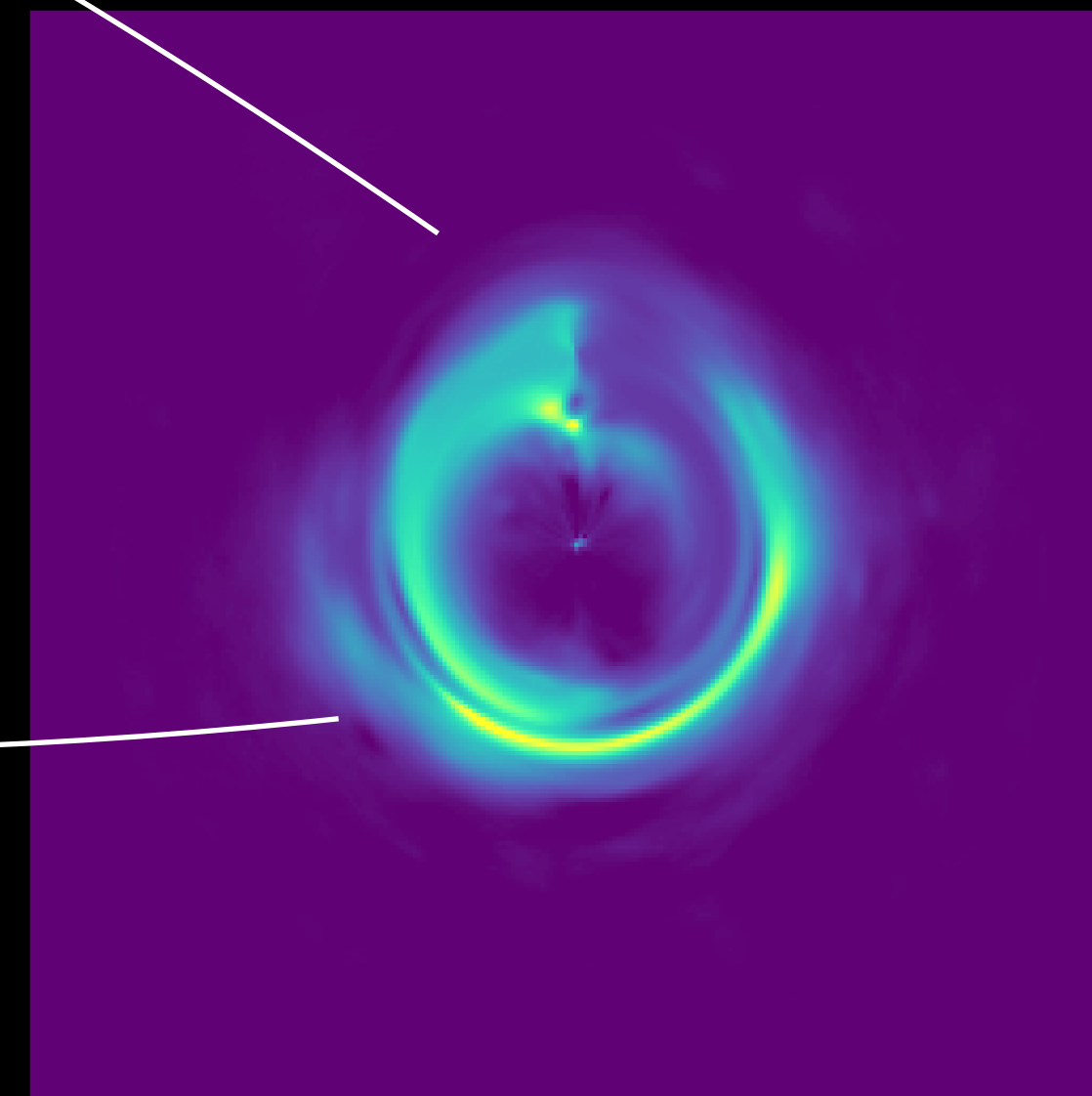


Source galaxy



DM halo mass profile

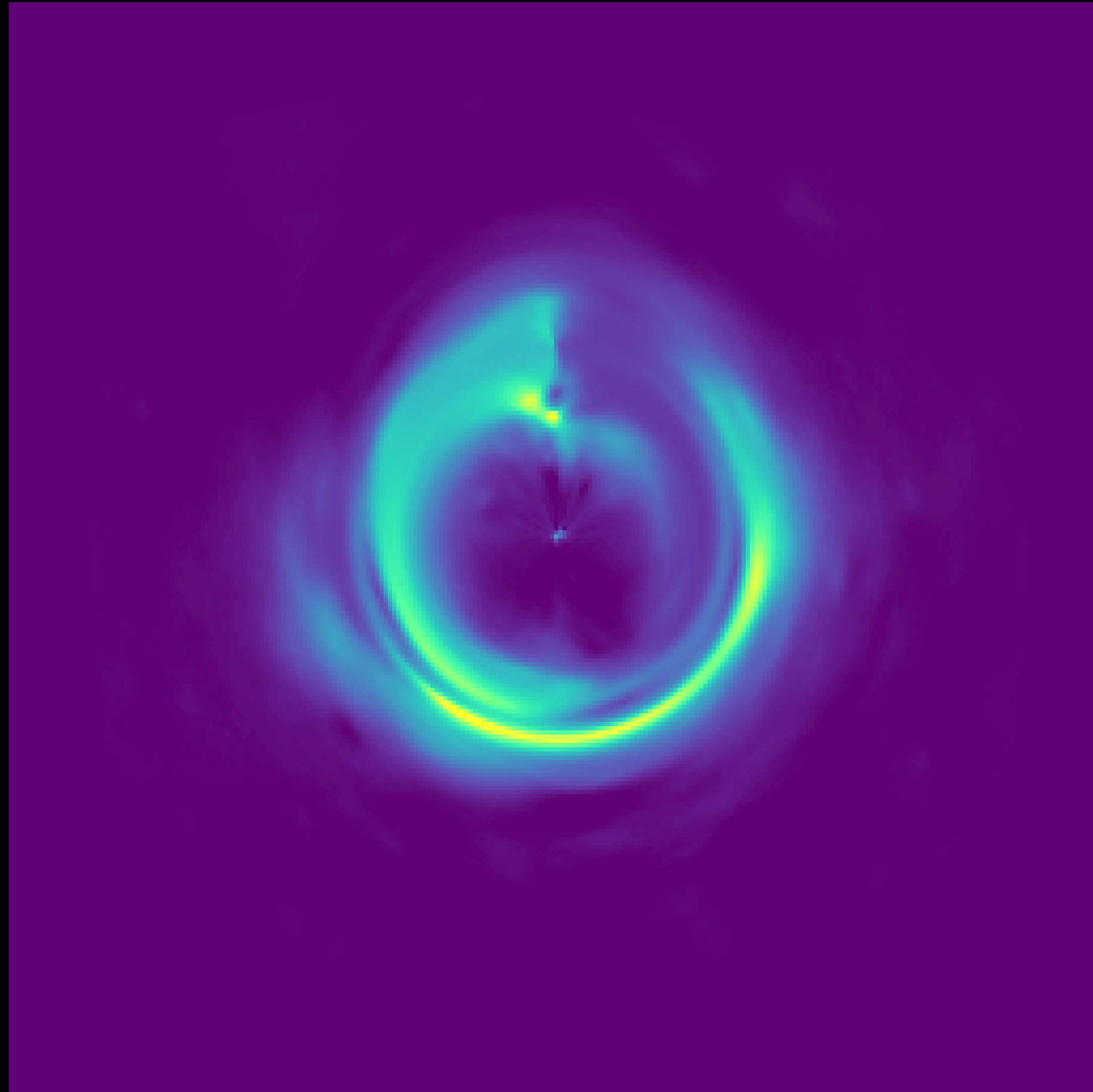
$$\nabla \cdot \alpha = 2\kappa$$



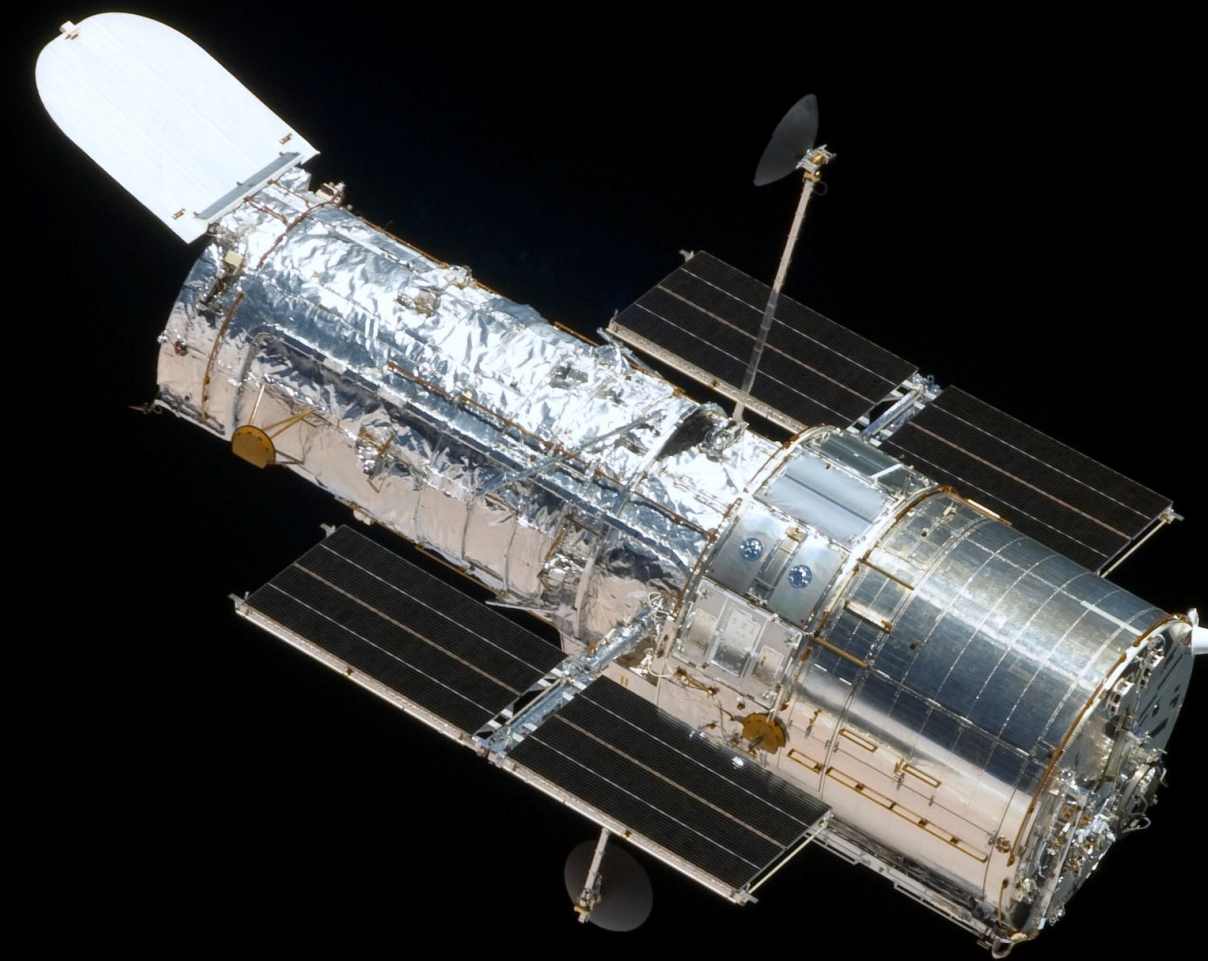
$$y = x - \alpha(\mathbf{x})$$

The likelihood

Instrument response



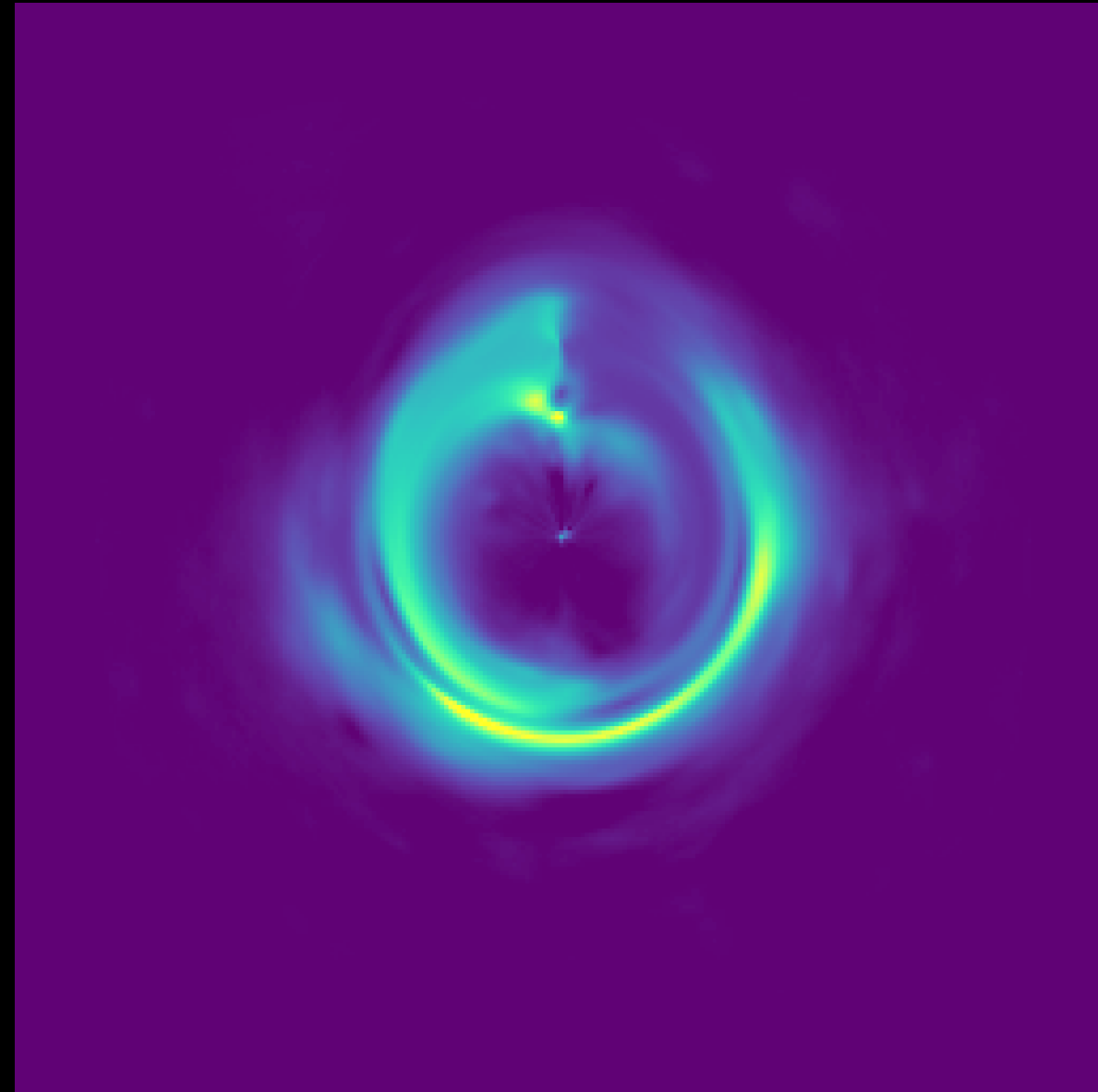
Lensed galaxy signal



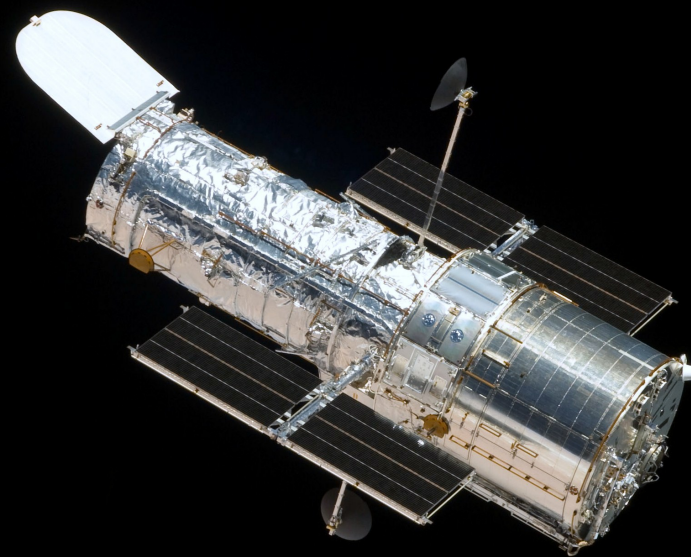
Hubble Space Telescope

The likelihood

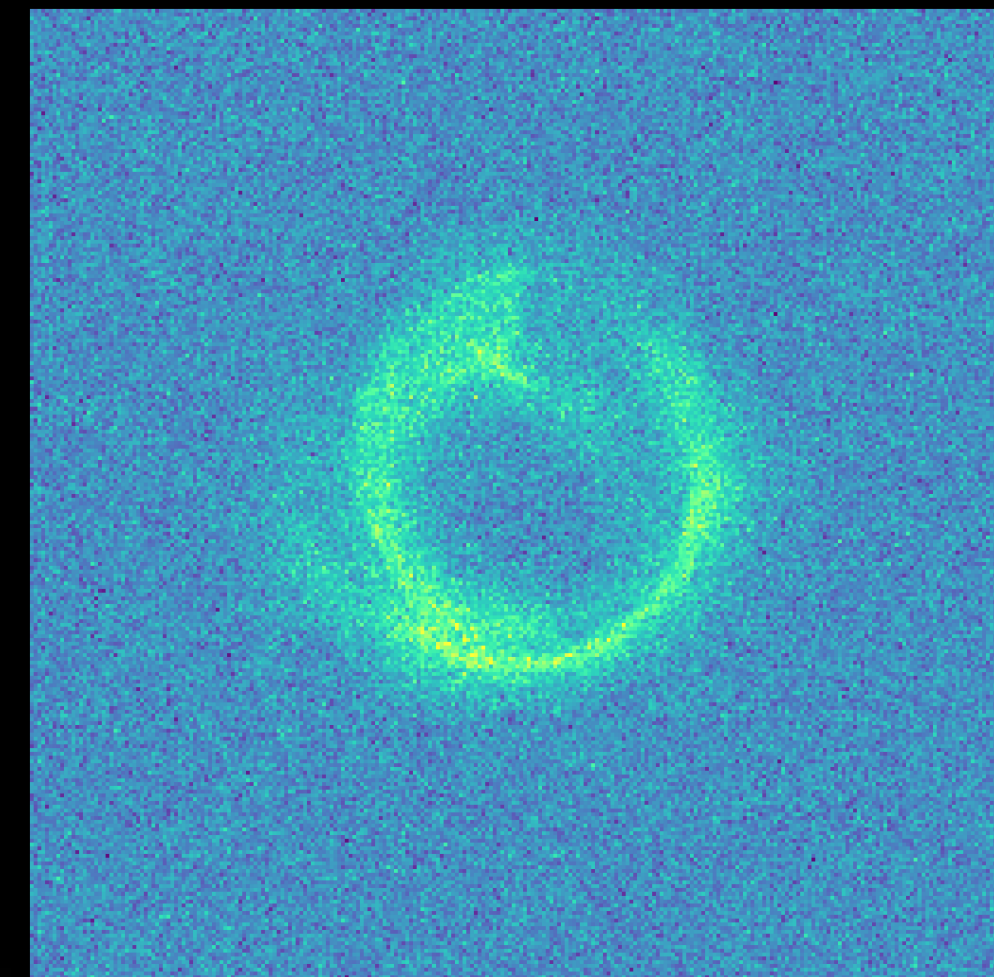
Instrument response



Lensed galaxy signal



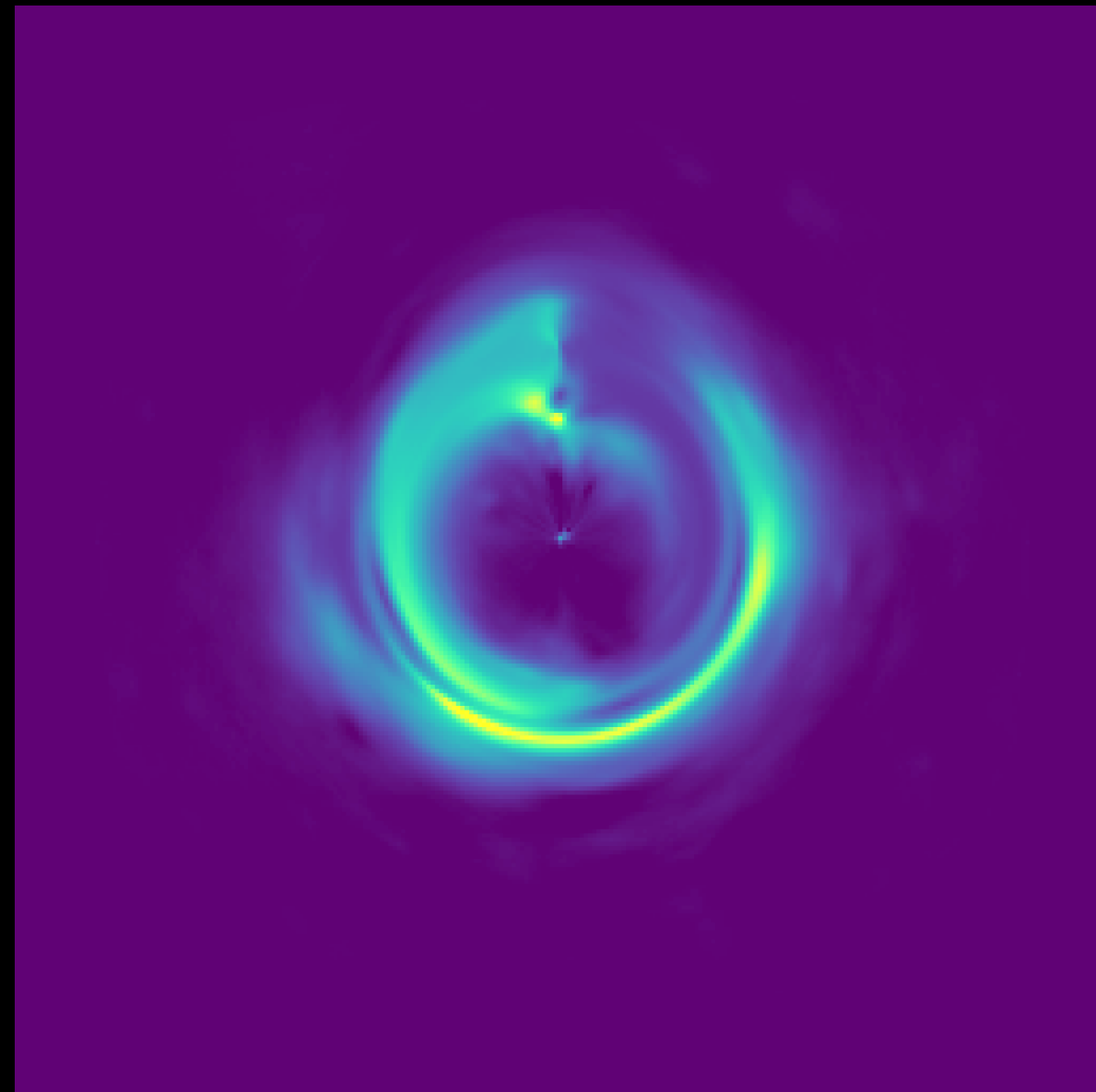
Hubble Space Telescope



Data on Earth

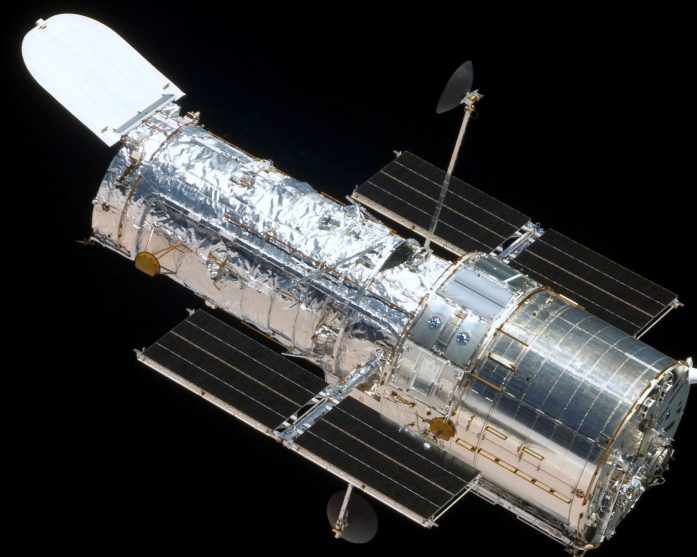
The likelihood

Instrument response



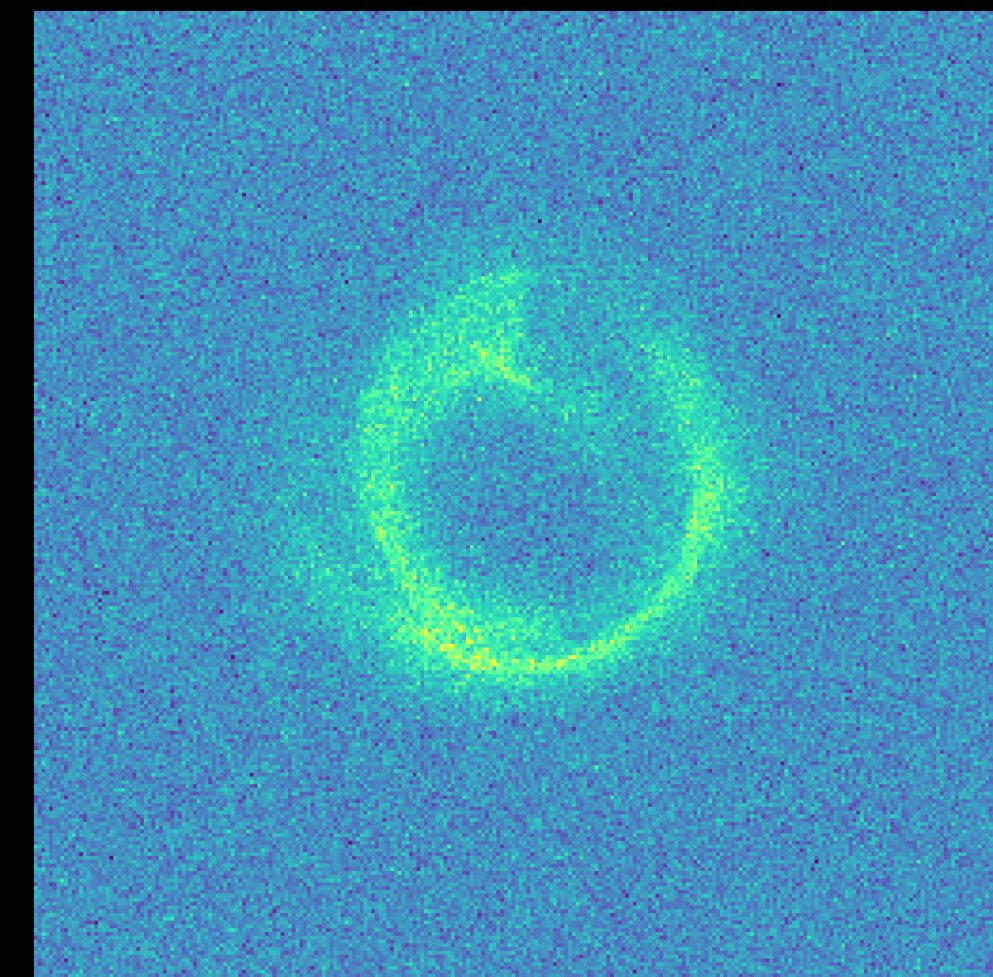
Lensed galaxy signal

$$L(\textcolor{red}{s})$$



Hubble Space Telescope

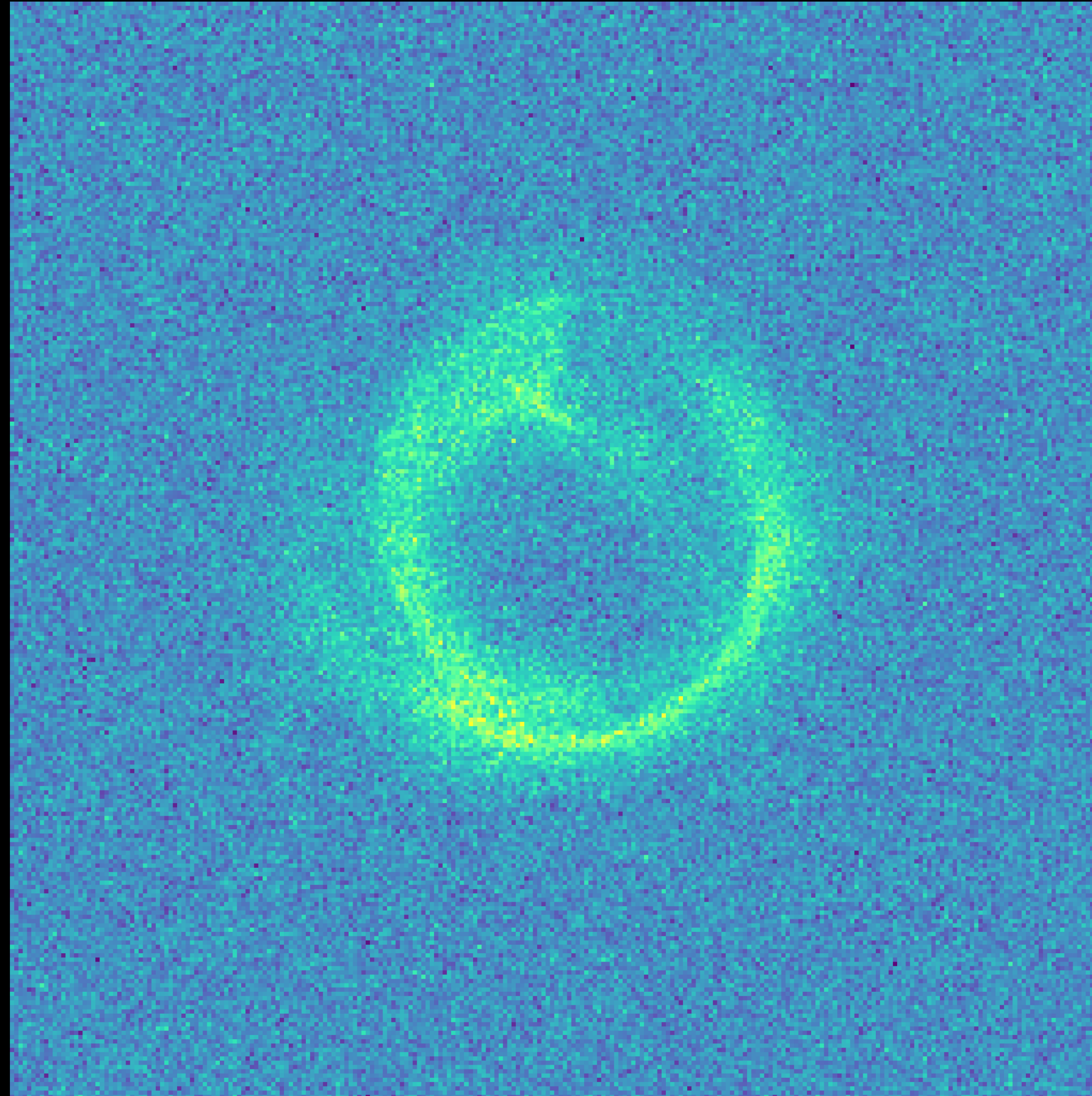
$$RL(\textcolor{red}{s})$$



Data on Earth

$$\textcolor{red}{P}(d | \textcolor{red}{s}) = \mathcal{G}(RL(\textcolor{red}{s}) - d, \sigma_h)$$

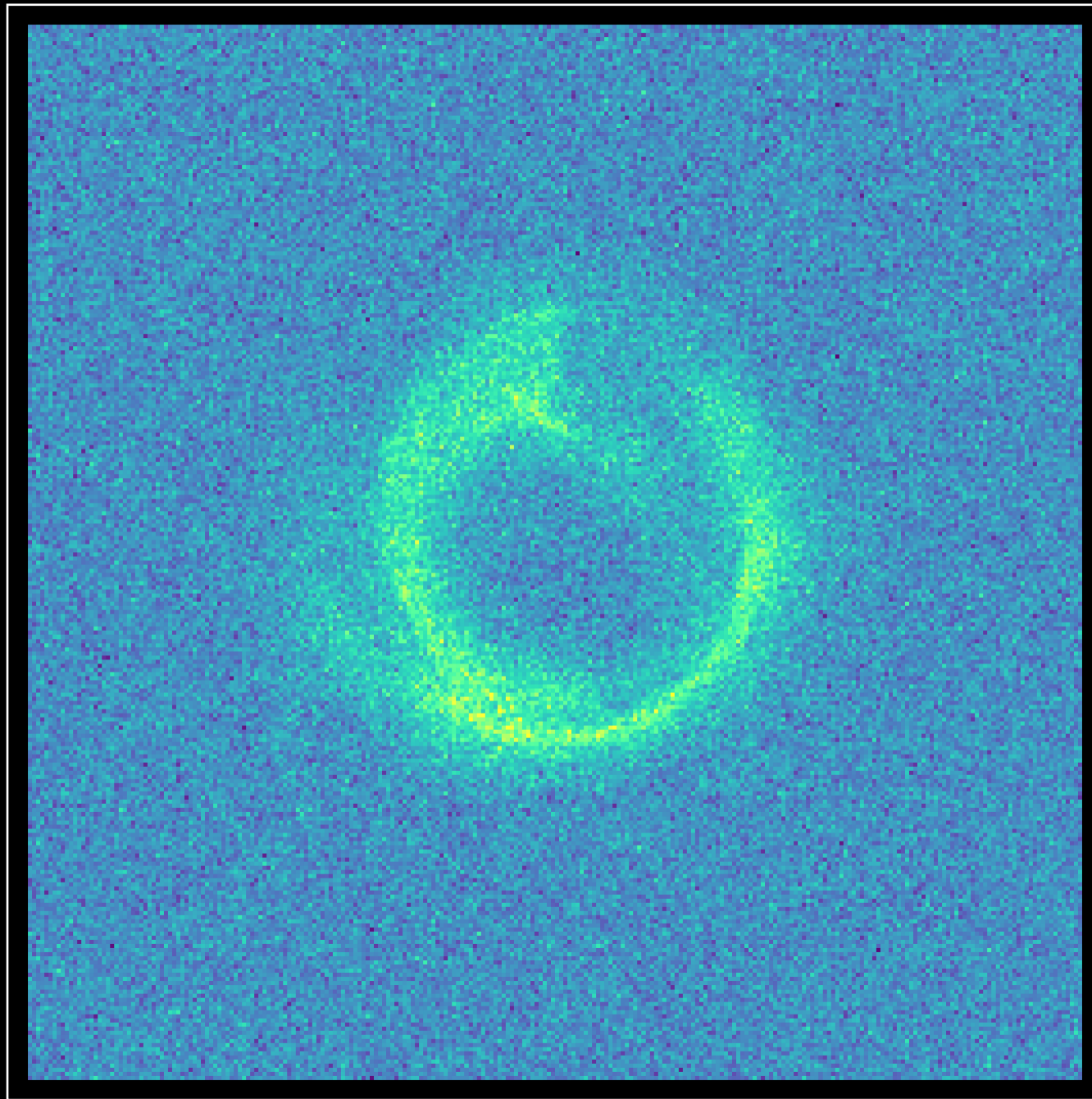
The data



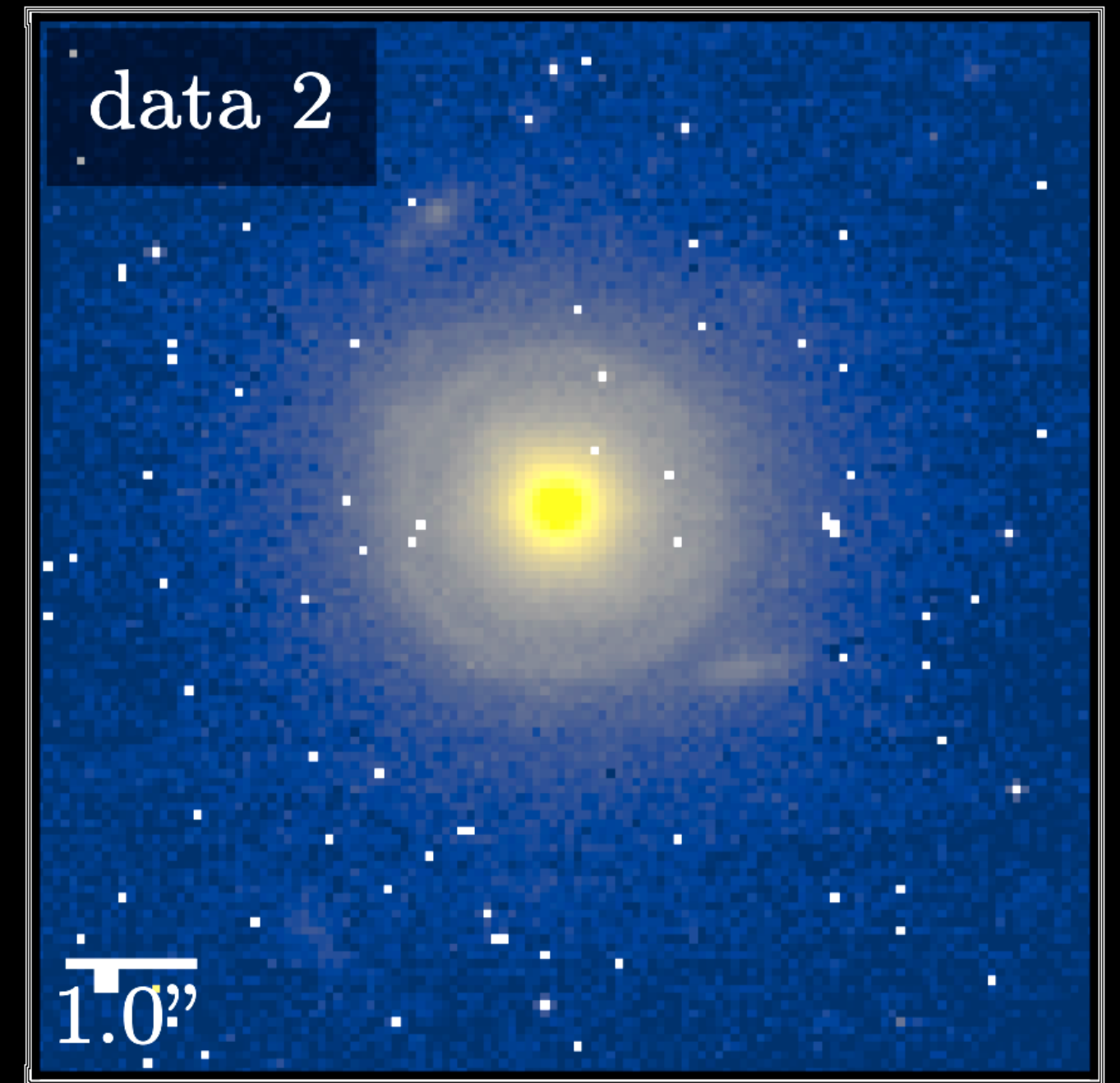
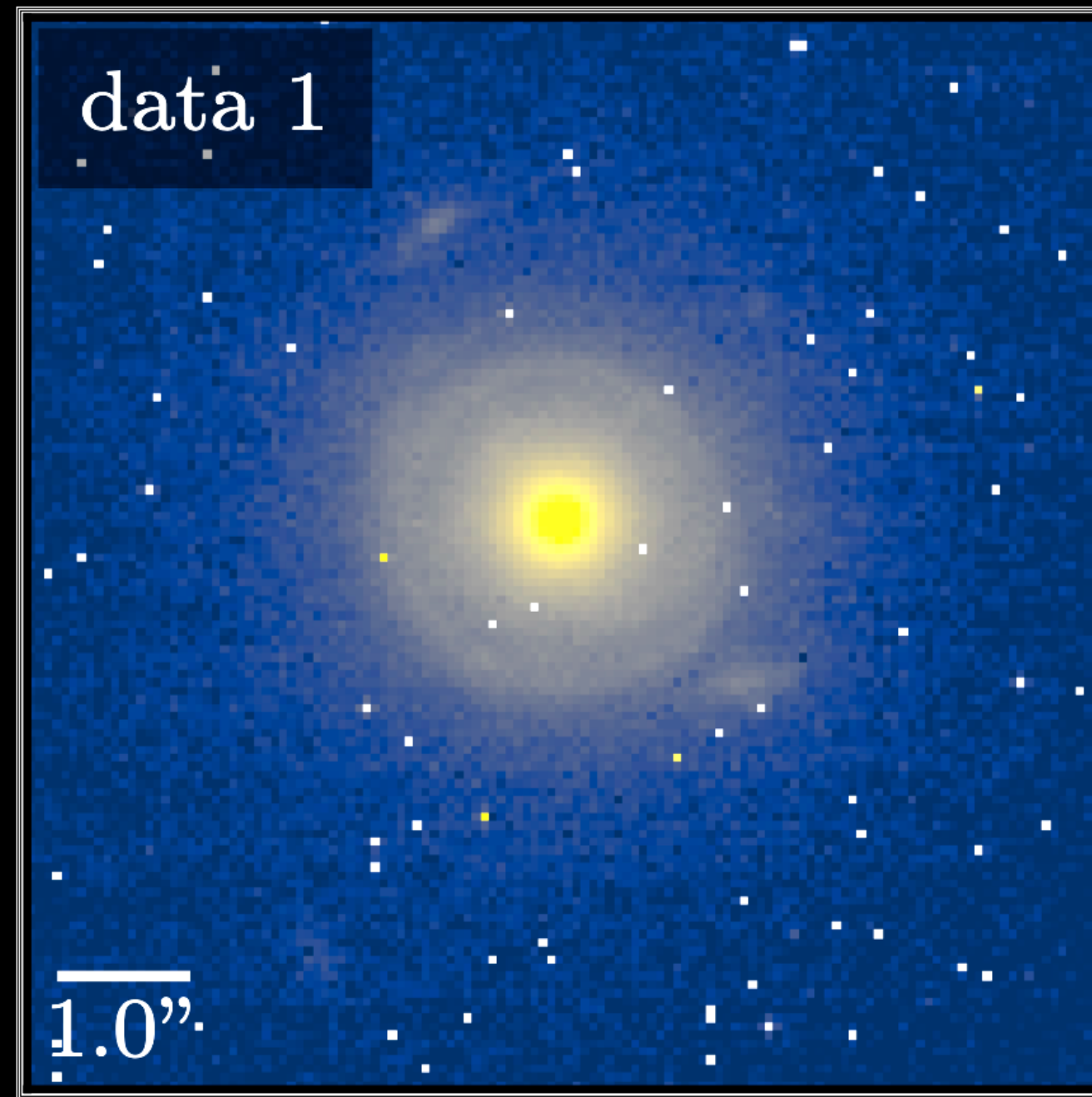
Simulated HST data

The likelihood

Instrument response



HST synthetic data

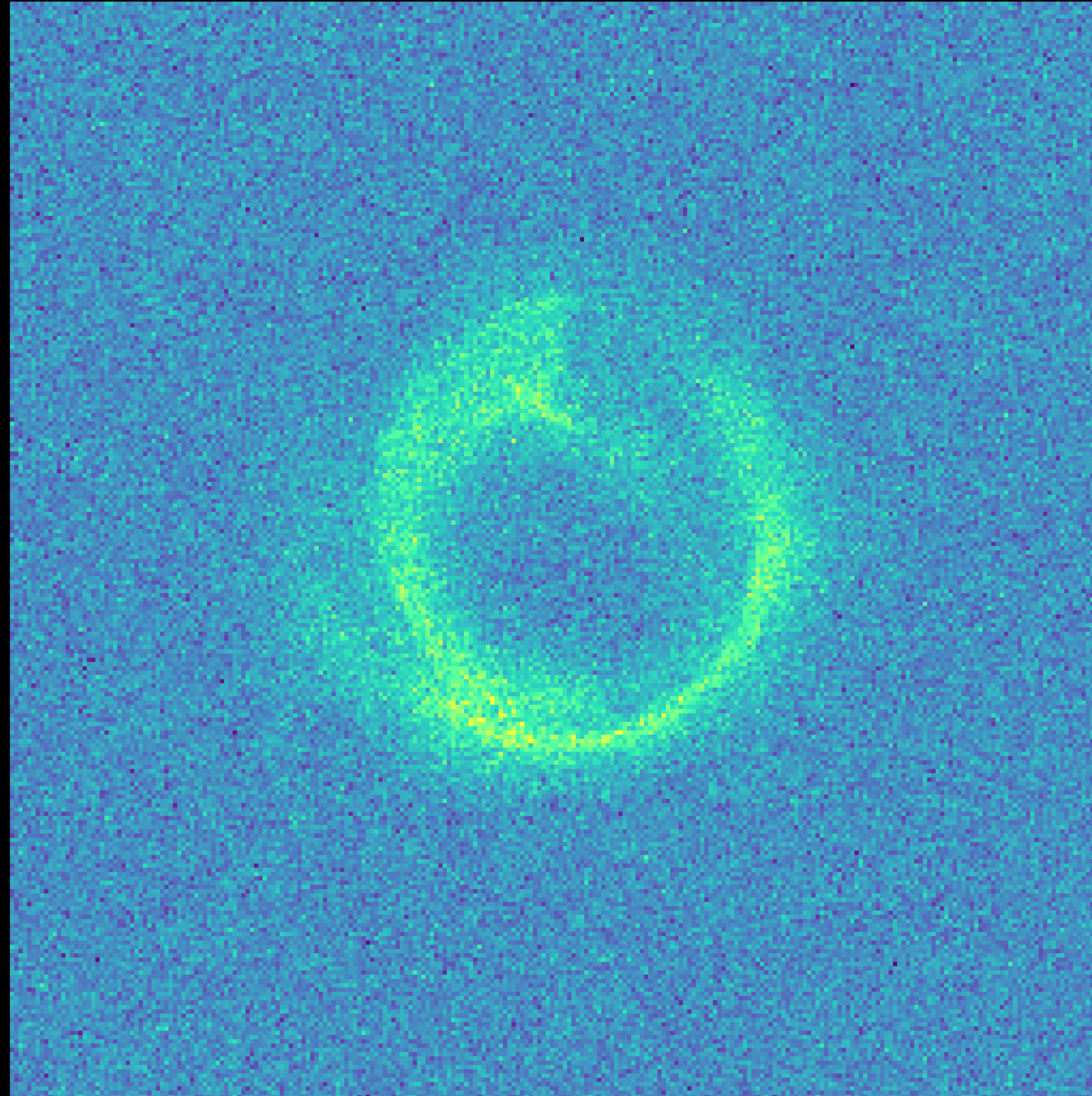


Real JWST lensing data

The Posterior

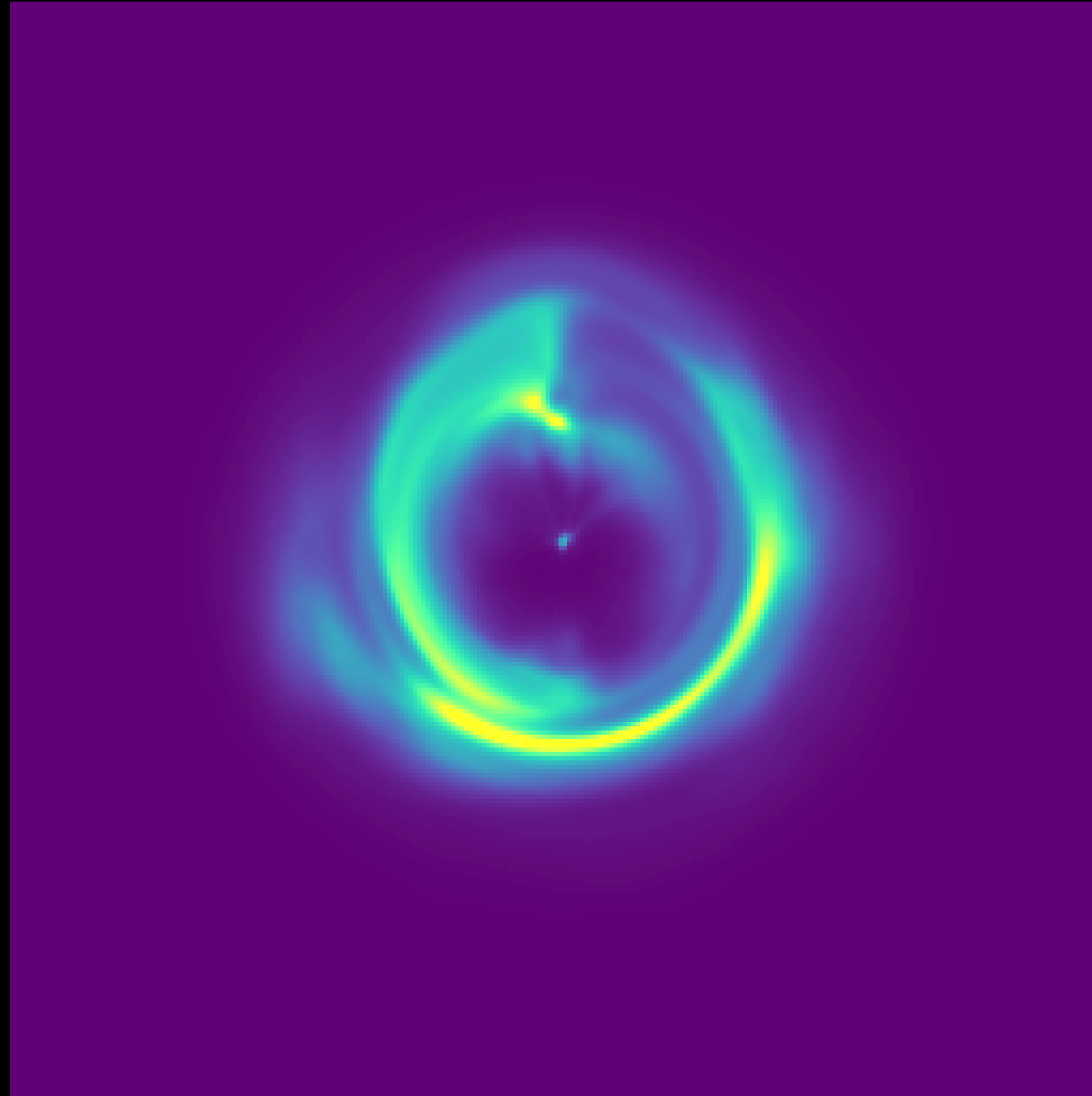
$$P(s \mid d) = \frac{P(d \mid s) P(s)}{P(d)}$$

The data



Simulated HST data

The reconstruction

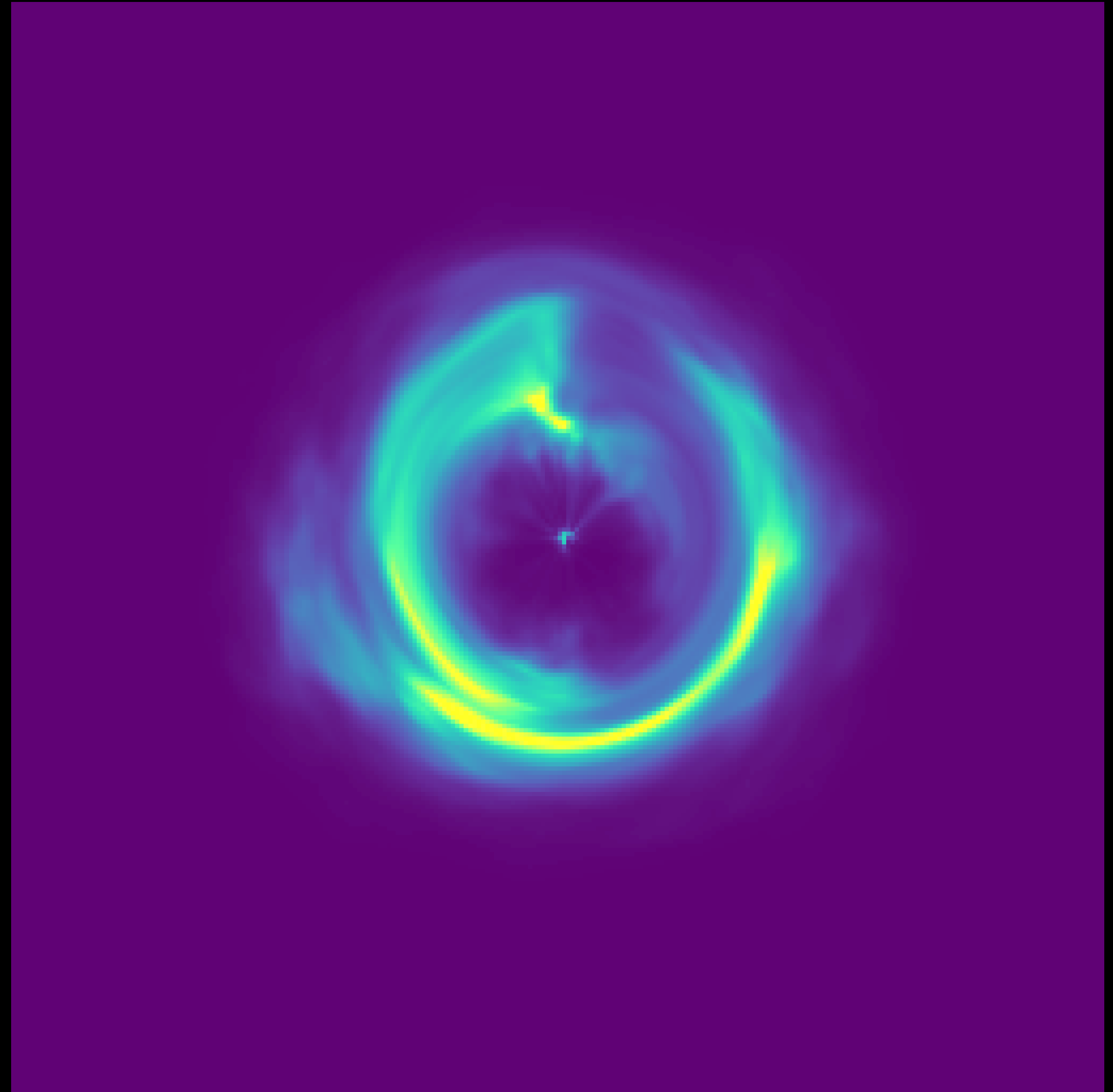


Lensed source light

Inference

Posterior (L_s)

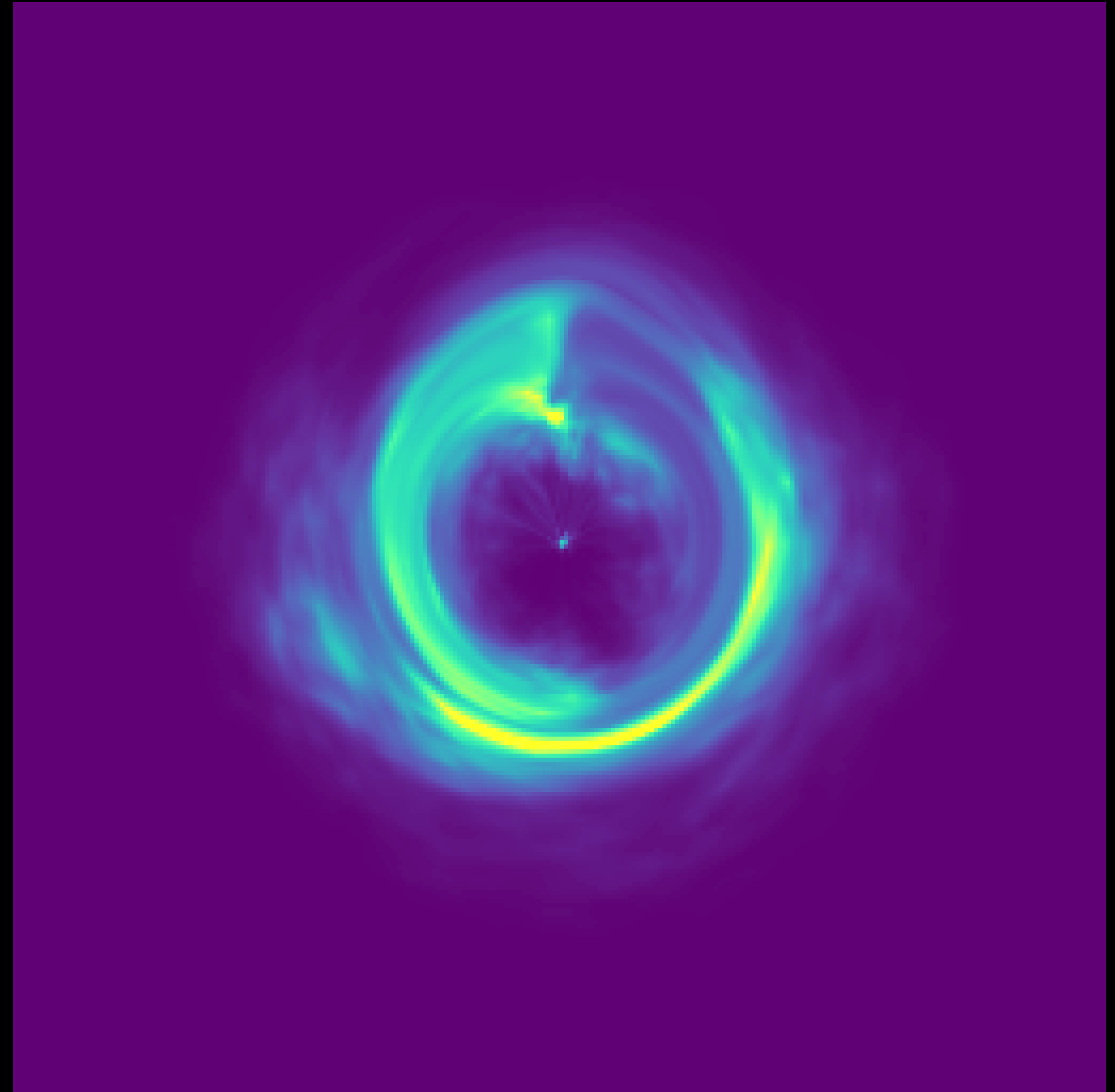
$$P(s | d)$$



Inference

Posterior (L_s)

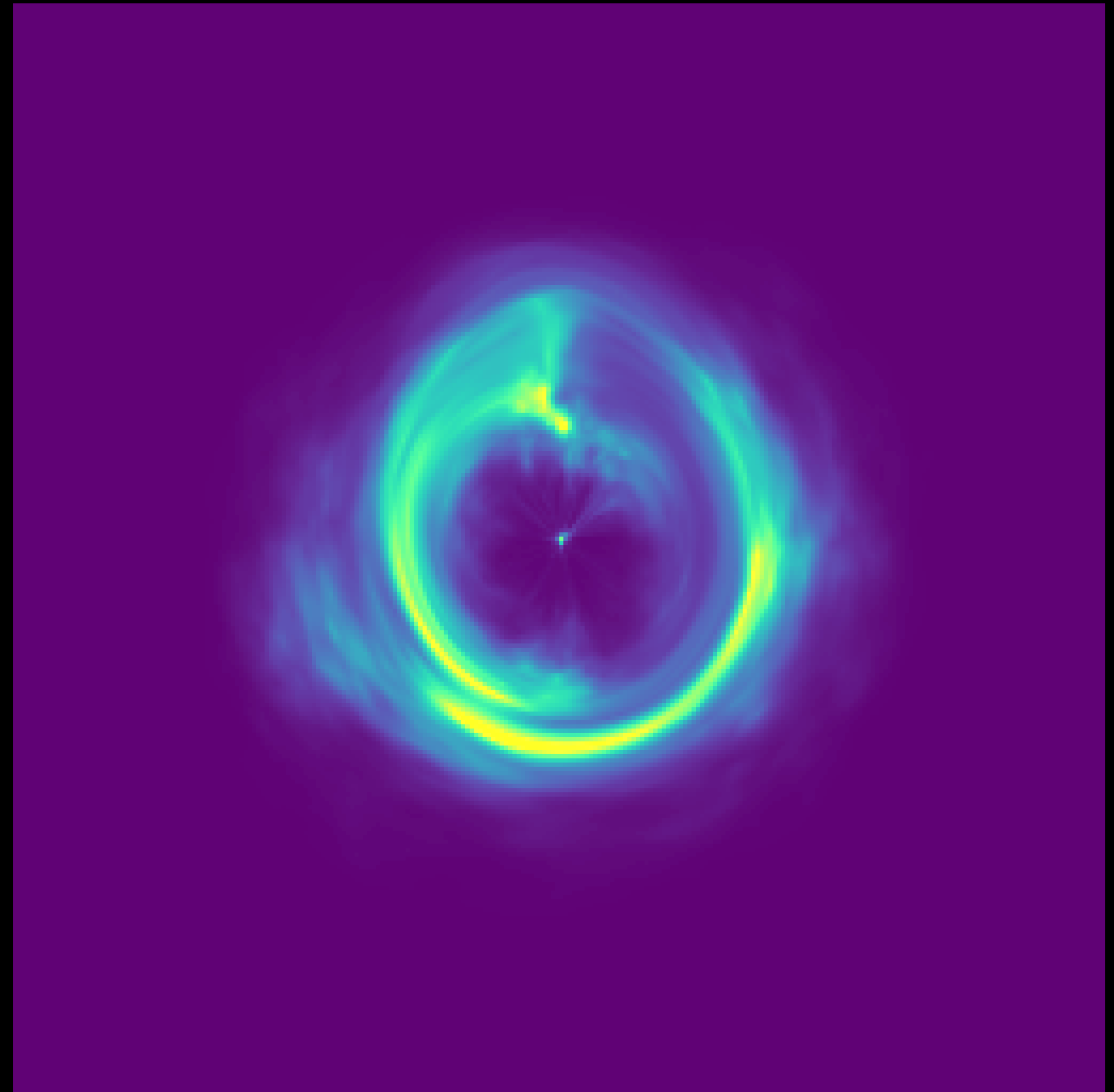
$$P(s | d)$$



Inference

Posterior (L_s)

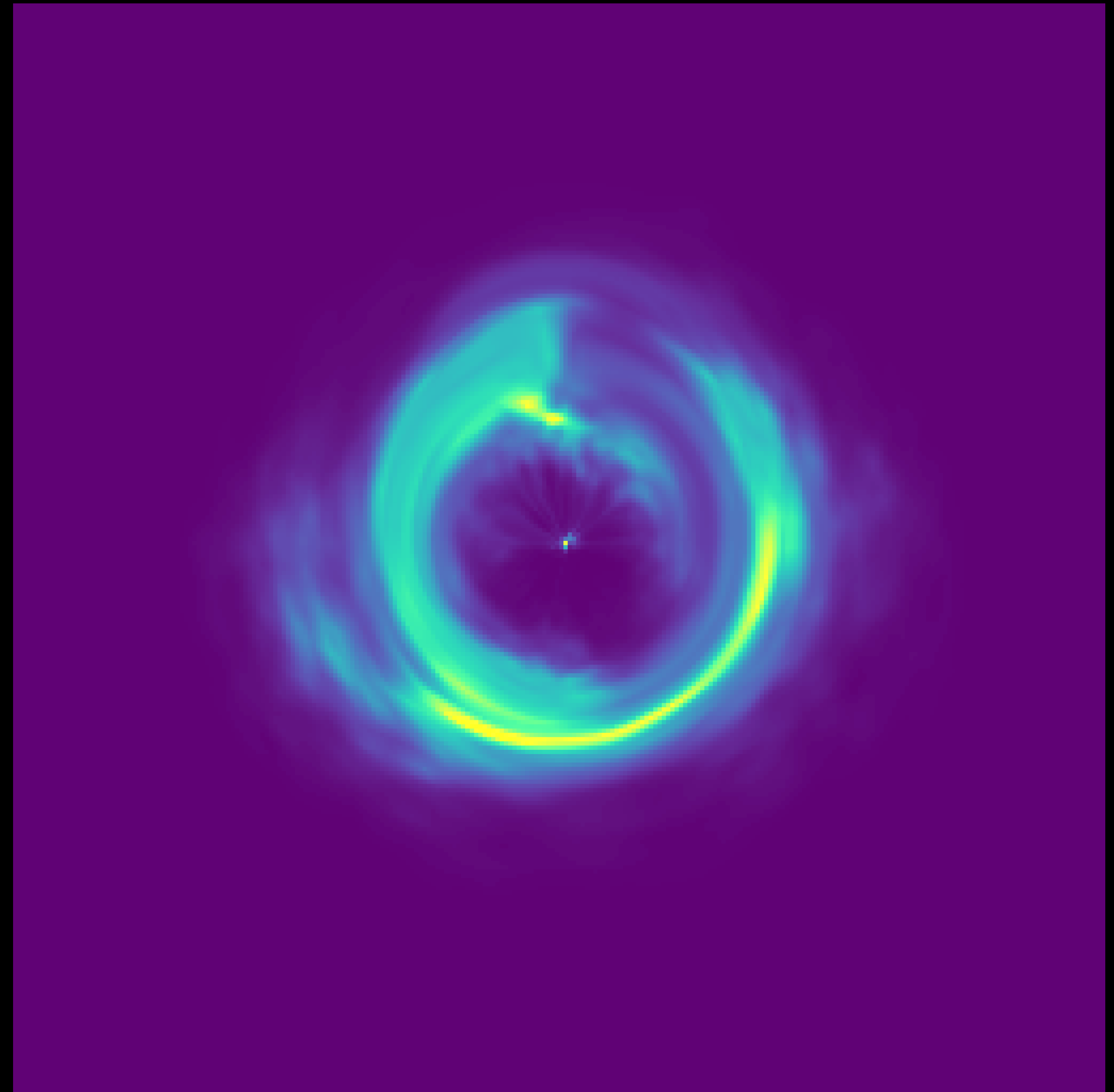
$$P(s | d)$$



Inference

Posterior (L_s)

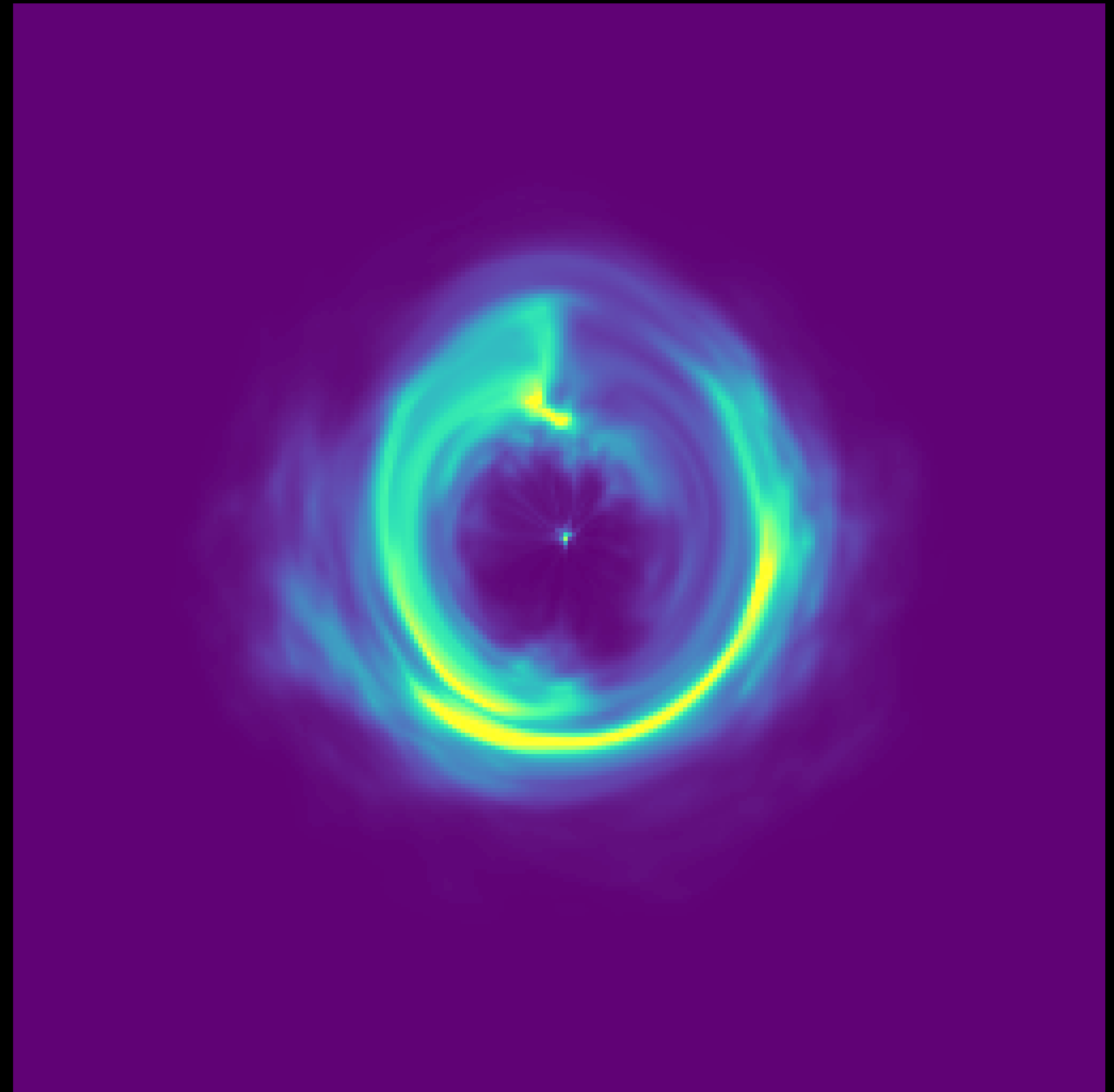
$$P(s | d)$$



Inference

Posterior (L_s)

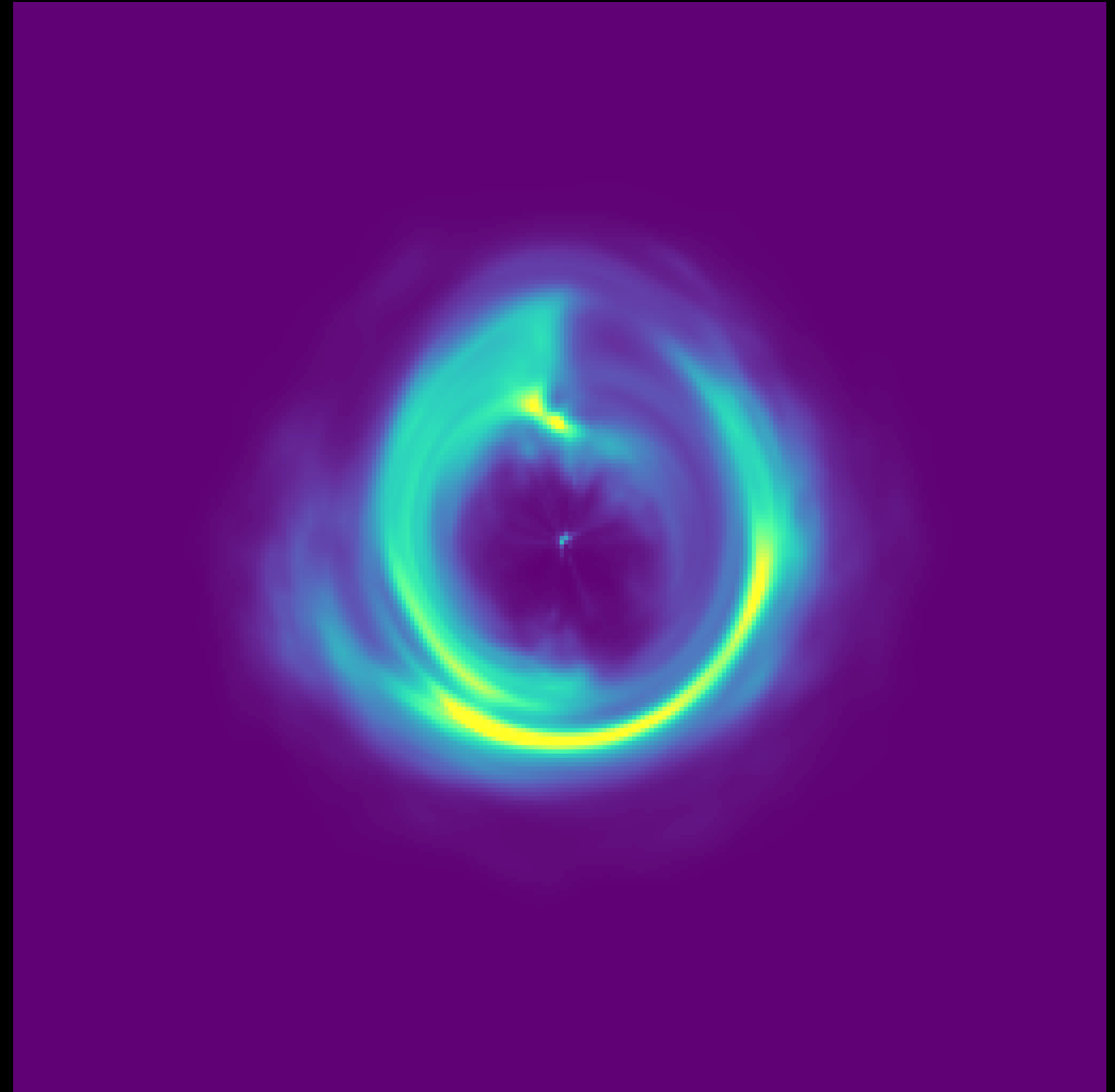
$$P(s | d)$$



Inference

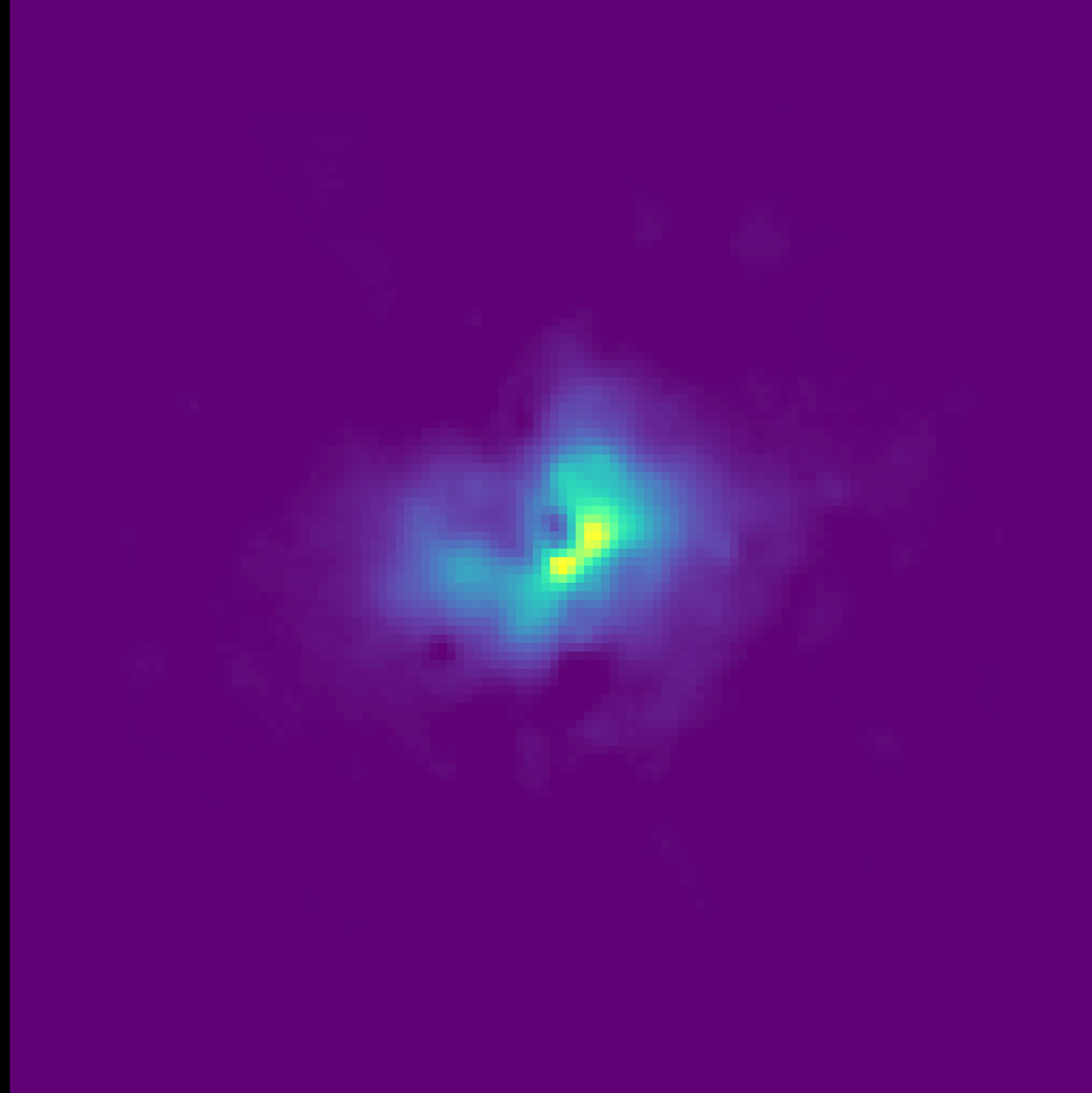
Posterior (L_s)

$$P(s \mid d)$$



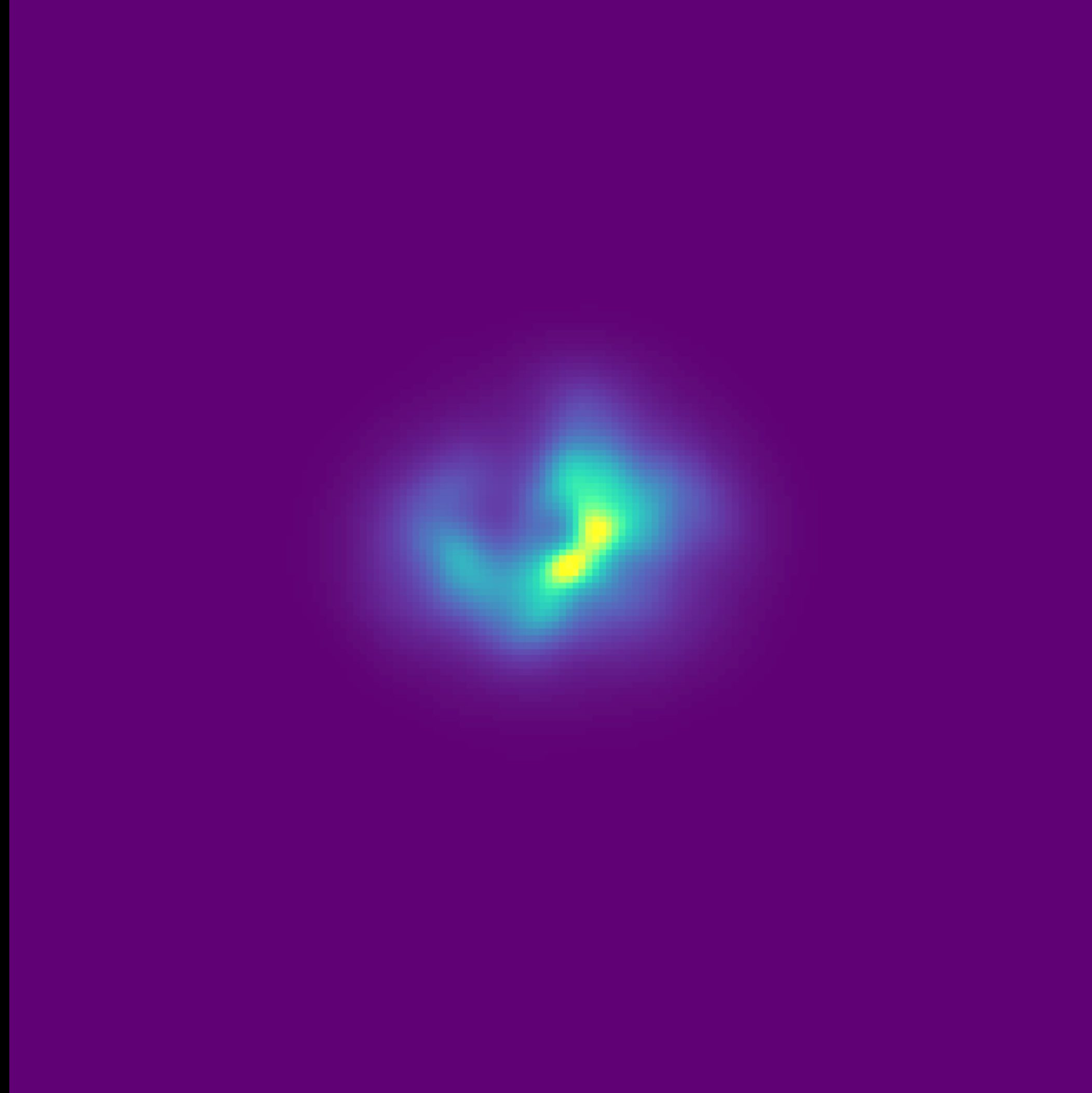
The source

Ground truth



The source

Reconstruction



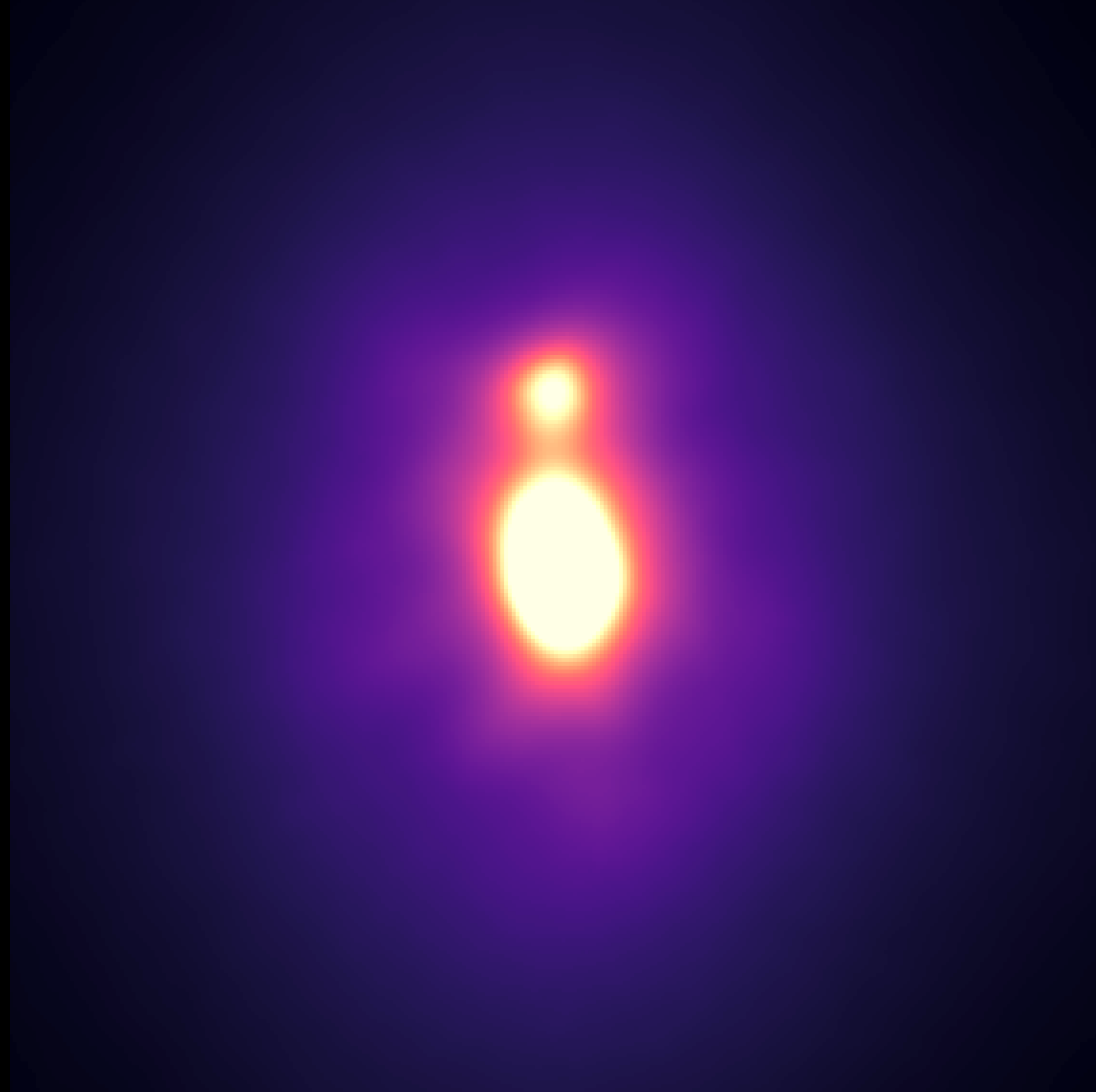
The convergence

Ground truth



The convergence

Reconstruction



So what's the problem?

The problem

The problem

Fields!

The problem

Fields!

→ allow for **non-trivial degeneracies!**

The problem

Fields!

→ allow for **non-trivial degeneracies!**

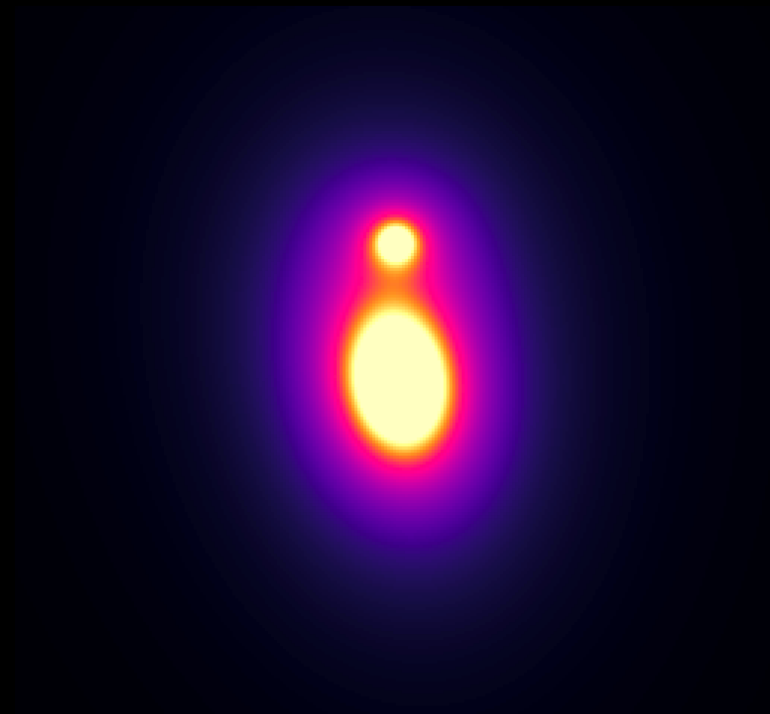
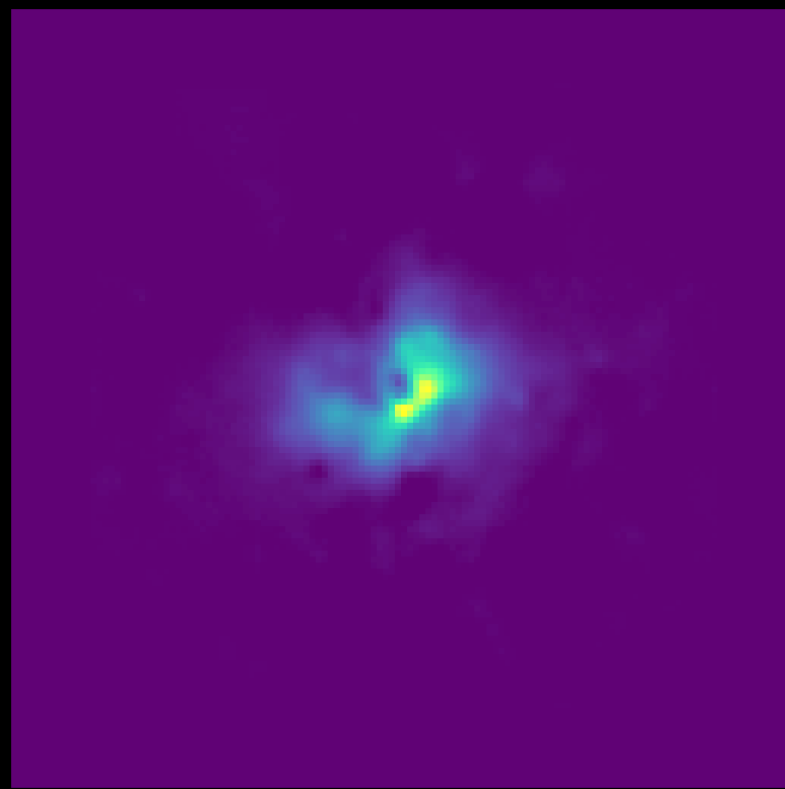
$$M_1(\theta_1)$$

The problem

Fields!

→ allow for **non-trivial degeneracies!**

$$M_1(\theta_1)$$

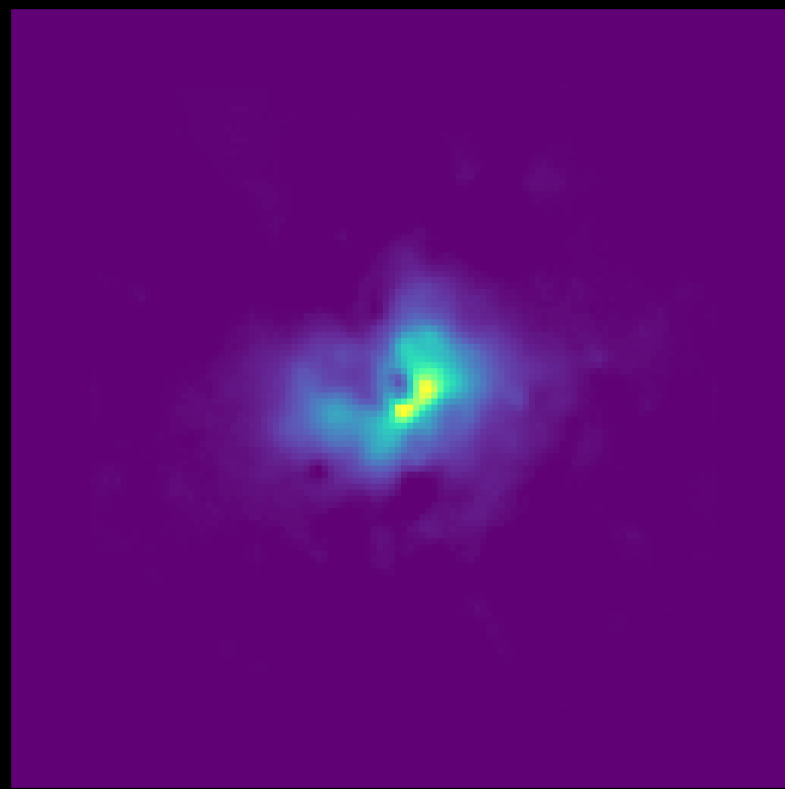


The problem

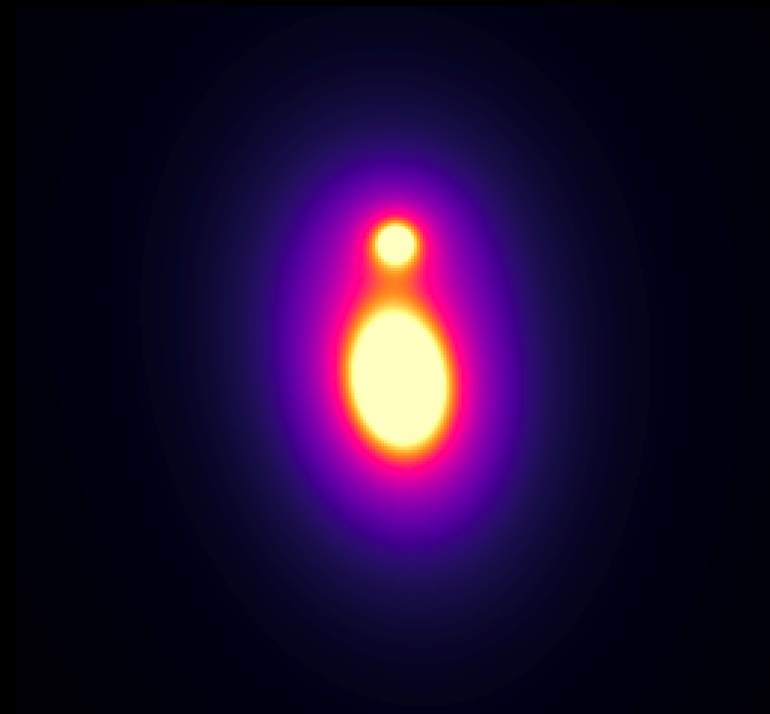
Fields!

→ allow for **non-trivial degeneracies!**

$M_1(\theta_1)$



$M_2(\theta_2)$

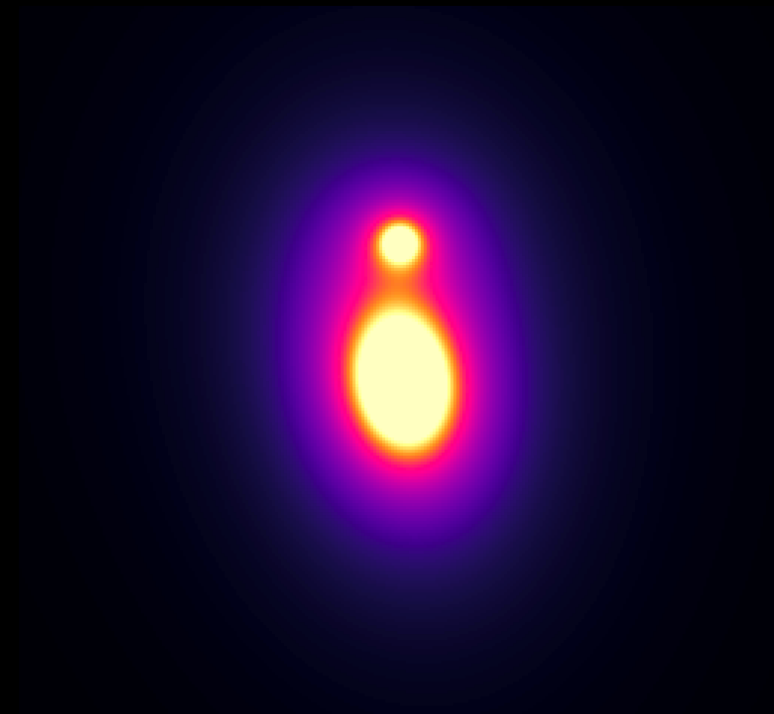
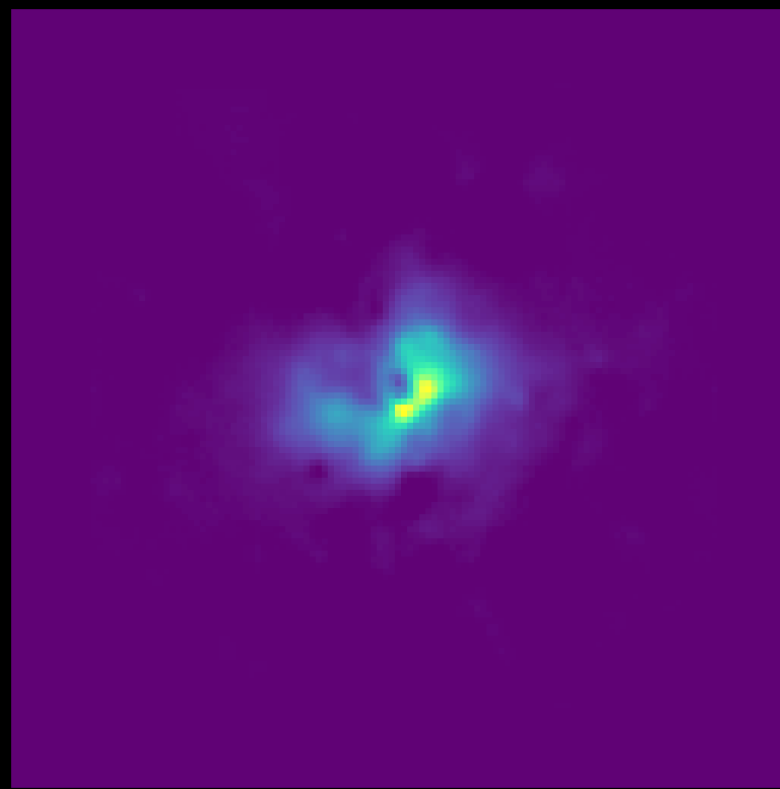


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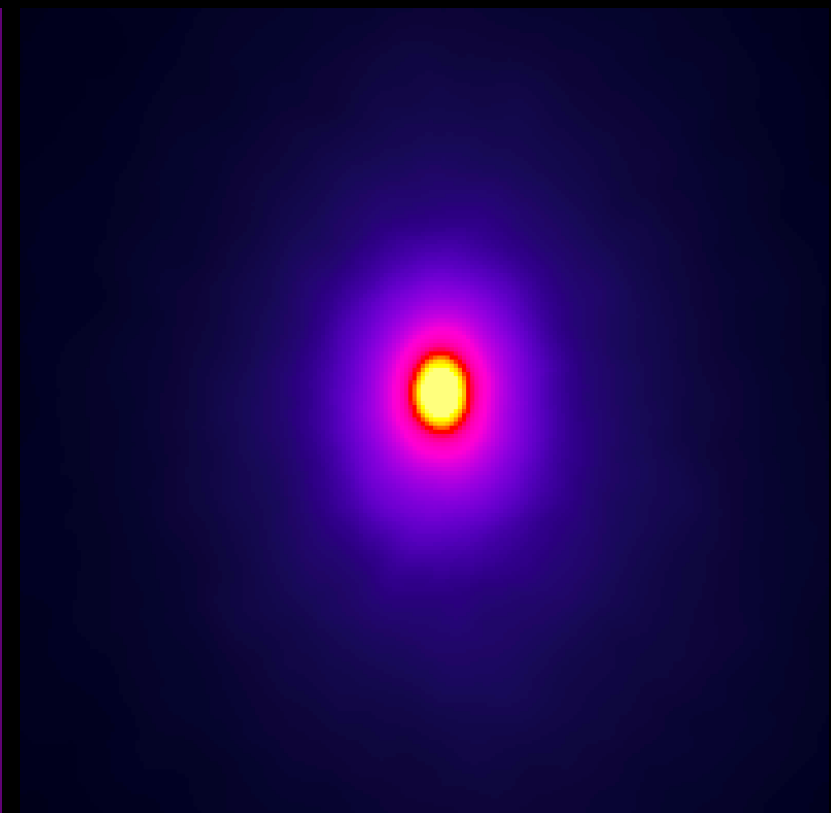
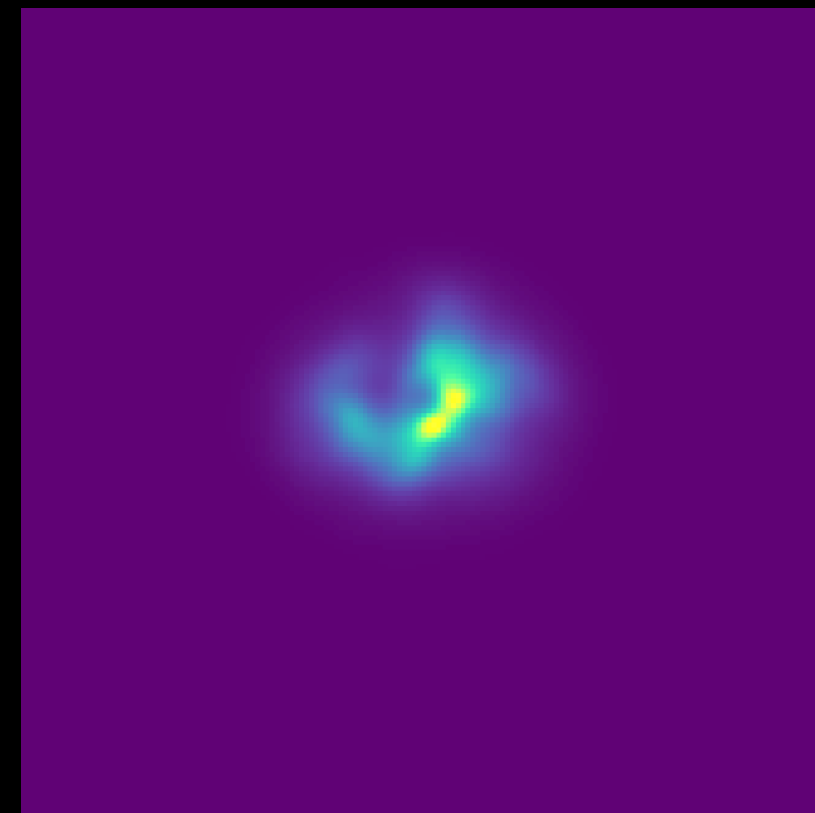
Fields!

→ allow for **non-trivial degeneracies!**

$M_1(\theta_1)$



$M_2(\theta_2)$

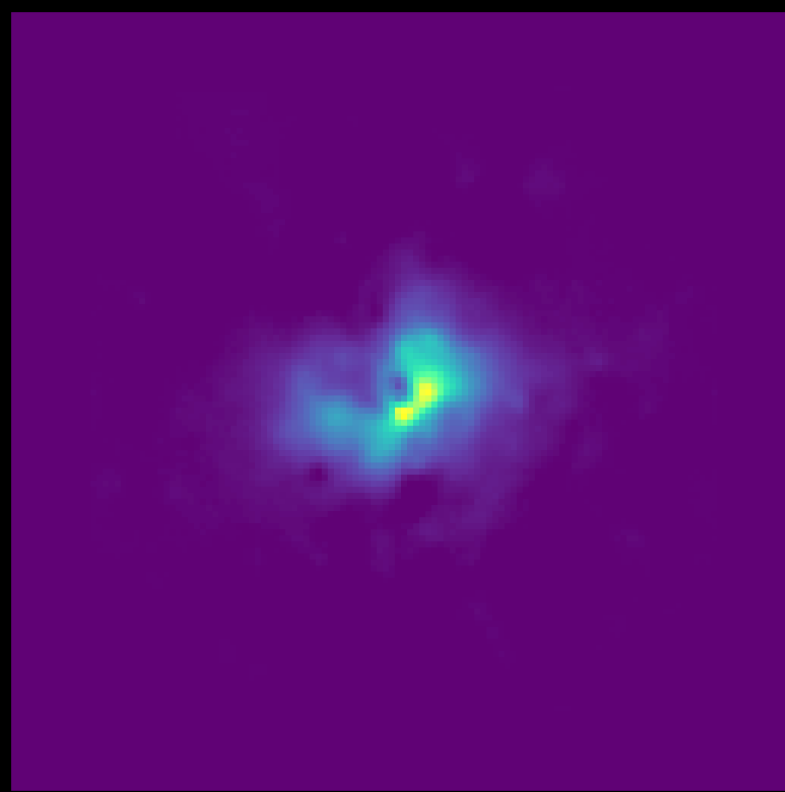


The problem

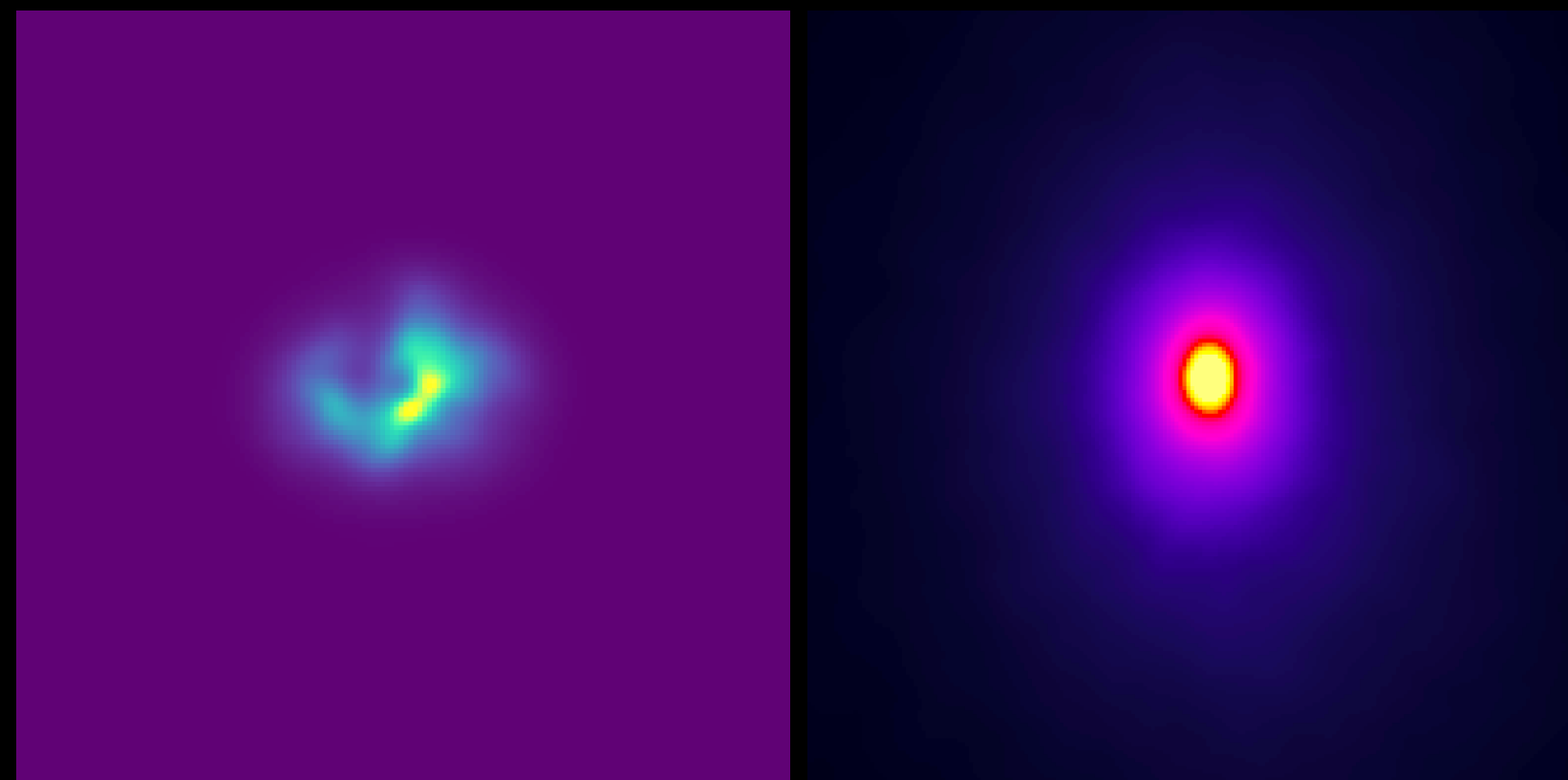
Fields!

→ allow for **non-trivial degeneracies!**

$M_1(\theta_1)$



$M_2(\theta_2)$



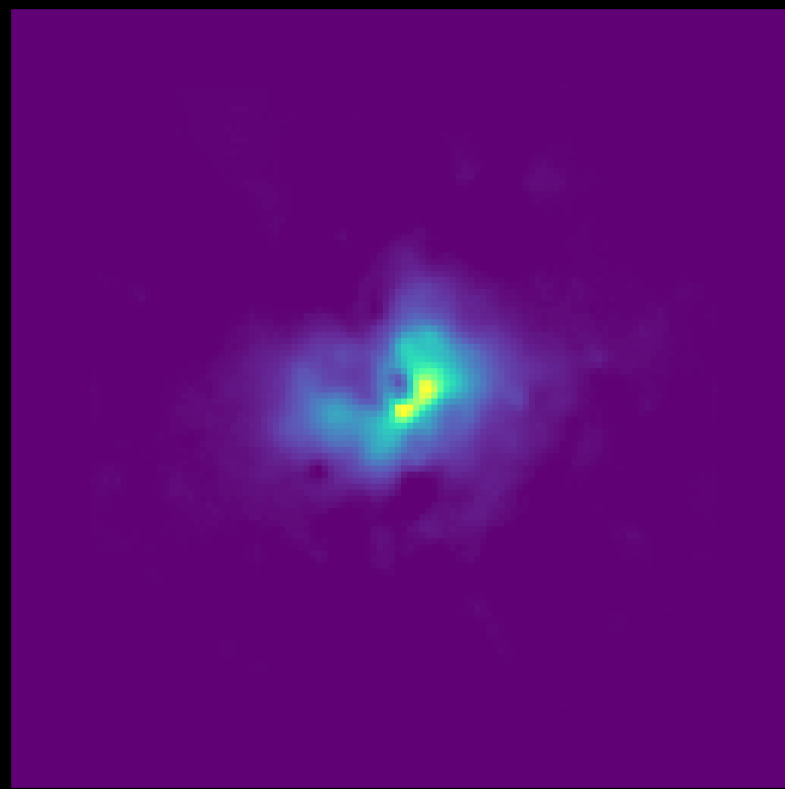
$$\chi_{\text{red}}^2 \simeq 1.01$$

The problem

Fields!

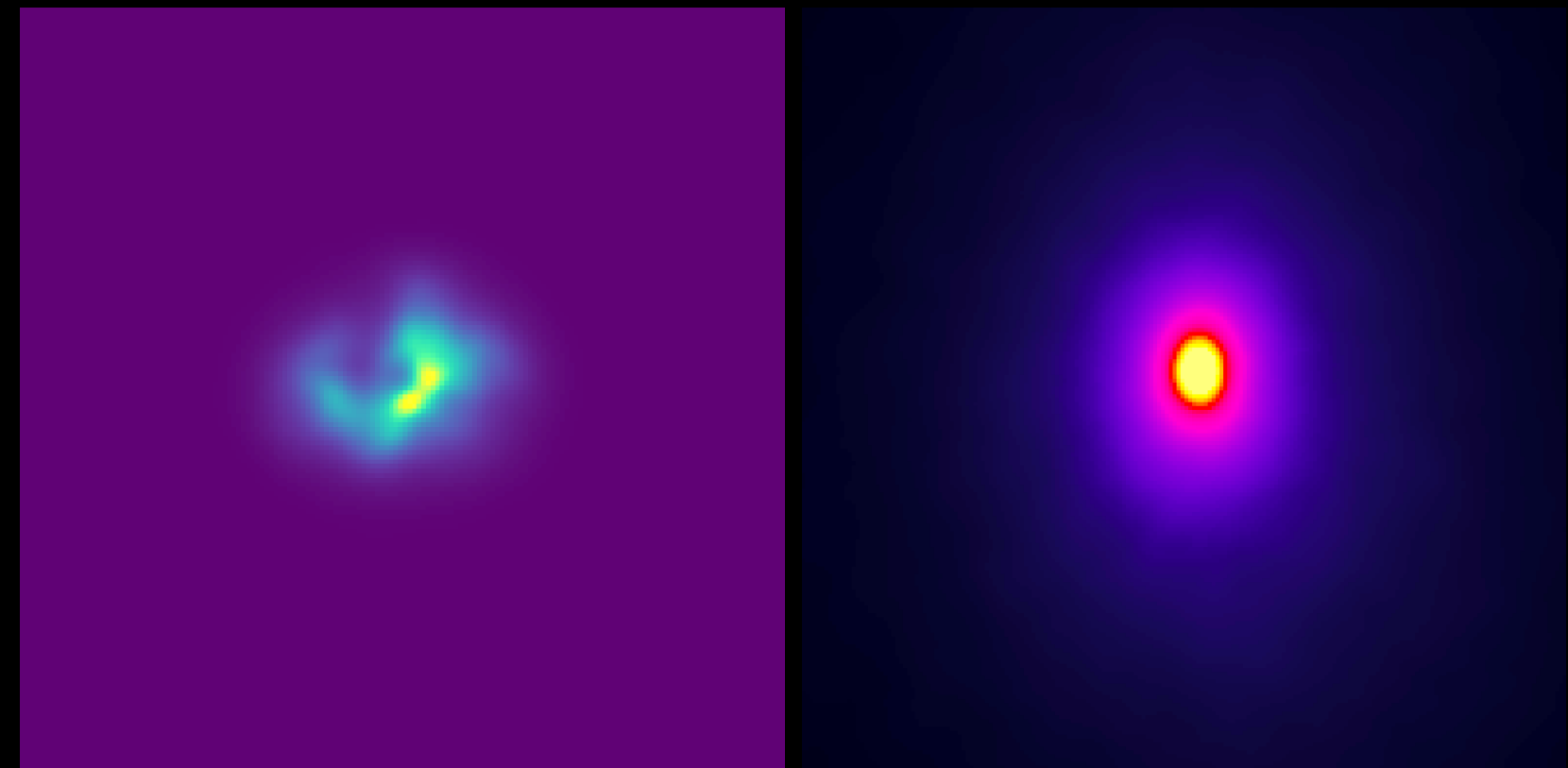
→ allow for **non-trivial degeneracies!**

$M_1(\theta_1)$



$$\chi^2_{\text{red}} \simeq 1.01$$

$M_2(\theta_2)$



$$\chi^2_{\text{red}} \simeq 1.02$$

Model choice

Model choice

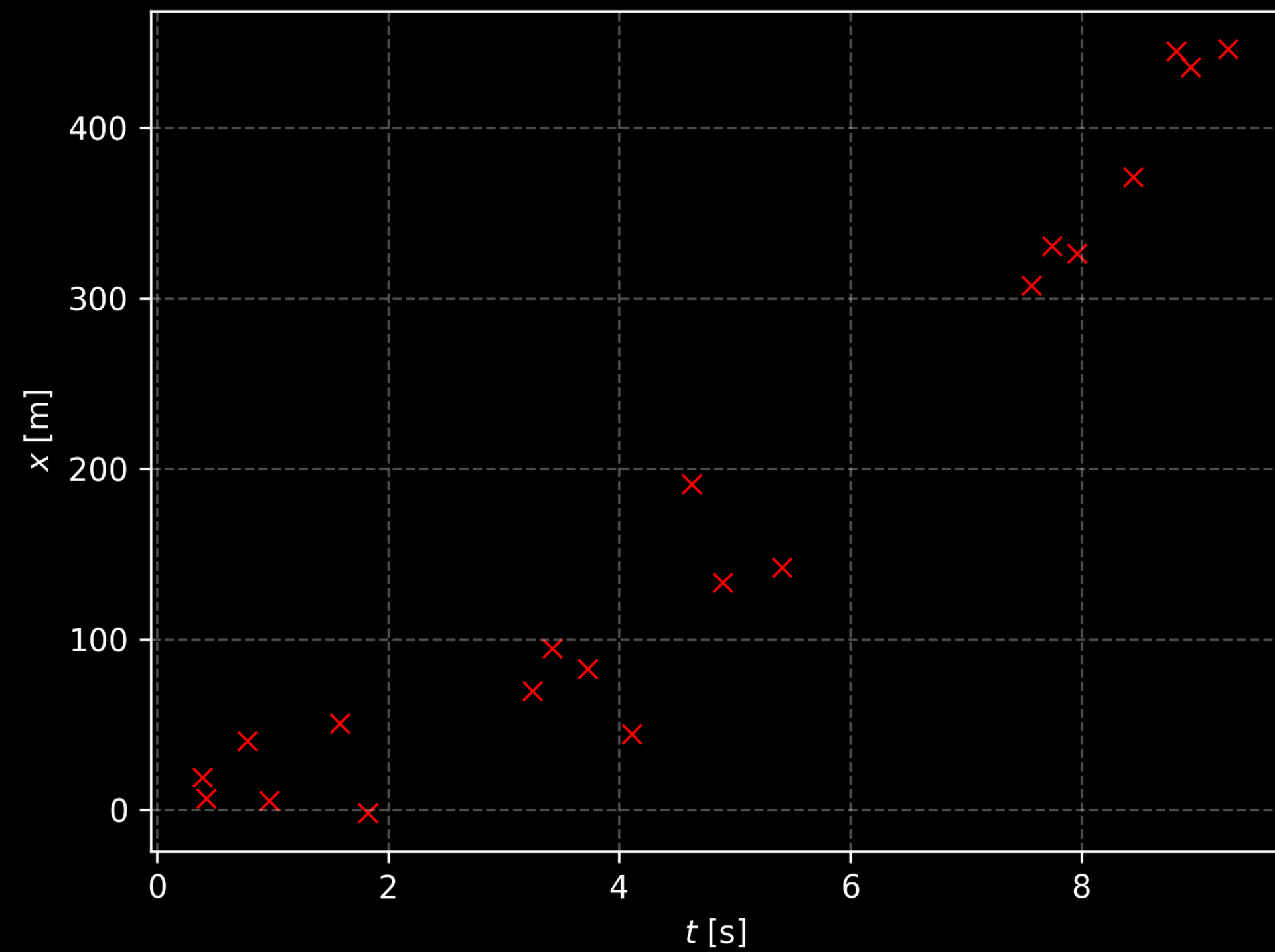


The solution

Can we let the data decide?

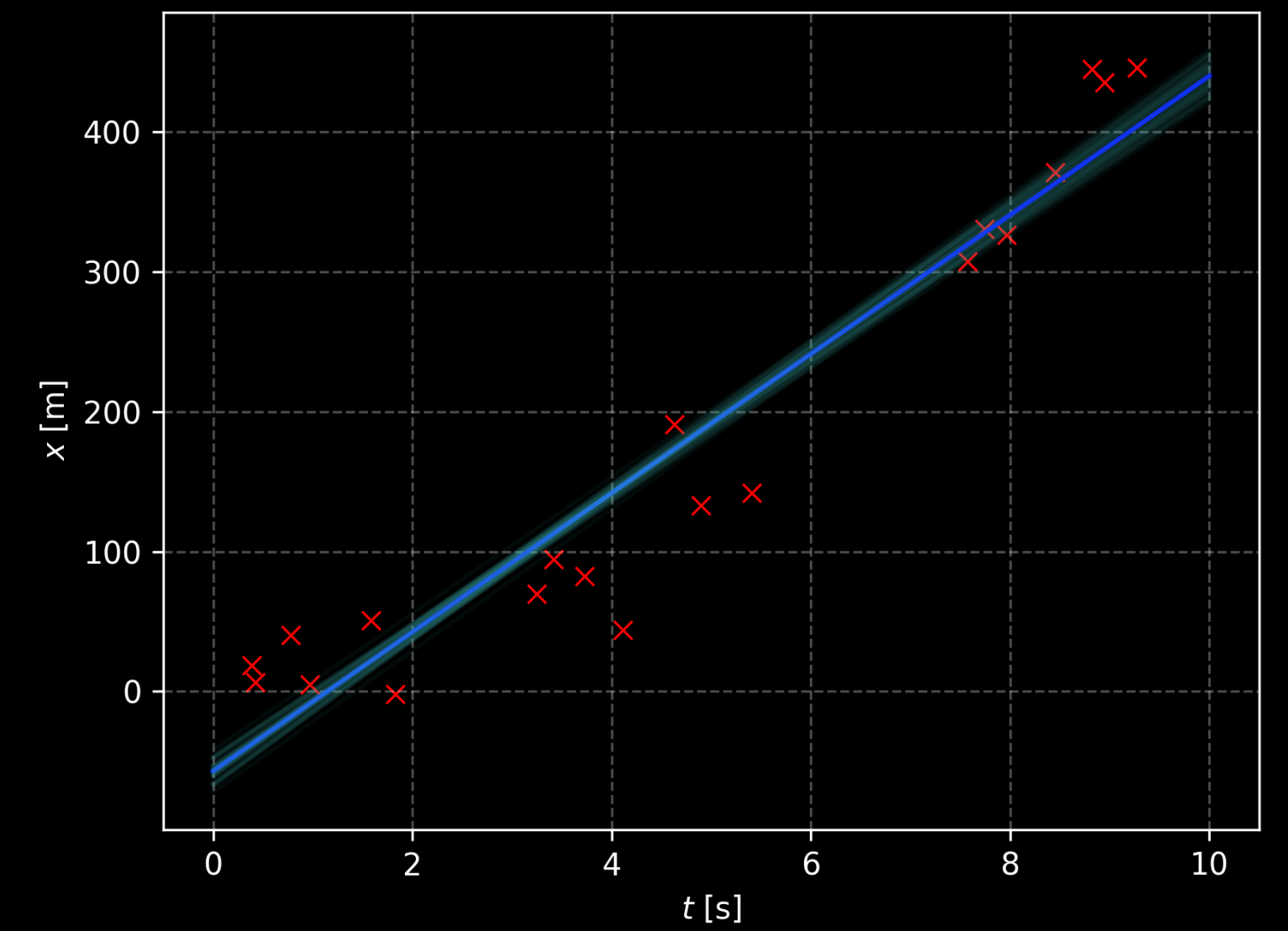
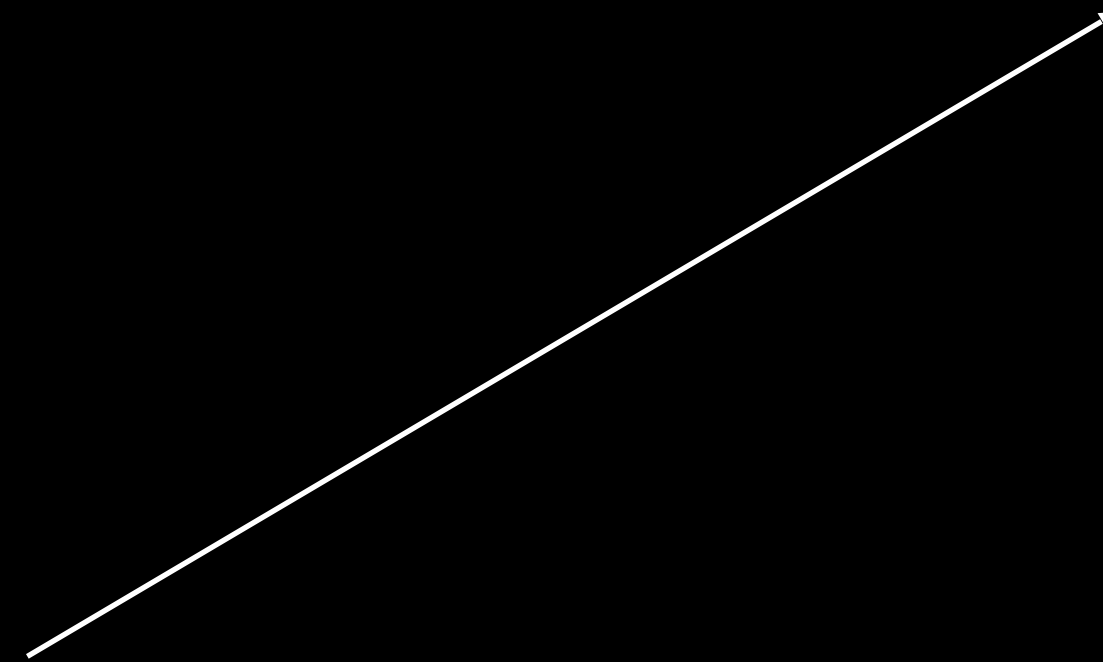
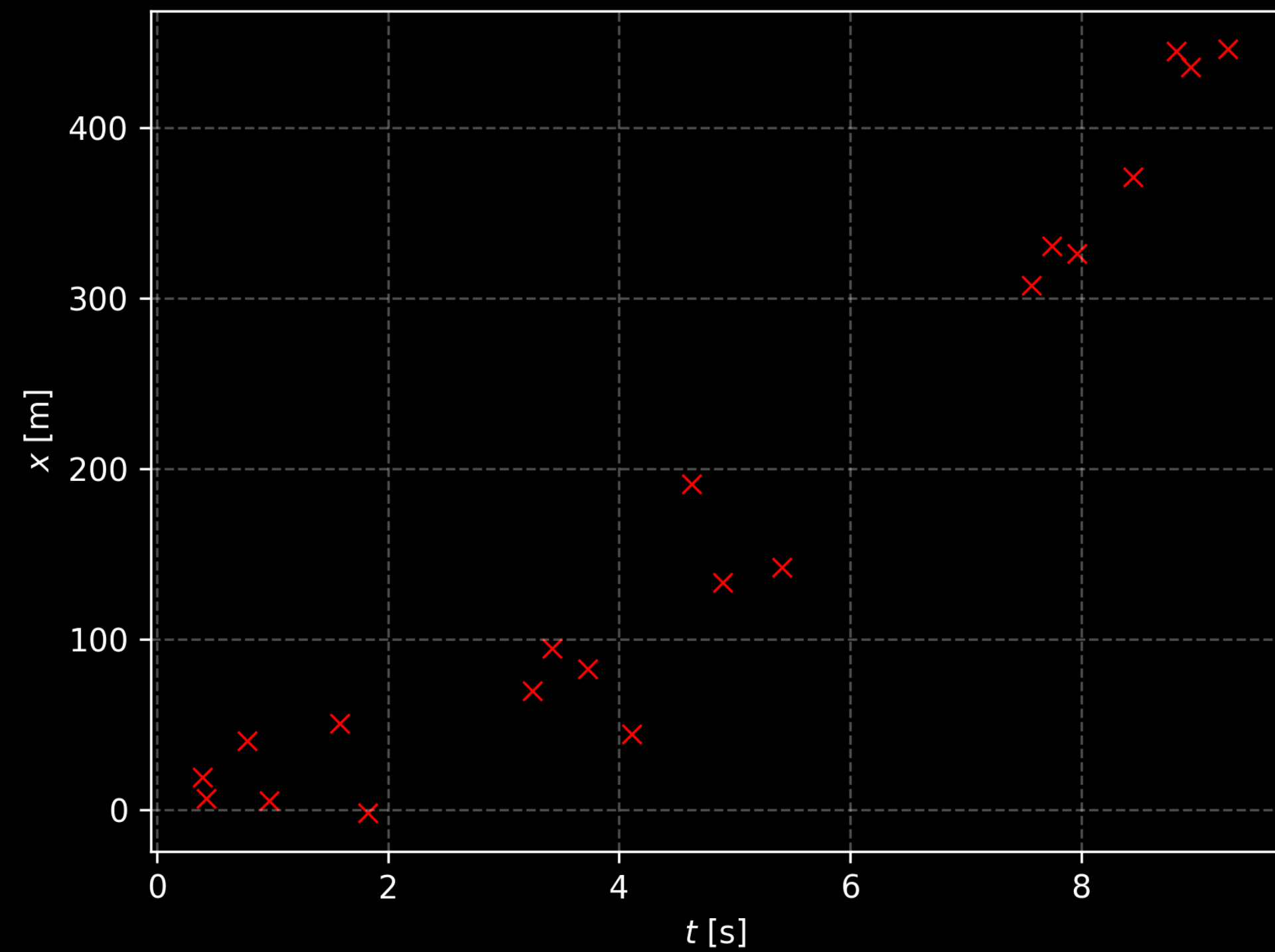
The solution

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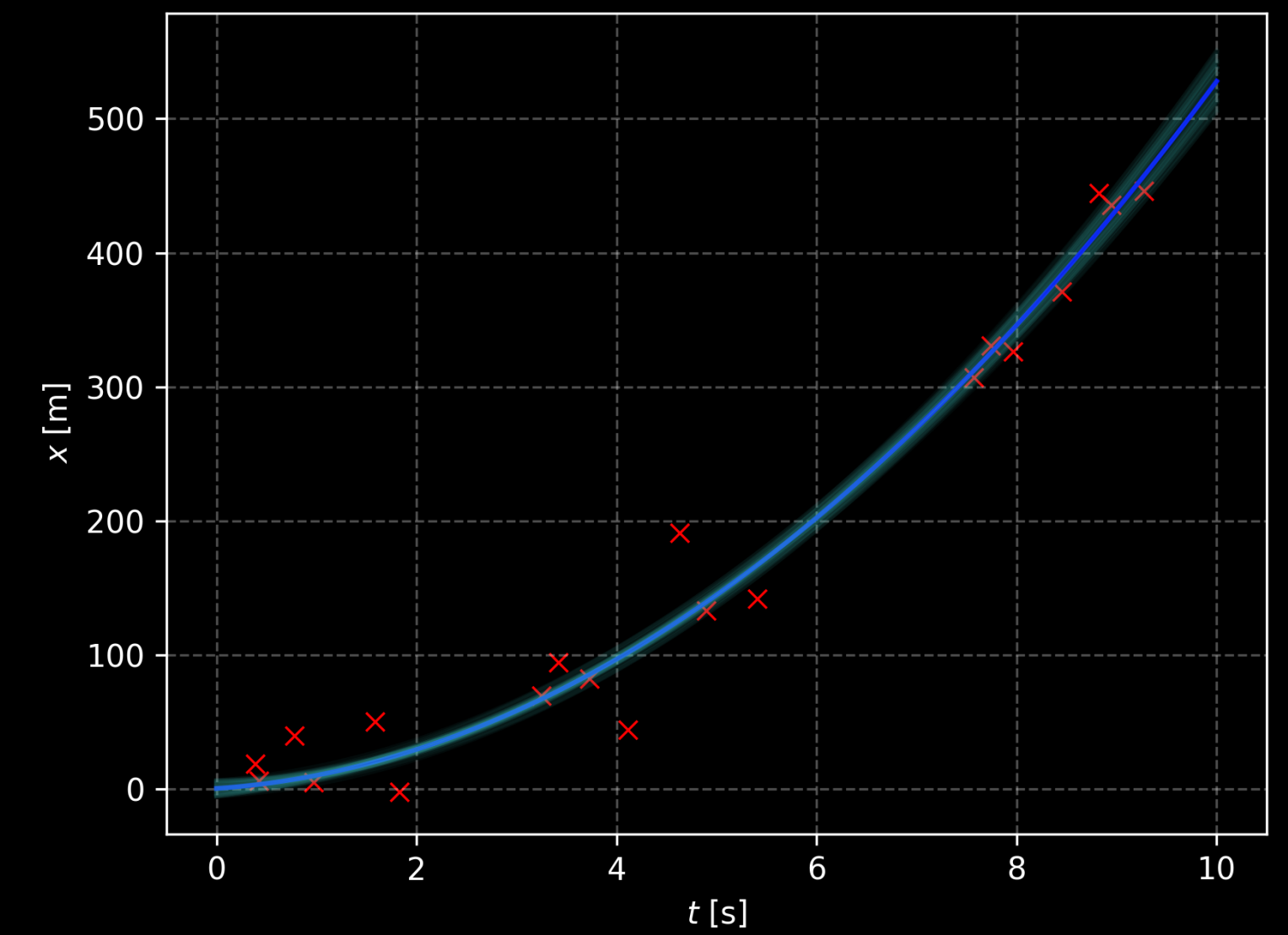
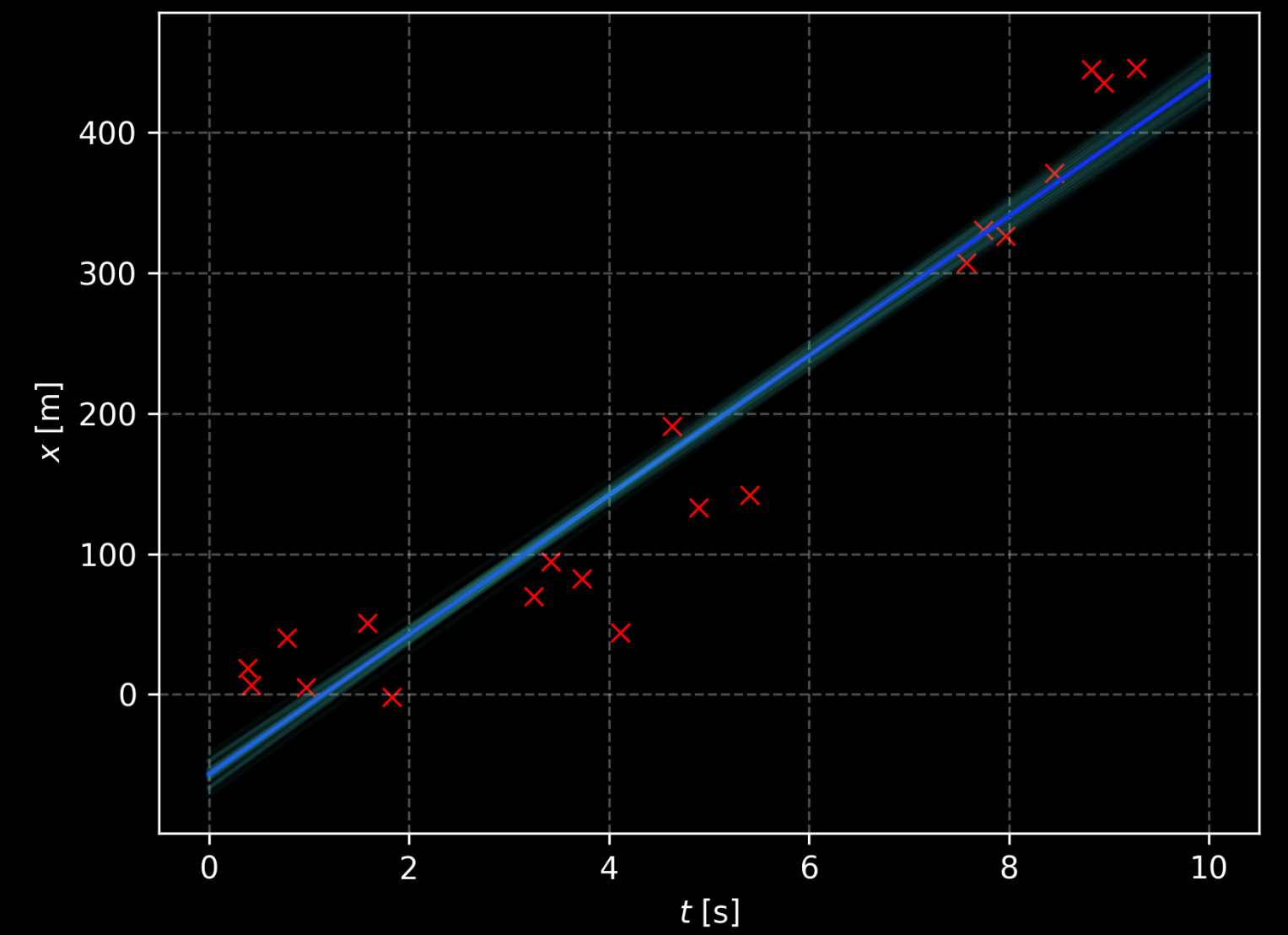
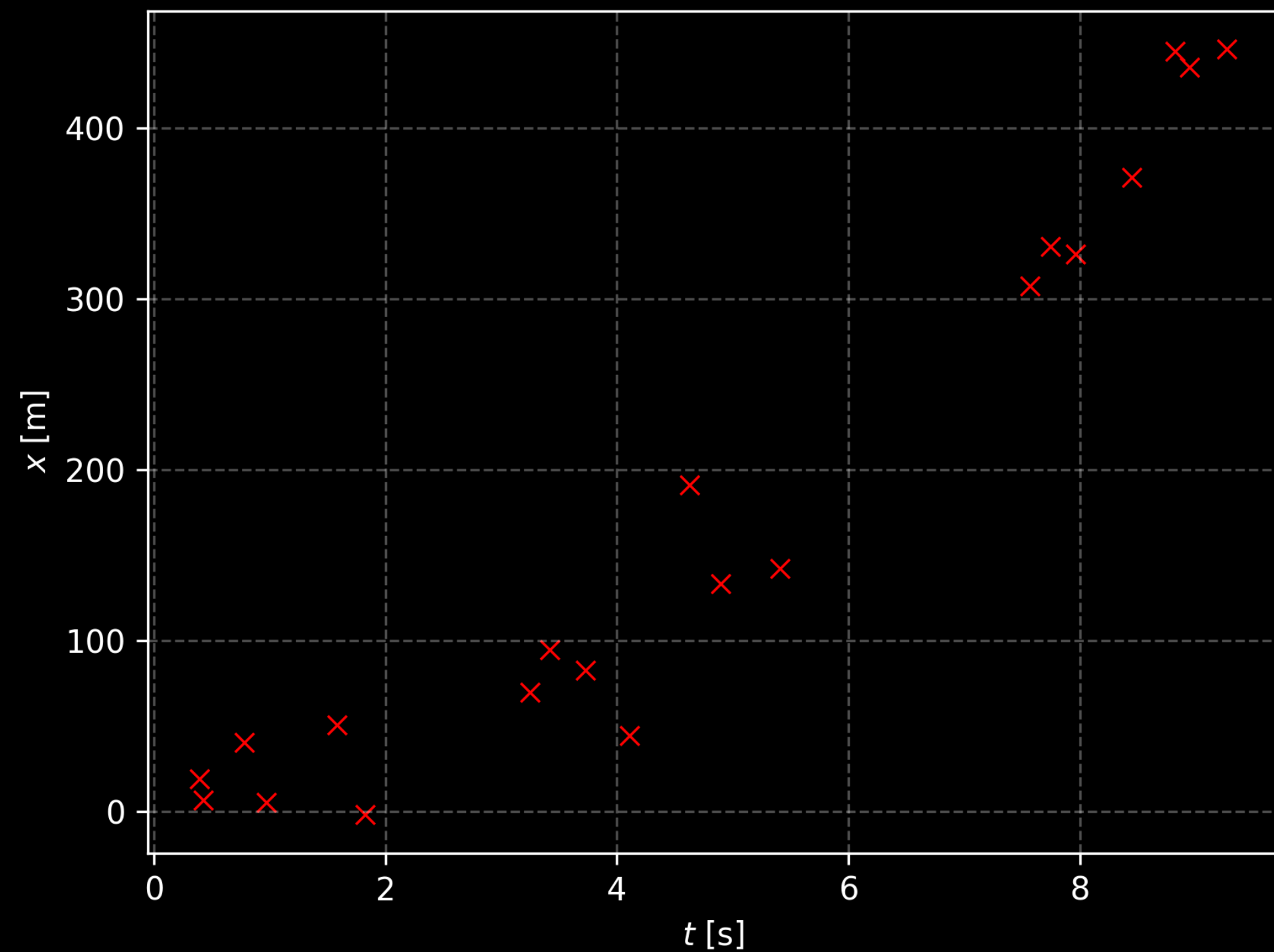
The solution

Can we let the data decide?



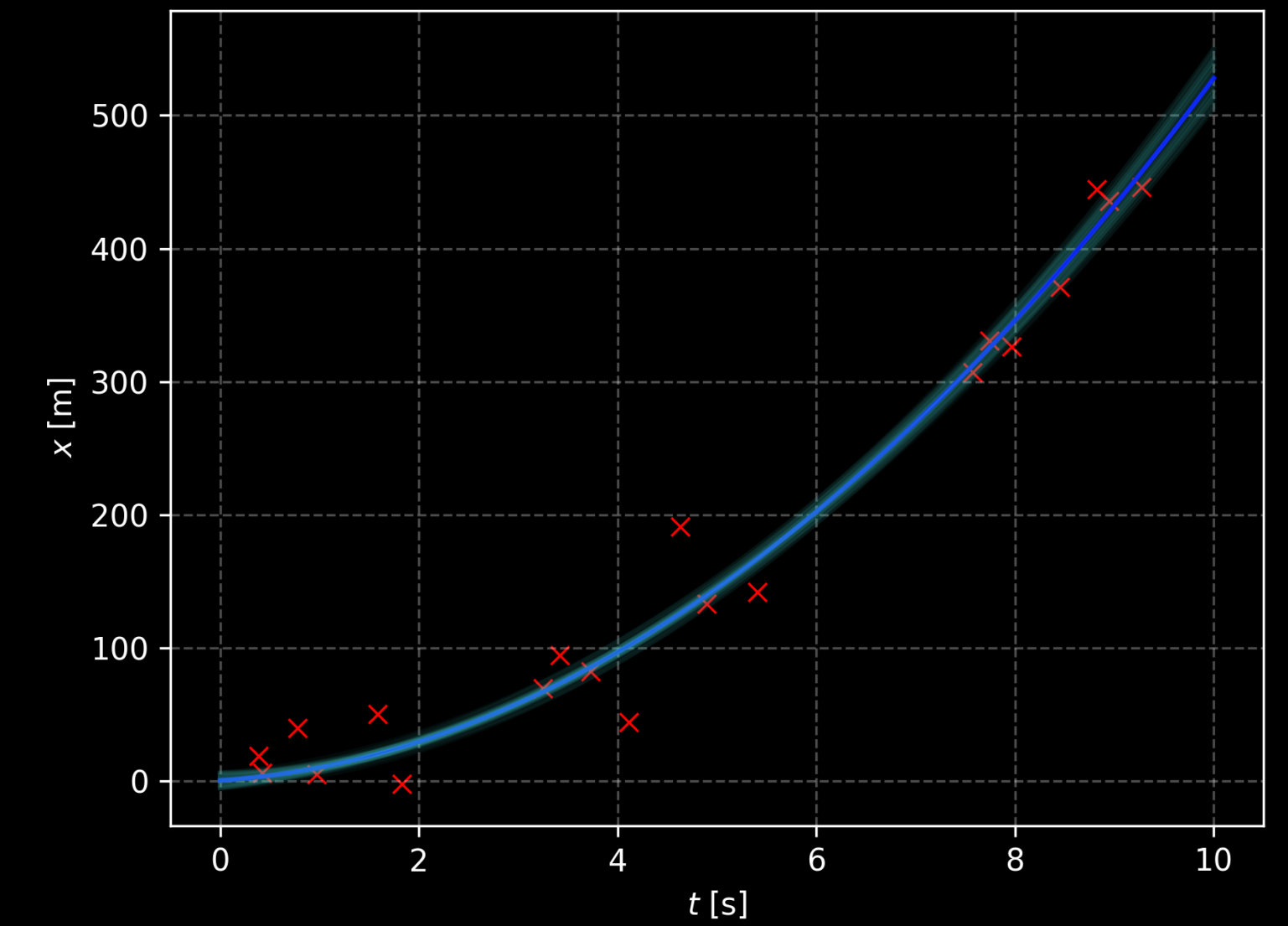
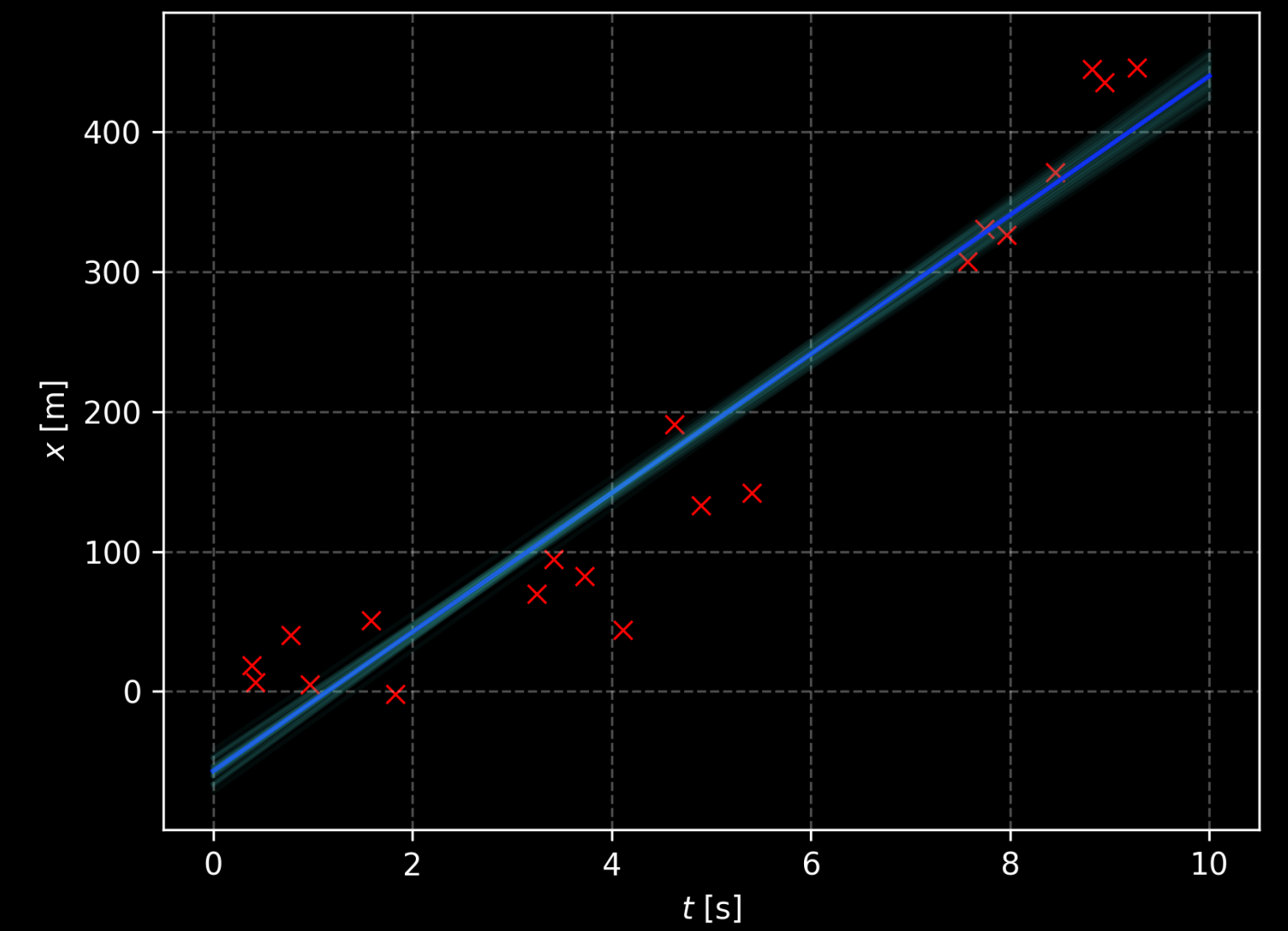
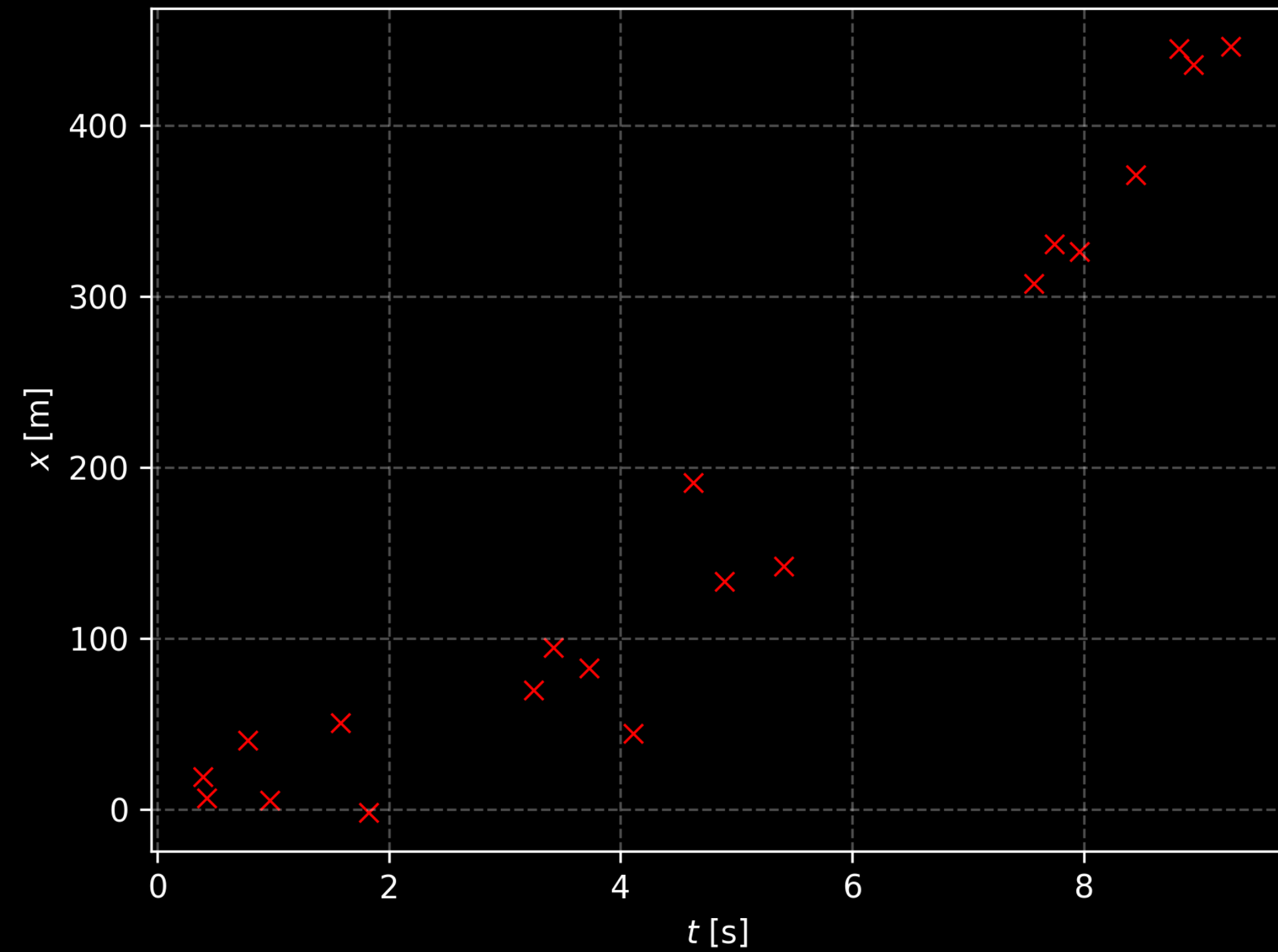
The solution

Can we let the data decide?



The solution

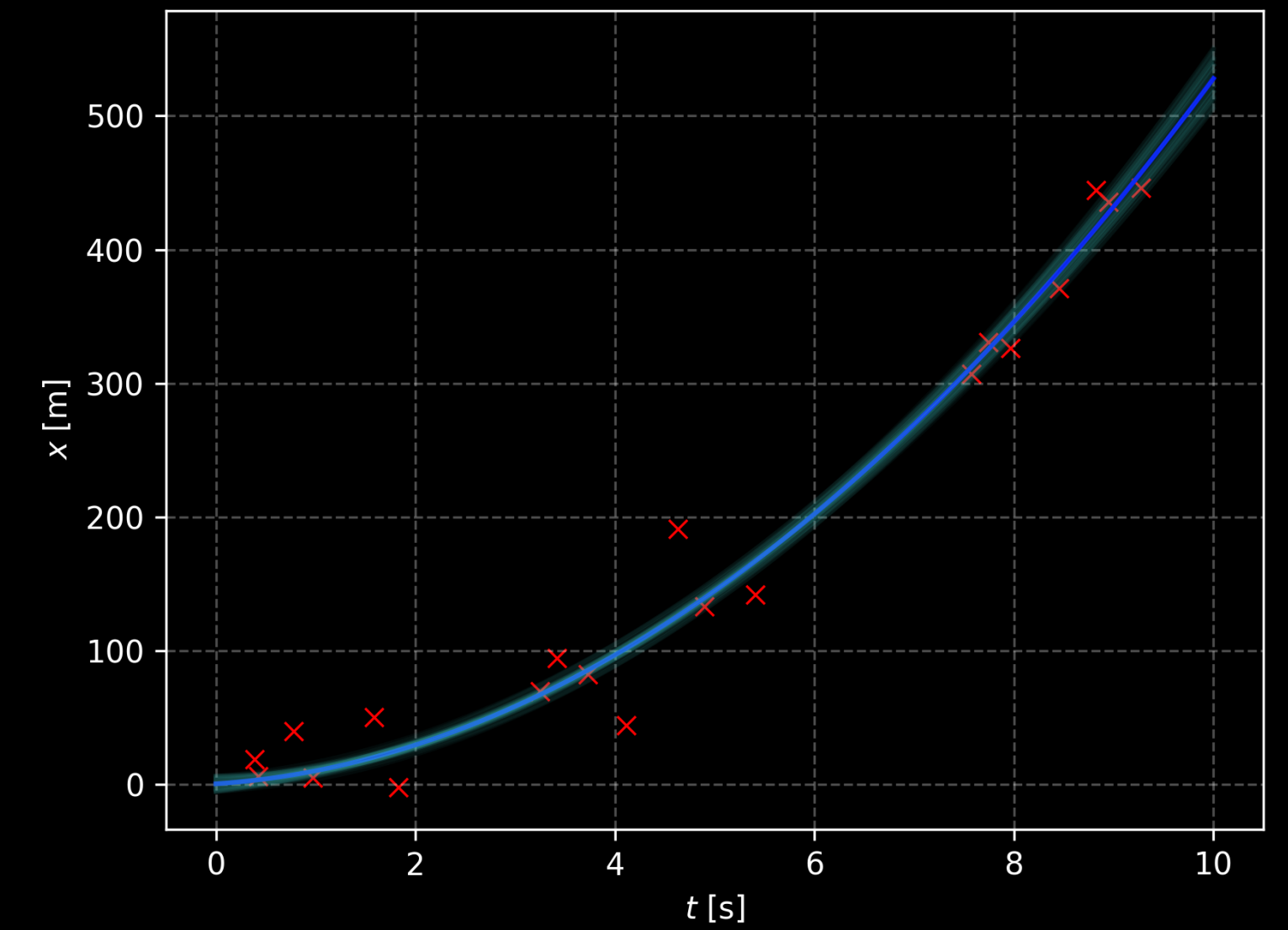
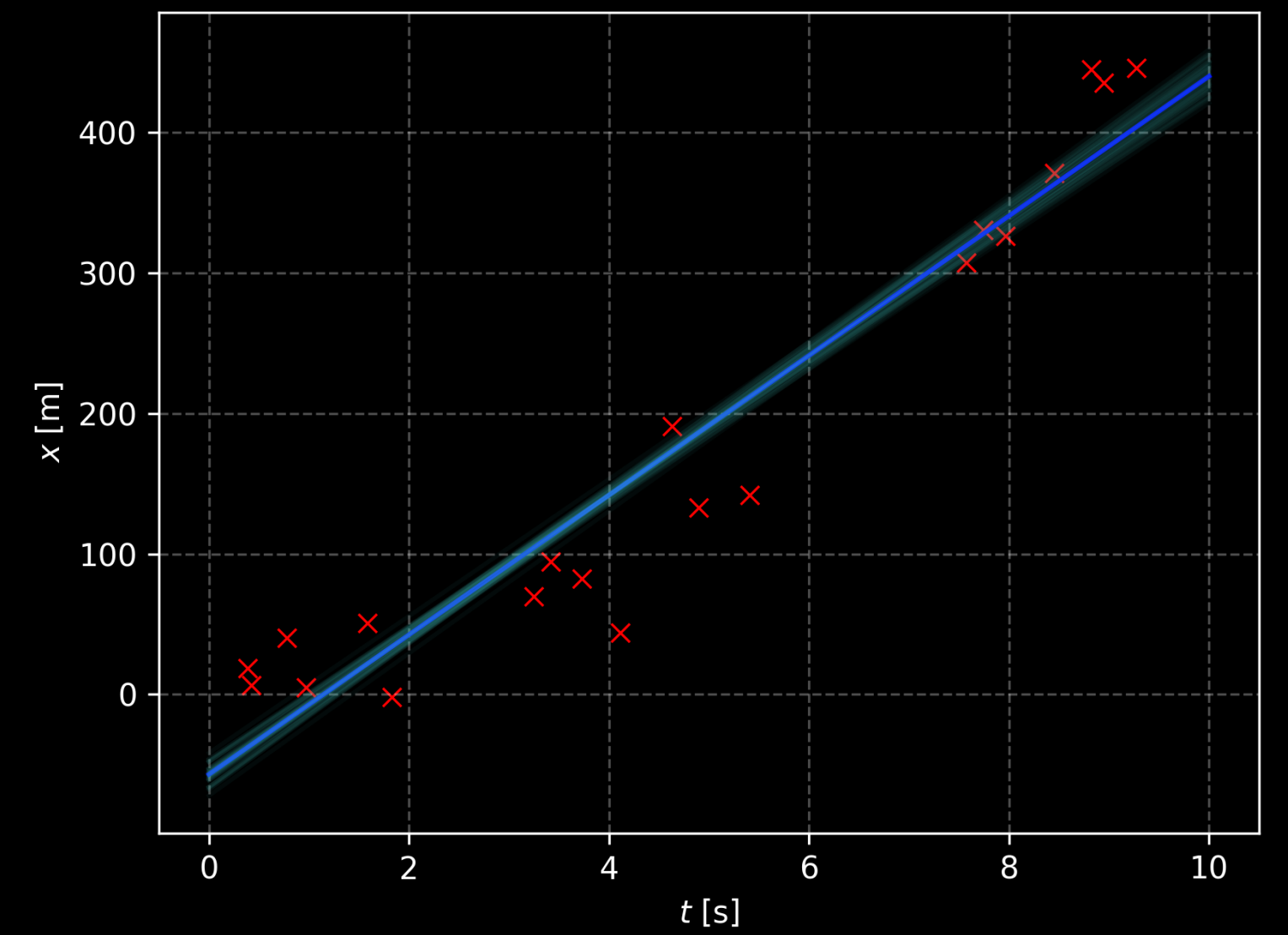
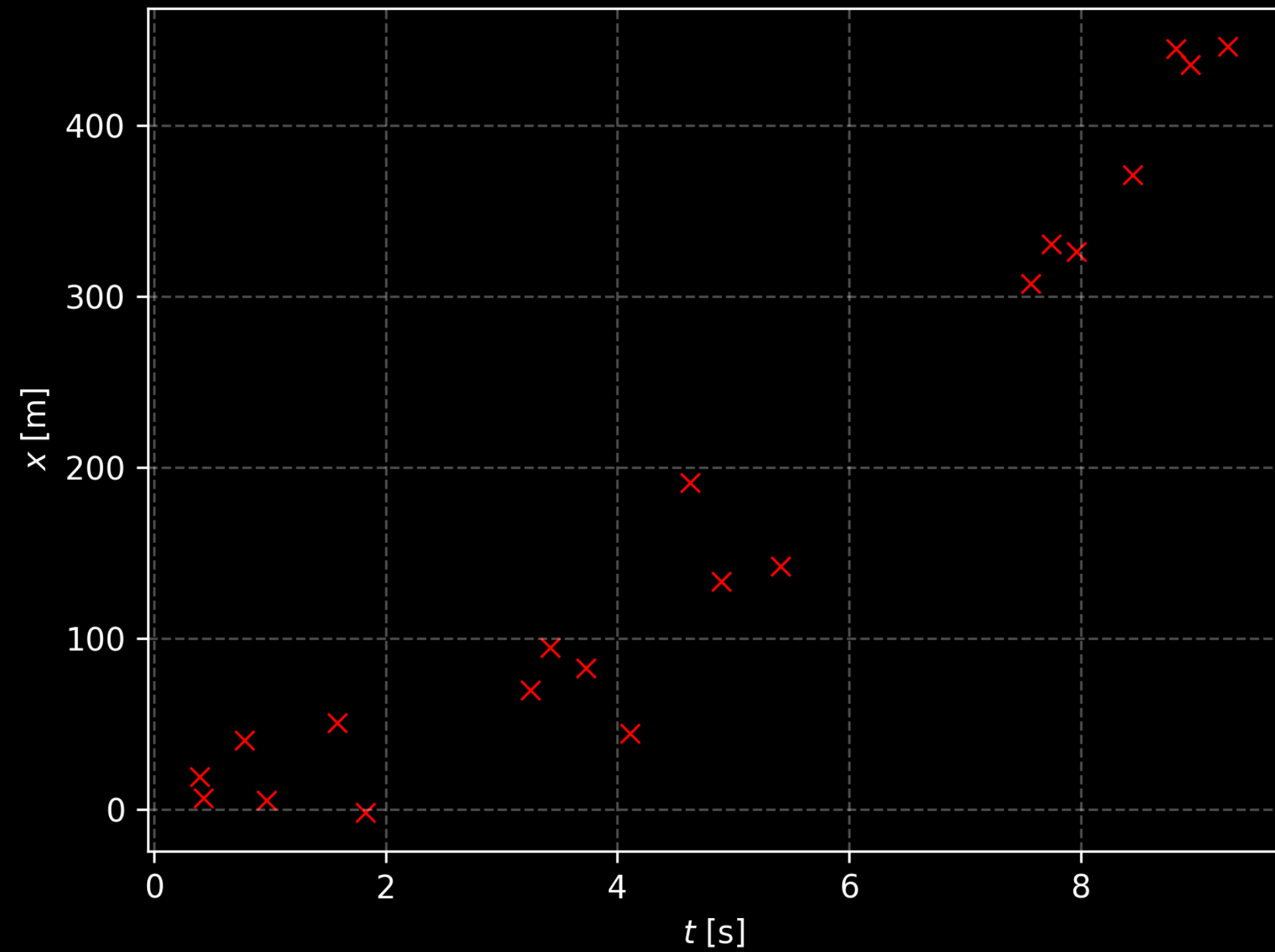
Can we let the data decide?



The solution

Can we let the data decide?

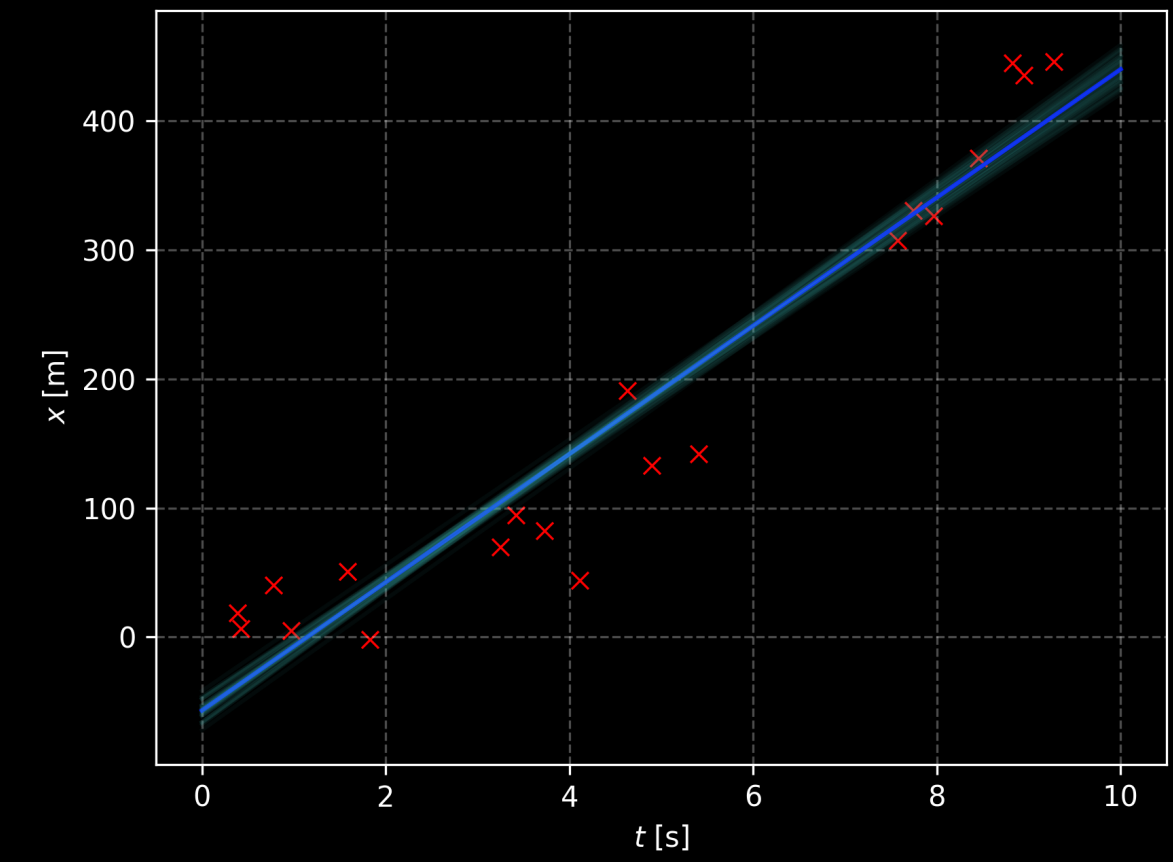
??



Comparing models

Comparing models

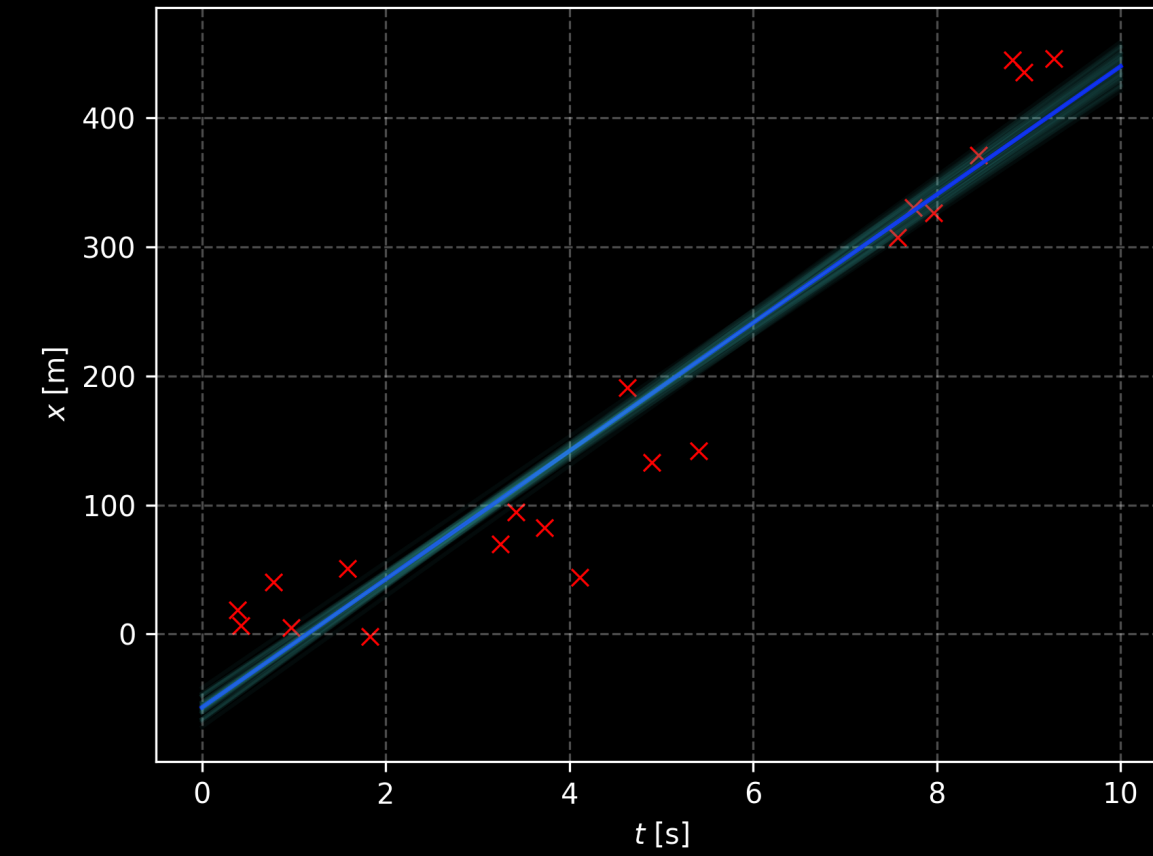
$$M_1(\theta_1)$$



Comparing models

$$M_1(\theta_1)$$

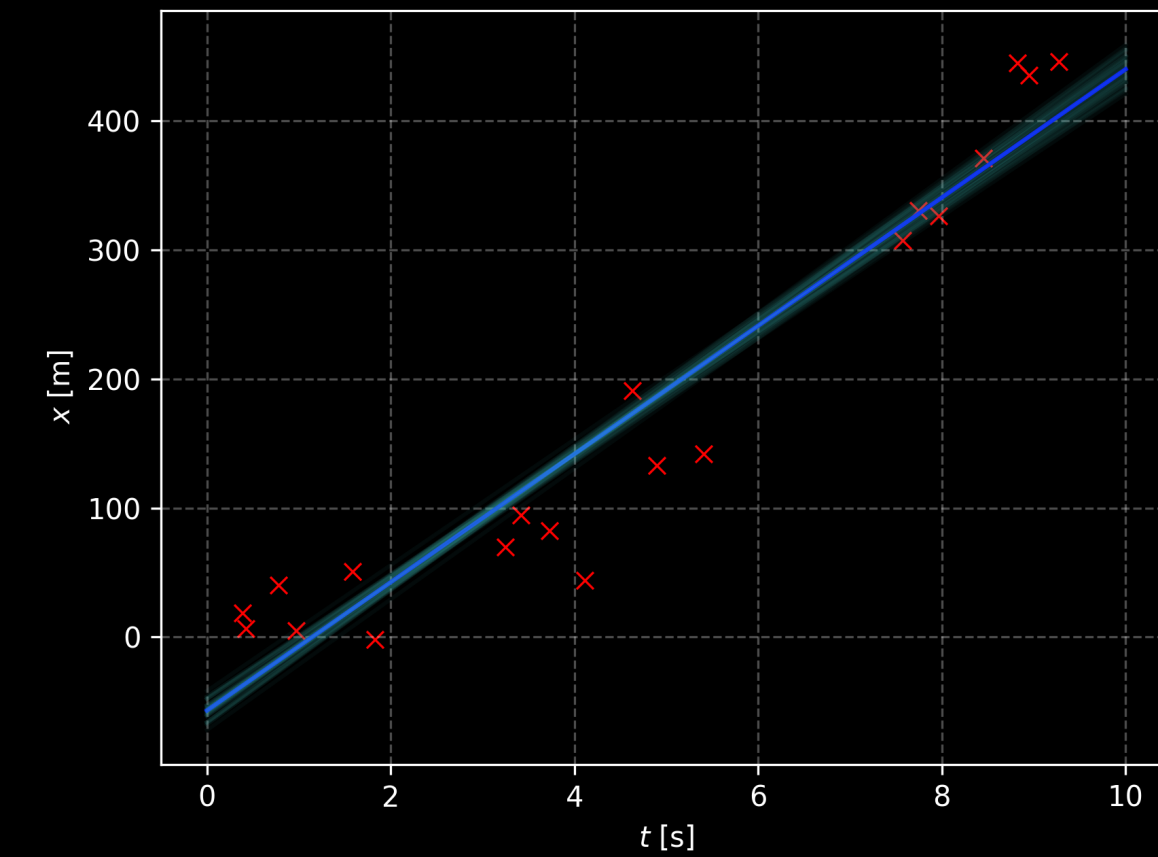
$$P(M_1 | d) = \frac{P(d | M_1) P(M_1)}{P(d)}$$



Comparing models

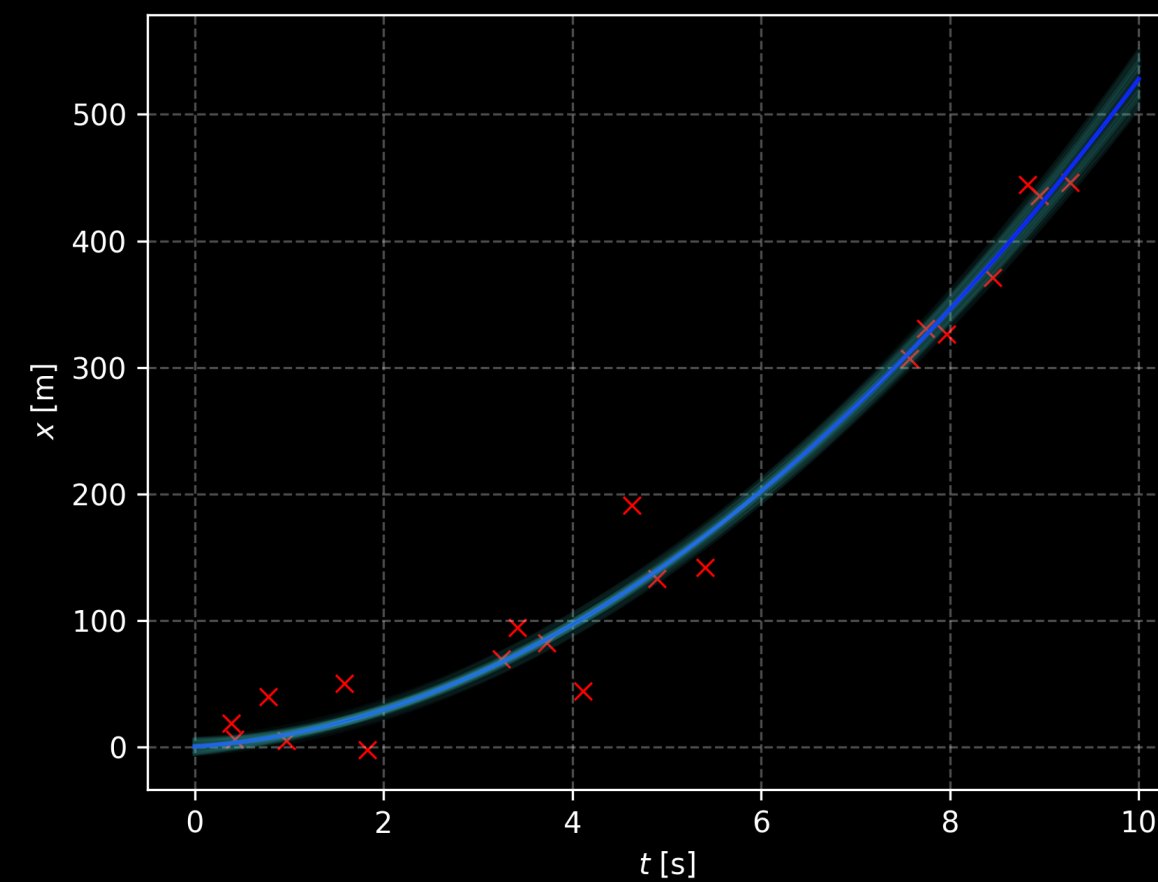
$M_1(\theta_1)$

$$P(M_1 | d) = \frac{P(d | M_1) P(M_1)}{P(d)}$$



$M_2(\theta_2)$

$$P(M_1 | d) = \frac{P(d | M_1) P(M_1)}{P(d)}$$

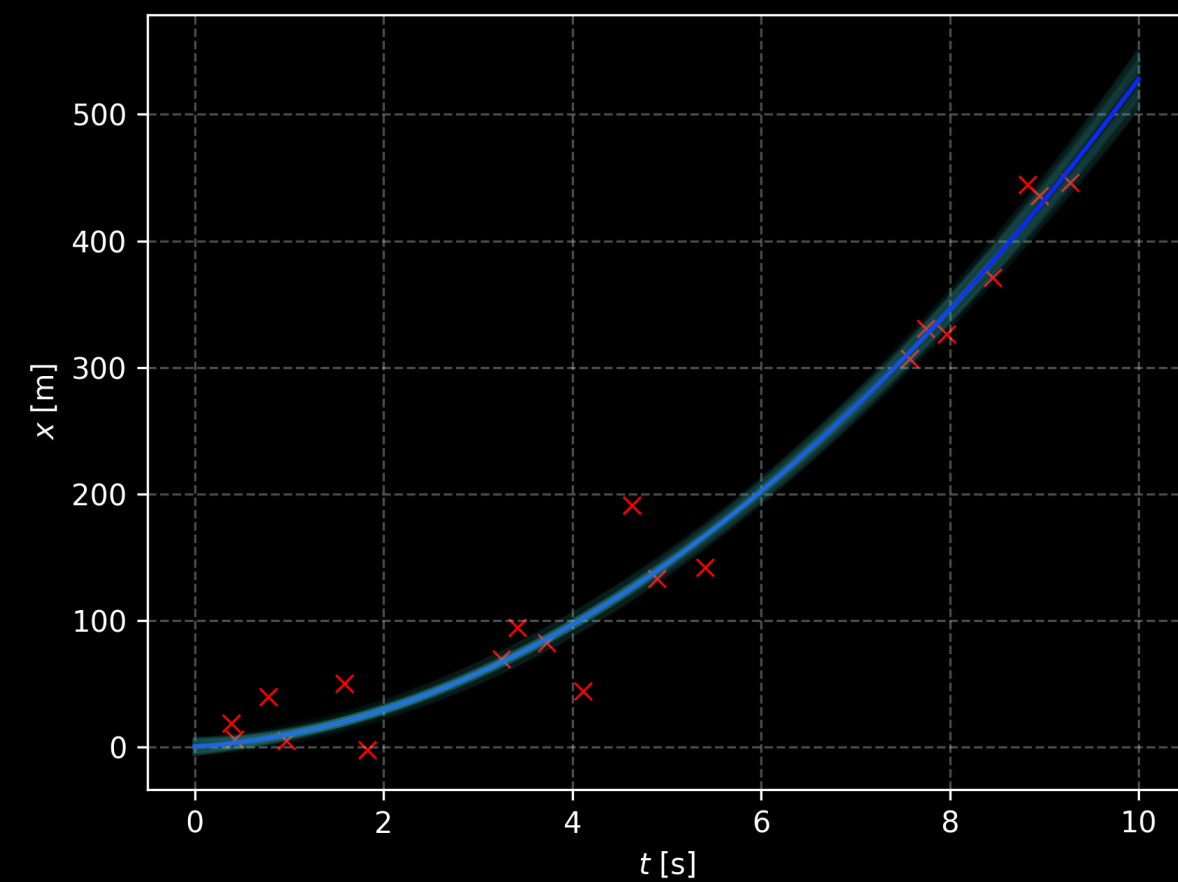
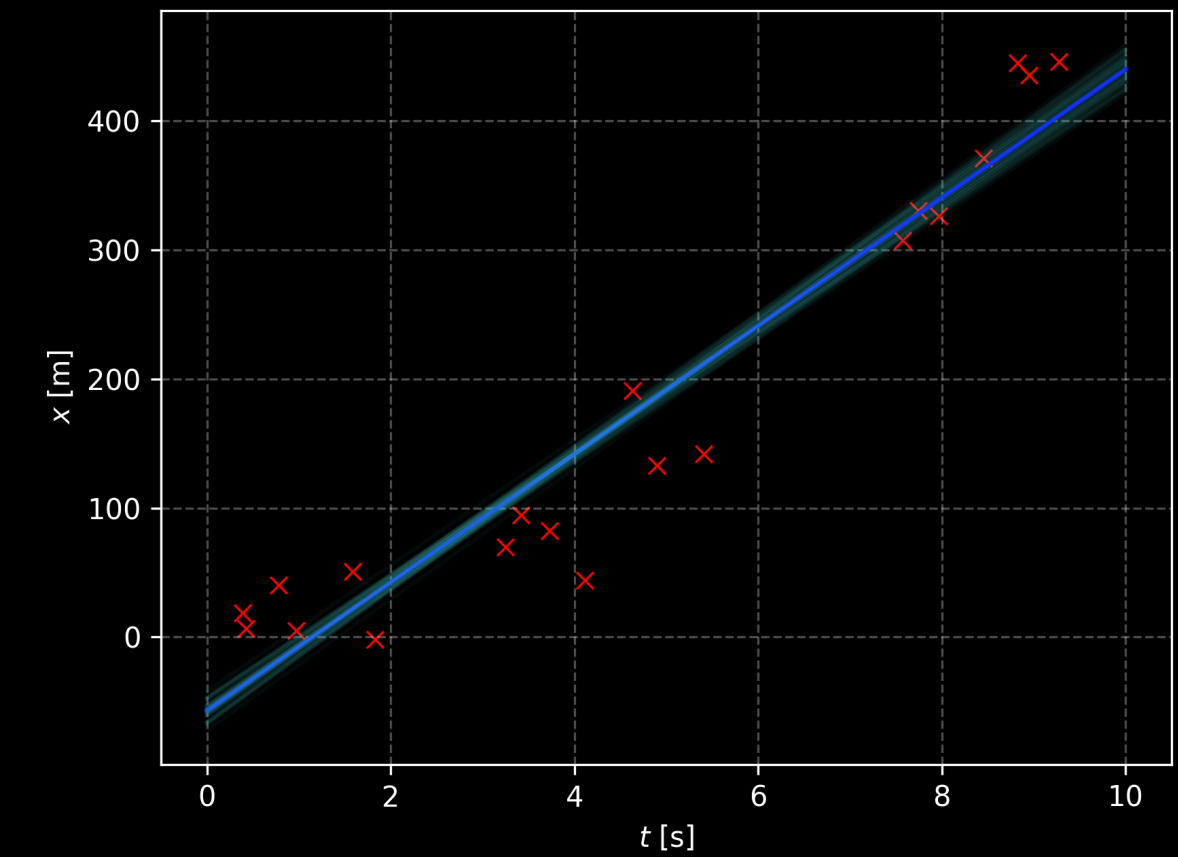


Comparing models

$$M_1(\theta_1) \quad P(M_1 | d) = \frac{P(d | M_1) P(M_1)}{P(d)}$$

$$M_2(\theta_2) \quad P(M_2 | d) = \frac{P(d | M_2) P(M_2)}{P(d)}$$

$$\mathcal{O}(M_1, M_2) := \frac{p(d | M_1) \cancel{p(M_1)}}{p(d | M_2) \cancel{p(M_2)}} = \frac{p(d | M_1)}{p(d | M_2)}$$

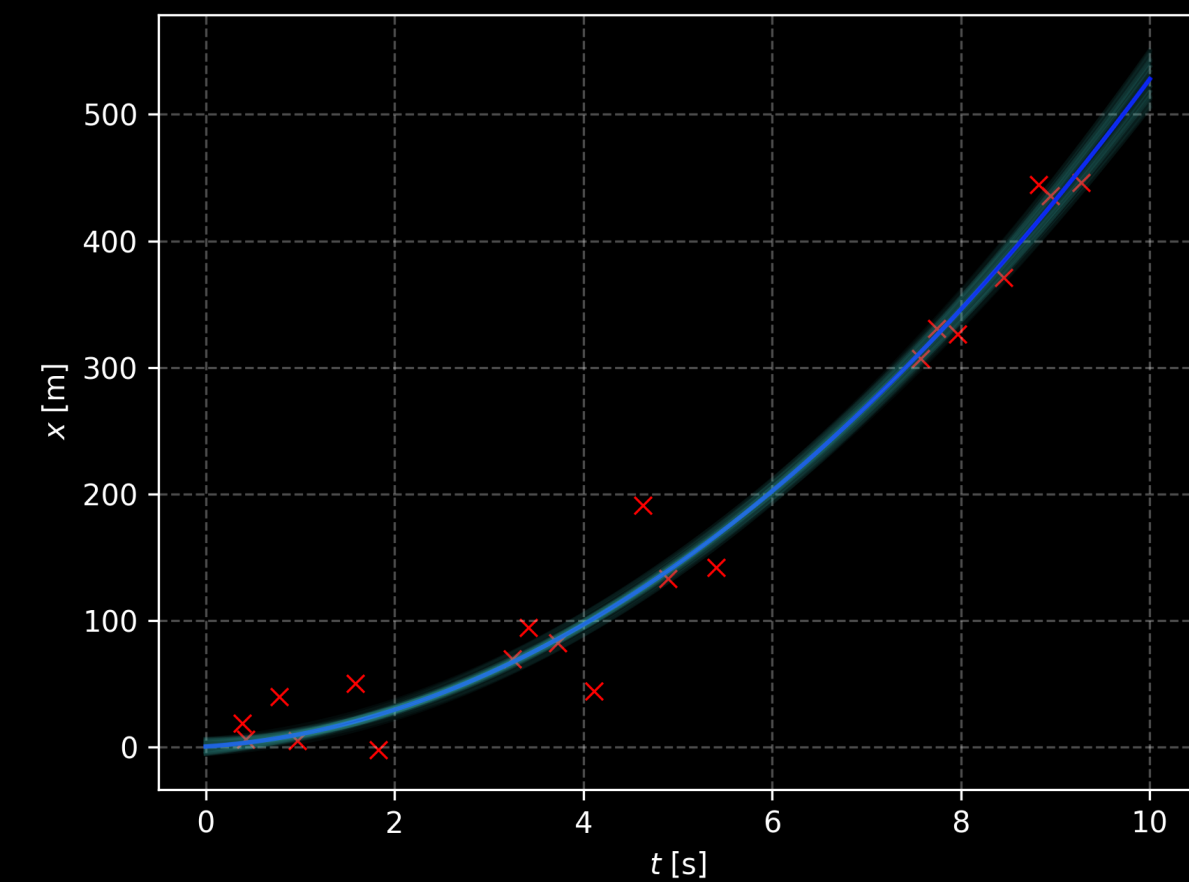
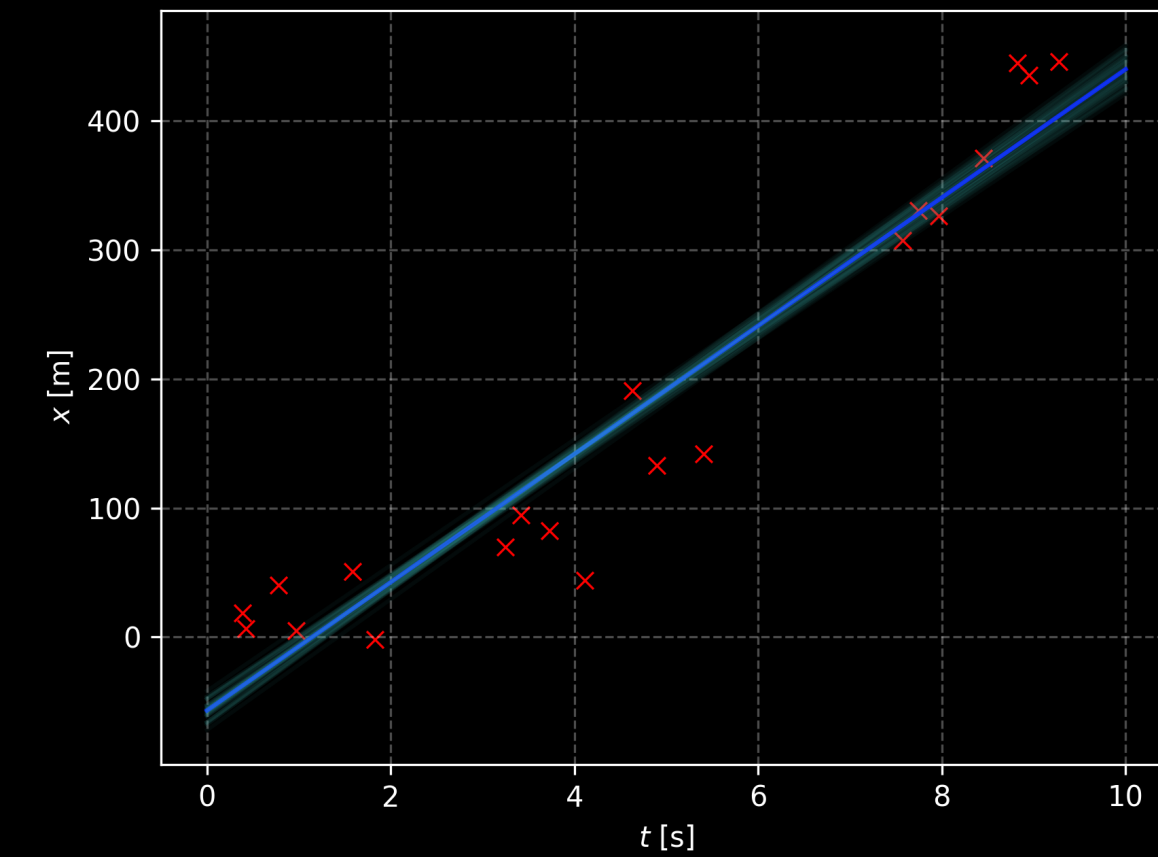


Comparing models

$$M_1(\theta_1) \quad P(M_1 | d) = \frac{P(d | M_1) P(M_1)}{P(d)}$$

$$M_2(\theta_2) \quad P(M_2 | d) = \frac{P(d | M_2) P(M_2)}{P(d)}$$

$$\mathcal{O}(M_1, M_2) := \frac{p(d | M_1) \cancel{p(M_1)}}{p(d | M_2) \cancel{p(M_2)}} = \frac{p(d | M_1)}{p(d | M_2)} \simeq e^{-24}$$



Comparing models

$$M_1(\theta_1)$$

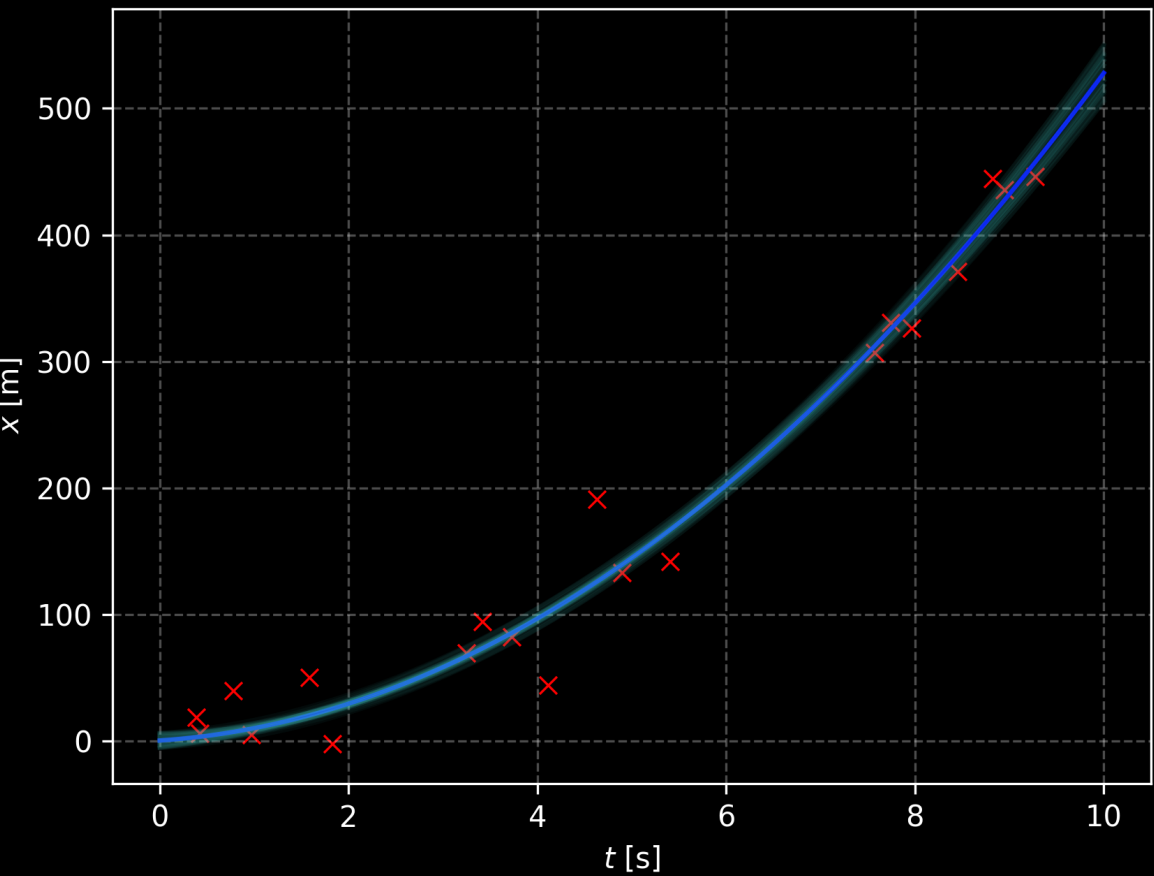
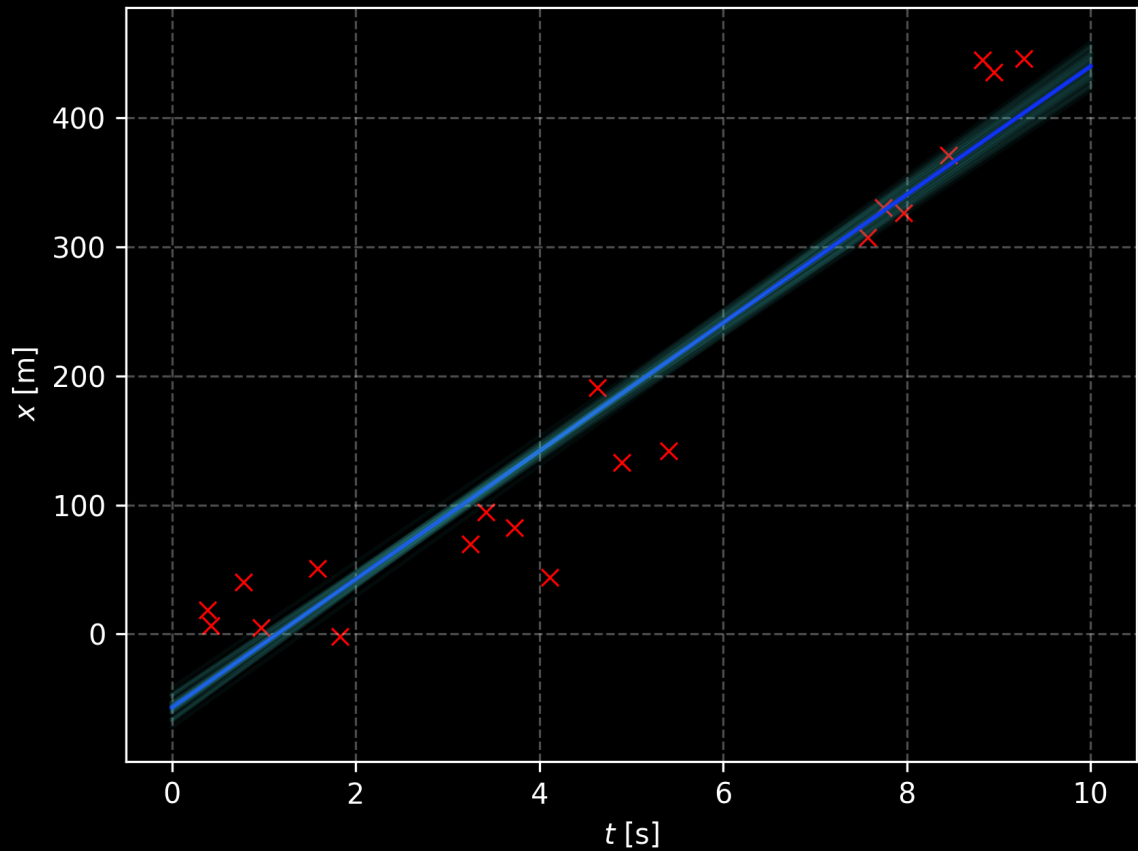
$$P(M_1 \mid d) = \frac{P(d \mid M_1) P(M_1)}{P(d)}$$



$$M_2(\theta_2)$$

$$P(M_1 \mid d) = \frac{P(d \mid M_1) P(M_1)}{P(d)}$$

$$\mathcal{O}(M_1, M_2) := \frac{p(d \mid M_1) \cancel{p(M_1)}}{p(d \mid M_2) \cancel{p(M_2)}} = \frac{p(d \mid M_1)}{p(d \mid M_2)} \simeq e^{-24}$$



So what's the problem?

The Evidence

$$P(s | d) = \frac{P(d | s) P(s)}{P(d)}$$

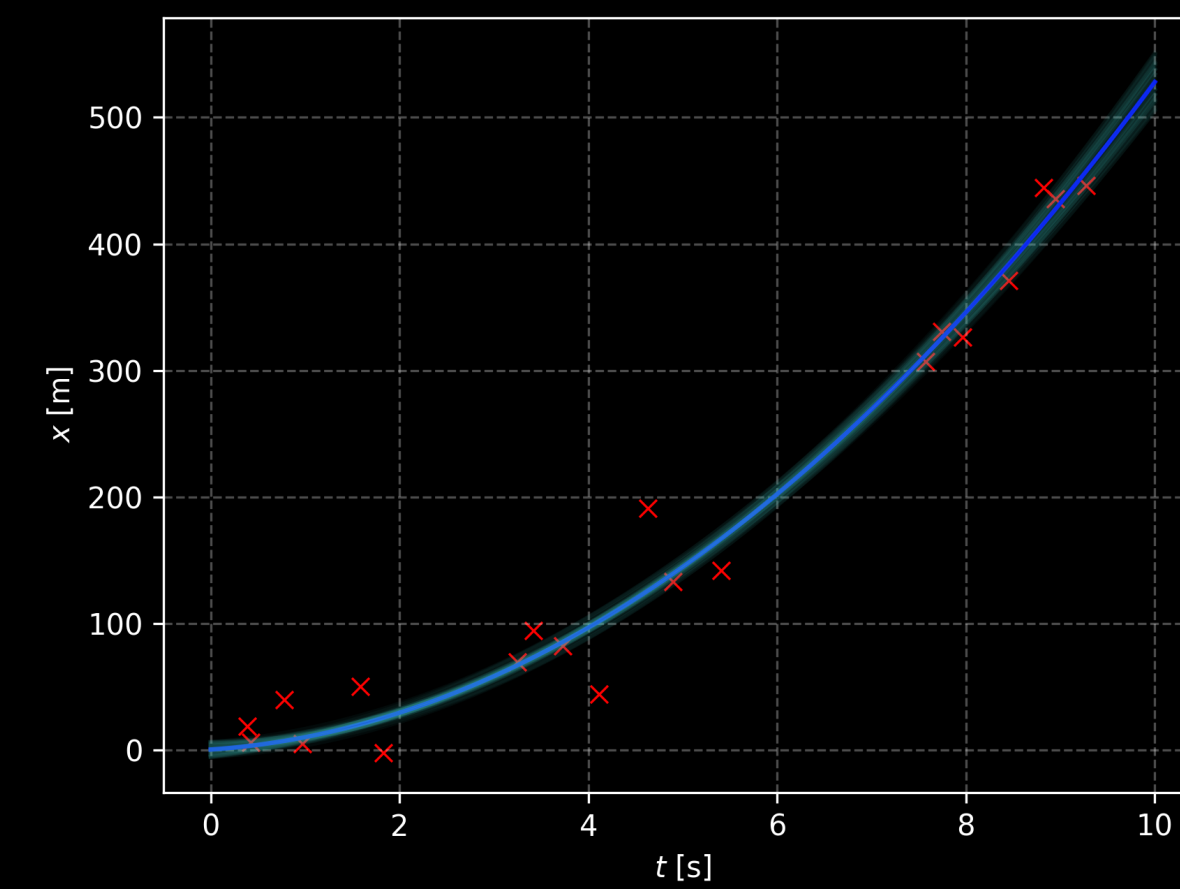
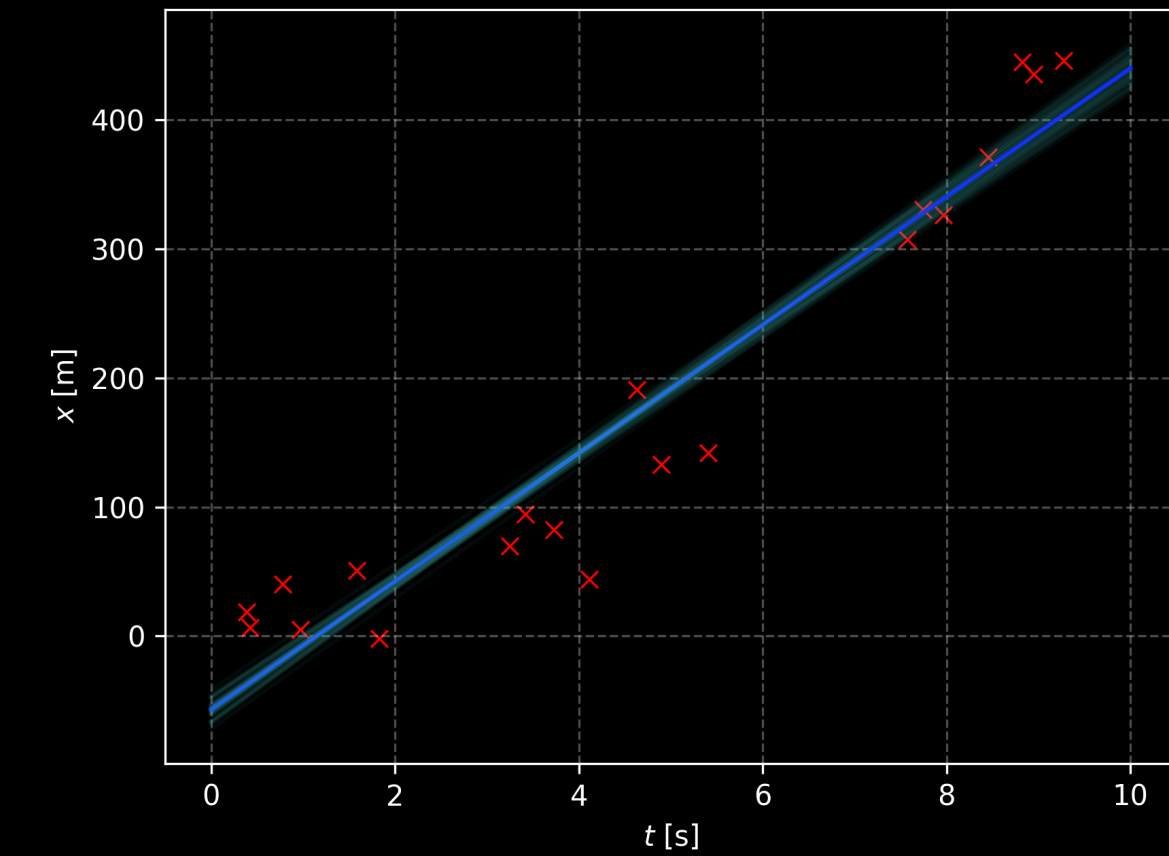
Evidence in HD

Evidence in HD

$$p(d|M) = \int p(d|\theta, M) p(\theta|M) d\theta$$

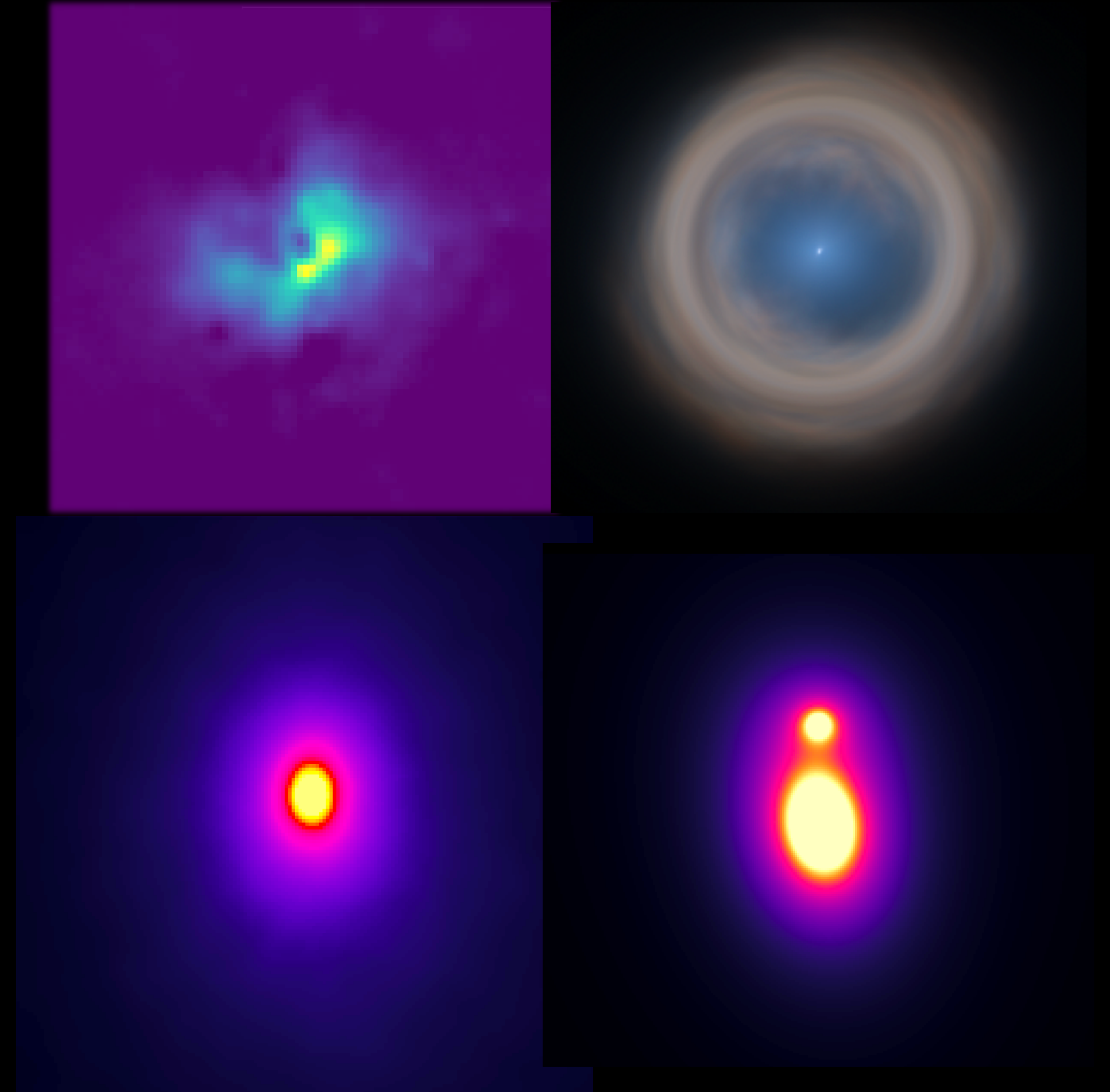
Evidence in HD

$$p(d|M) = \int p(d|\theta, M) p(\theta|M) d\theta$$



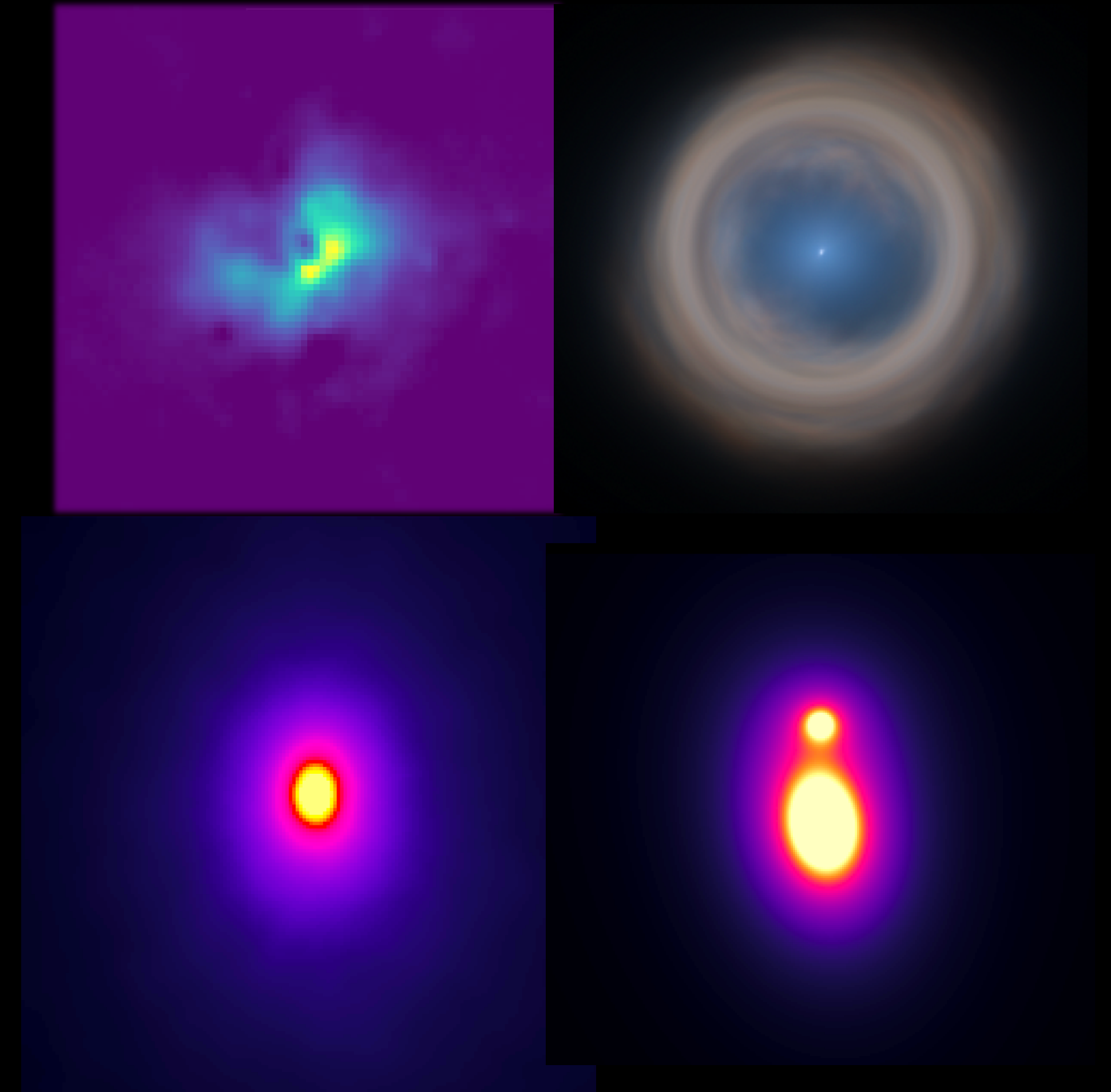
Evidence in HD

$$p(d|M) = \int p(d|\theta, M) p(\theta|M) d\theta$$



Evidence in HD

$$p(d|M) = \int p(d|\theta, M) p(\theta|M) d\theta$$



$$\dim(\theta) \sim 10^6 - 10^9$$

Model choice

For fields :)

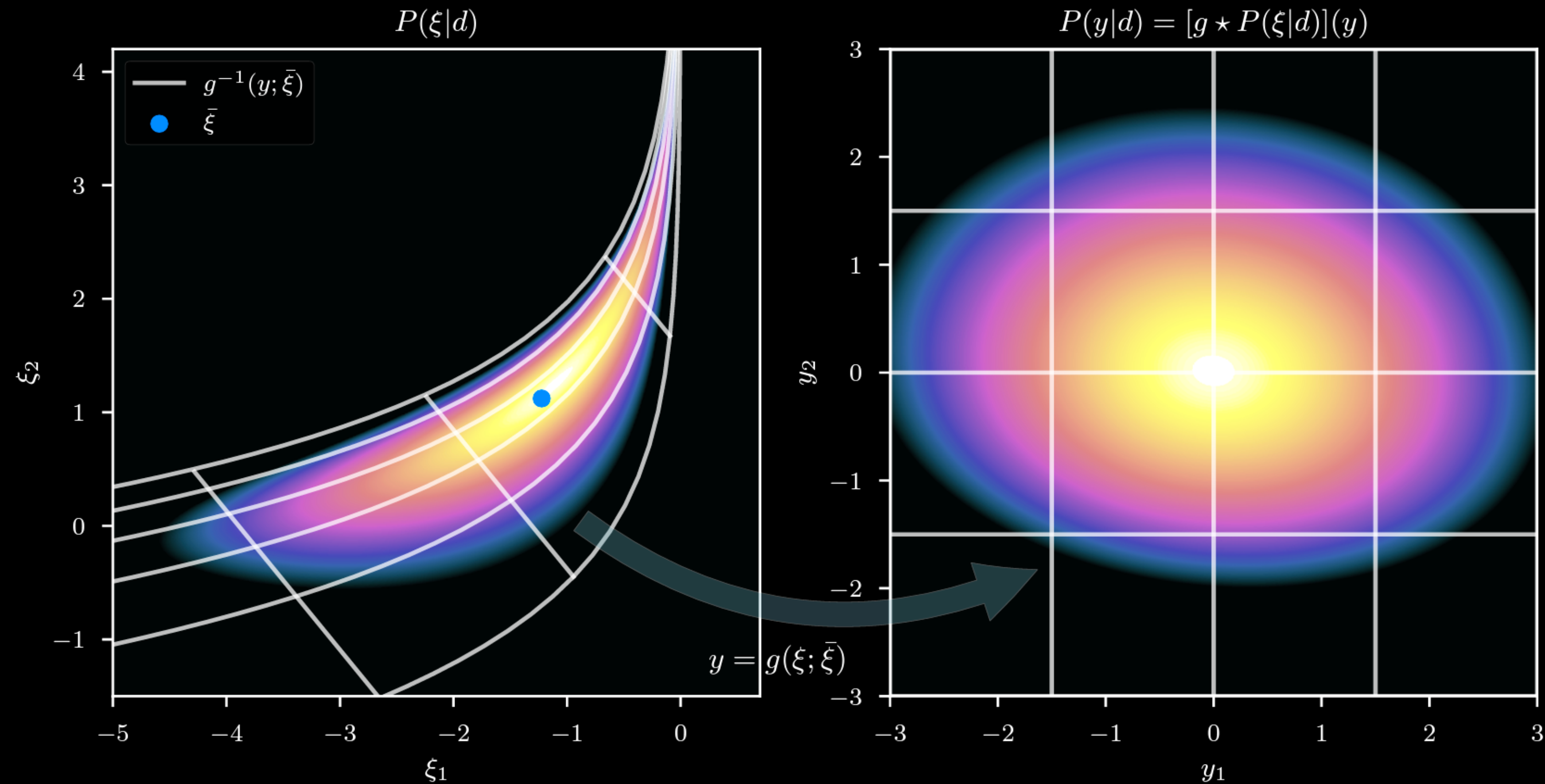
Model choice

For fields :)



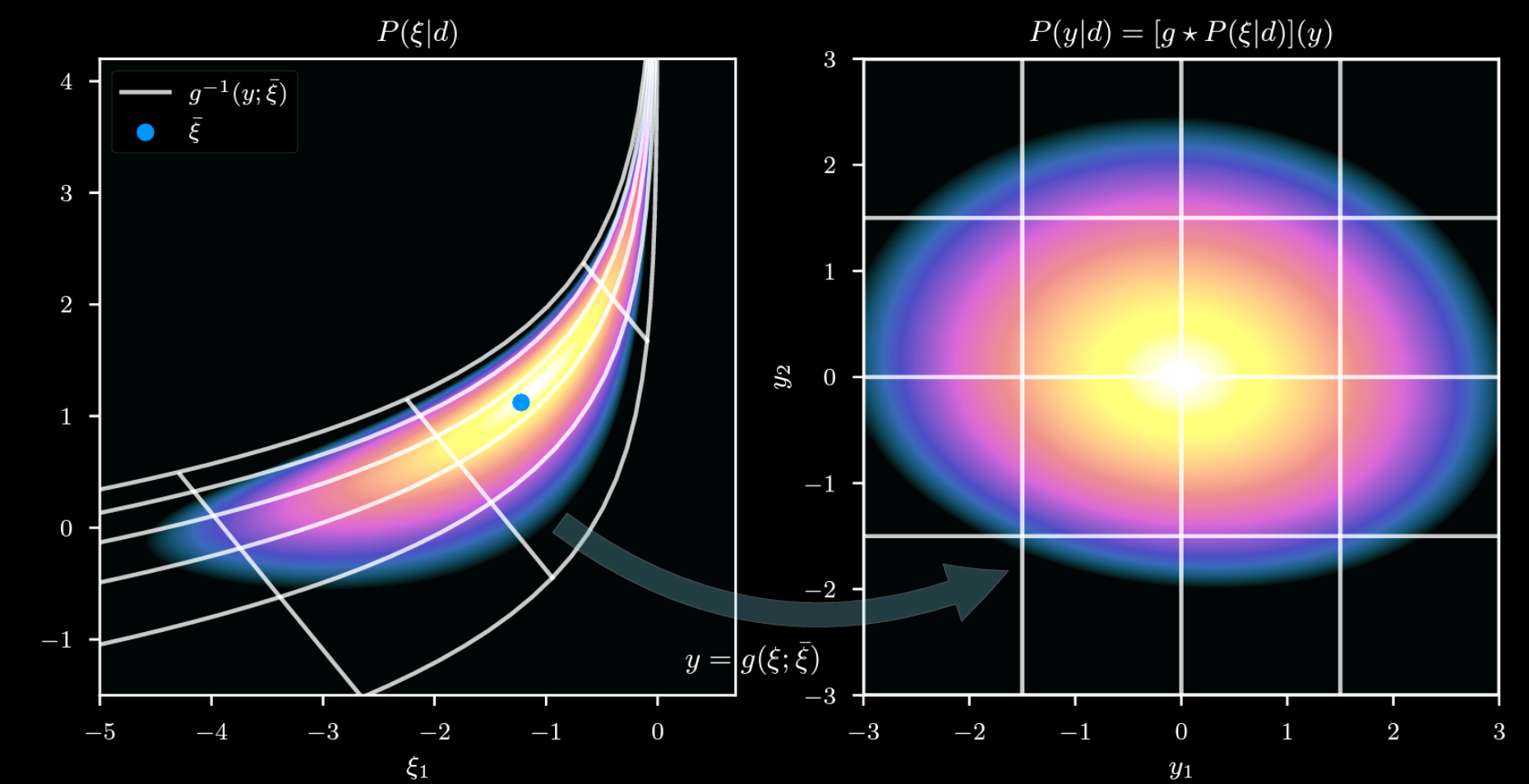
Inference

geometric Variational Inference



Inference

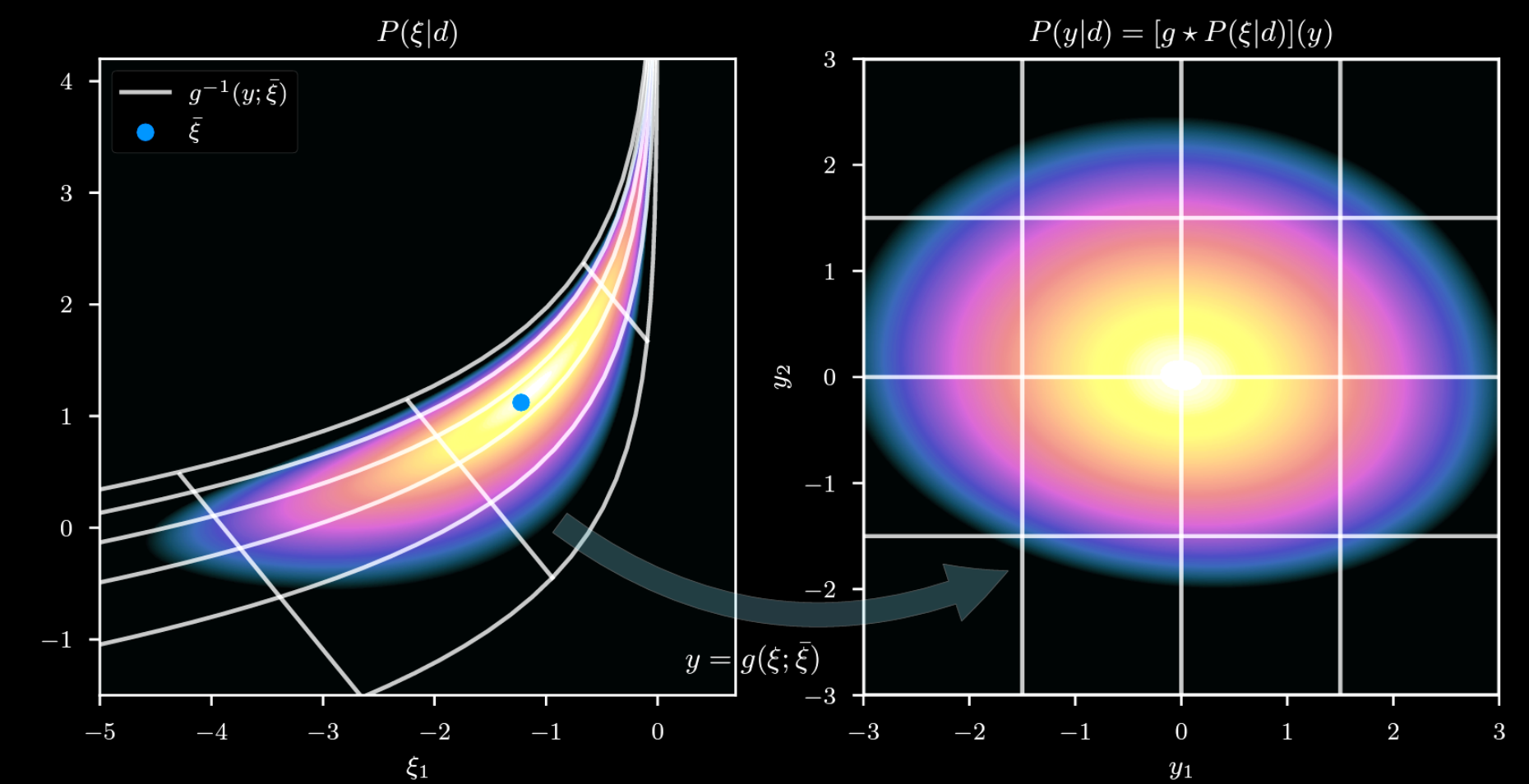
geometric Variational Inference



Inference

geometric Variational Inference

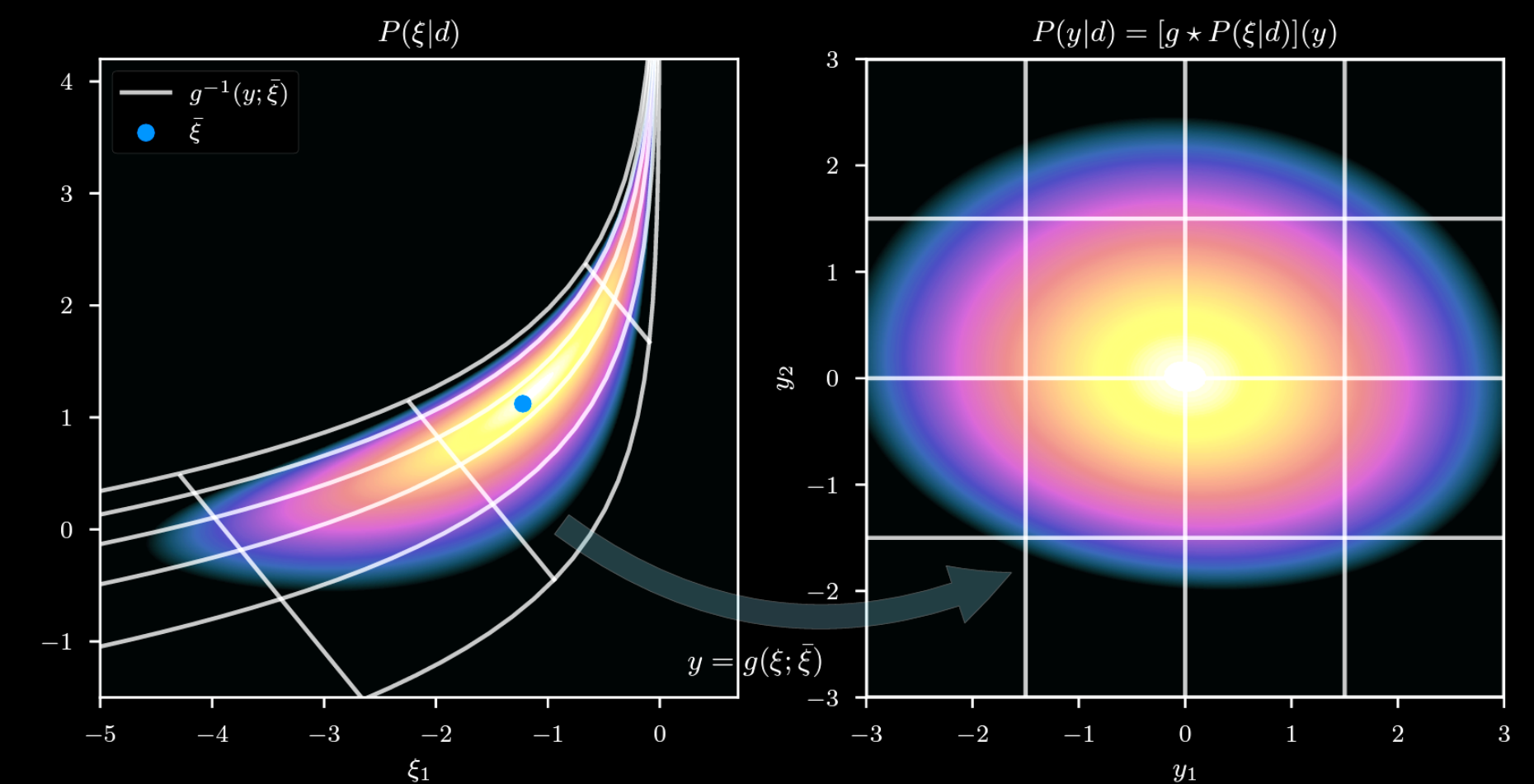
$$D_{\text{KL}}(q_\phi \parallel p) = -\mathbb{E}_{q_\phi}[\mathcal{H}_q(\theta)] + \mathbb{E}_{q_\phi}[\mathcal{H}_p(\theta, d \mid M)] + \log p(d \mid M)$$



Inference

geometric Variational Inference

$$D_{\text{KL}}(q_\phi \parallel p) = \underbrace{-\mathbb{E}_{q_\phi}[\mathcal{H}_q(\theta)] + \mathbb{E}_{q_\phi}[\mathcal{H}_p(\theta, d \mid M)] + \log p(d \mid M)}_{-\text{ELBO}}$$

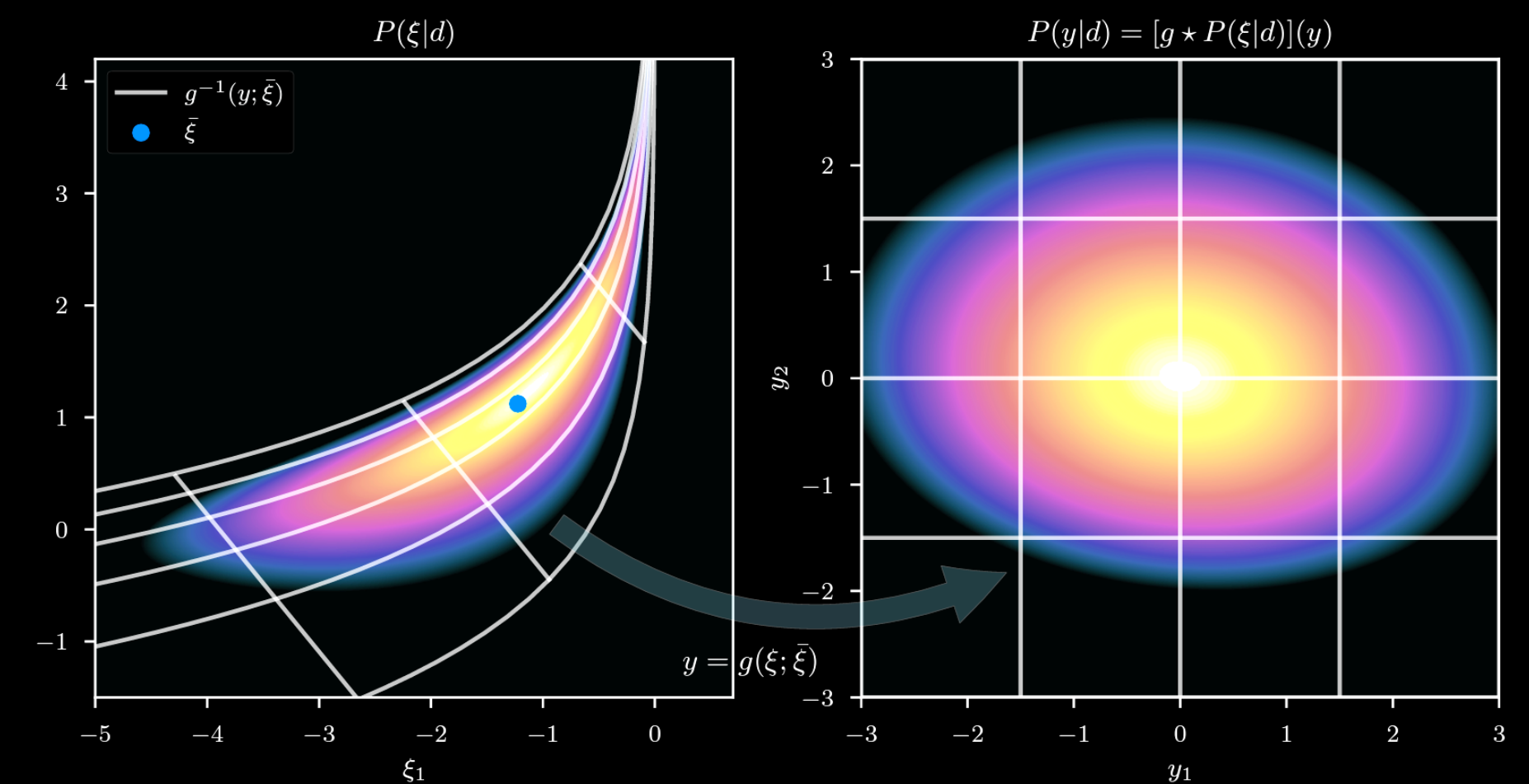


Inference

geometric Variational Inference

$$D_{\text{KL}}(q_\phi \parallel p) = \underbrace{-\mathbb{E}_{q_\phi}[\mathcal{H}_q(\theta)] + \mathbb{E}_{q_\phi}[\mathcal{H}_p(\theta, d \mid M)]}_{-\text{ELBO}} + \log p(d \mid M)$$

$$\log p(d \mid M) = D_{\text{KL}}(q_\phi \parallel p) + \text{ELBO}$$

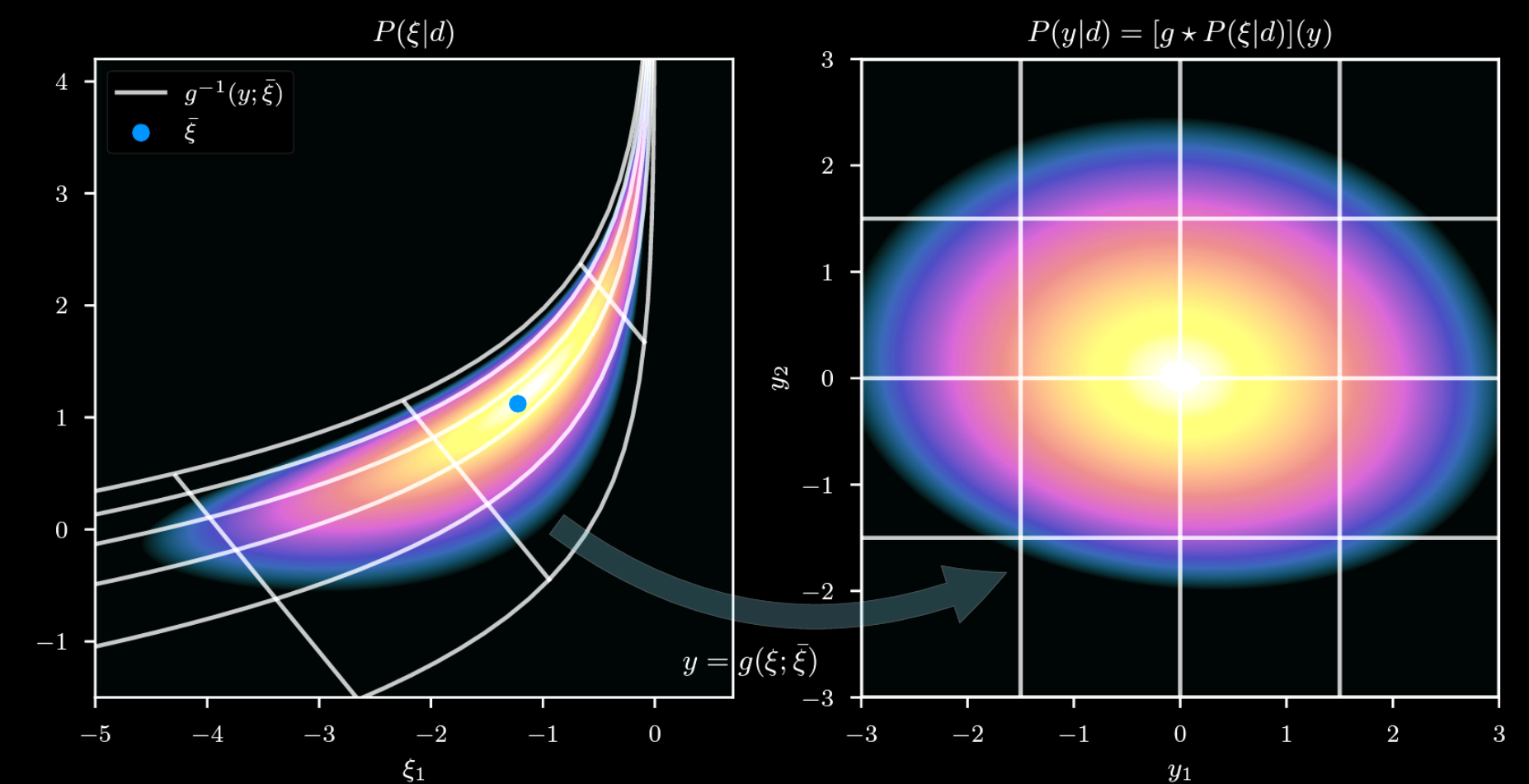


Inference

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$$D_{\text{KL}}(q_\phi \parallel p) = \underbrace{-\mathbb{E}_{q_\phi}[\mathcal{H}_q(\theta)] + \mathbb{E}_{q_\phi}[\mathcal{H}_p(\theta, d \mid M)]}_{-\text{ELBO}} + \log p(d \mid M)$$

$$\log p(d \mid M) \leq \text{ELBO}$$



Comparing models

Fields

Comparing models

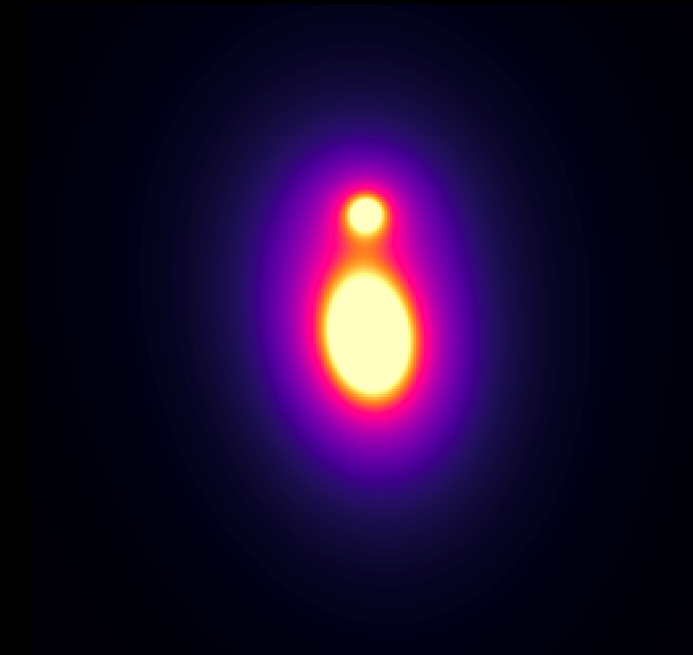
Fields

$$\log \mathcal{O}(M_1, M_2 | d) = [\text{ELBO}_1 - \text{ELBO}_2] + [\mathcal{D}_{\text{KL},1} - \mathcal{D}_{\text{KL},2}] + \log \frac{p(M_1)}{p(M_2)}$$

Comparing models

Fields

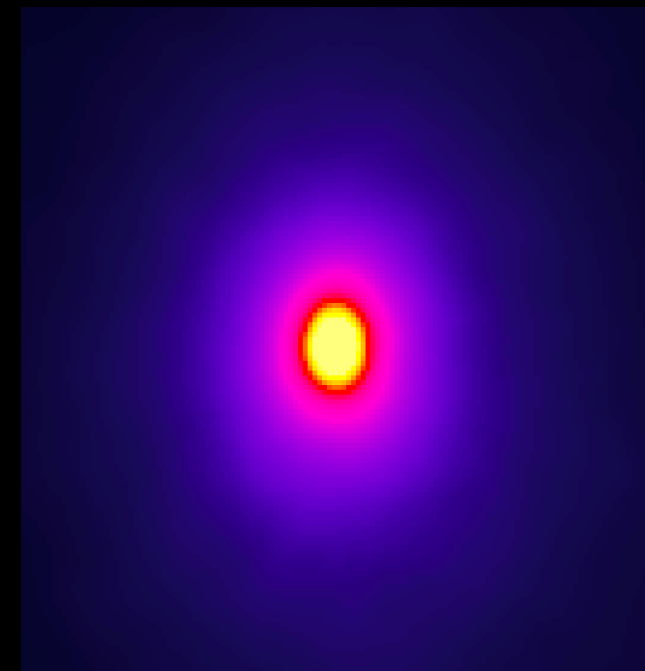
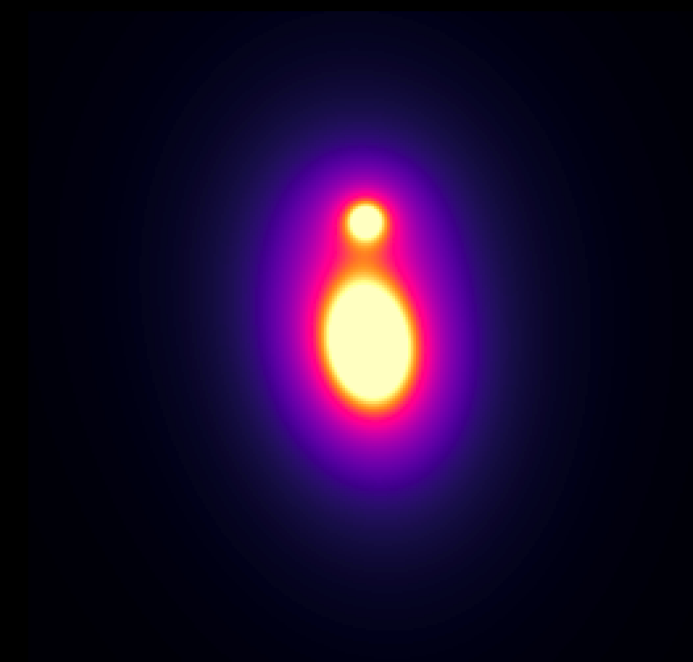
$$\log \mathcal{O}(M_1, M_2 | d) = [\text{ELBO}_1 - \text{ELBO}_2] + [\mathcal{D}_{\text{KL},1} - \mathcal{D}_{\text{KL},2}] + \log \frac{p(M_1)}{p(M_2)}$$



Comparing models

Fields

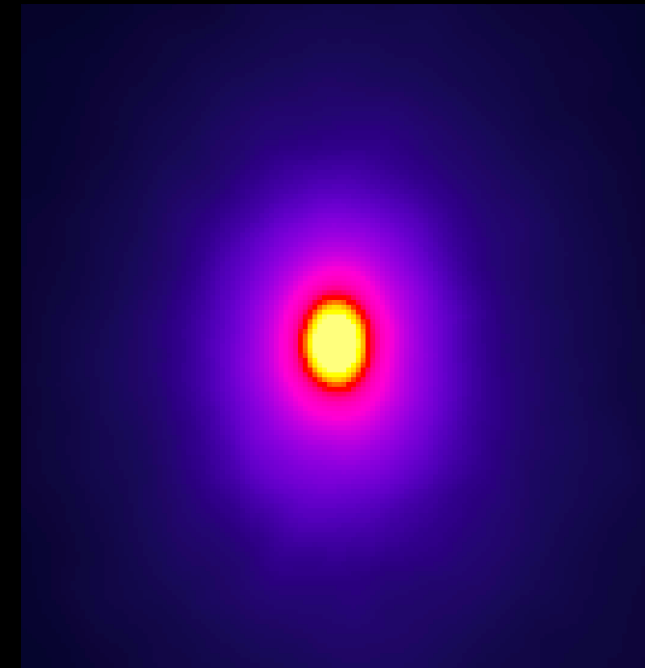
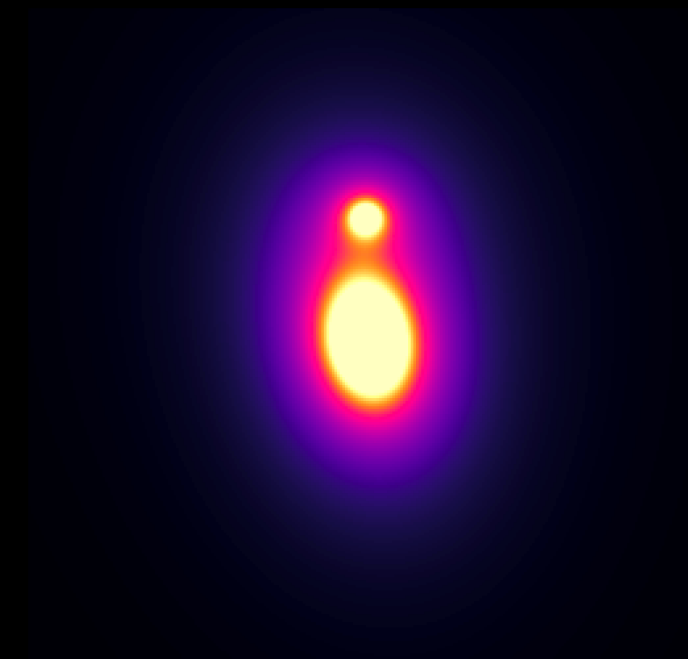
$$\log \mathcal{O}(M_1, M_2 | d) = [\text{ELBO}_1 - \text{ELBO}_2] + [\mathcal{D}_{\text{KL},1} - \mathcal{D}_{\text{KL},2}] + \log \frac{p(M_1)}{p(M_2)}$$



Comparing models

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$$\log \mathcal{O}(M_1, M_2 | d) = [\text{ELBO}_1 - \text{ELBO}_2] + [\mathcal{D}_{\text{KL},1} - \mathcal{D}_{\text{KL},2}] + \log \frac{p(M_1)}{p(M_2)}$$

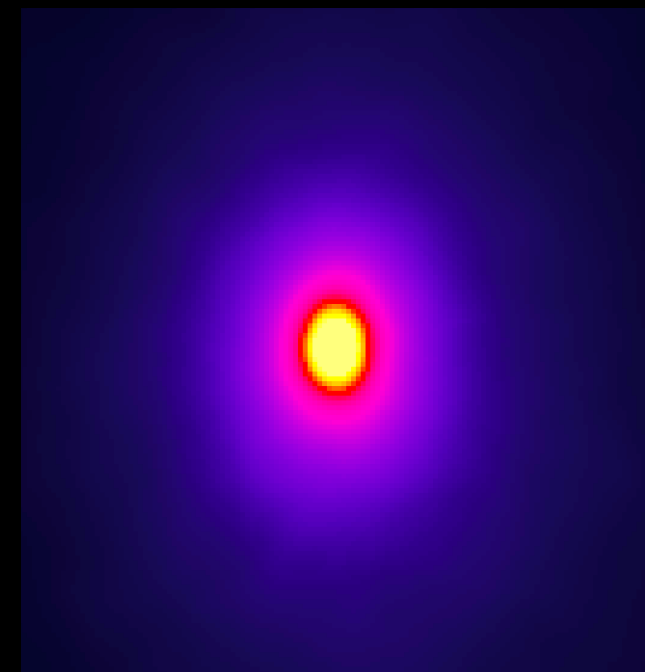
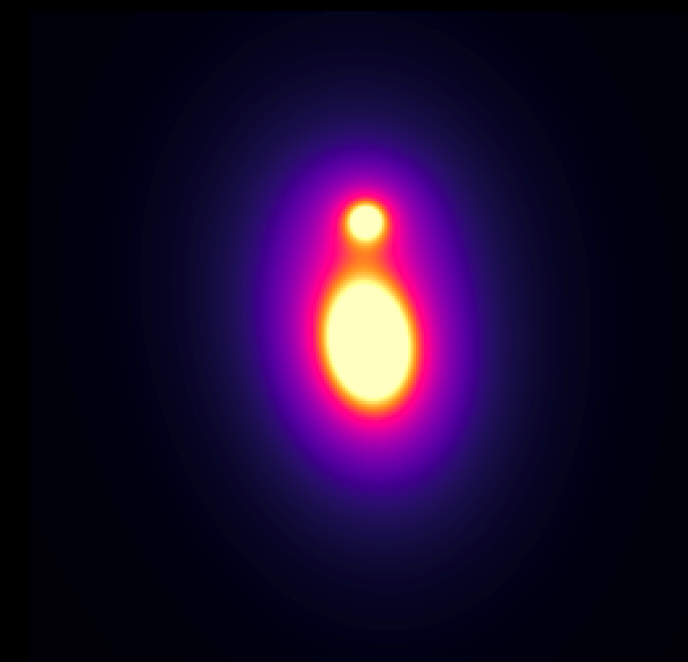


$$\log \mathcal{O}(M_1, M_2 | d) \simeq 28$$

Comparing models

Fields

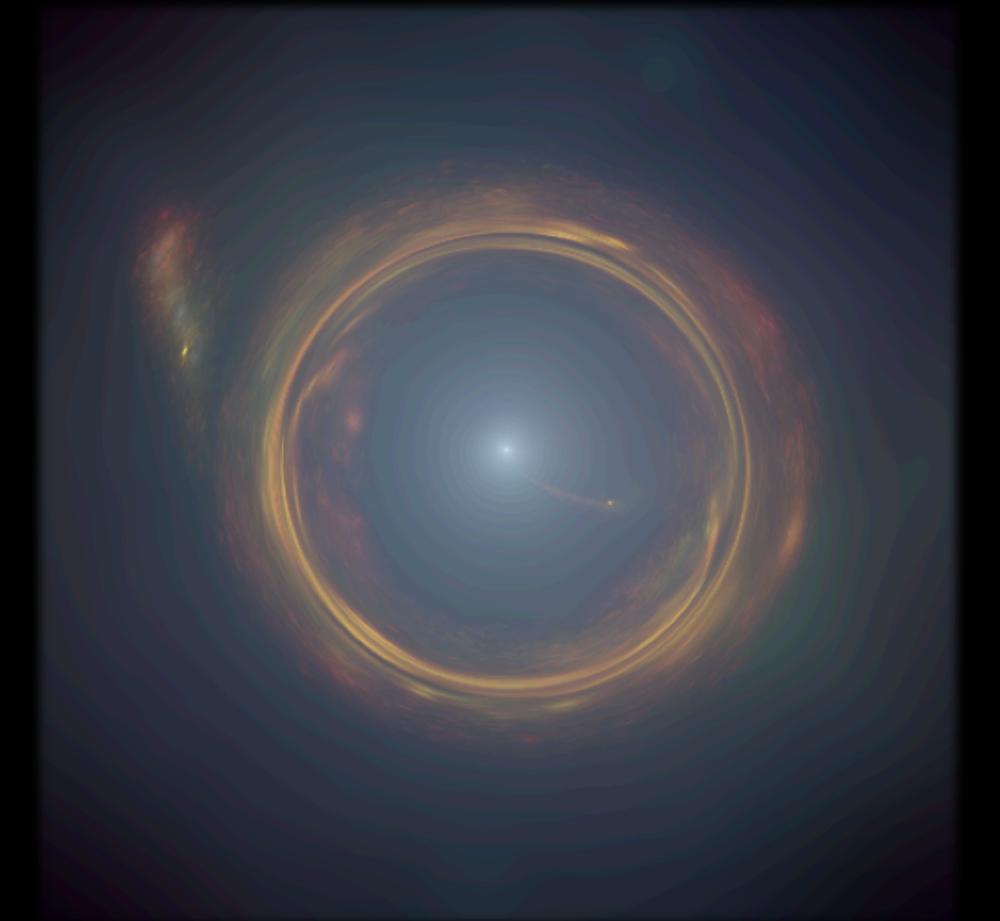
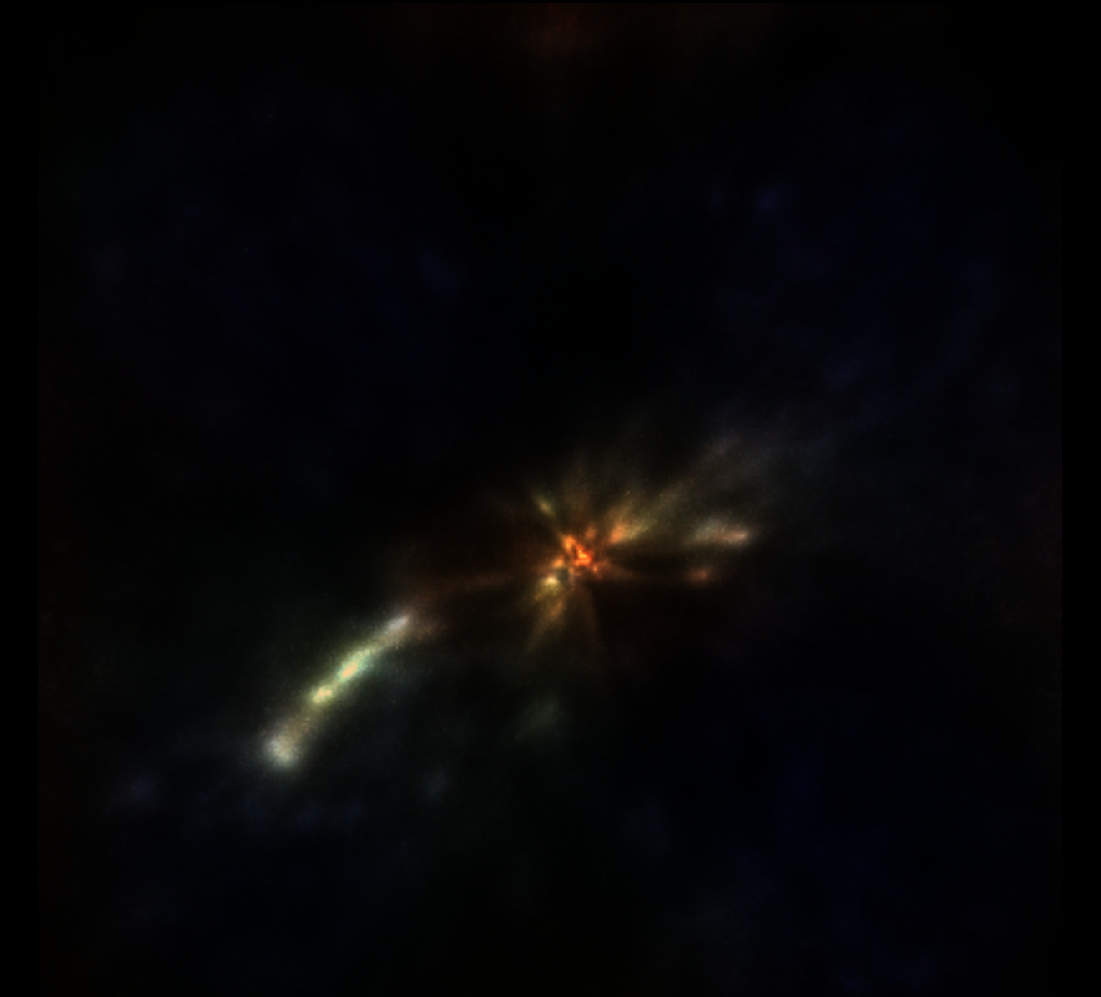
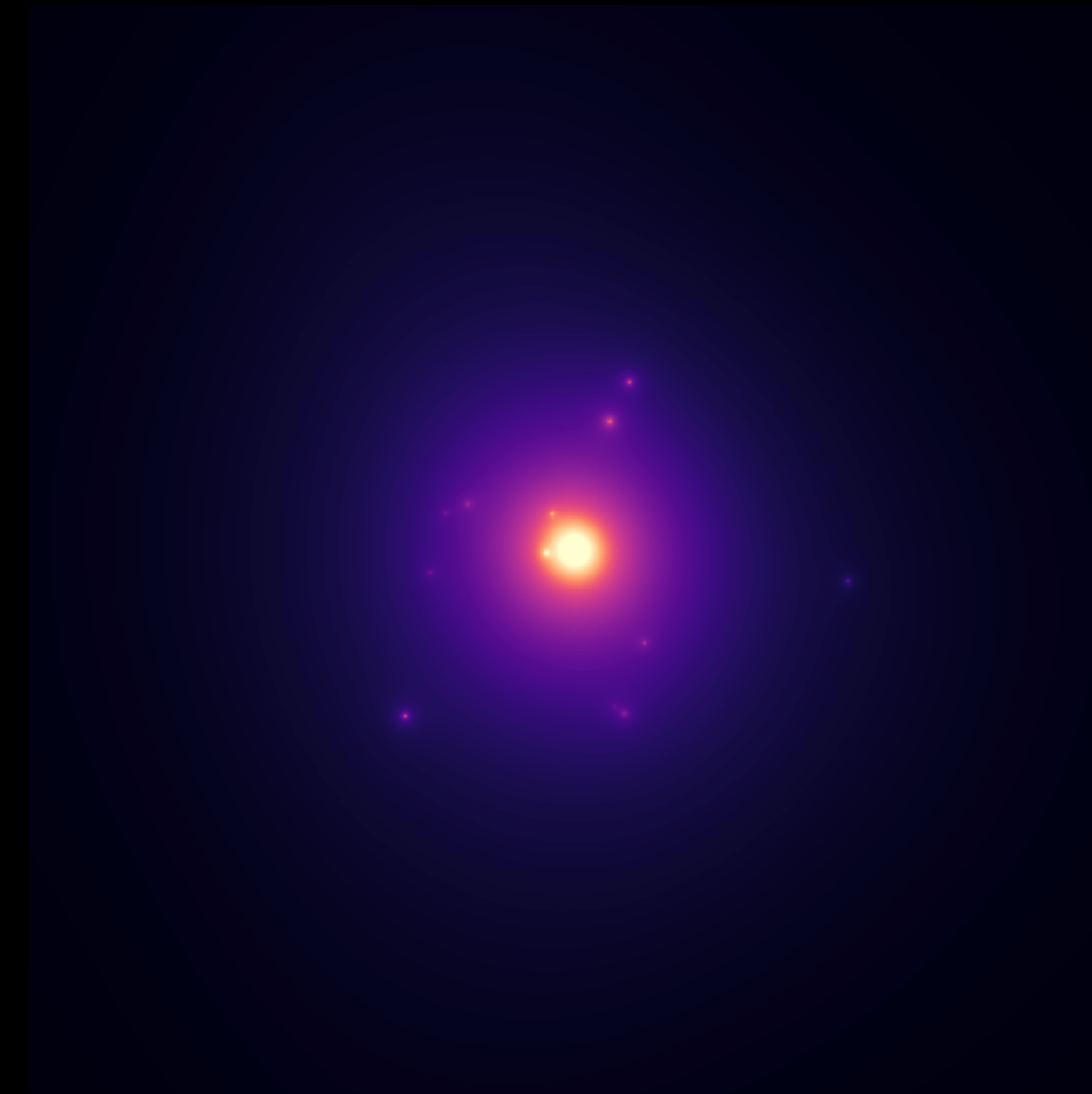
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$$\log \mathcal{O}(M_1, M_2 | d) \simeq 28$$

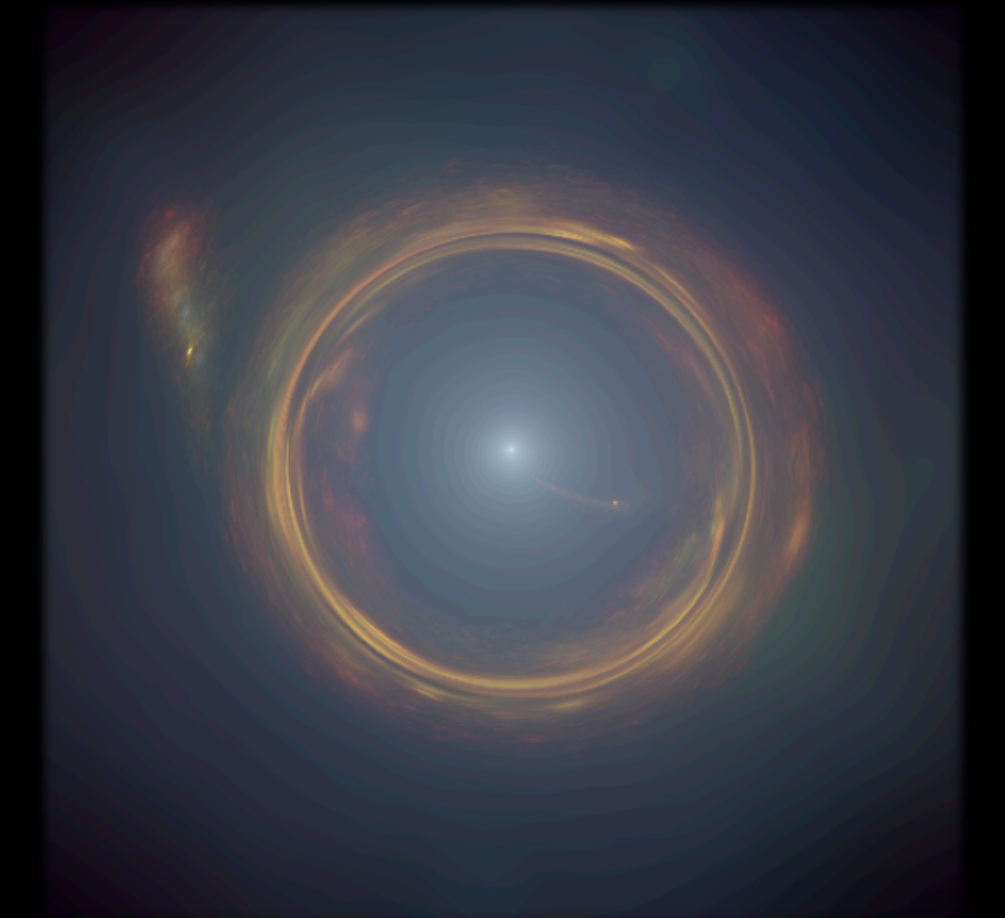
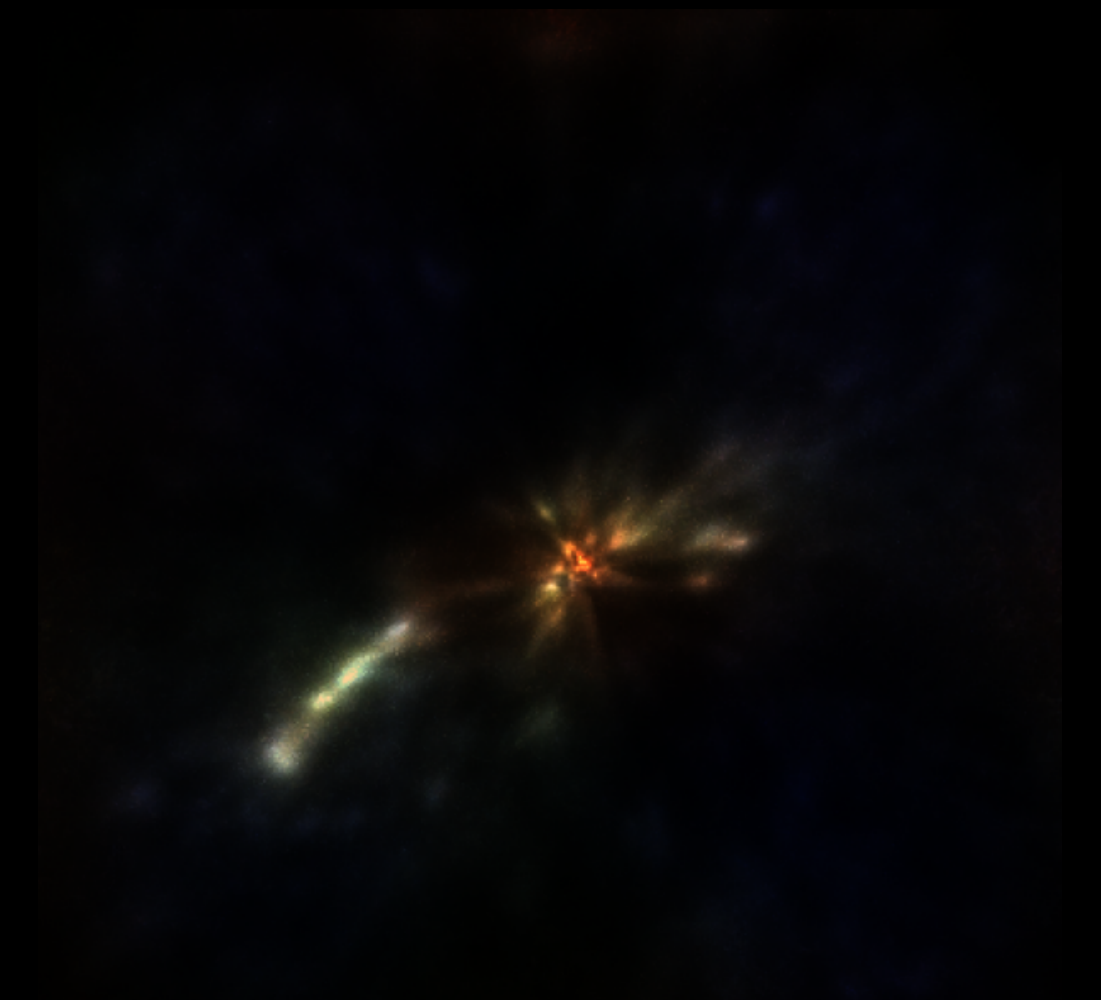
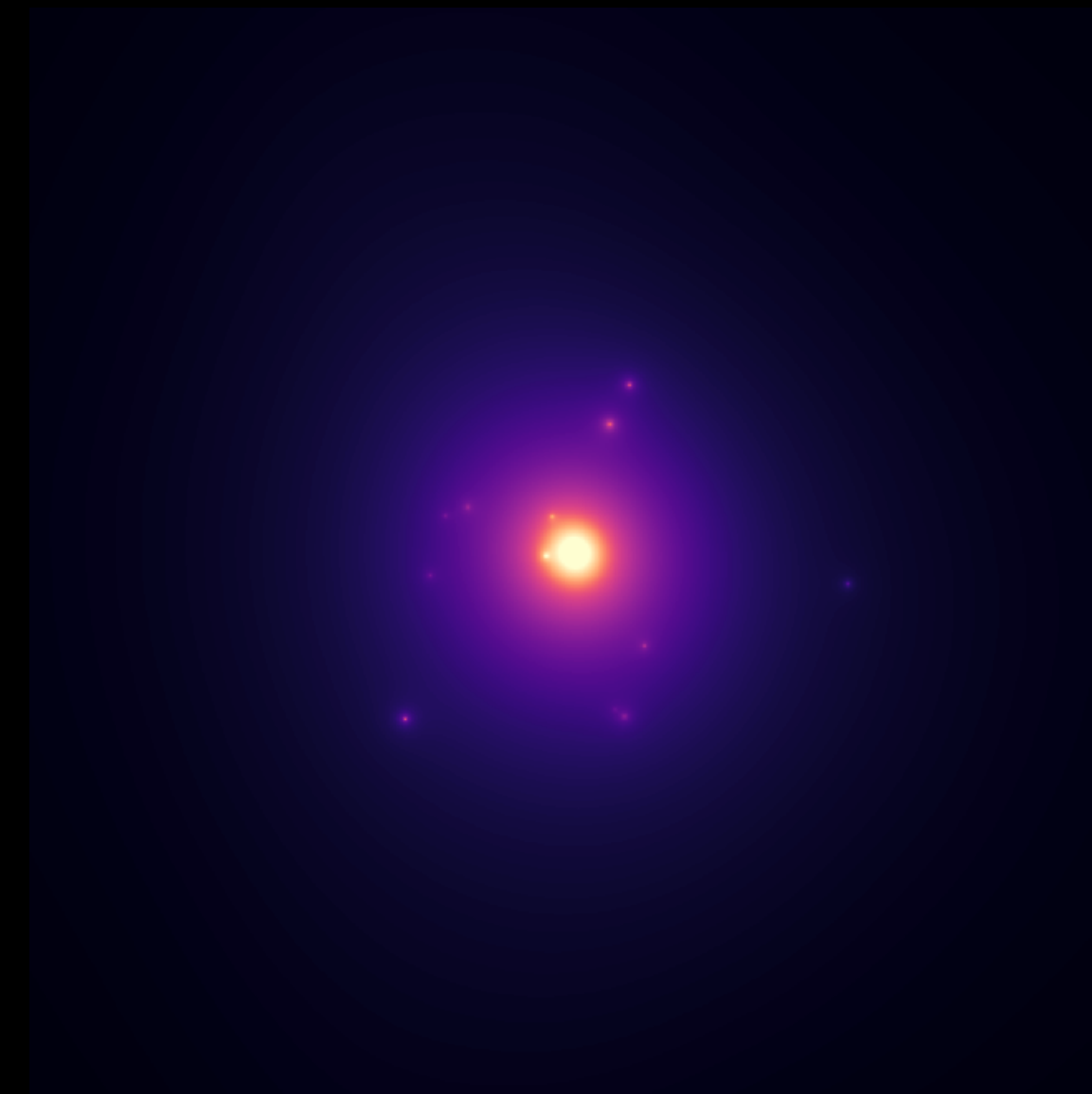


Take home



Take home

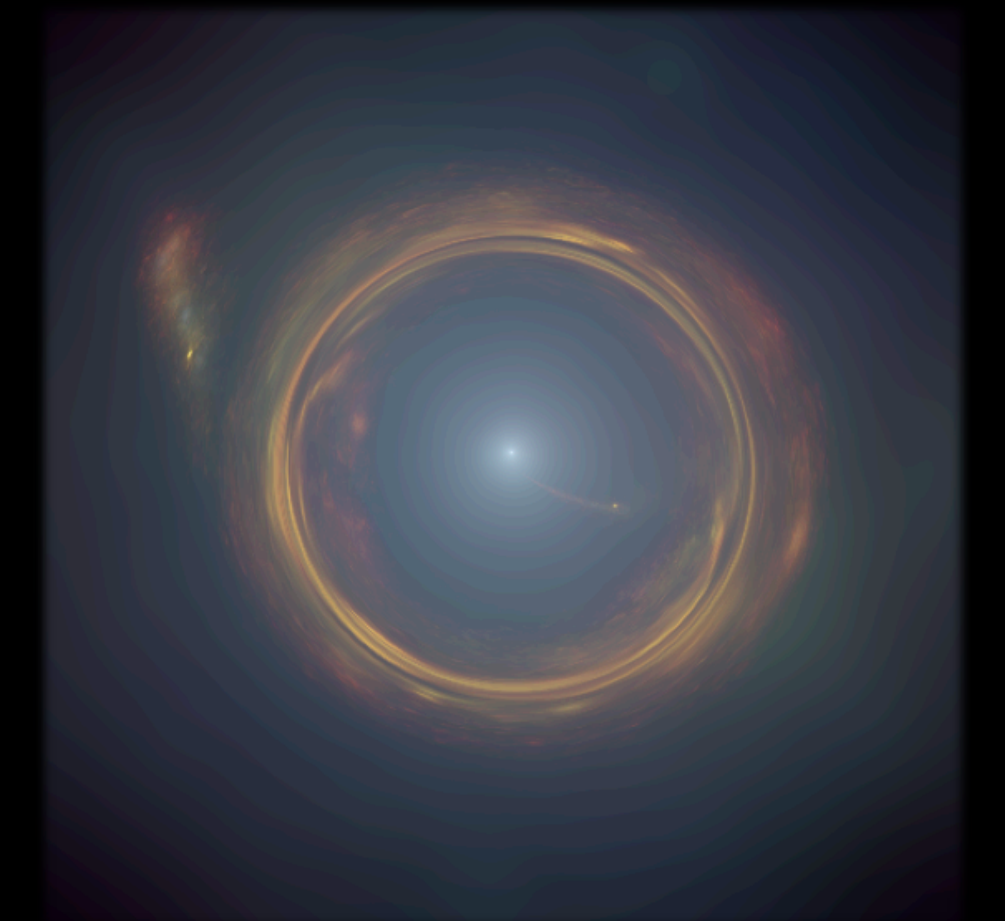
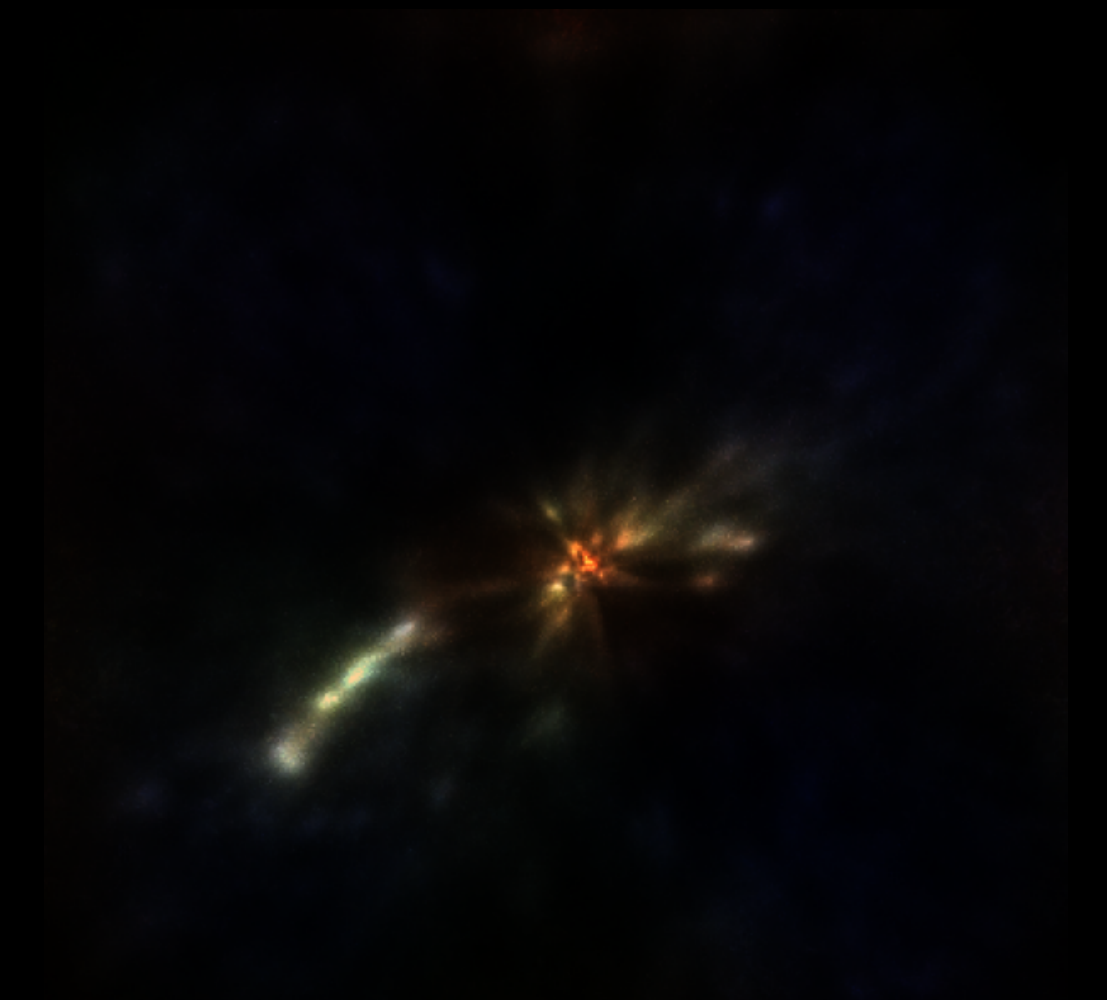
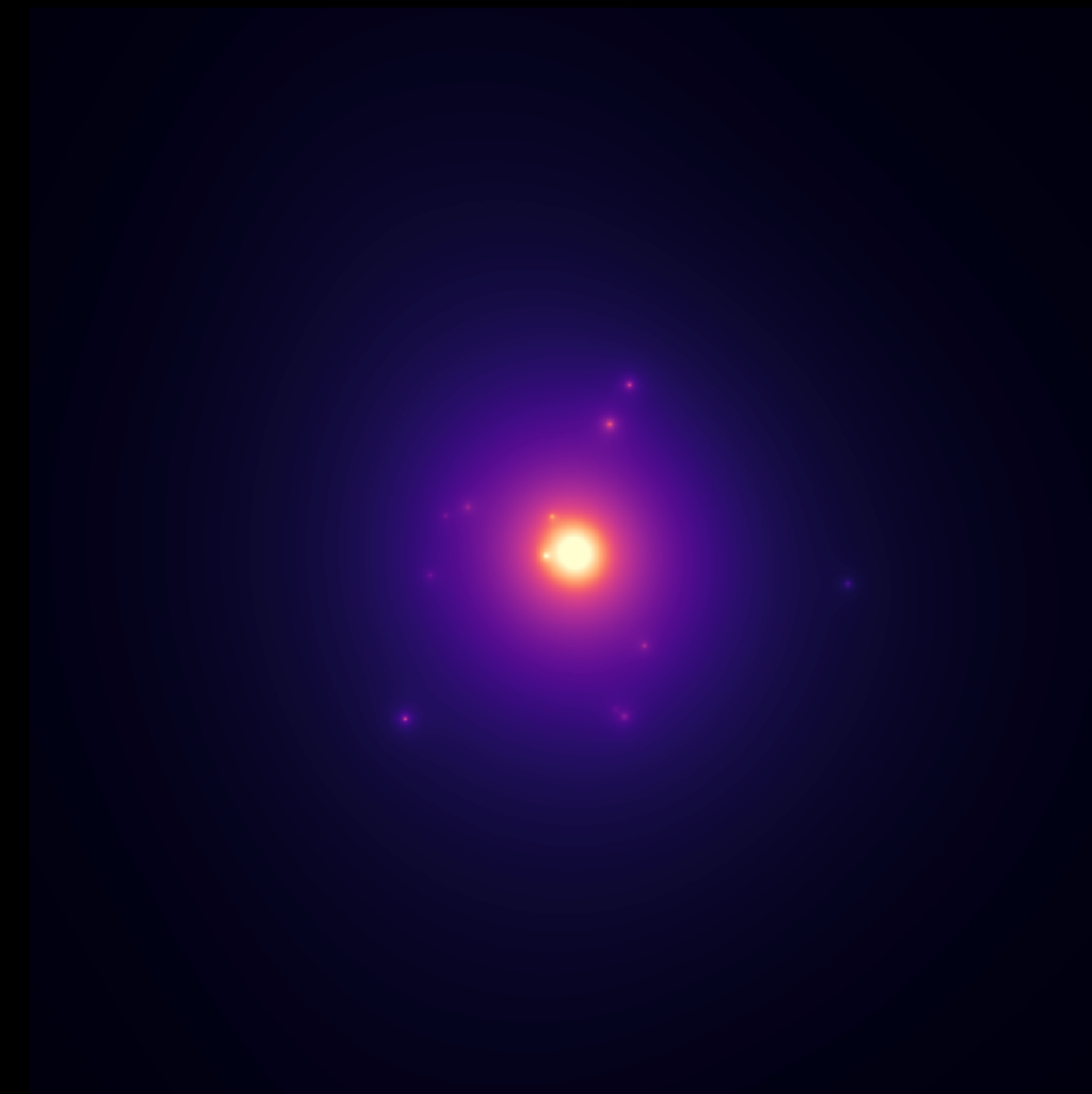
Model choice in high dimension is possible!



Take home

Model choice in high dimension is possible!

Applications:

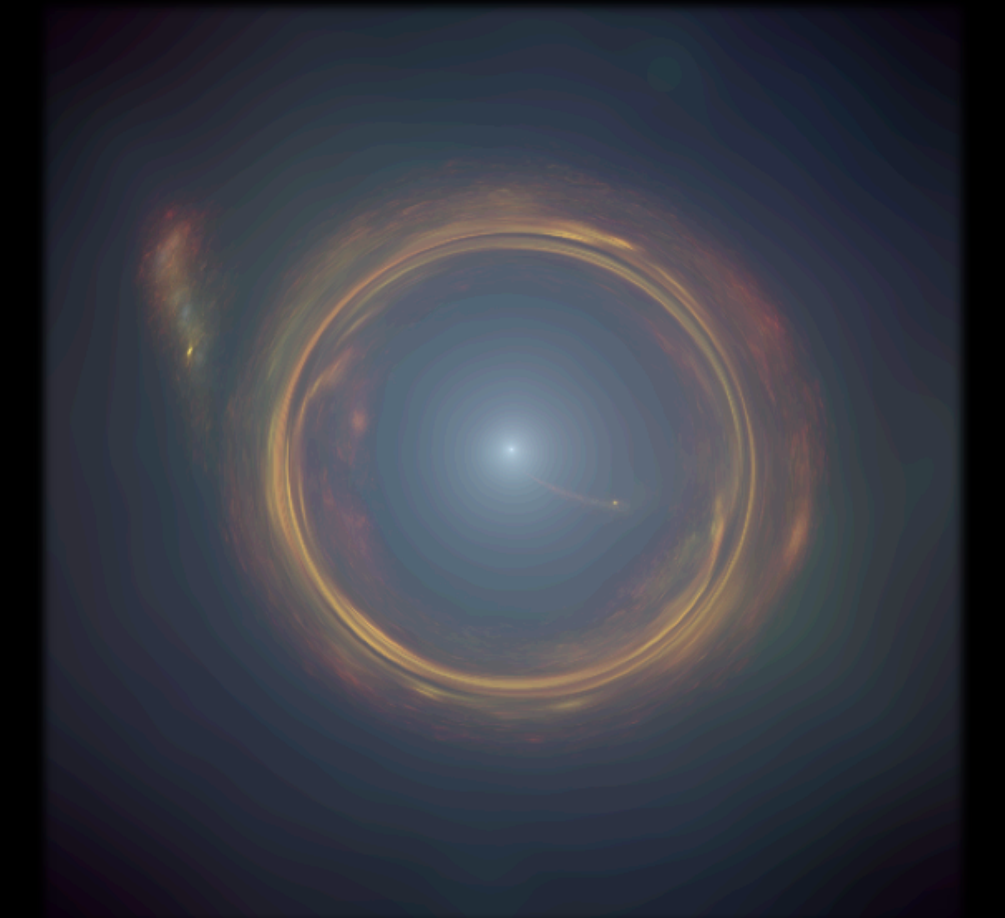
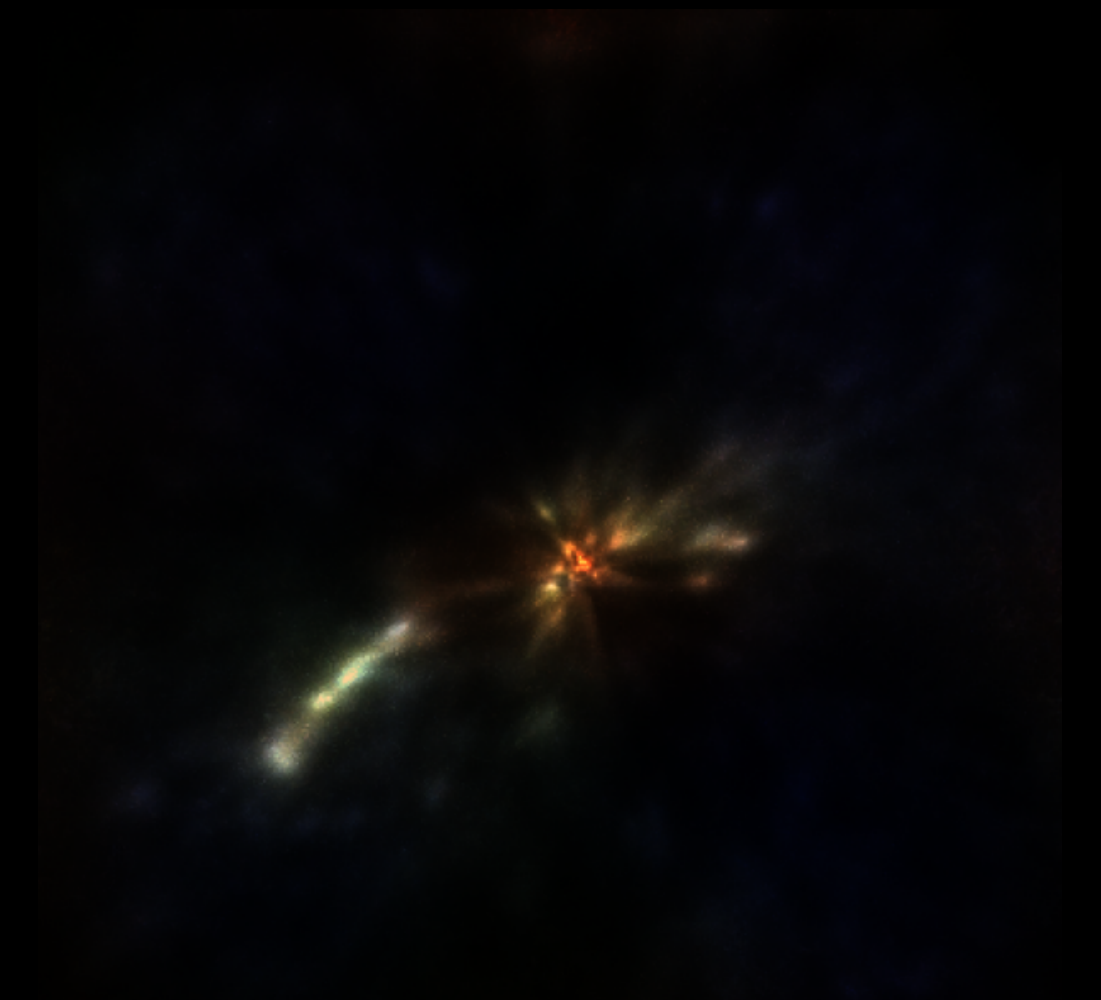
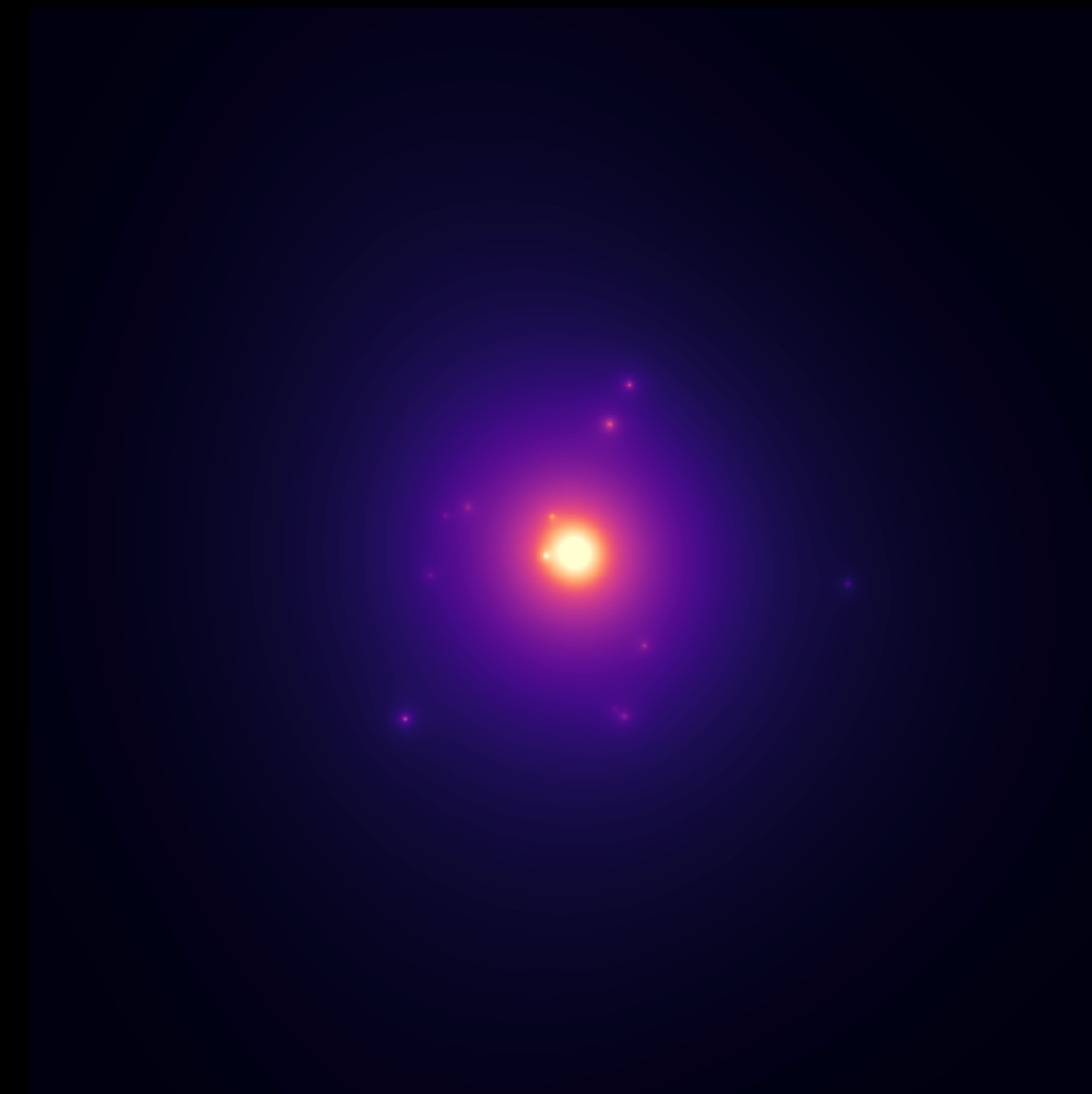


Take home

Model choice in high dimension is possible!

Applications:

- Astrophysics

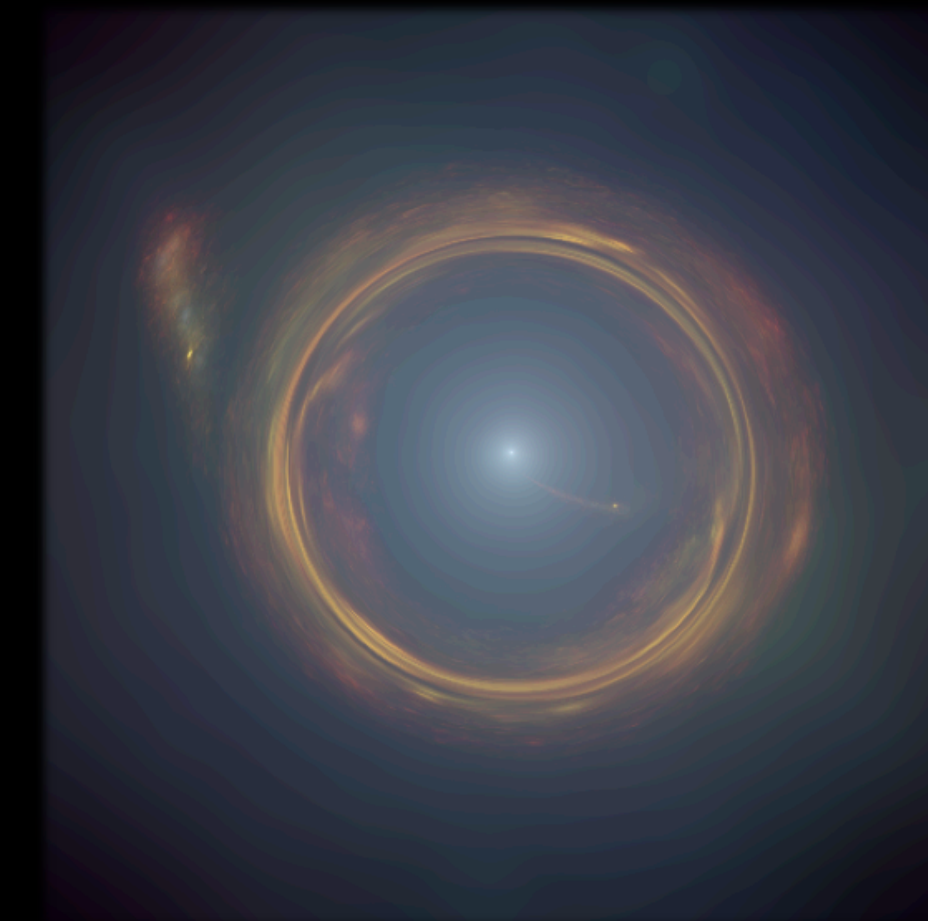
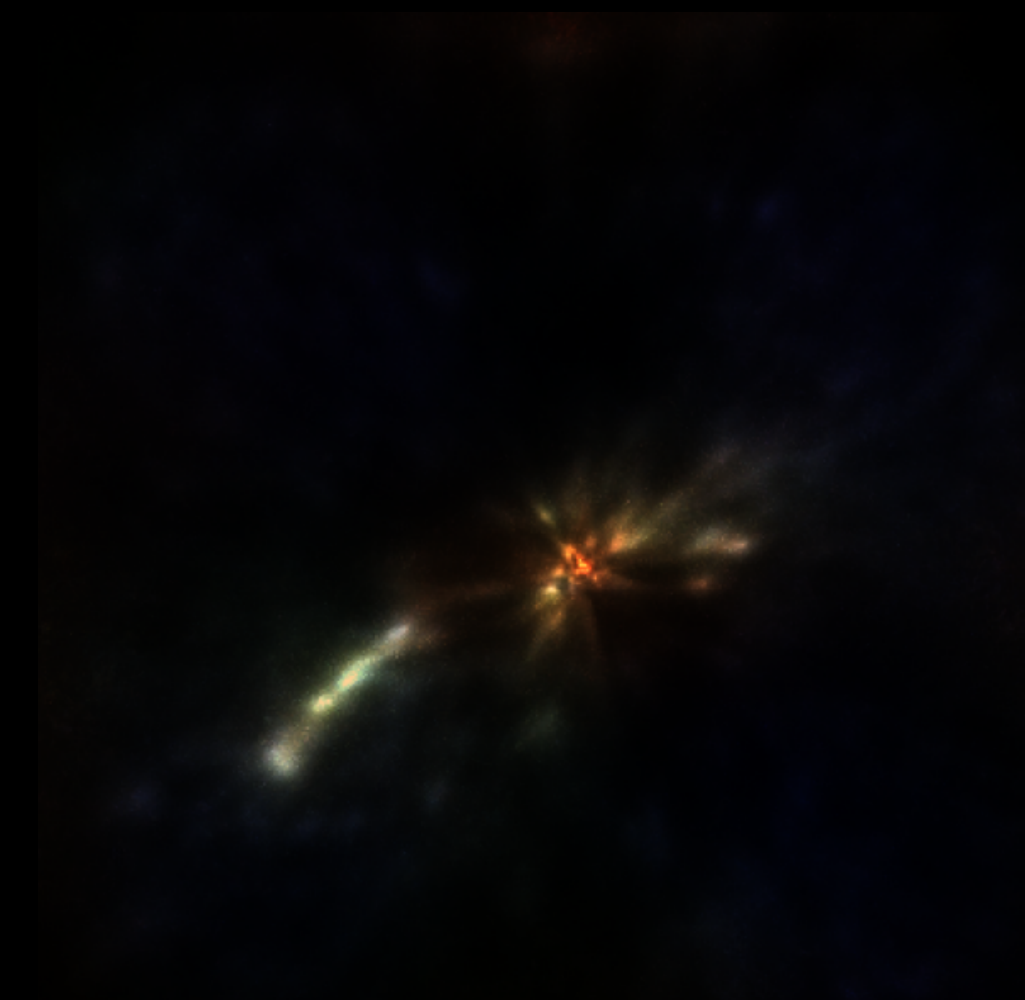
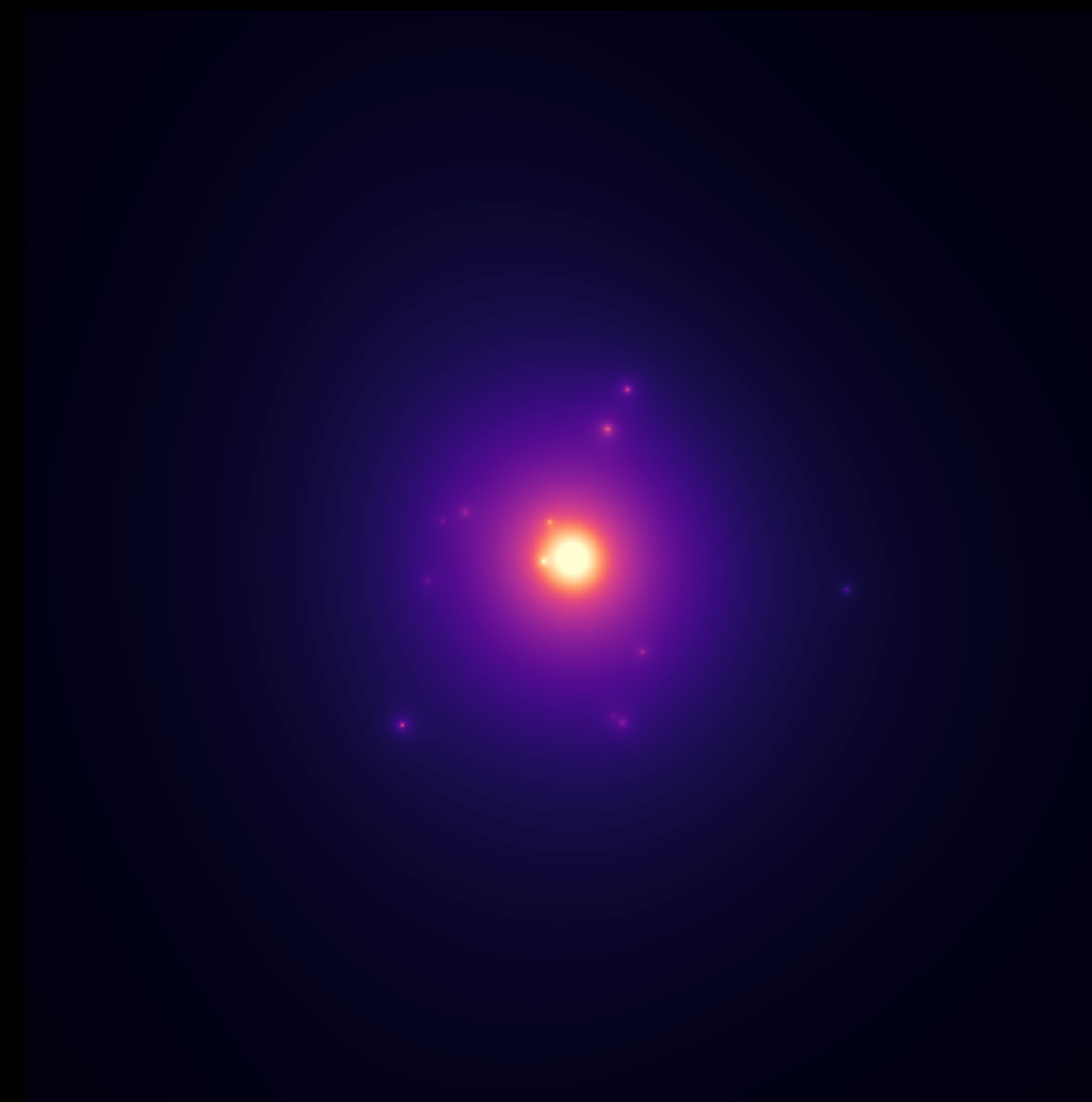


Take home

Model choice in high dimension is possible!

Applications:

- Astrophysics
- Causal inference

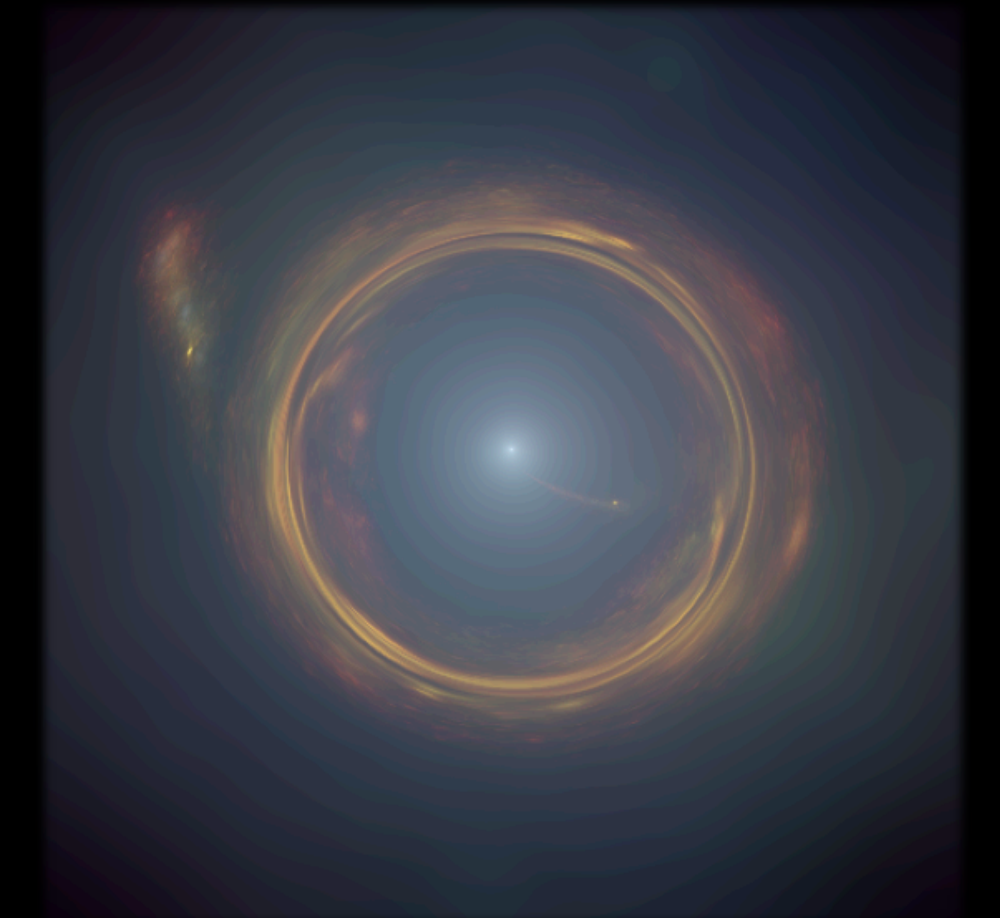
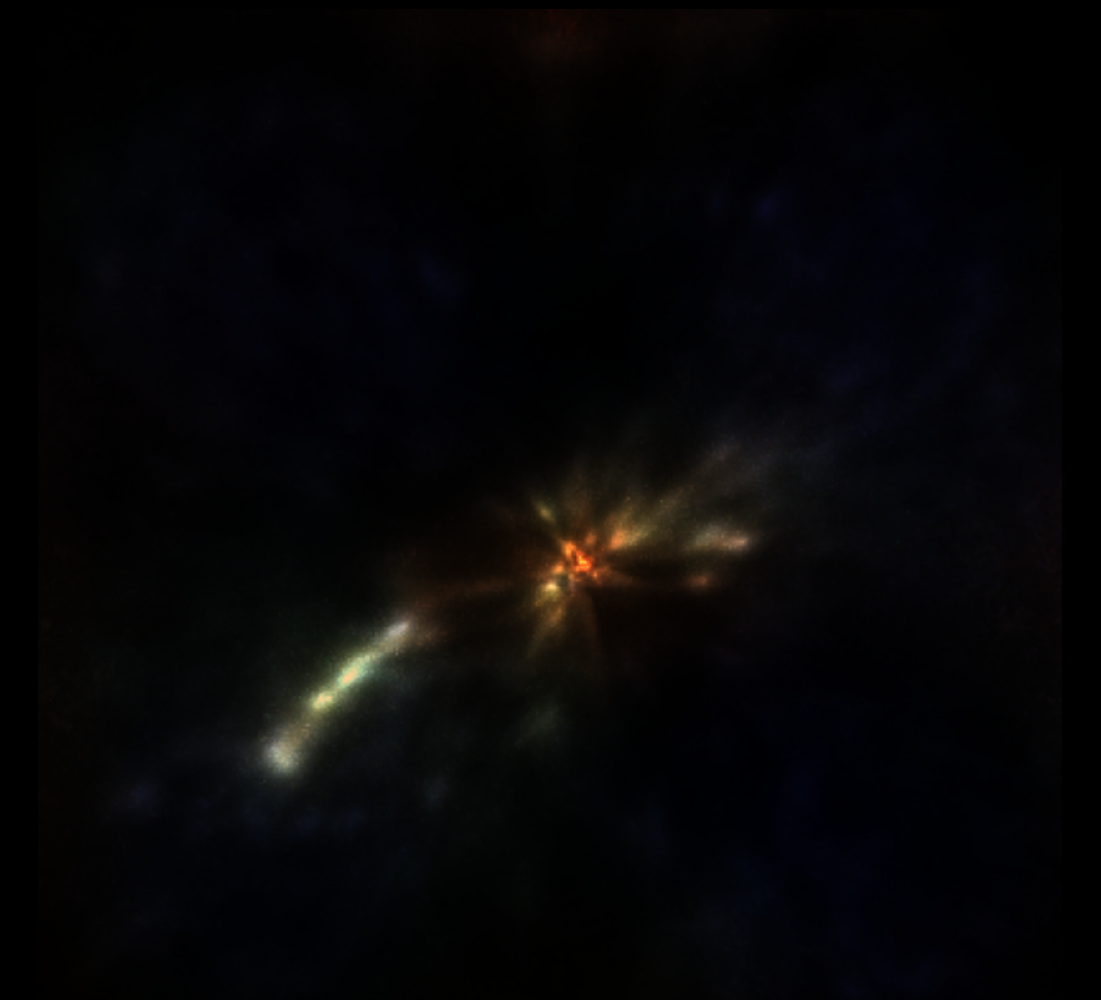
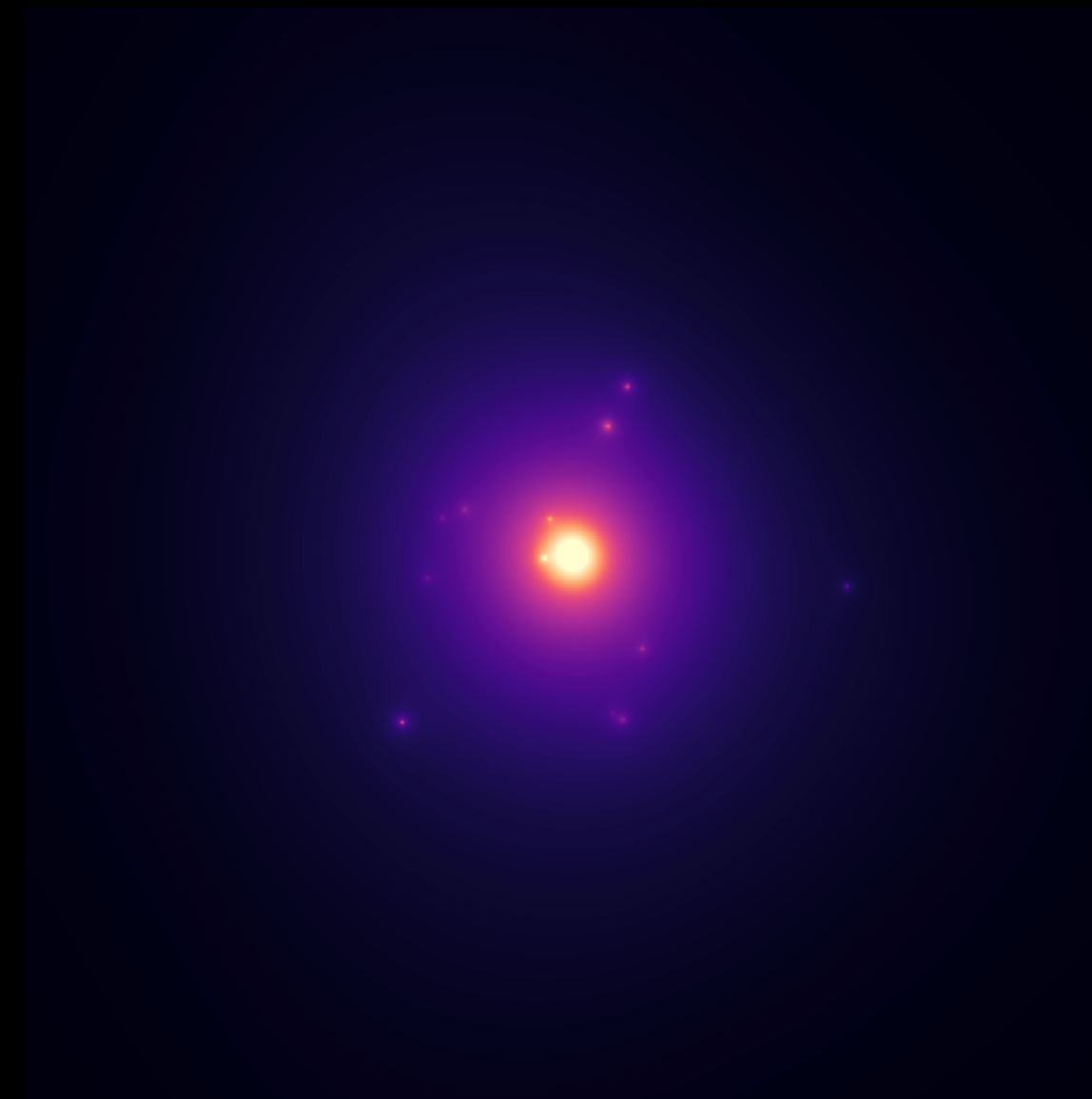


Take home

Model choice in high dimension is possible!

Applications:

- Astrophysics
- Causal inference
- Statistics

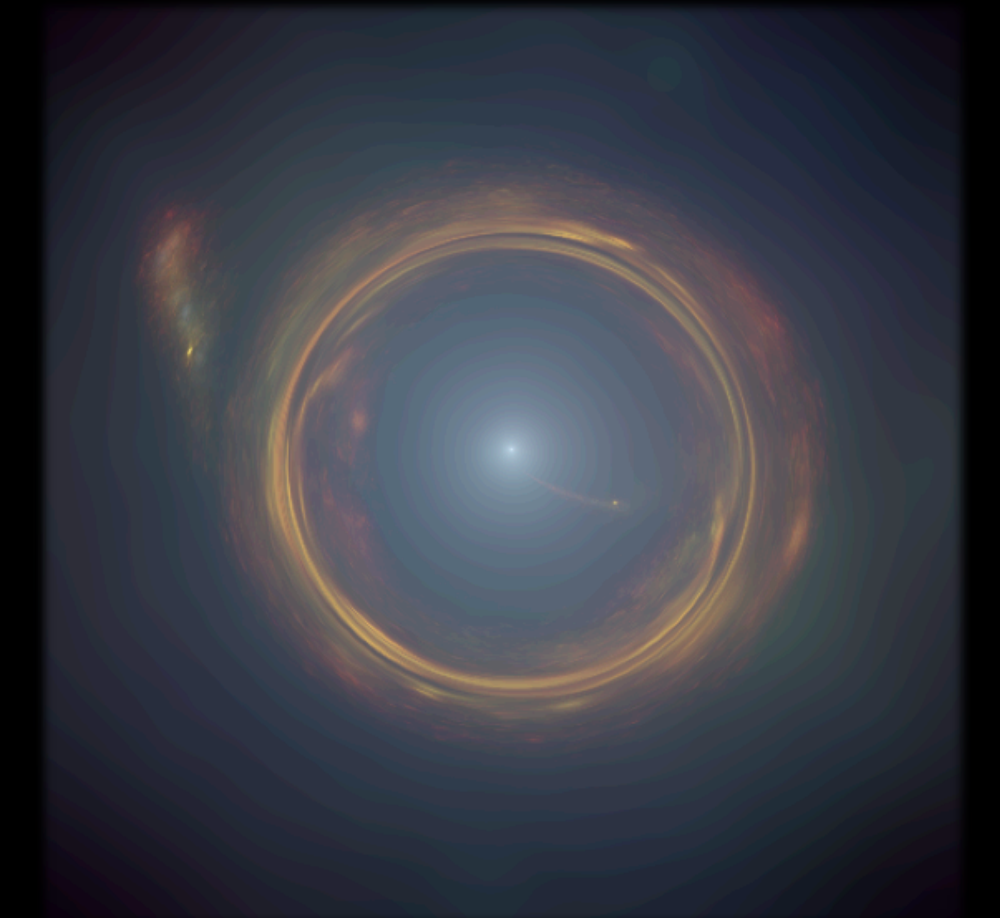
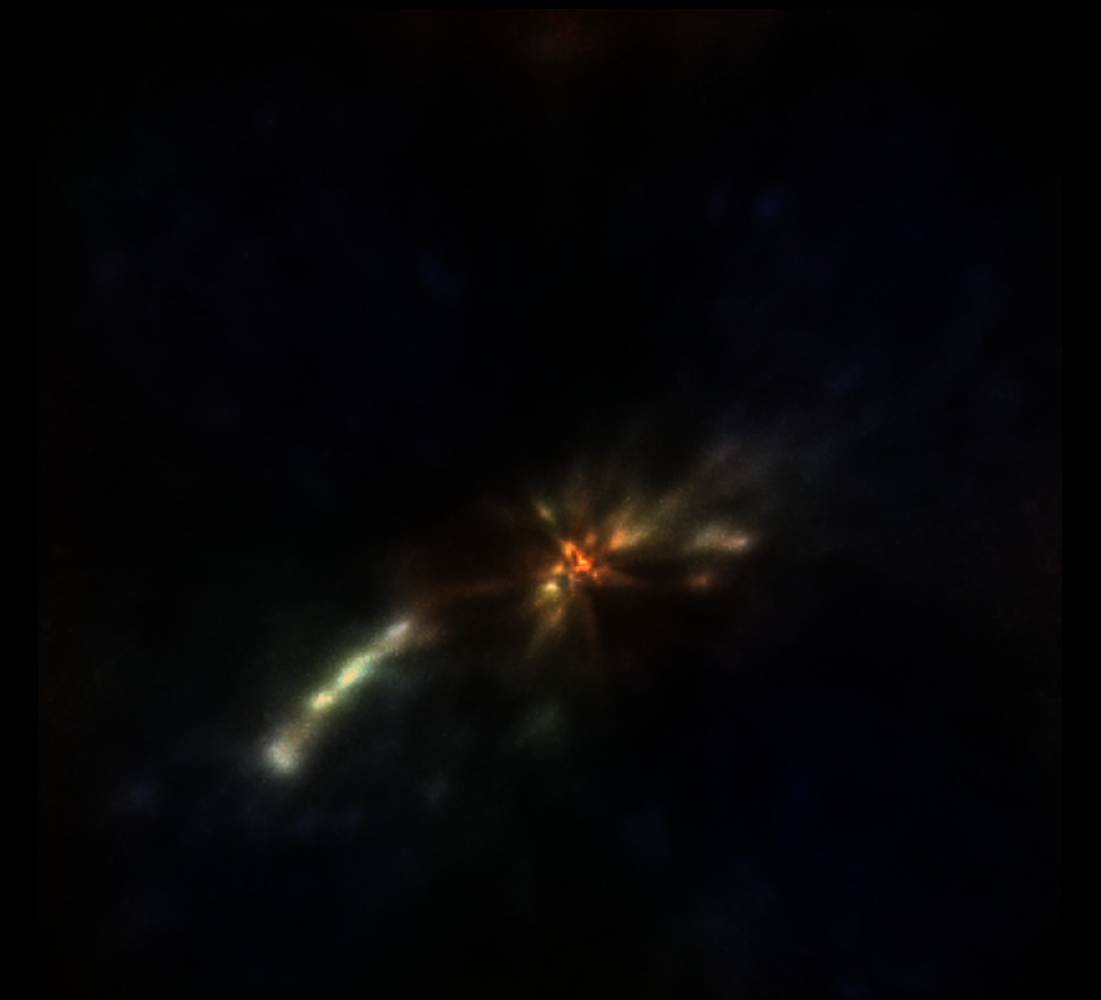
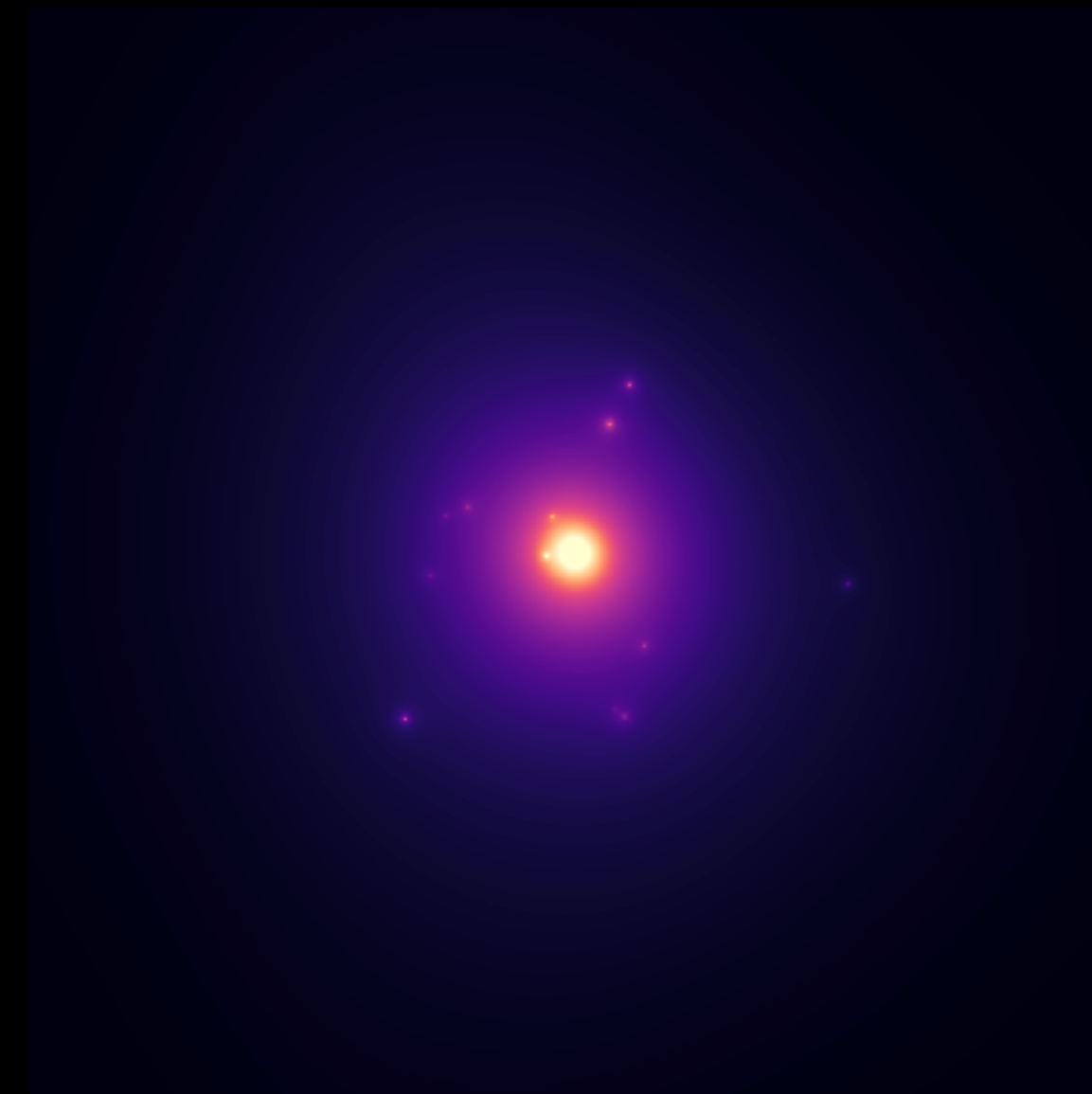


Take home

Model choice in high dimension is possible!

Applications:

- Astrophysics
- Causal inference
- Statistics
- ...



Some advertisement...

Some advertisement...

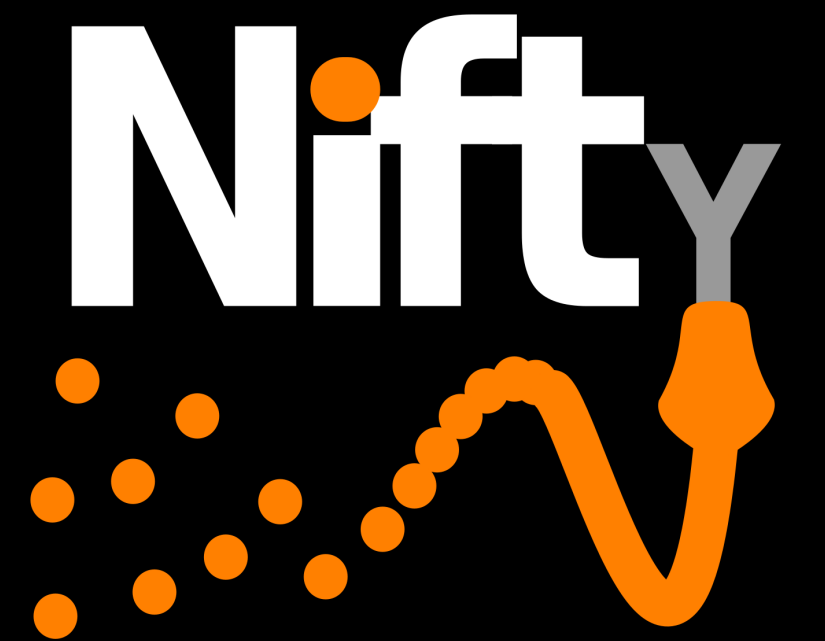
- **NIFTy (Numerical Information Field Theory)**
High-dimensional inference & Gaussian processes + ELBO

Some advertisement...

- **NIFTy (Numerical Information Field Theory)**
High-dimensional inference & Gaussian processes + ELBO
- **J-UBIK (Universal Bayesian Imaging Kit)**
Sky models & astronomical instrument responses

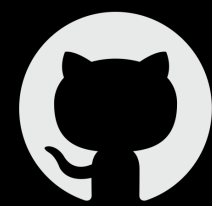
Some advertisement...

- **NIFTy (Numerical Information Field Theory)**
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- **LensCharm**
Charming strong-lensing signal reconstructions



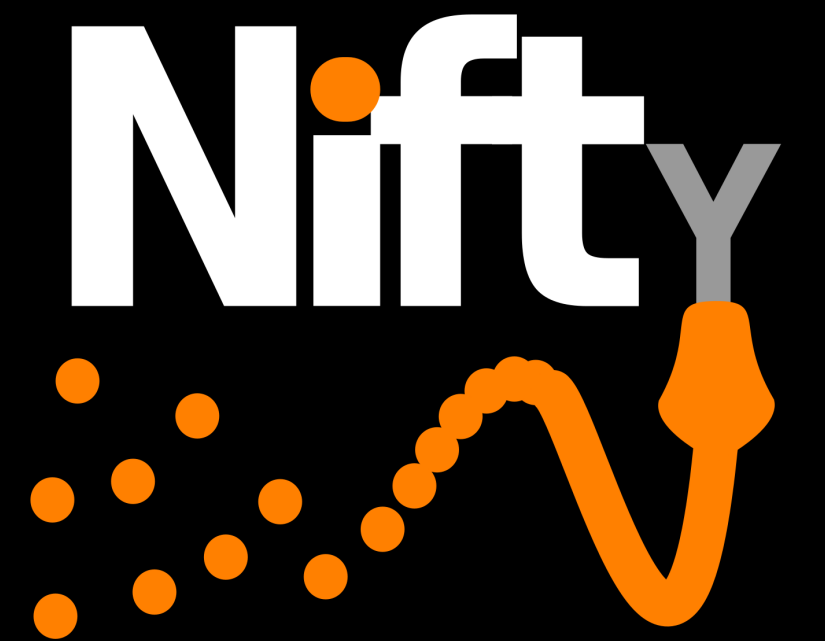
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<https://github.com/NIFTy-PPL/NIFTy>

<https://github.com/NIFTy-PPL/J-UBIK>



Some advertisement...

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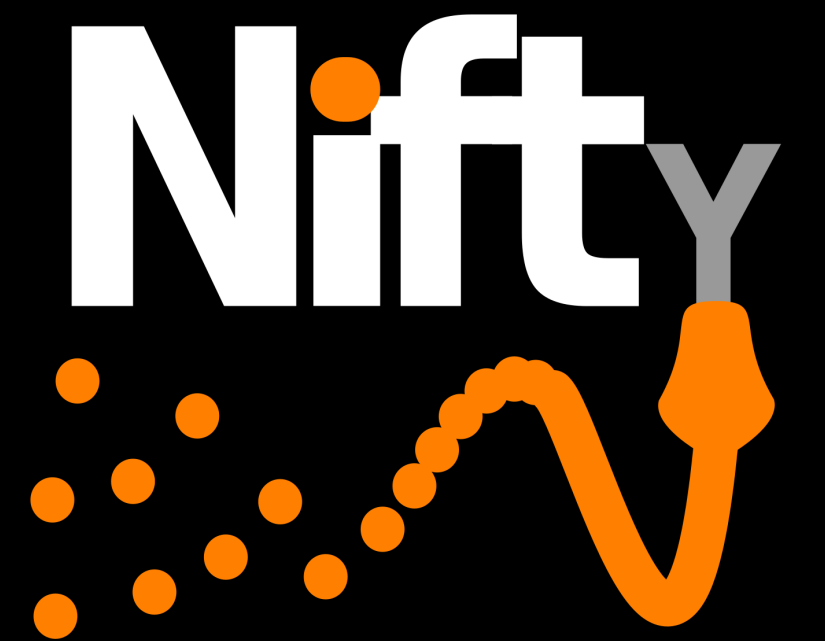


<https://github.com/NIFTy-PPL/NIFTy>

<https://github.com/NIFTy-PPL/J-UBIK>



GitLab <https://gitlab.mpcdf.mpg.de/ift/lenscharm>



Thank you!

matteani@mpa-garching.mpg.de

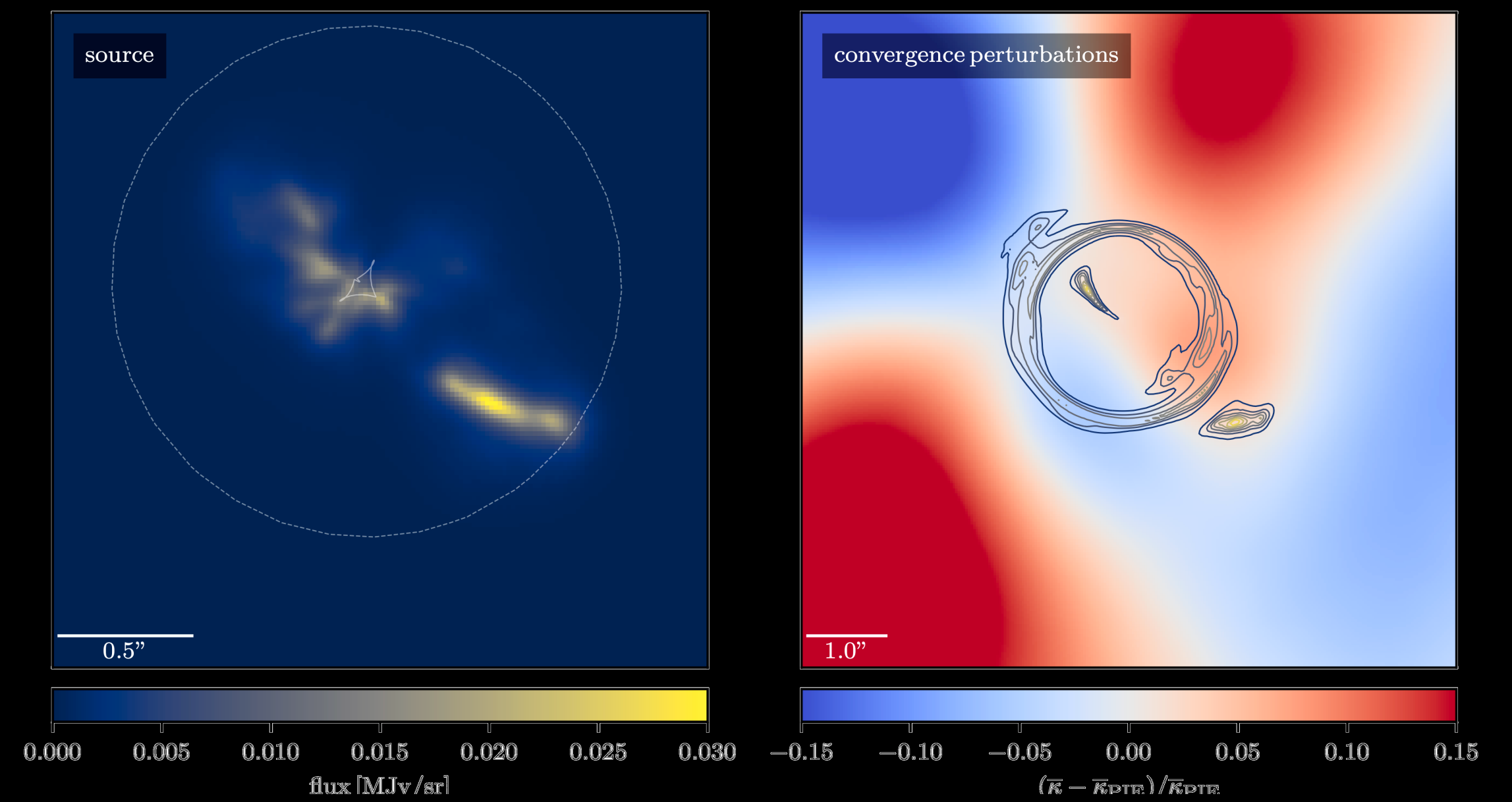
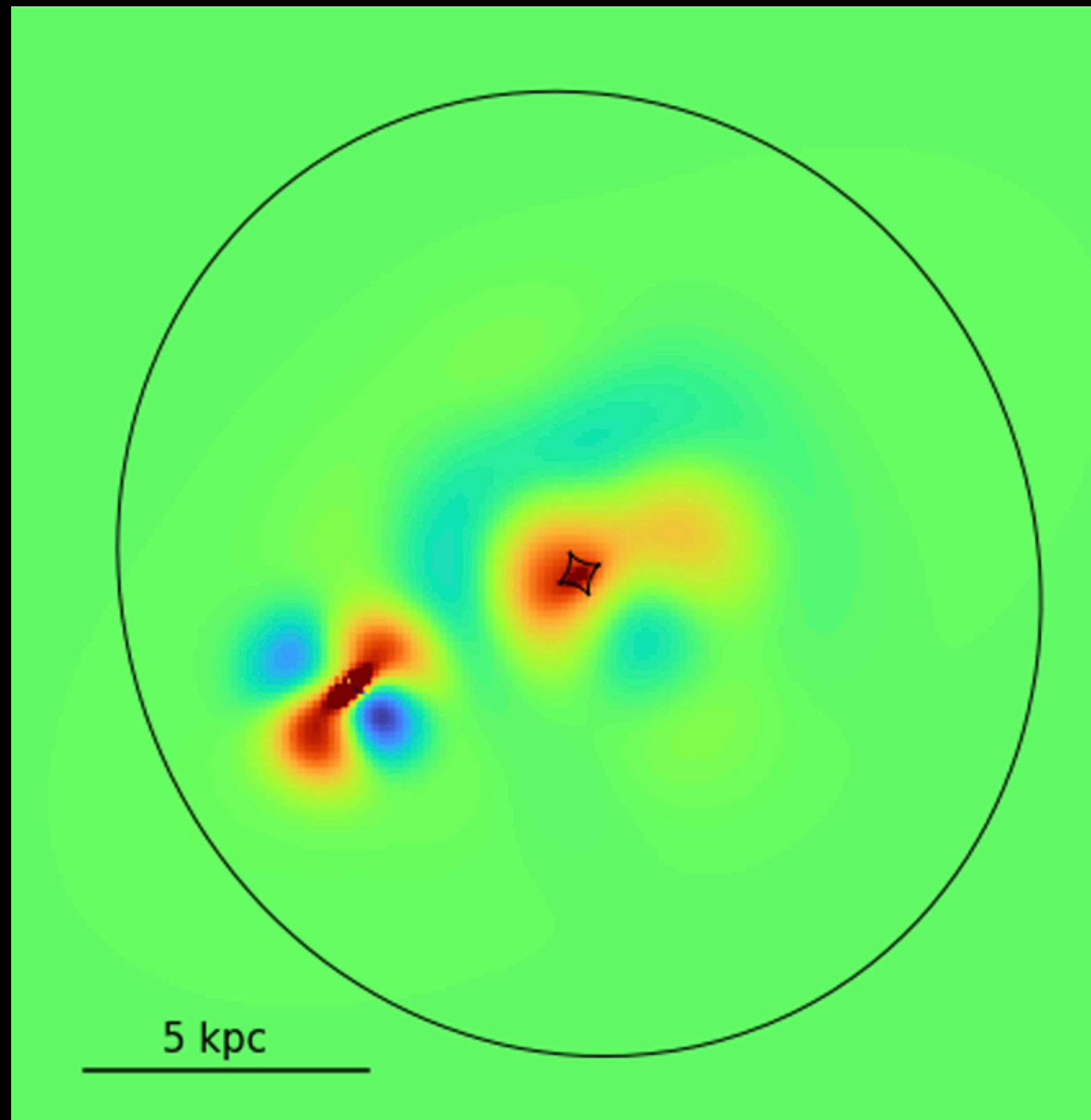
MAX PLANCK INSTITUTE
FOR ASTROPHYSICS



Backup

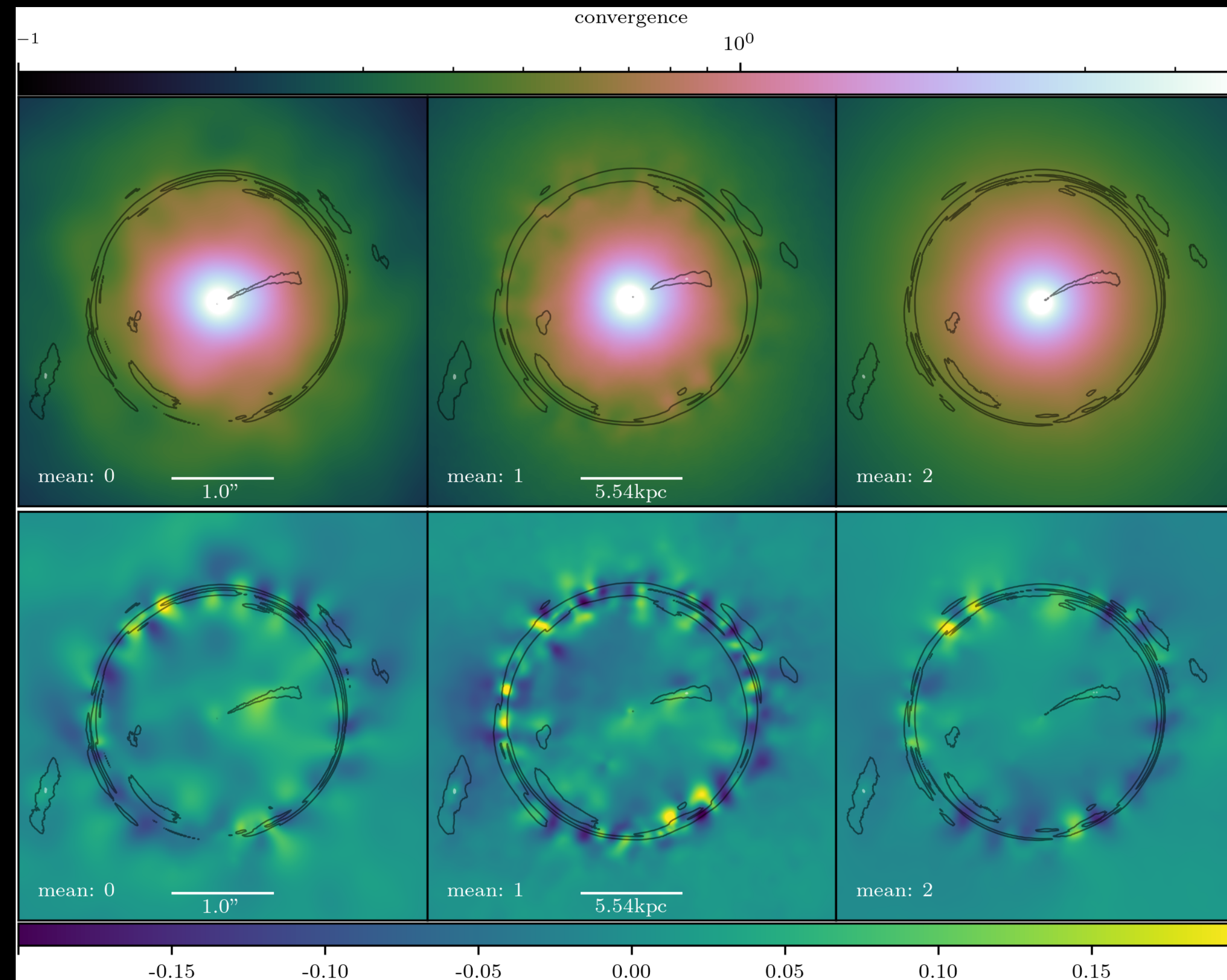
Backup

SPT0418-47



Backup

SPT0418-47 (convergence)



LensCharm

Why should you use it?

LensCharm

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- **Nonparametric mass distribution models**
- **Nonparametric source galaxy models**
- **Instrumental effects** are explicitly modeled

LensCharm

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- **Uncertainties!**

LensCharm

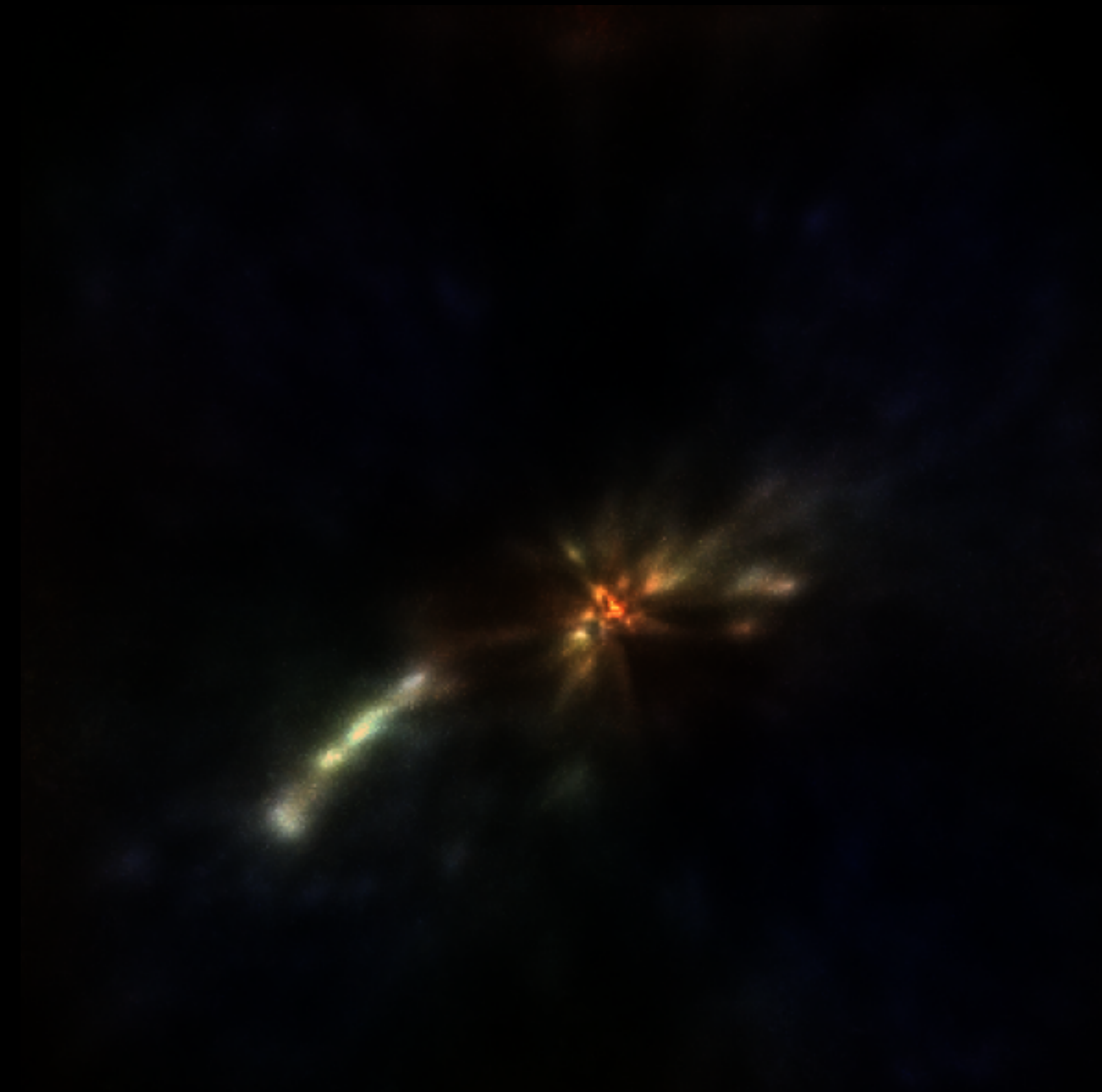
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LensCharm

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LensCharm 2.0

LensCharm 2.0

Colors!

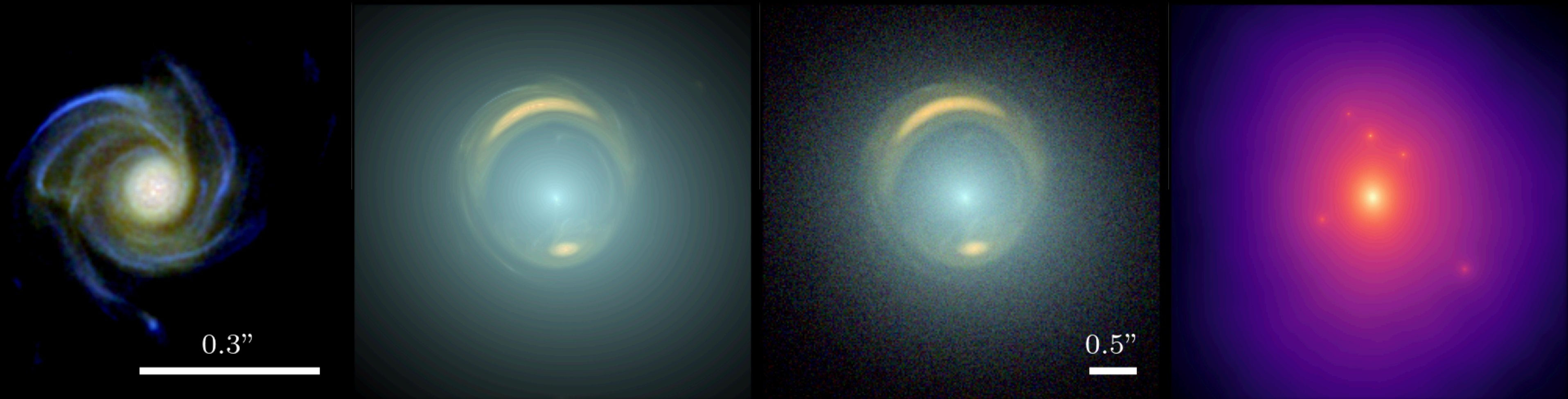
LensCharm 2.0

Colors!



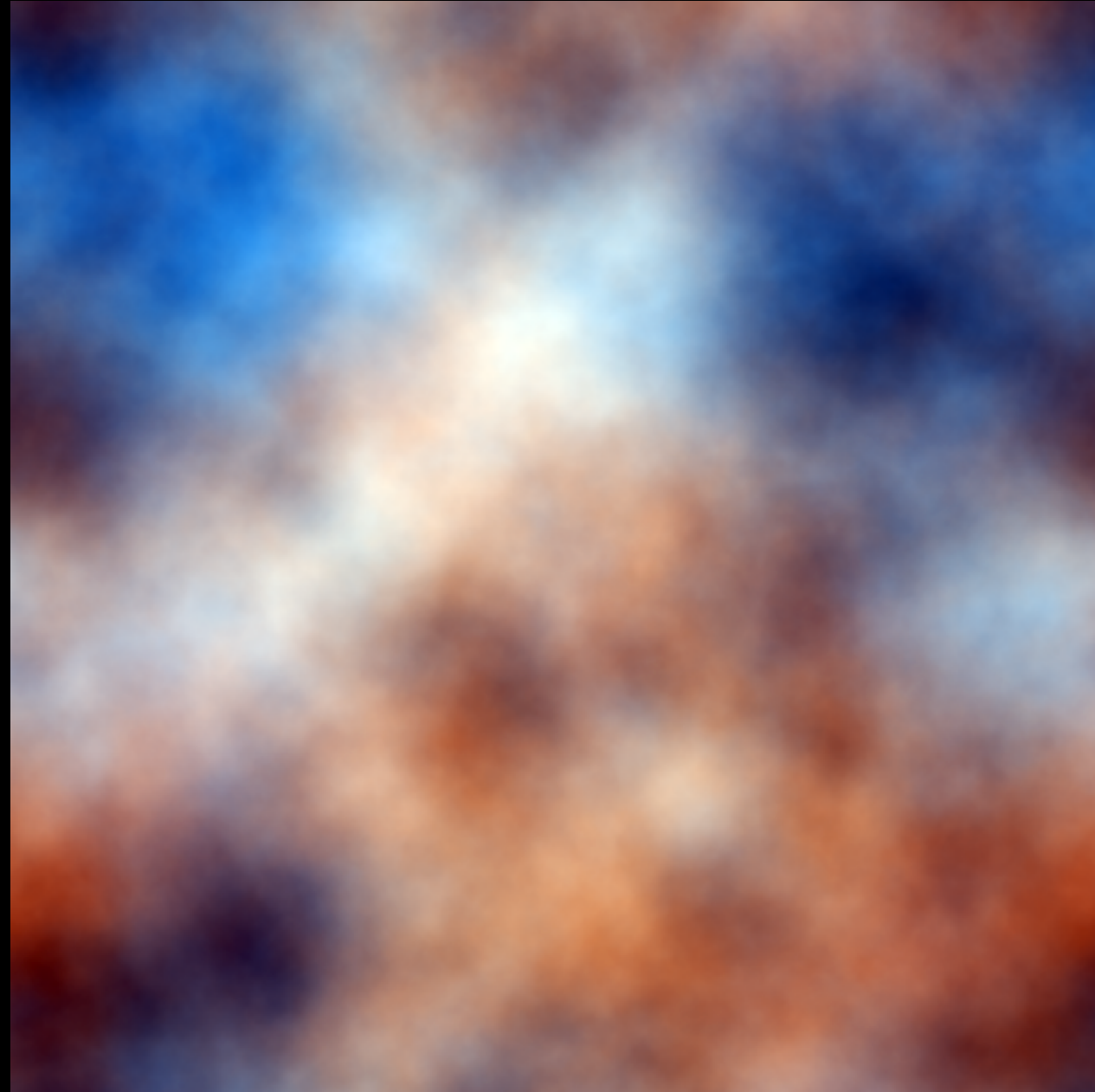
LensCharm 2.0

Colors!



LensCharm 2.0

Multi-frequency sky model

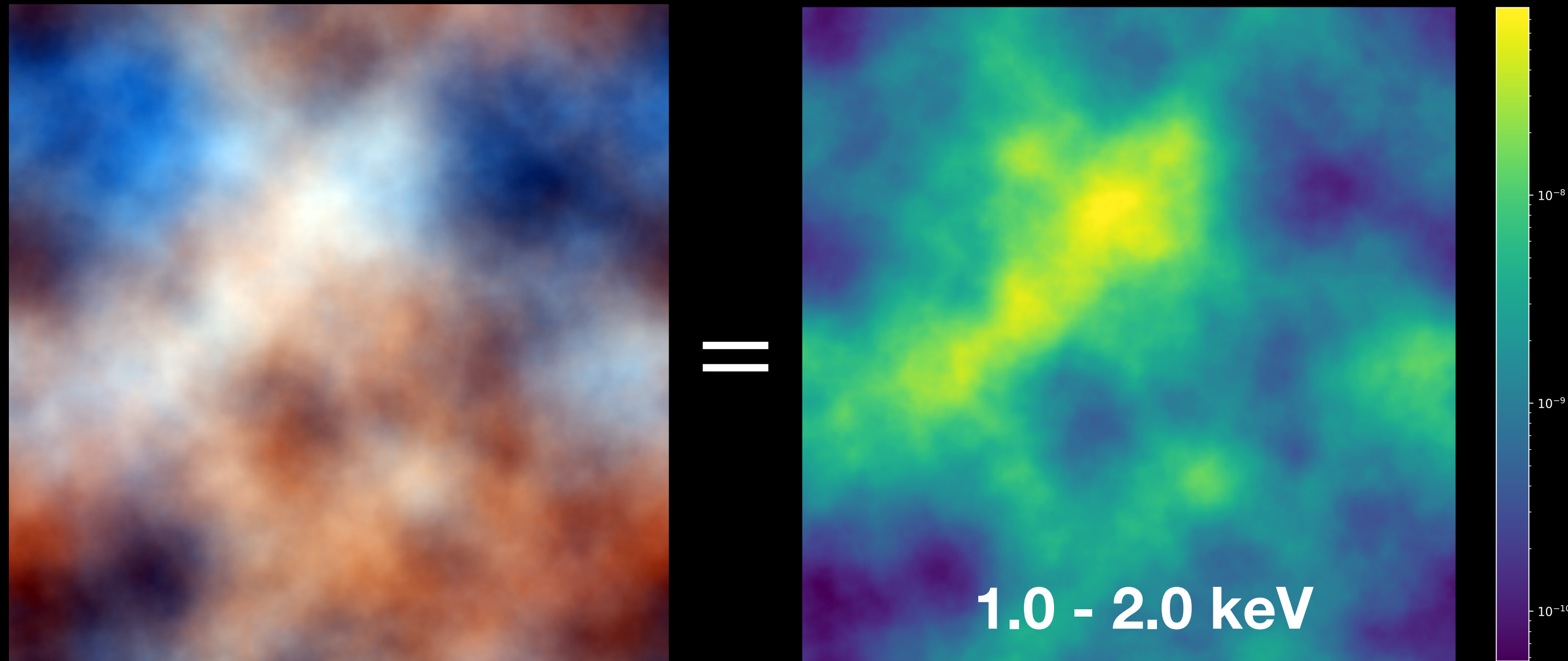


=

$$I(\mathbf{x}, \nu) =$$

LensCharm 2.0

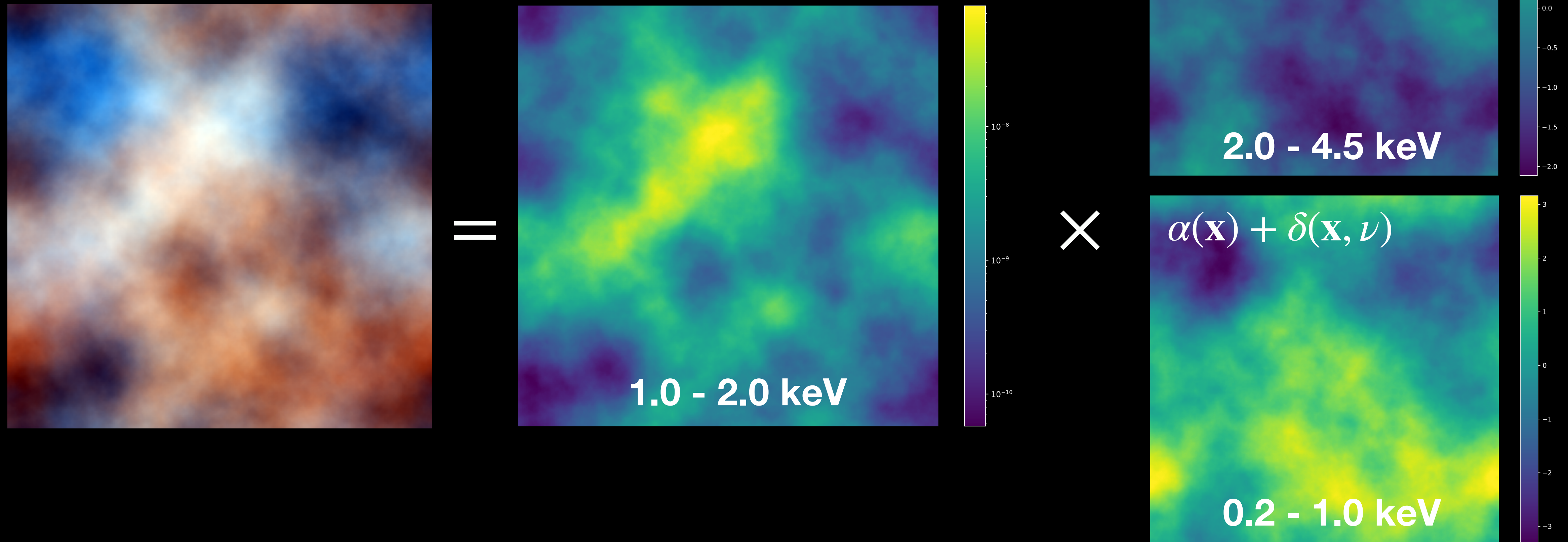
Multi-frequency sky model



$$I(\mathbf{x}, \nu) = I_0(\mathbf{x})$$

LensCharm 2.0

Multi-frequency sky model



$$I(\mathbf{x}, \nu) = I_0(\mathbf{x}) \times \left(\frac{\nu}{\nu_0} \right)^{\alpha(\mathbf{x}) + \delta(\mathbf{x}, \nu)}$$

LensCharm 2.0

Multi-frequency sky model

LensCharm 2.0

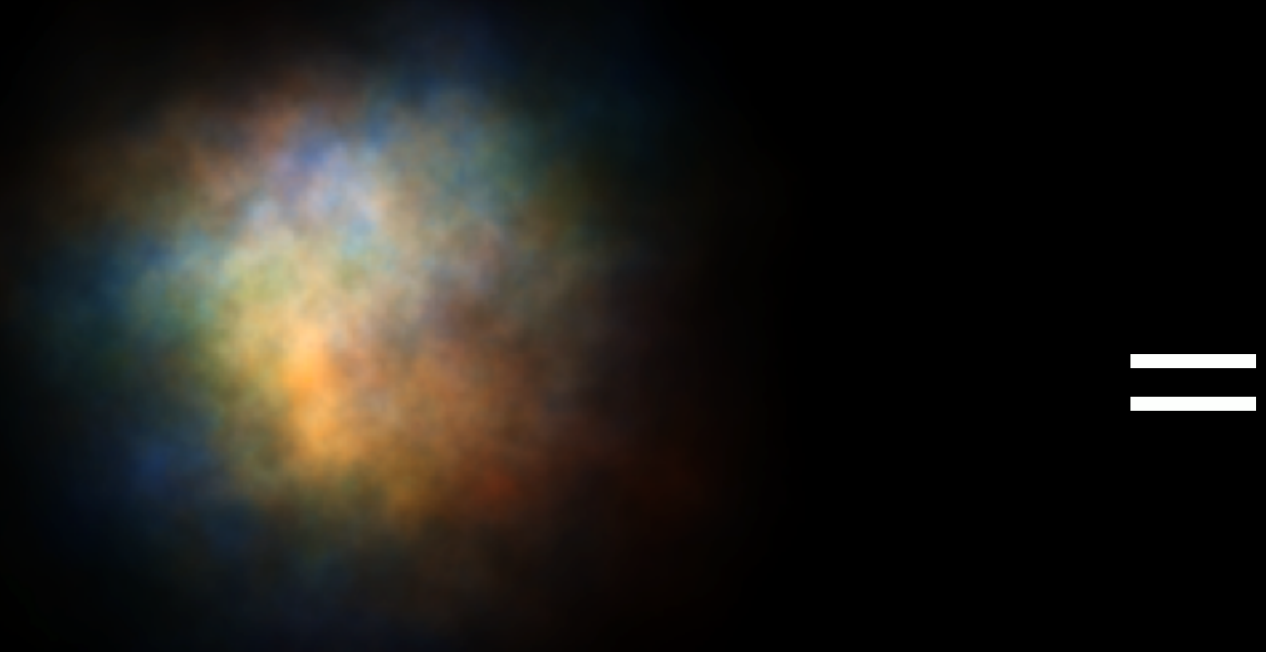
Multi-frequency sky model



$$I(\mathbf{x}, \nu)$$

LensCharm 2.0

Multi-frequency sky model



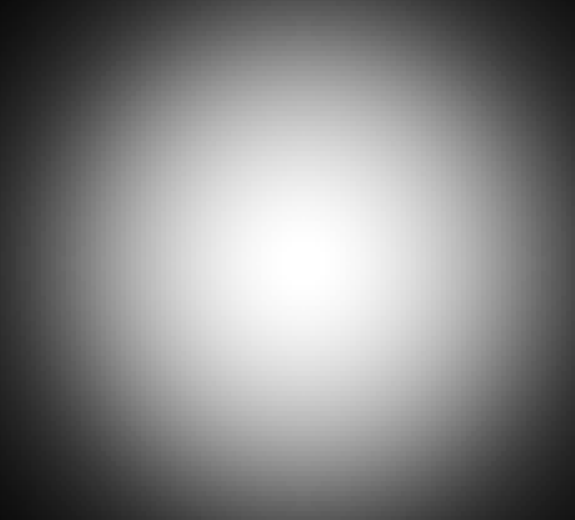
$$I(\mathbf{x}, \nu)$$

LensCharm 2.0

Multi-frequency sky model



=



$I(\mathbf{x}, \nu)$

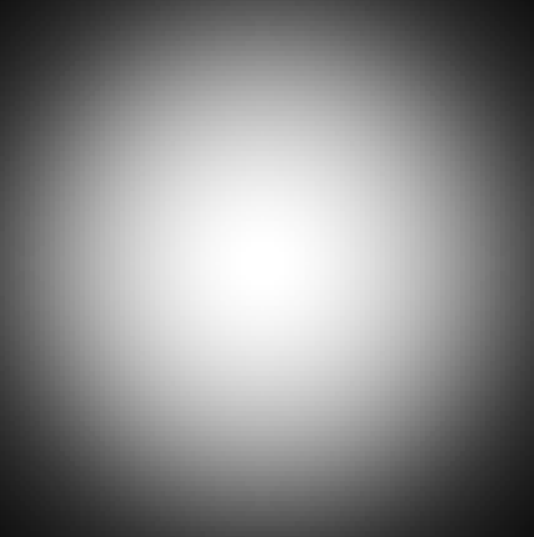
I_0^{para}

LensCharm 2.0

Multi-frequency sky model



=



$I(\mathbf{x}, \nu)$

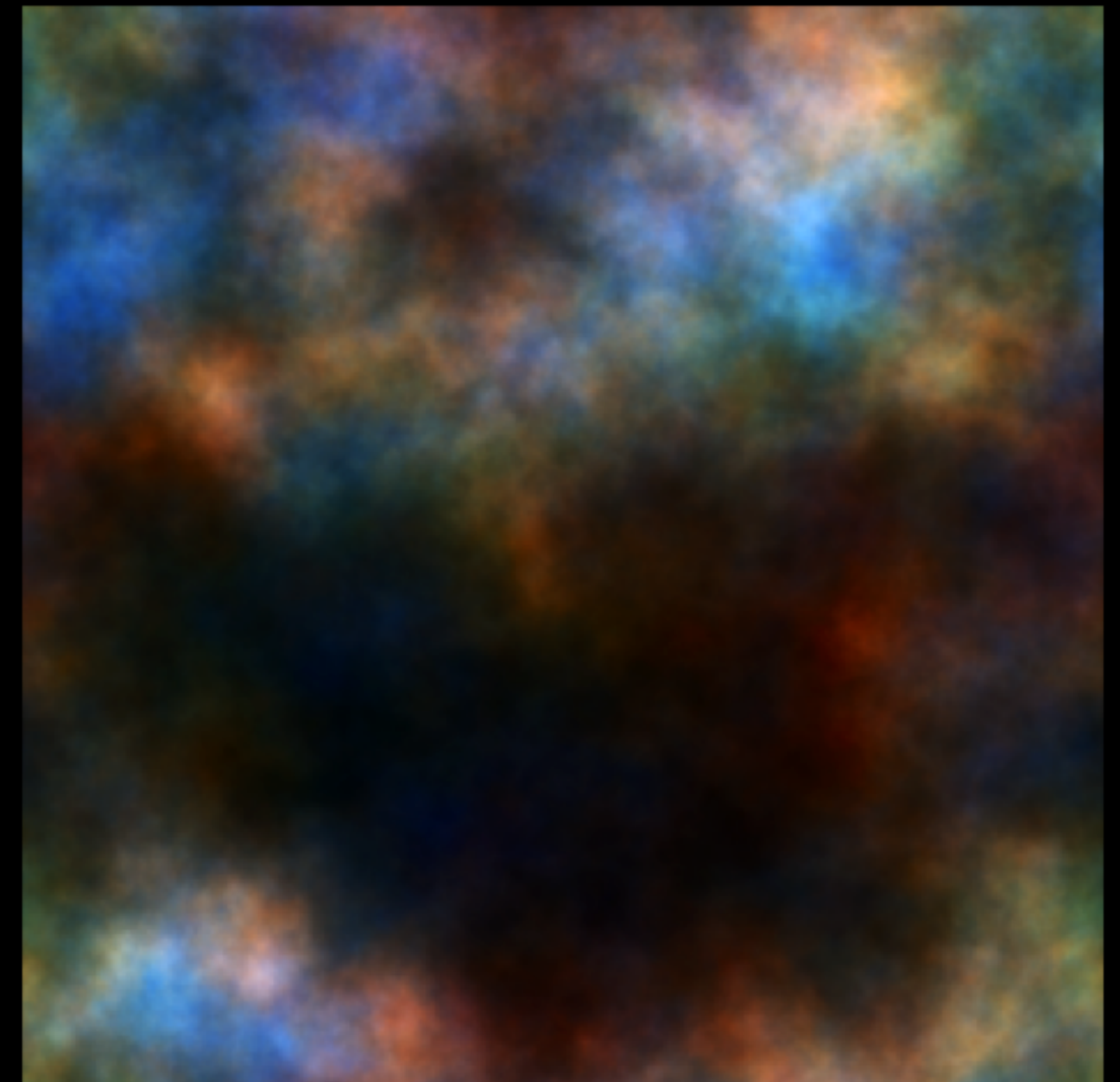
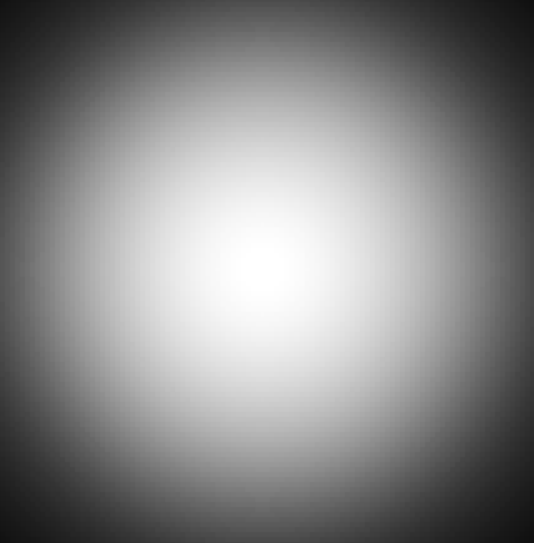
I_0^{para}

LensCharm 2.0

Multi-frequency sky model



=



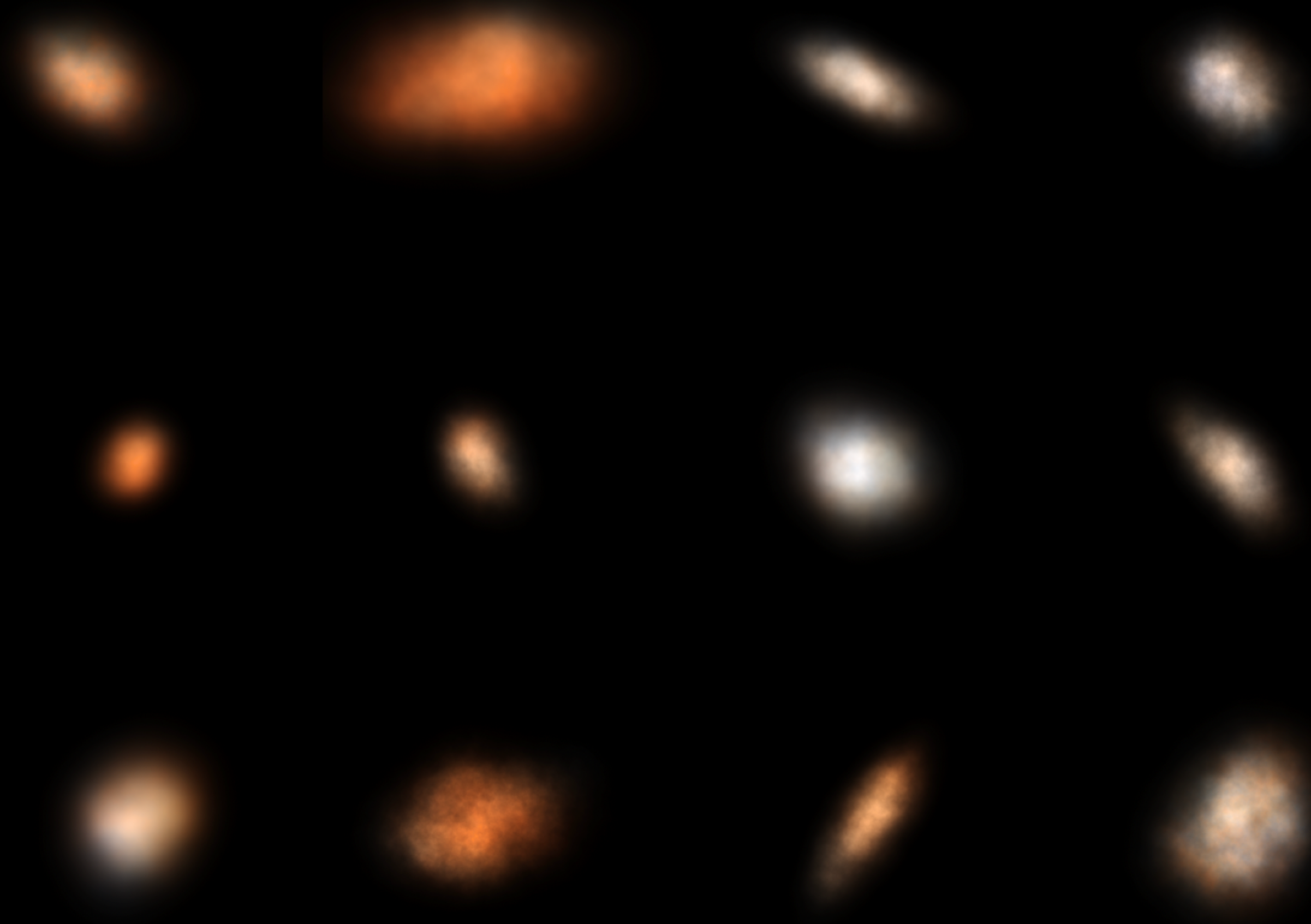
$$I(\mathbf{x}, \nu)$$

$$I_0^{\text{para}}$$

$$I_0(\mathbf{x}) \left(\frac{\nu}{\nu_0} \right)^{\alpha(\mathbf{x}) + \delta(\mathbf{x}, \nu)}$$

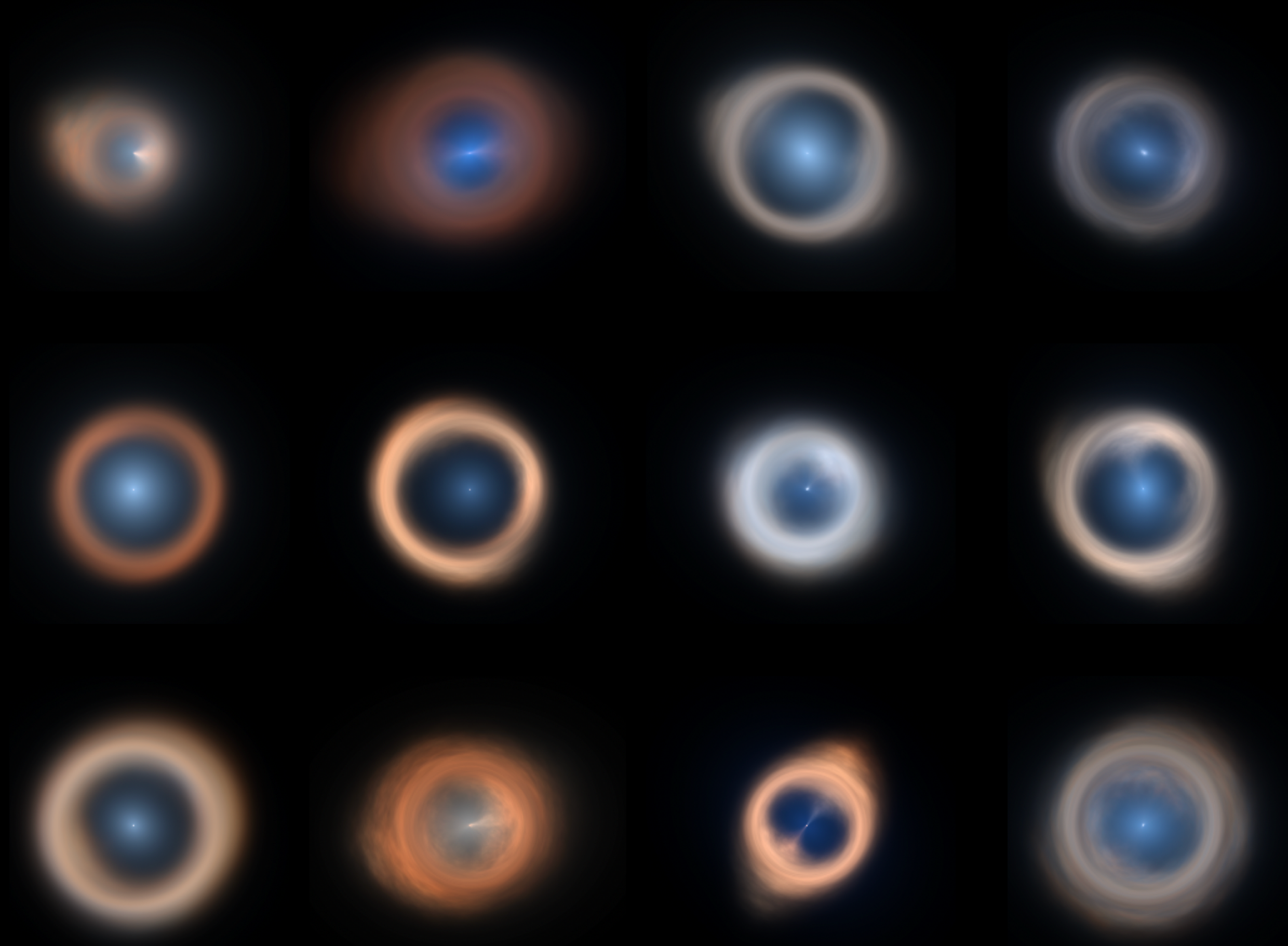
LensCharm 2.0

Multi-frequency sky model



LensCharm 2.0

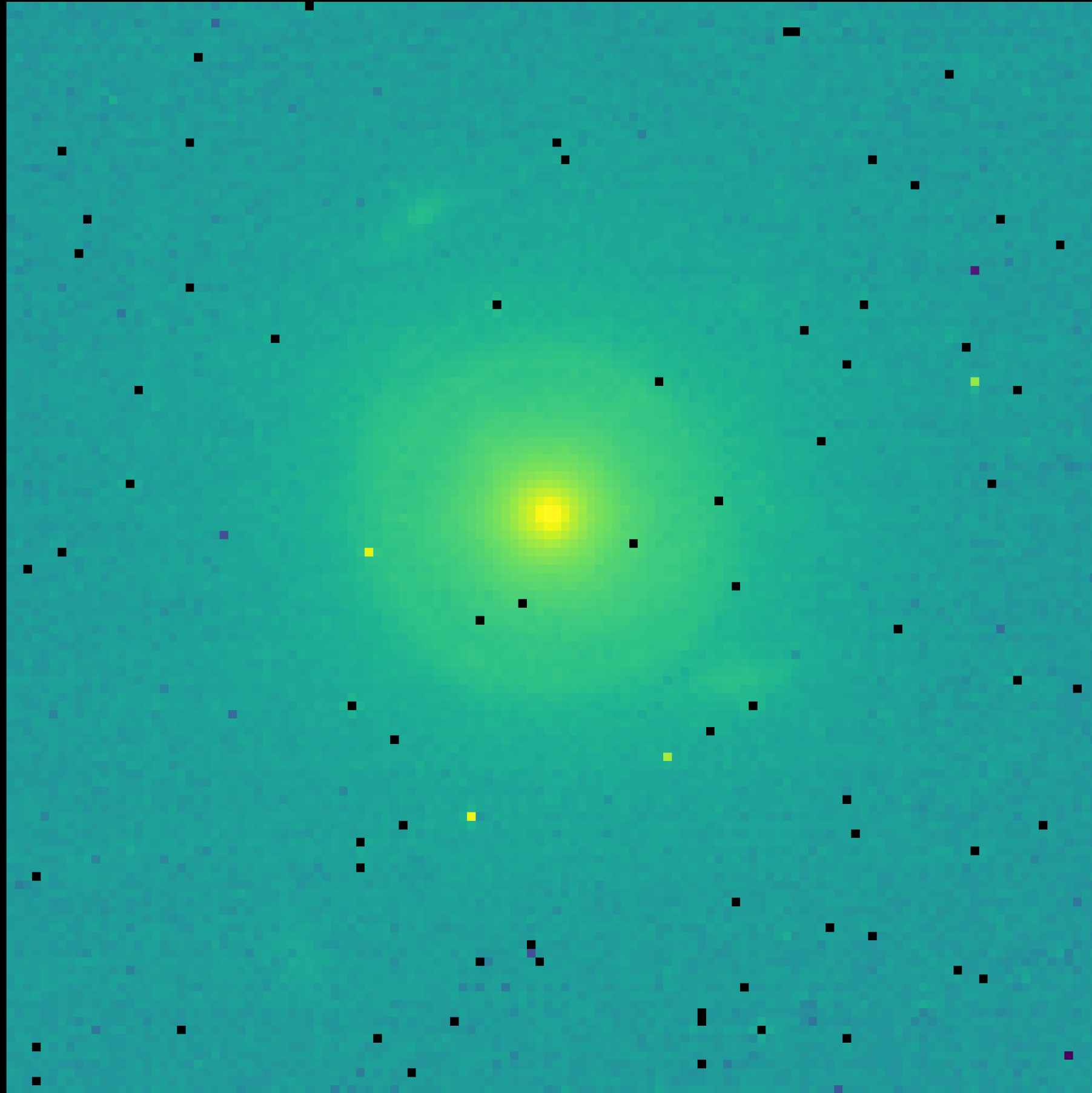
Multi-frequency sky model



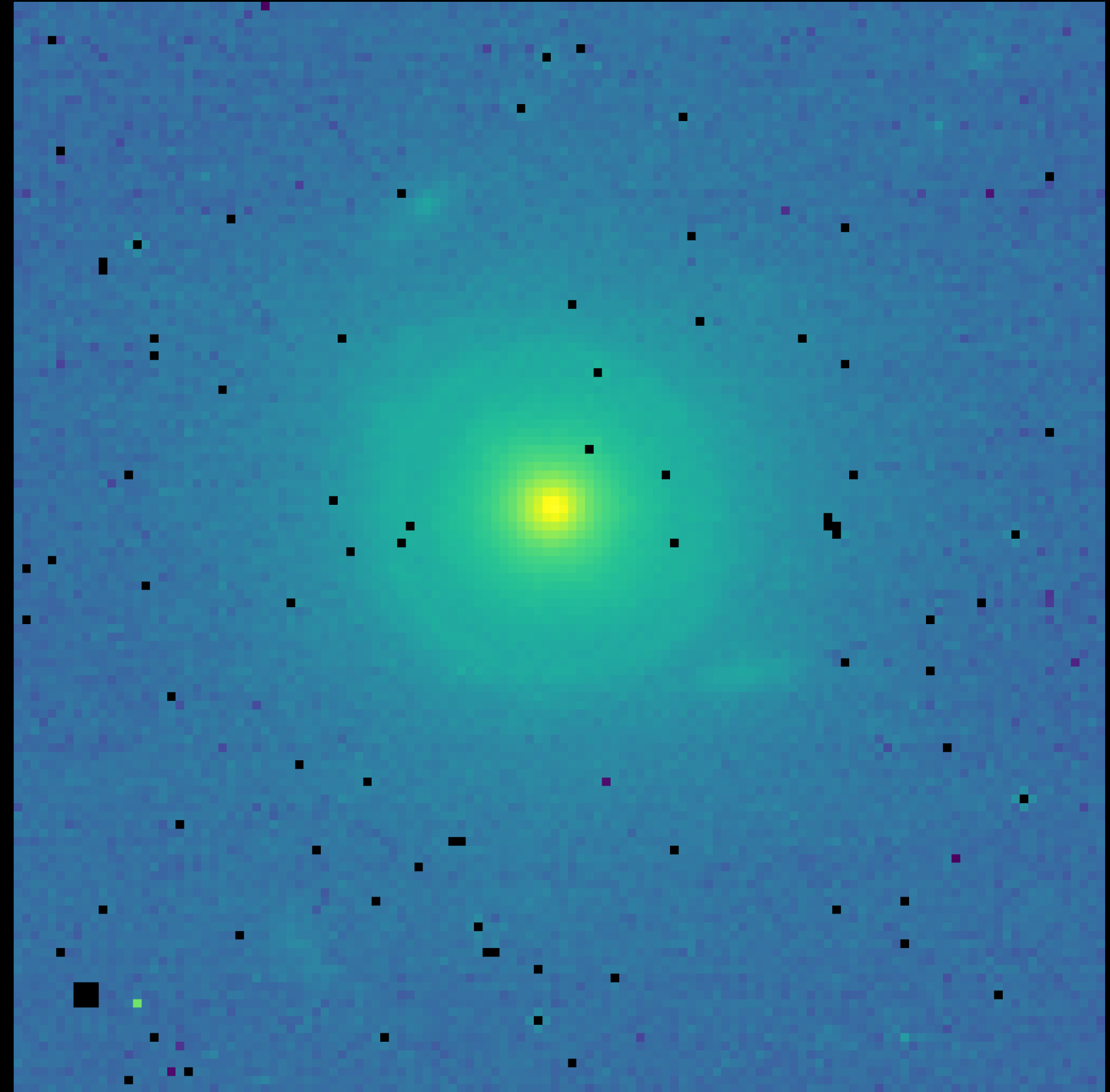
Let's get real

High-redshift galaxy reconstruction with LensCharm

SPT0418-47 ($z=4.22$) data



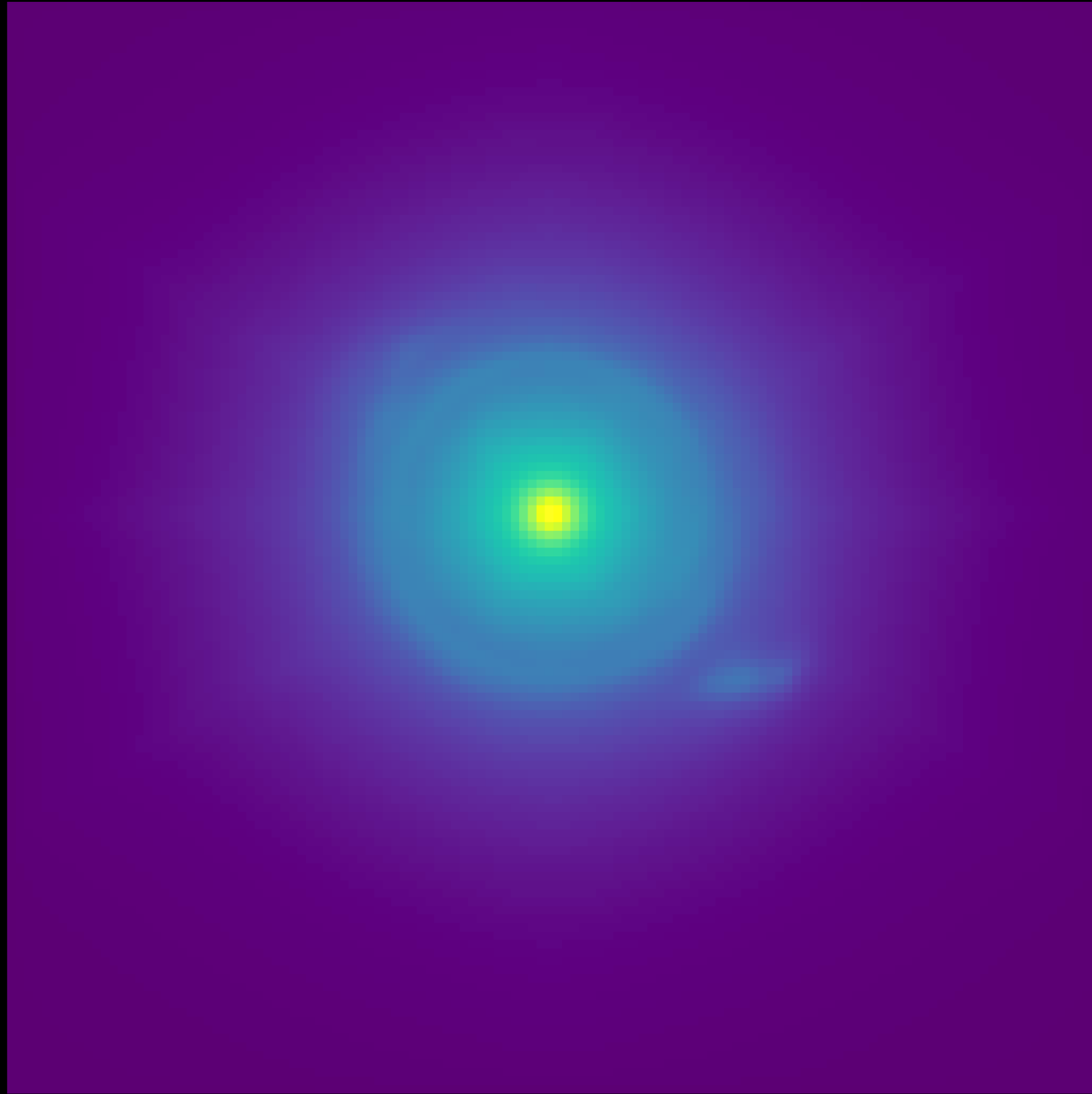
integration 1



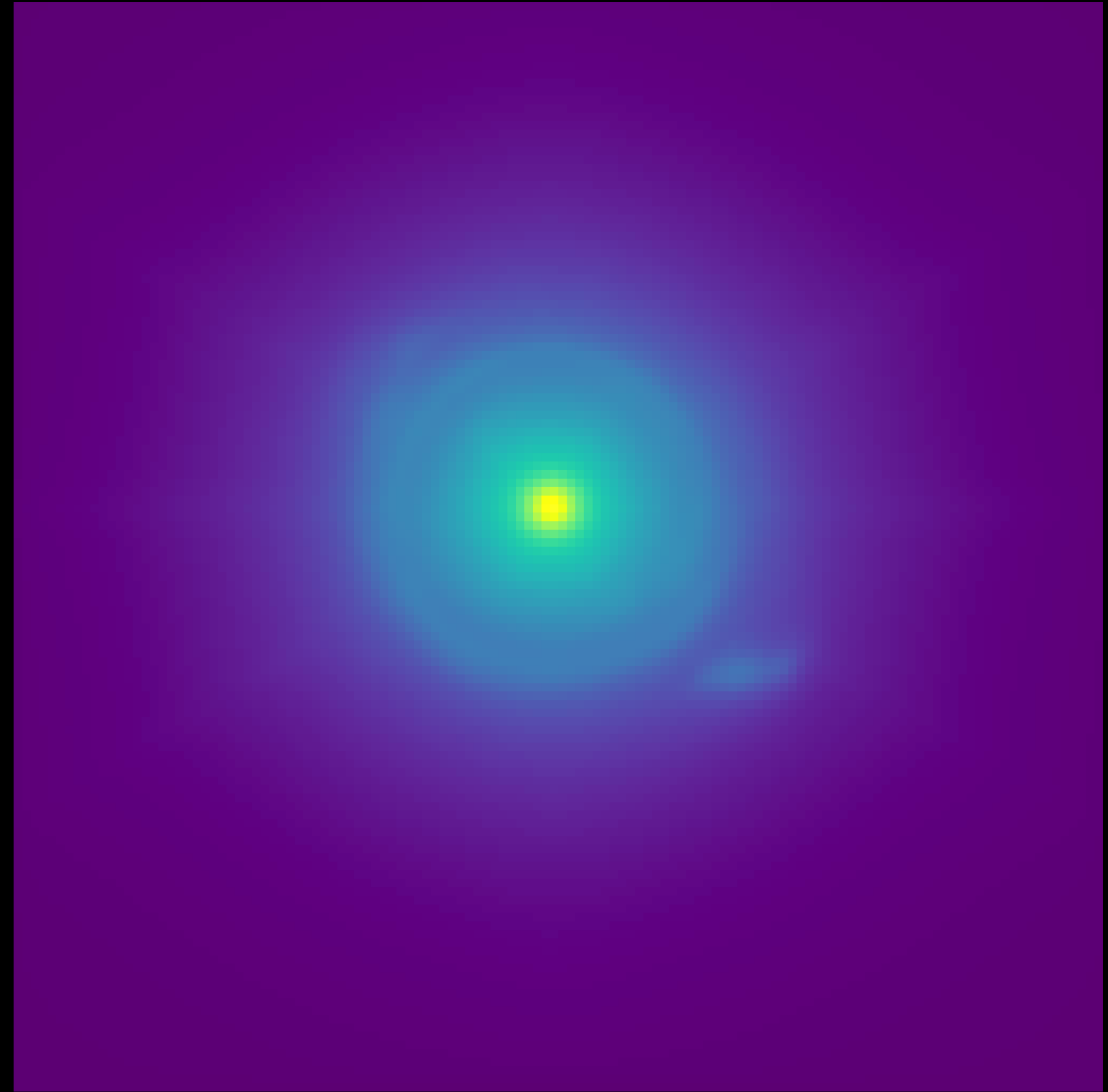
integration 2

High-redshift galaxy reconstruction with LensCharm

SPT0418-47 ($z=4.22$) signal response



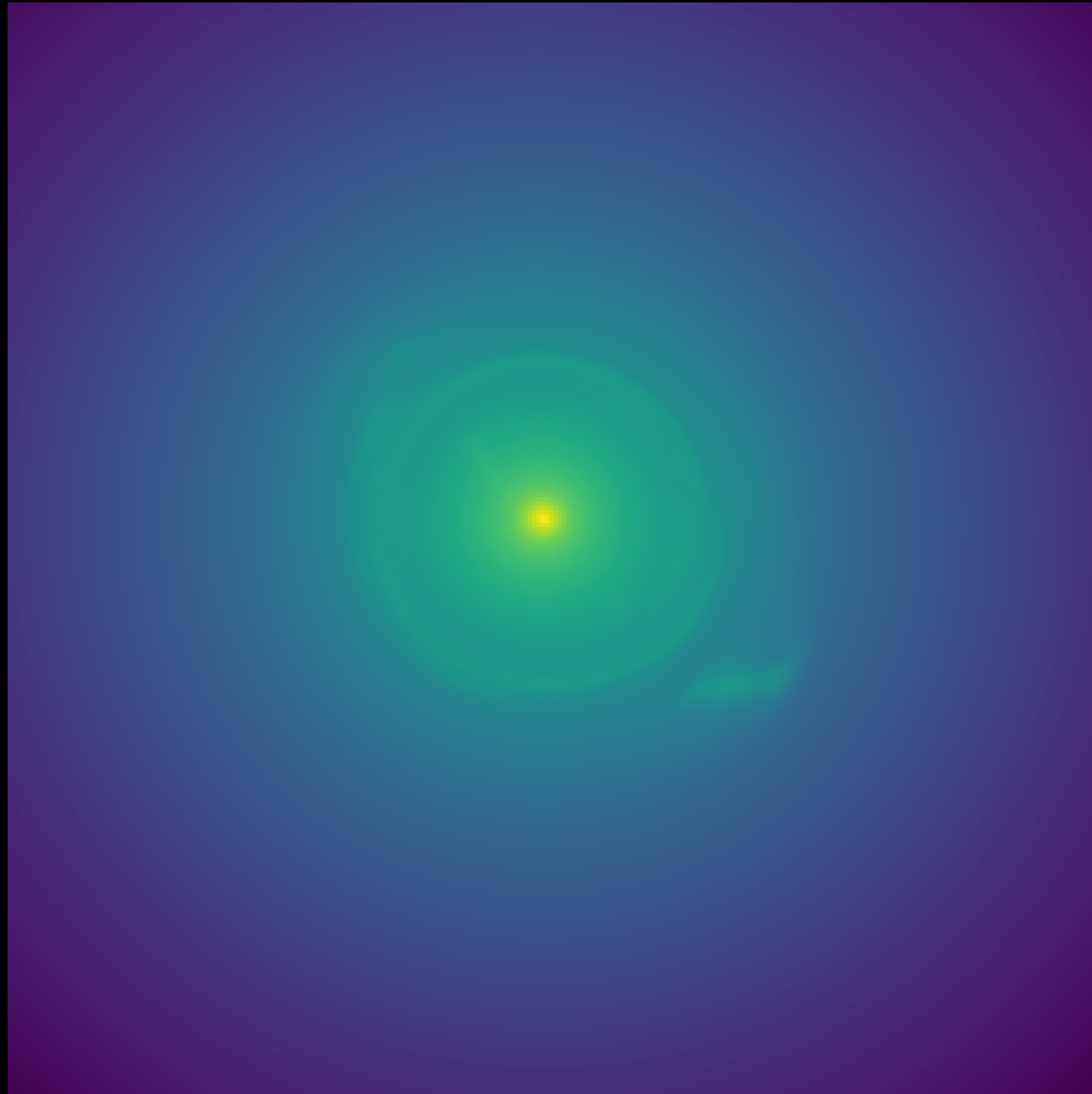
integration 1



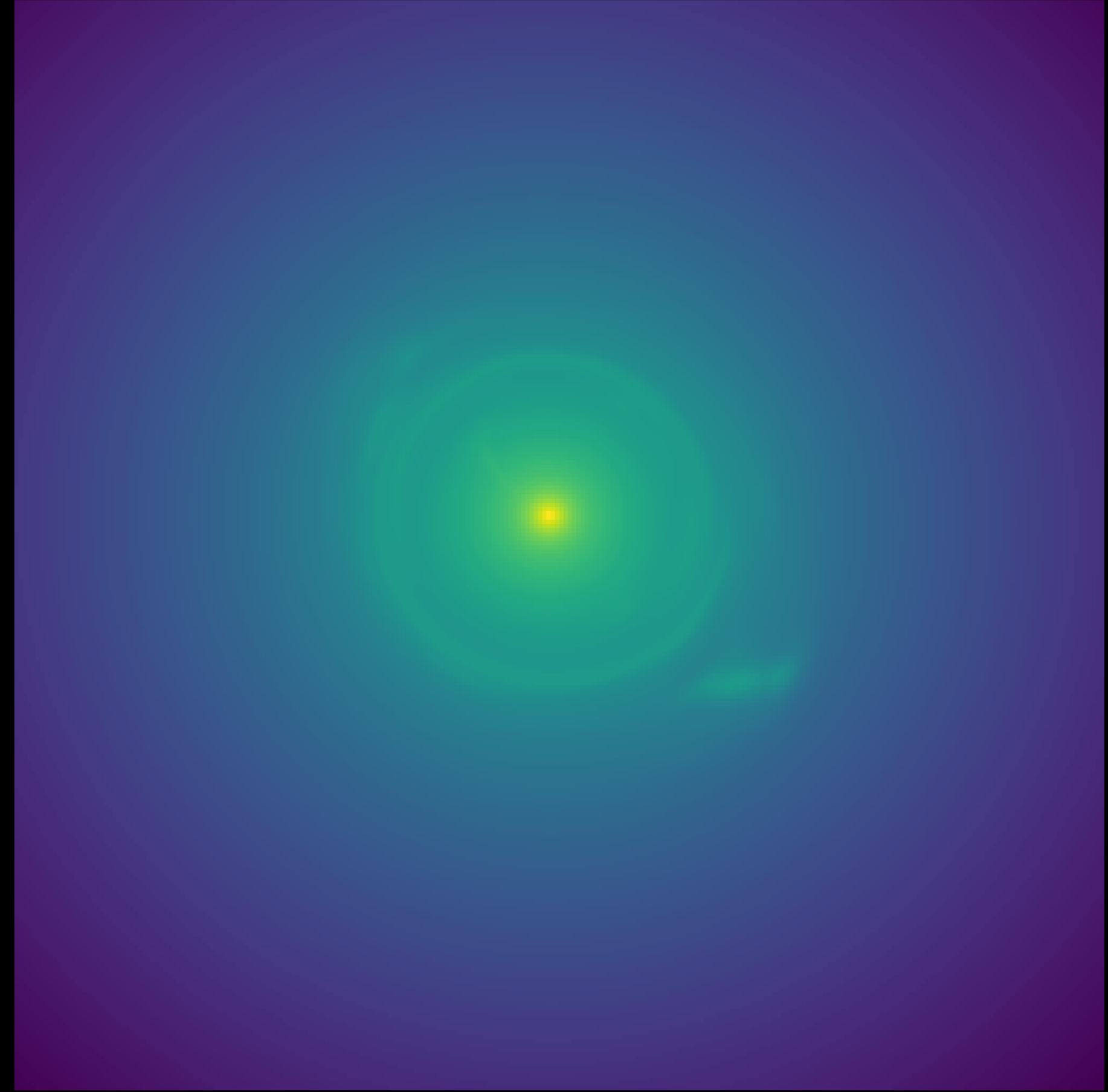
integration 2

High-redshift galaxy reconstruction with LensCharm

SPT0418-47 ($z=4.22$) lensed signal



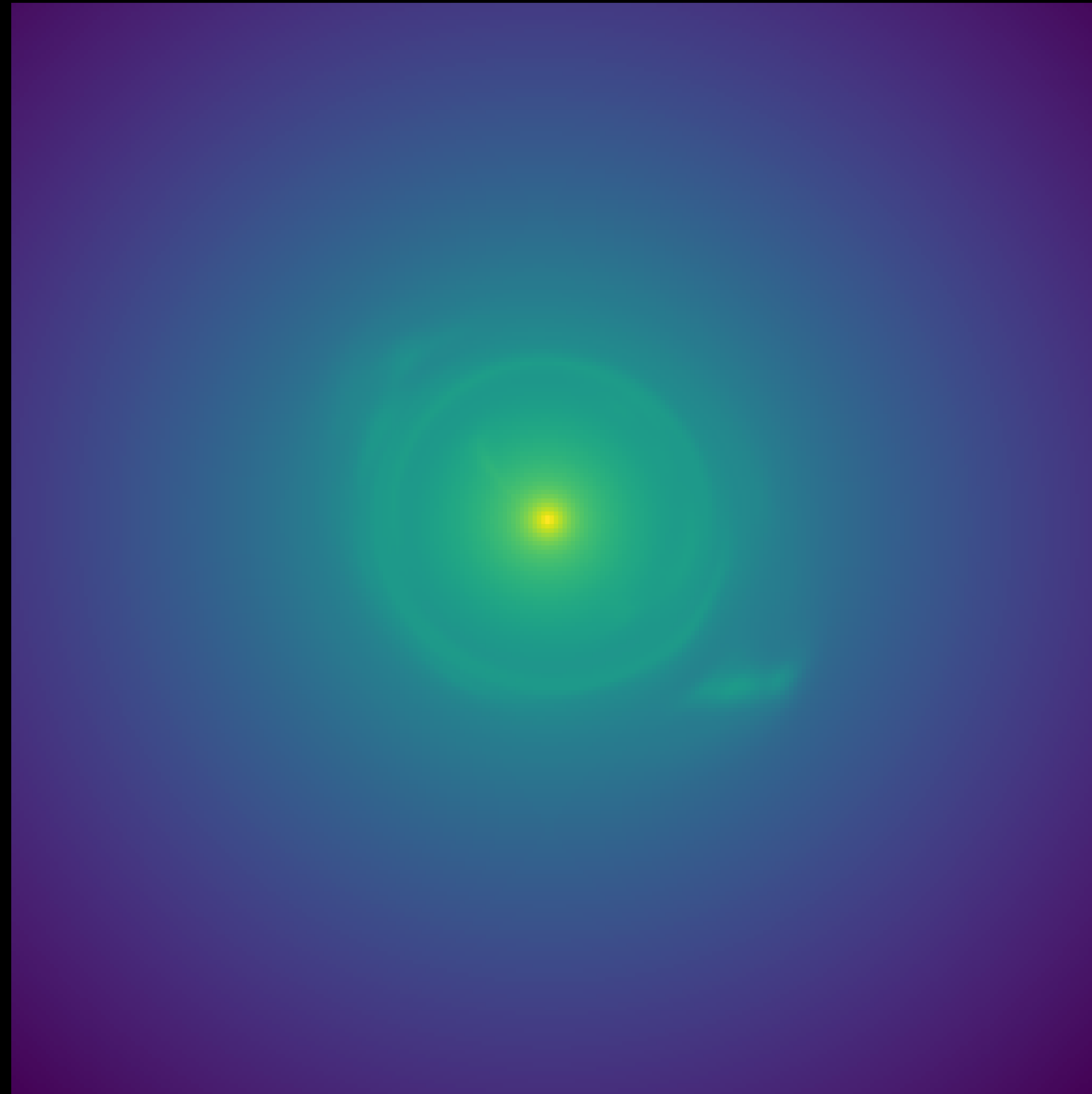
integration 1



integration 2

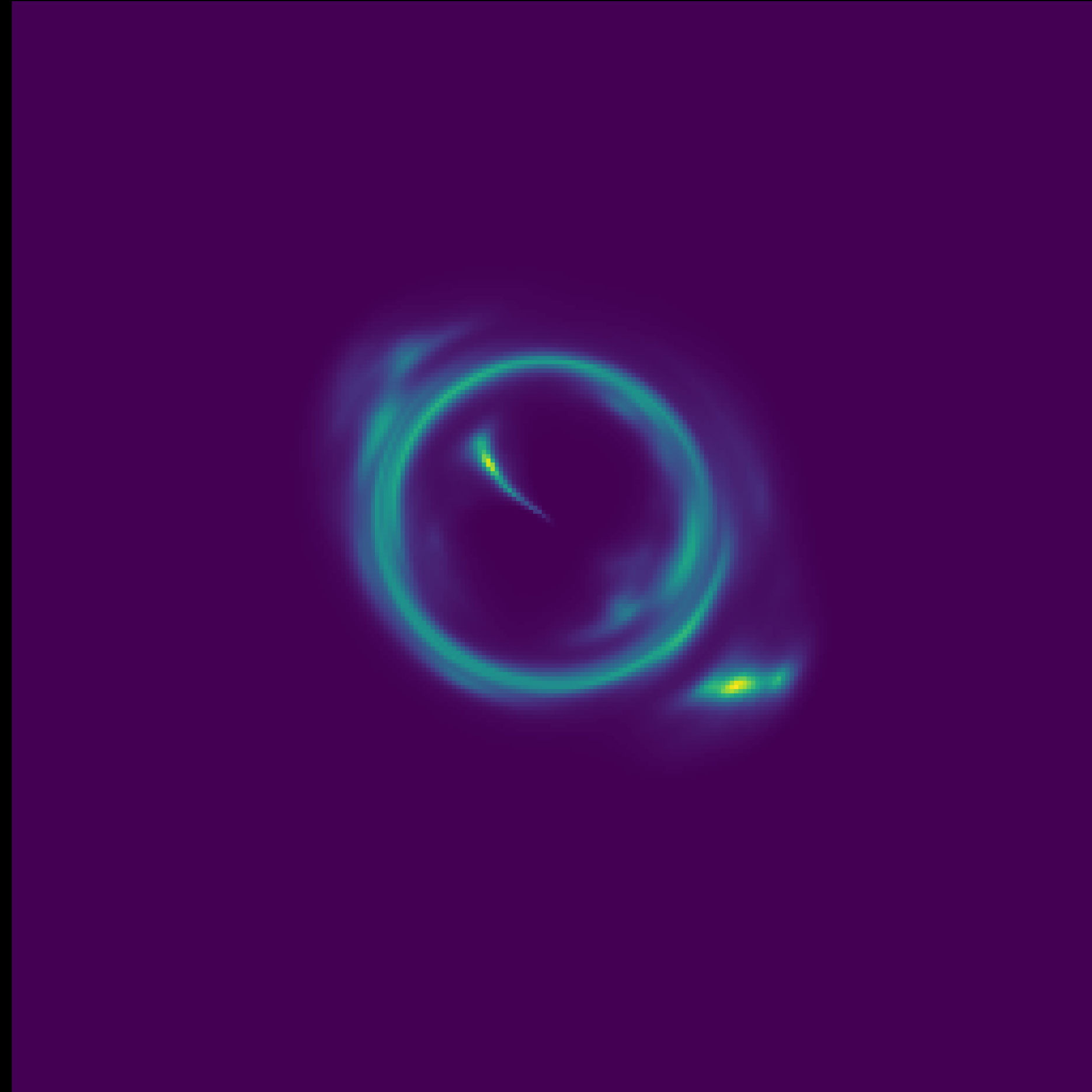
High-redshift galaxy reconstruction with LensCharm

SPT0418-47 ($z=4.22$) lensed signal



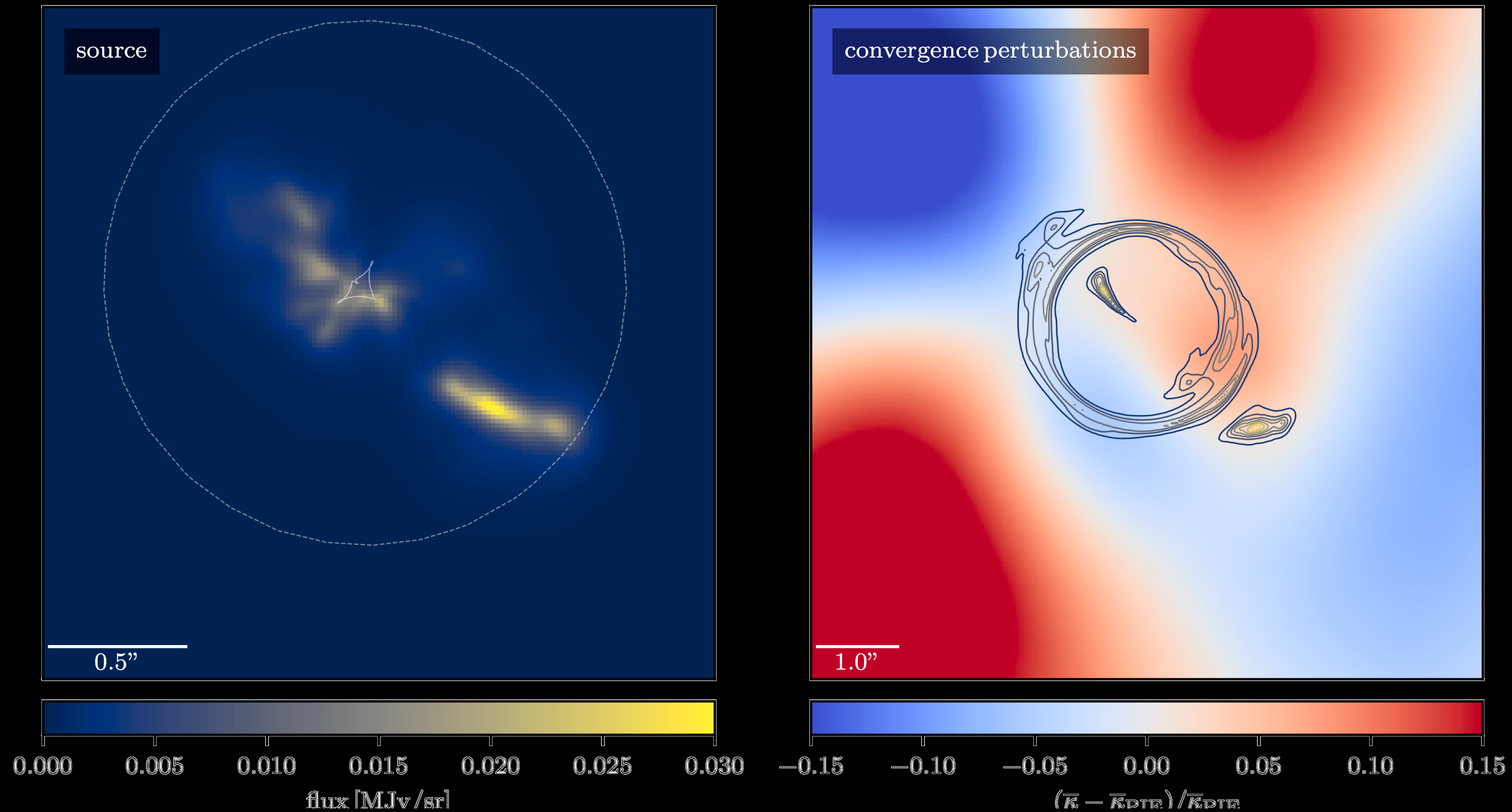
High-redshift galaxy reconstruction with LensCharm

SPT0418-47 ($z=4.22$) lensed signal



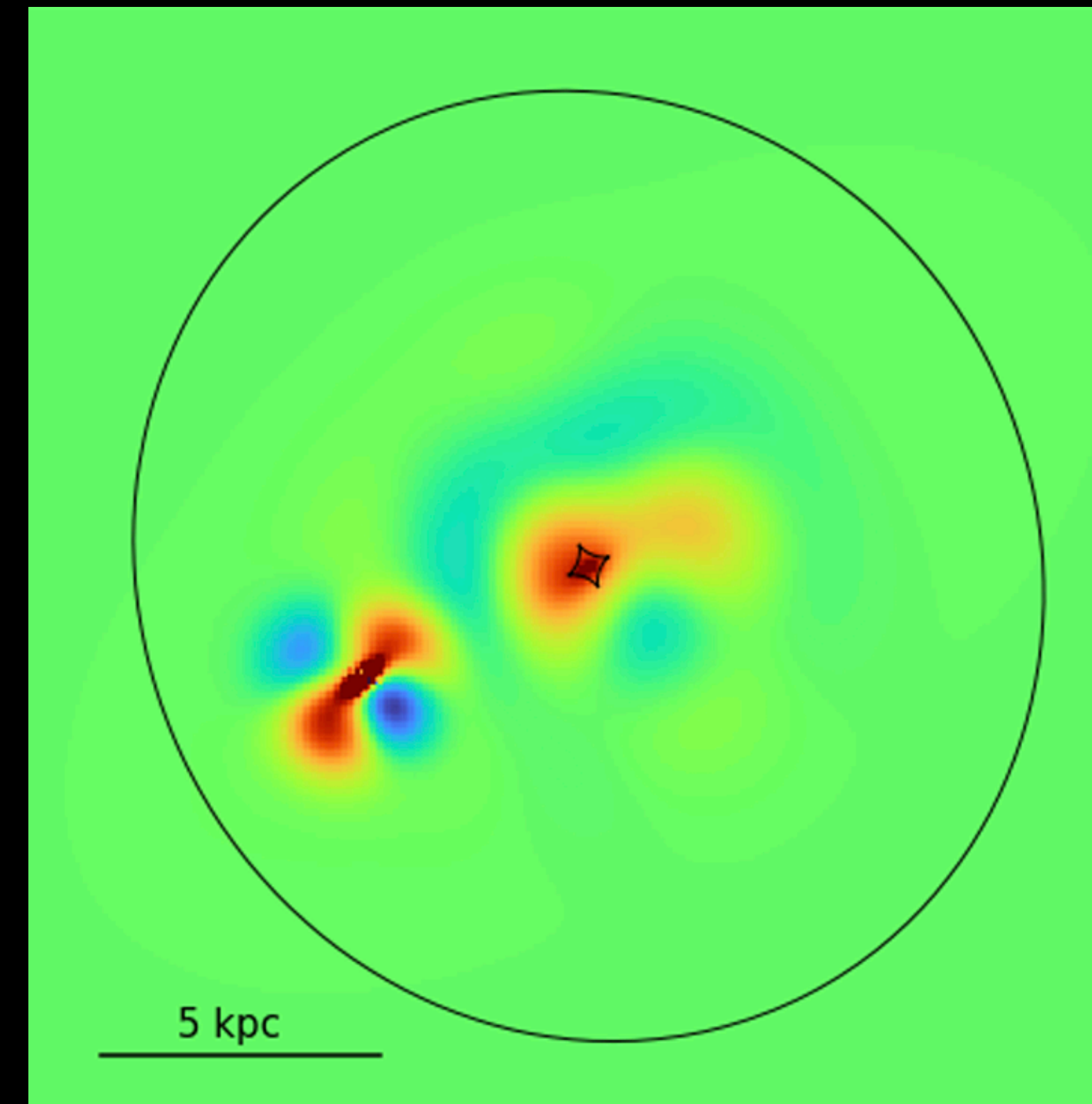
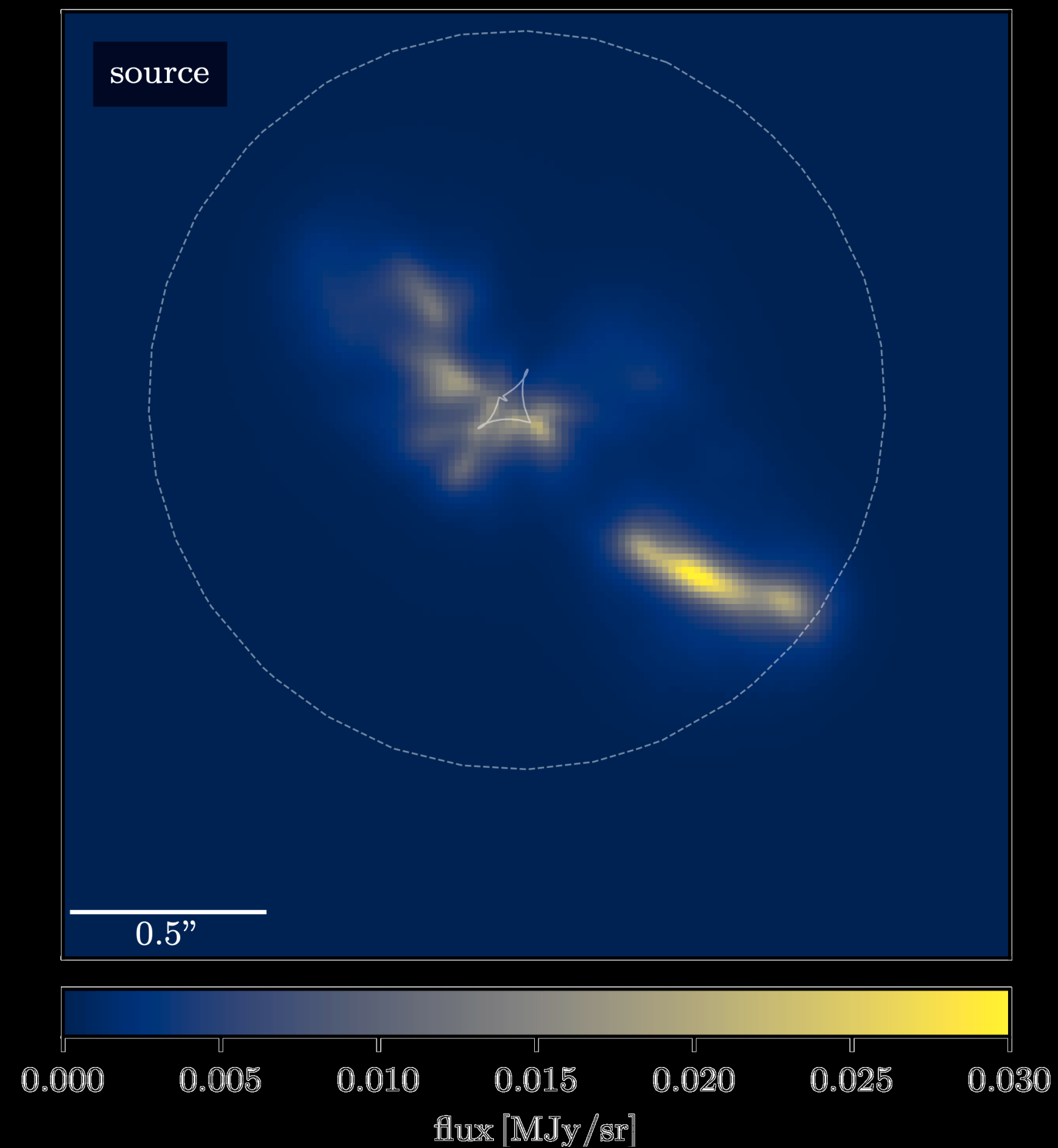
High-redshift galaxy reconstruction with LensCharm

SPT0418-47 ($z=4.22$) reconstruction



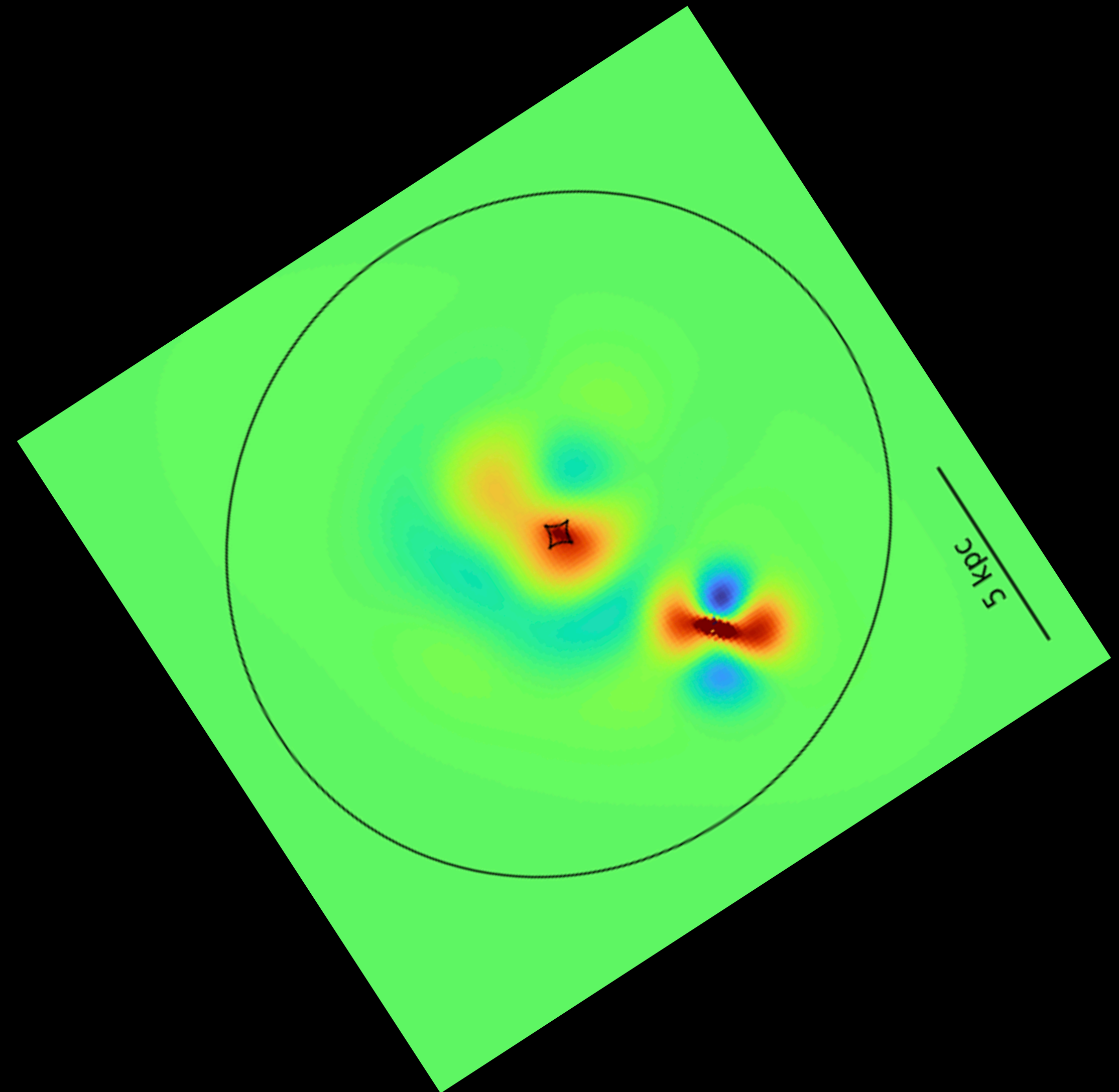
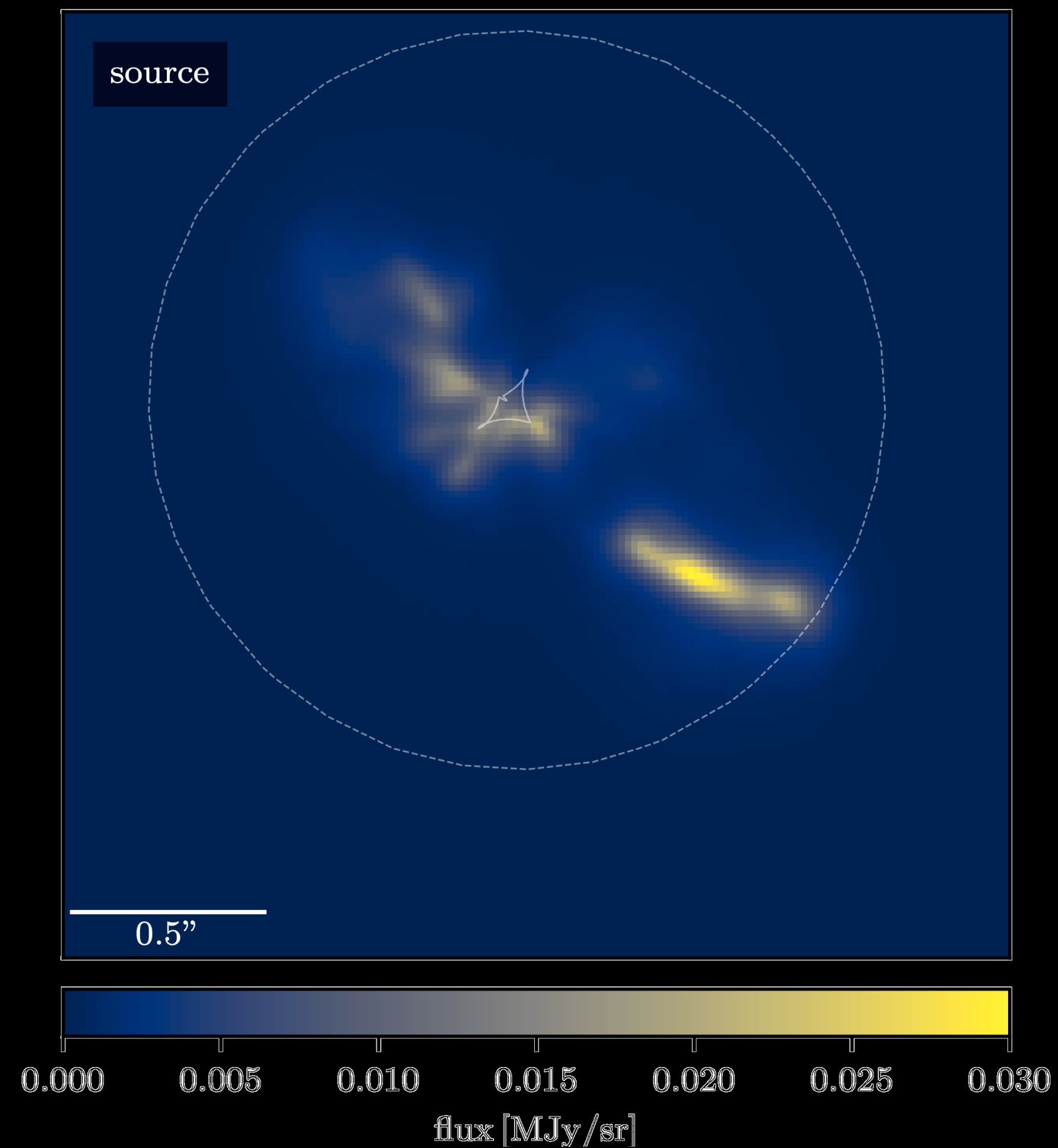
High-redshift galaxy reconstruction with LensCharm

SPT0418-47 ($z=4.22$) reconstruction



High-redshift galaxy reconstruction with LensCharm

SPT0418-47 ($z=4.22$) reconstruction

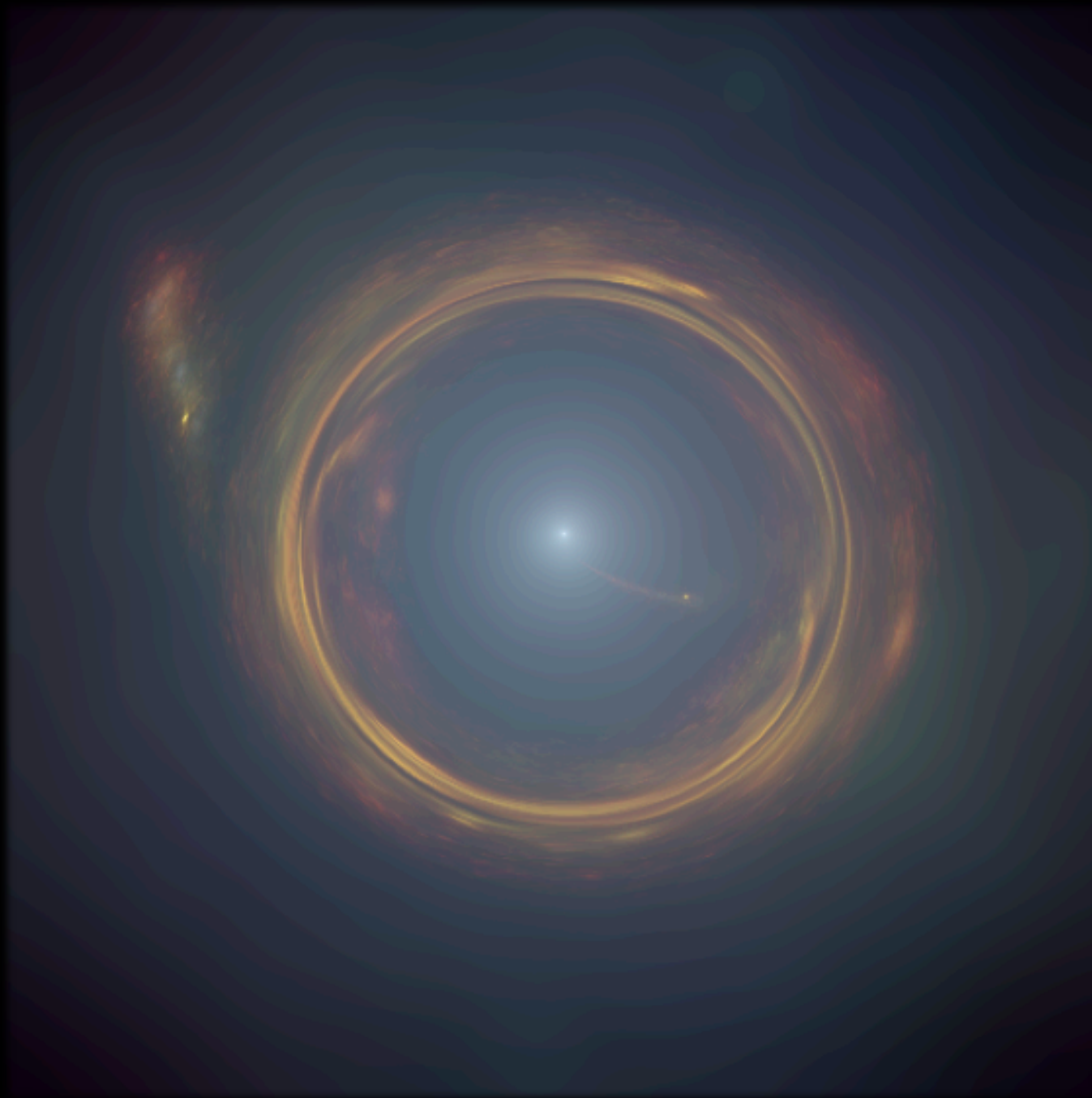


LensCharm 2.0

Multi-instrument, multifrequency!

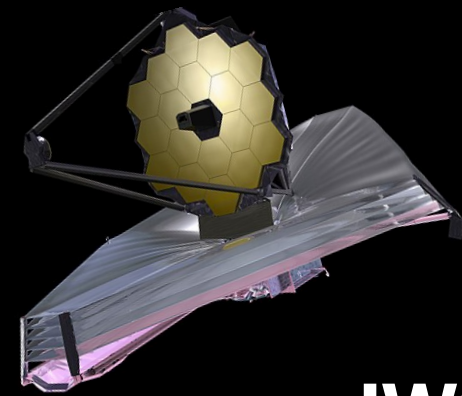
LensCharm 2.0

Multi-instrument, multifrequency!



LensCharm 2.0

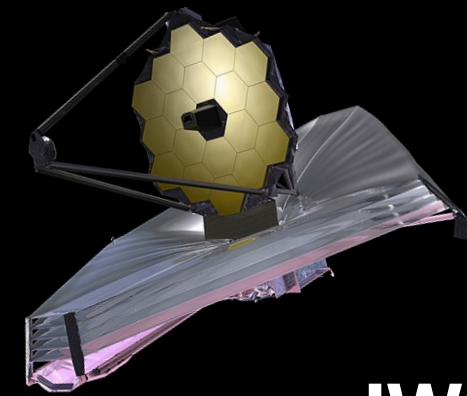
Multi-instrument, multifrequency!



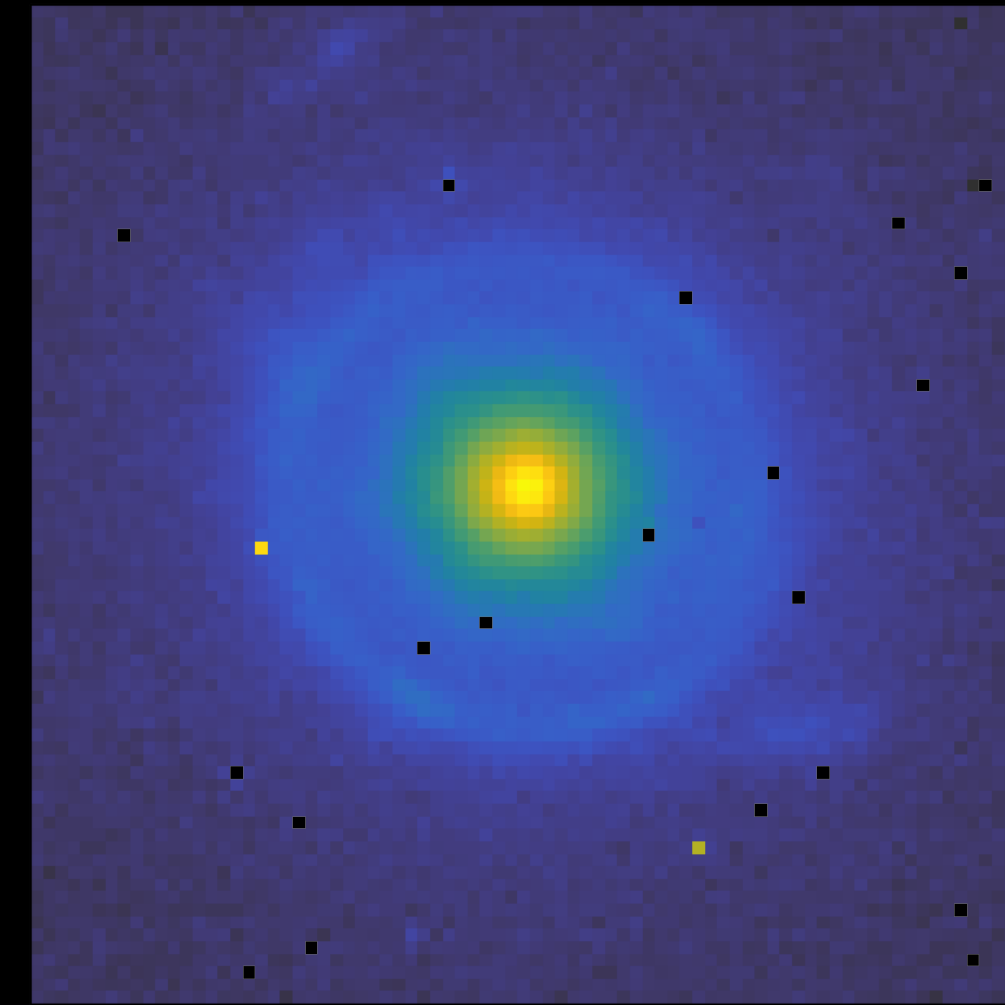
JWST

LensCharm 2.0

Multi-instrument, multifrequency!

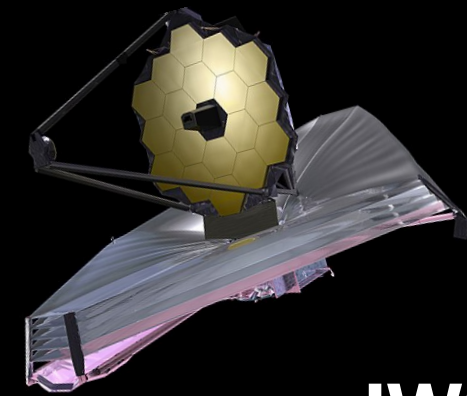


JWST

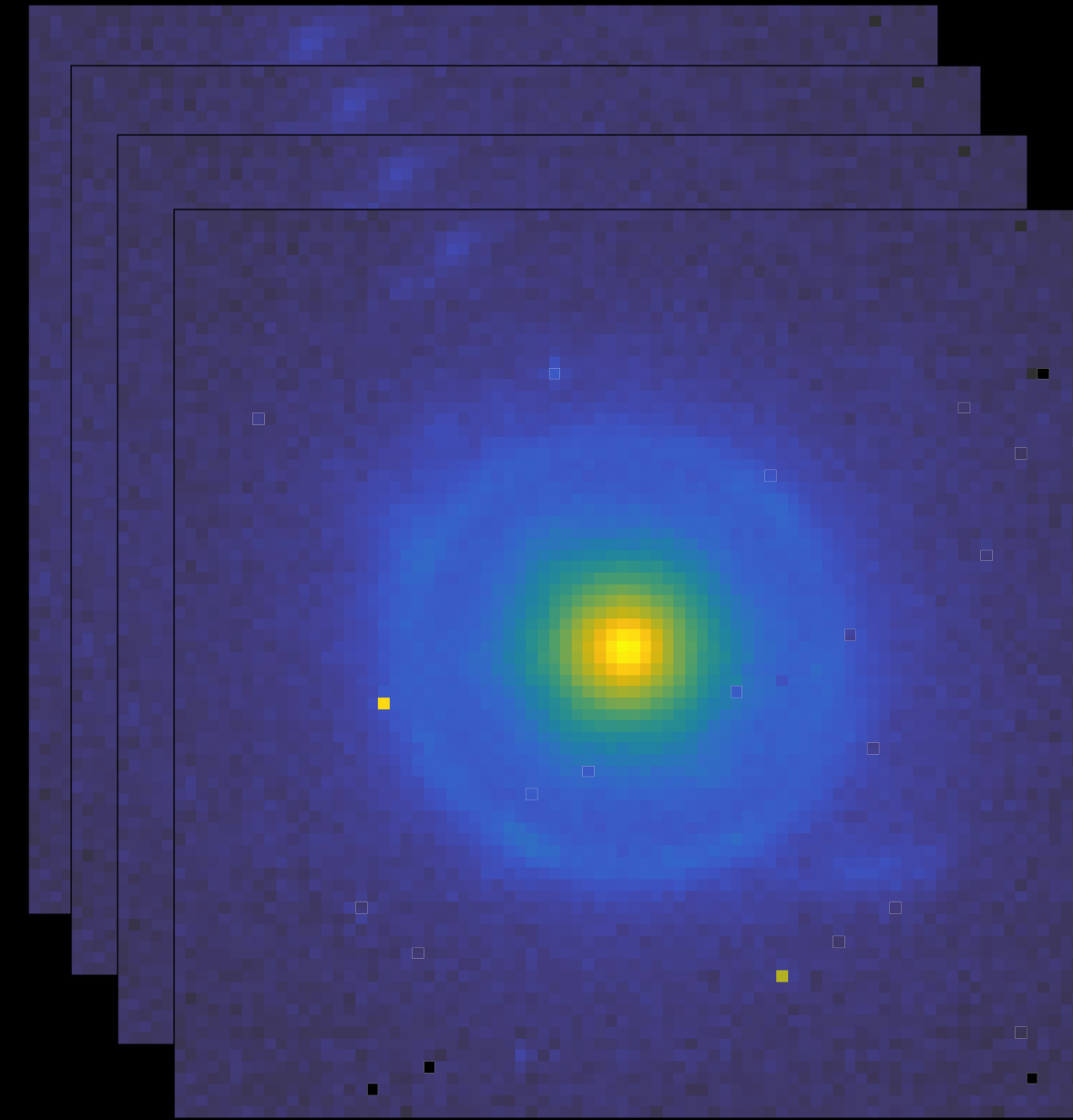


LensCharm 2.0

Multi-instrument, multifrequency!

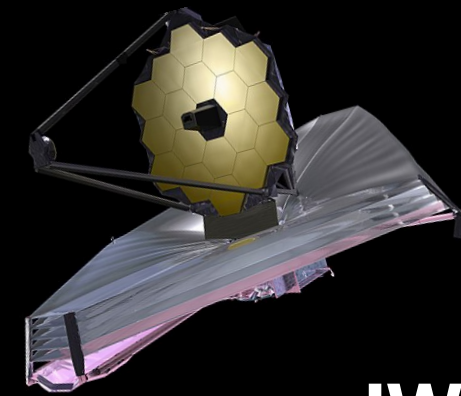


JWST

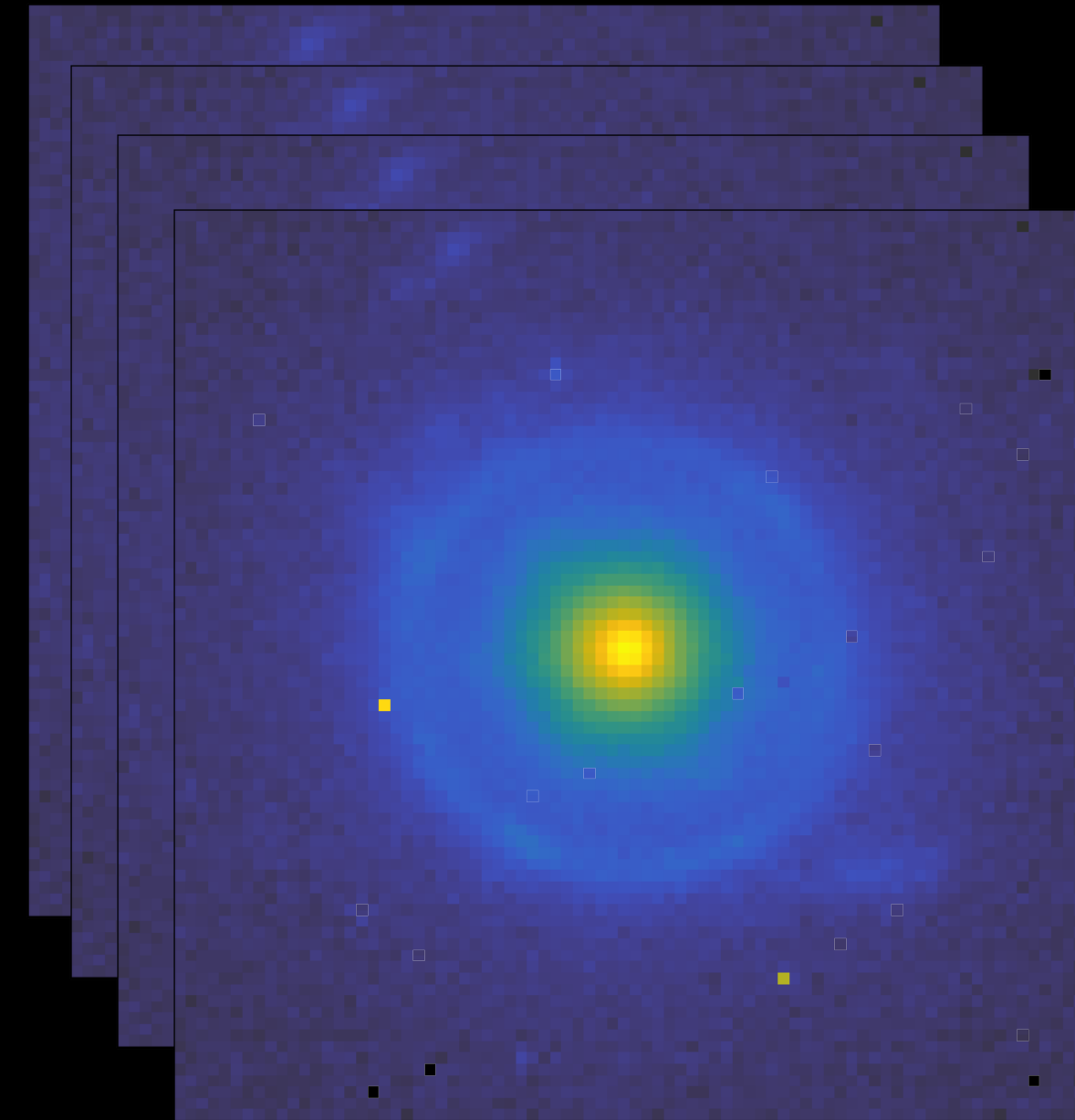


LensCharm 2.0

Multi-instrument, multifrequency!



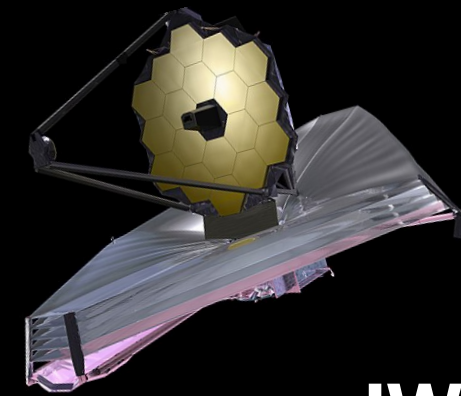
JWST



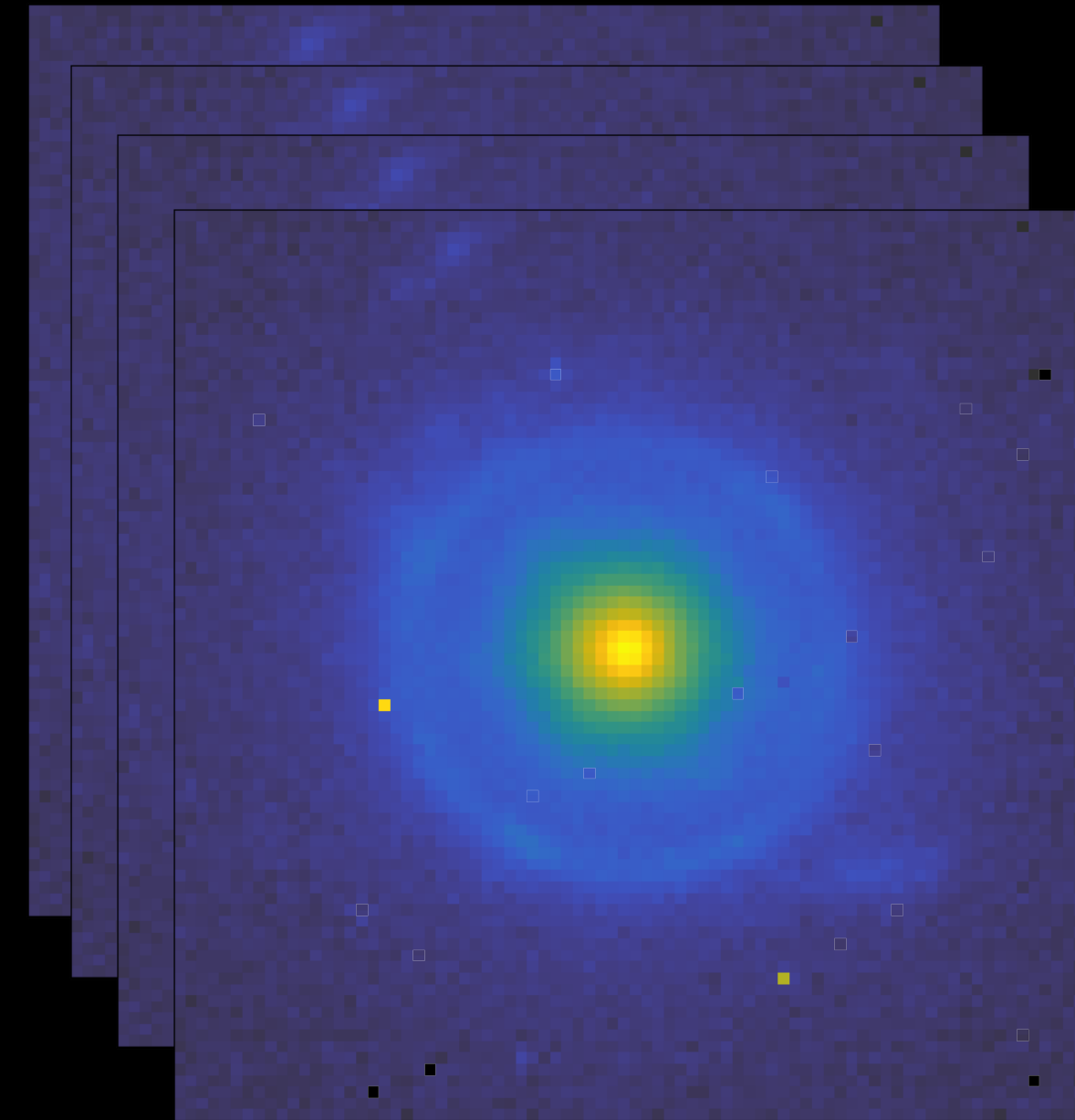
ALMA

LensCharm 2.0

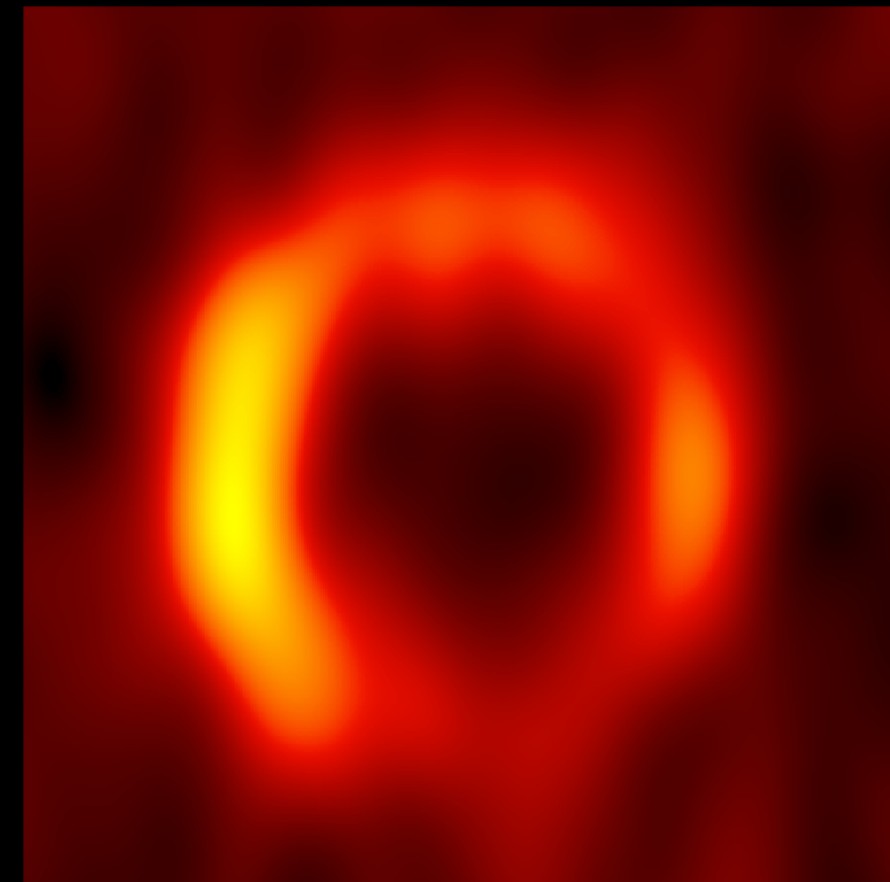
Multi-instrument, multifrequency!



JWST

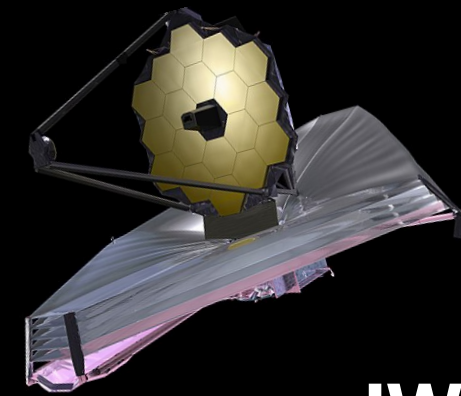


ALMA

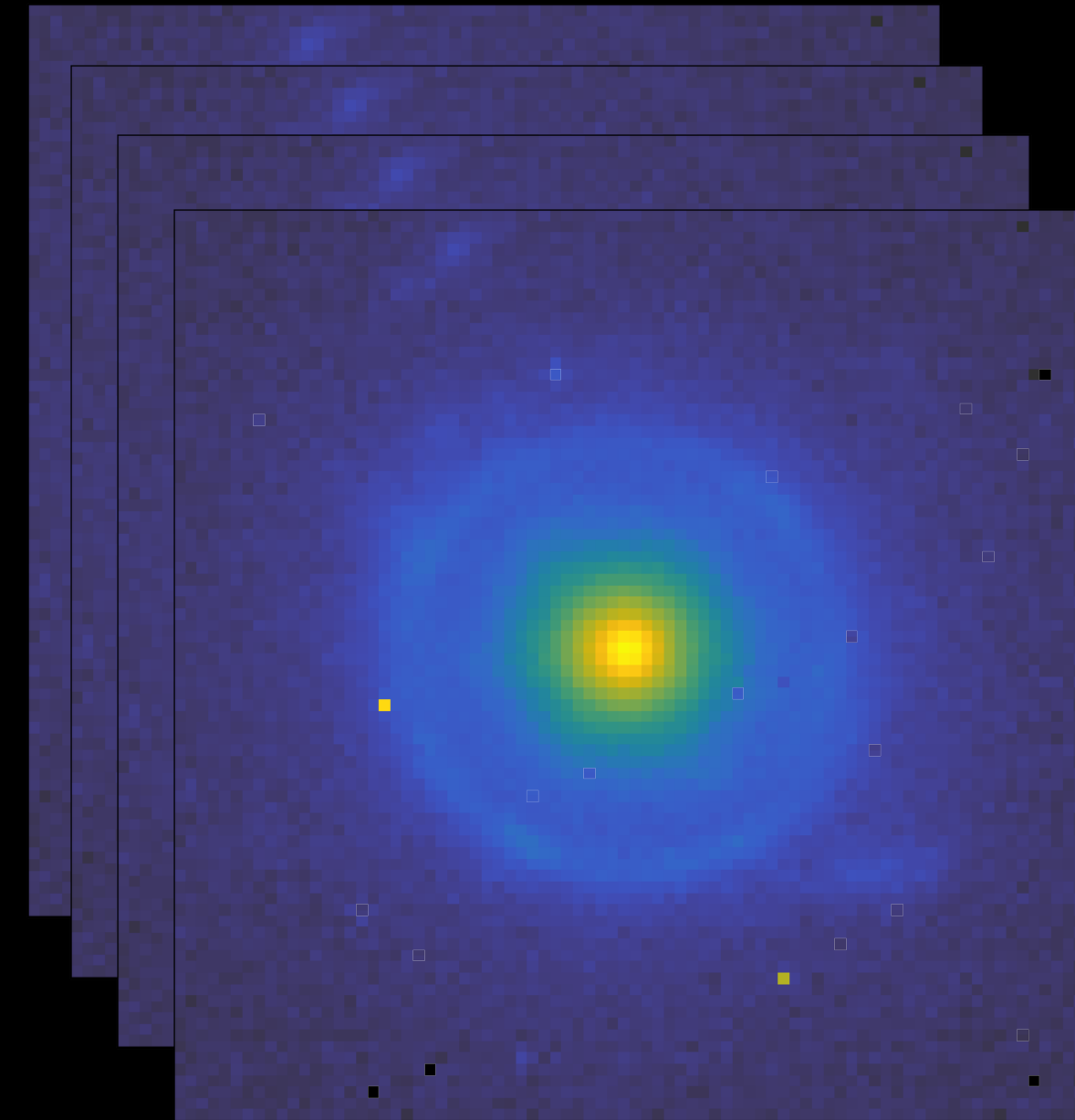


LensCharm 2.0

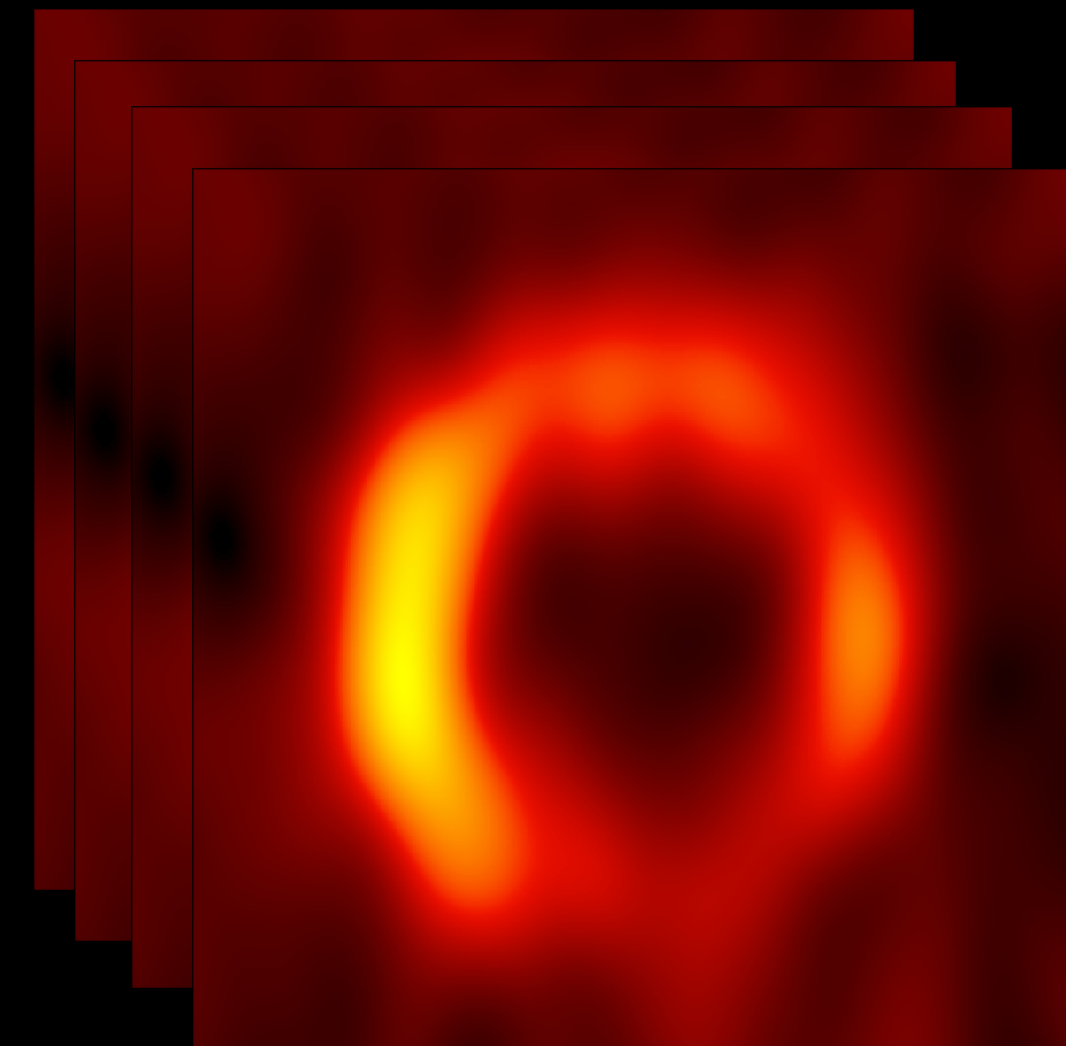
Multi-instrument, multifrequency!



JWST

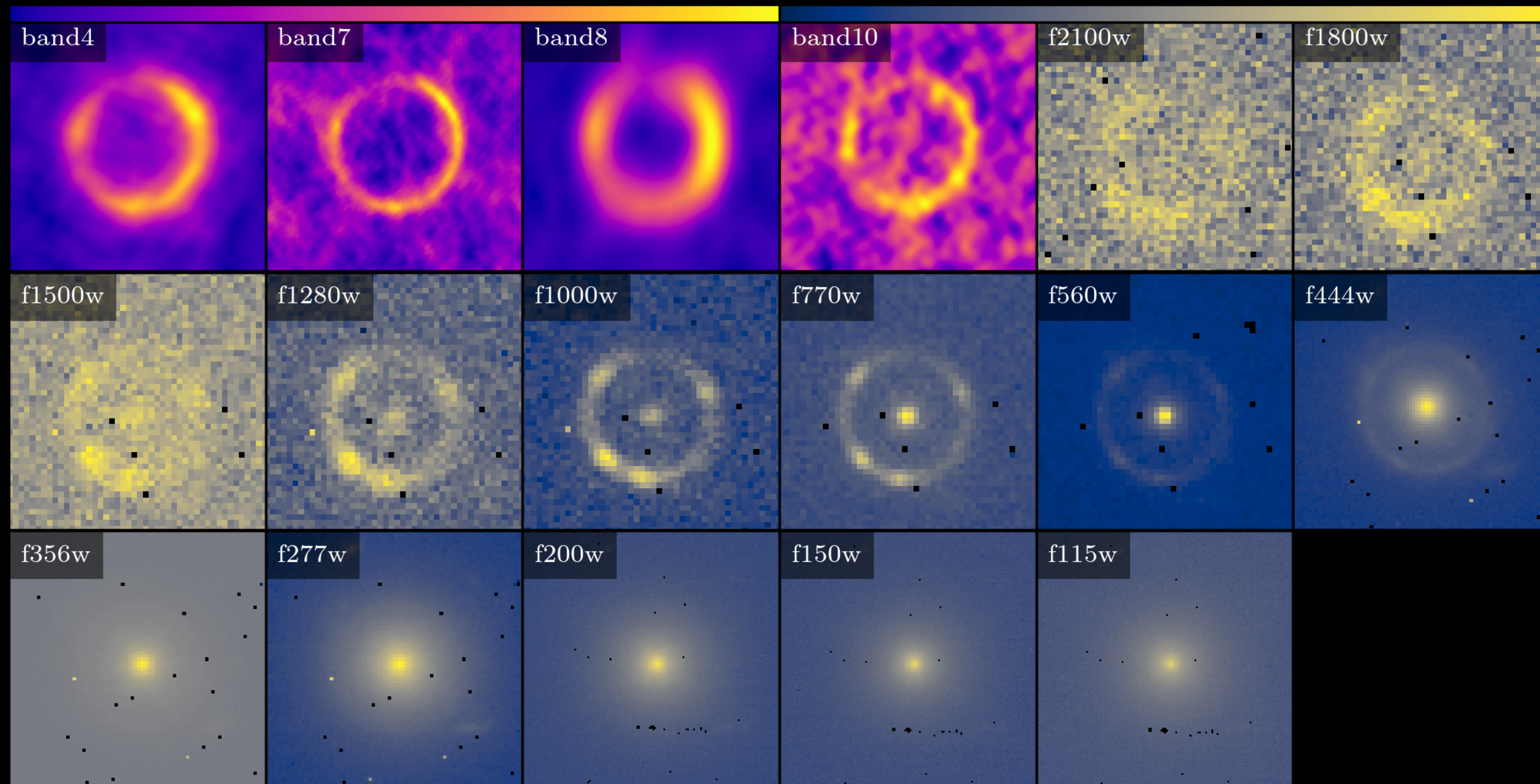


ALMA



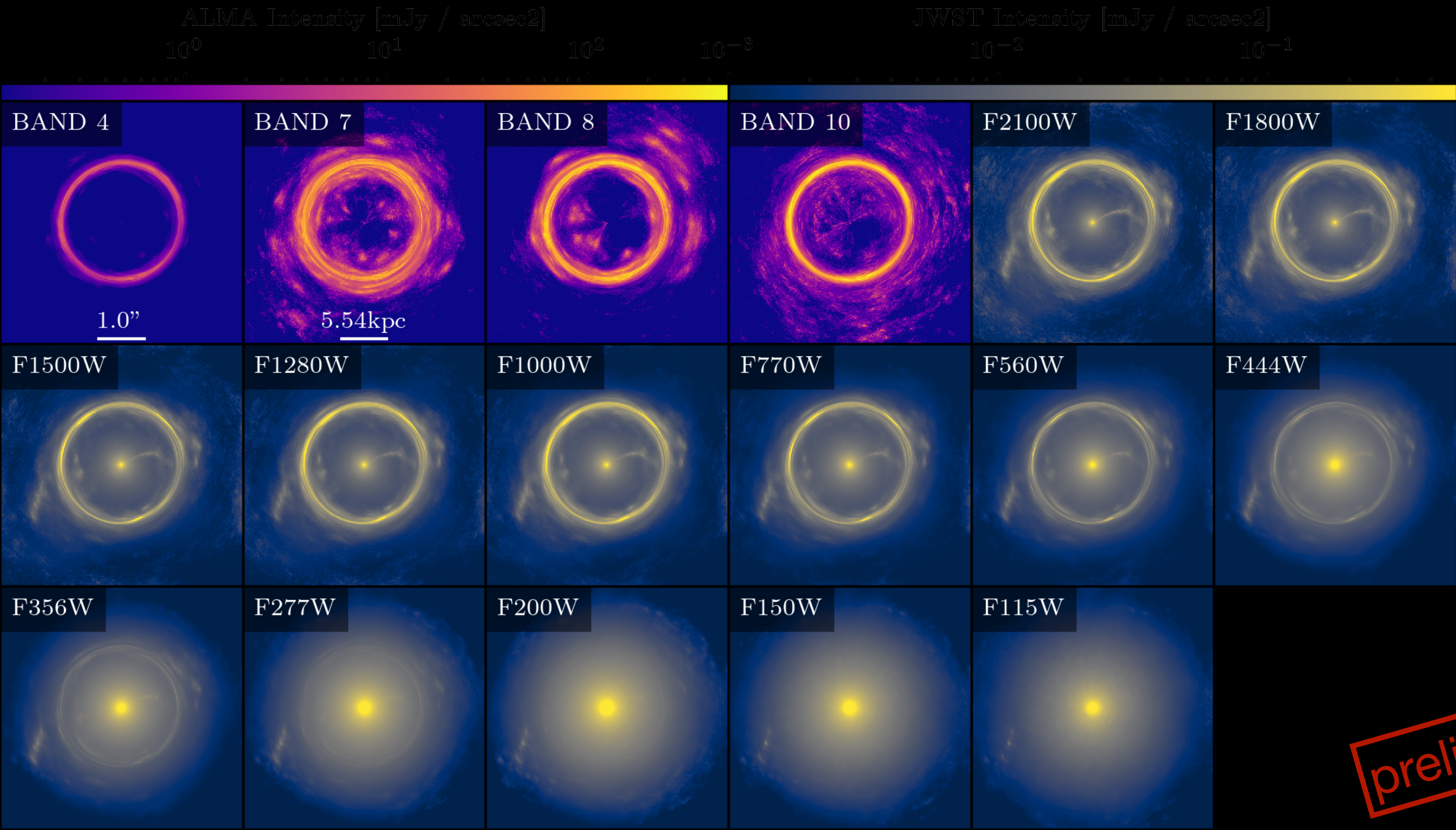
LensCharm 2.0

SPT 0418-47 ($z=4.22$)



LensCharm 2.0

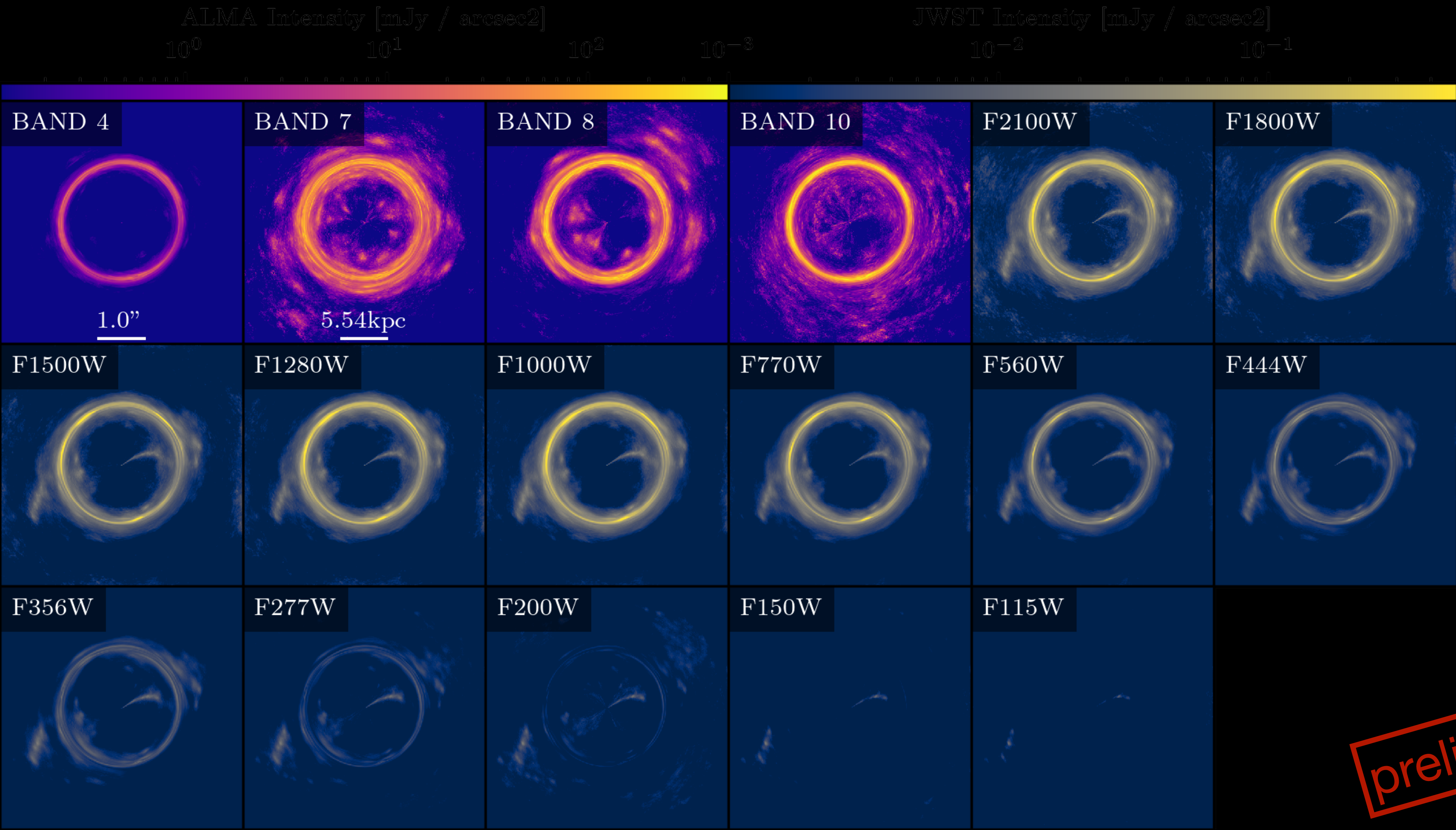
SPT 0418-47 (z=4.22)



preliminary

LensCharm 2.0

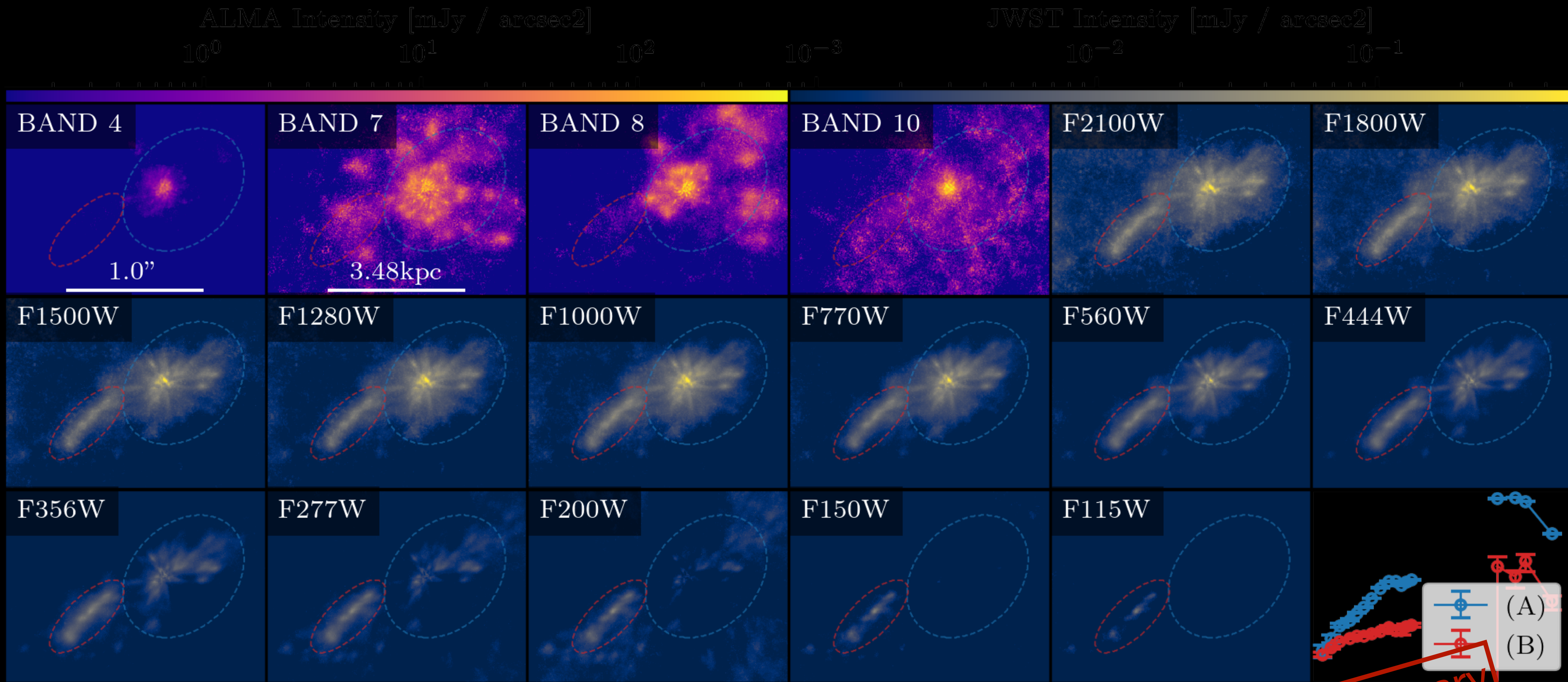
SPT 0418-47 (z=4.22)



preliminary

LensCharm 2.0

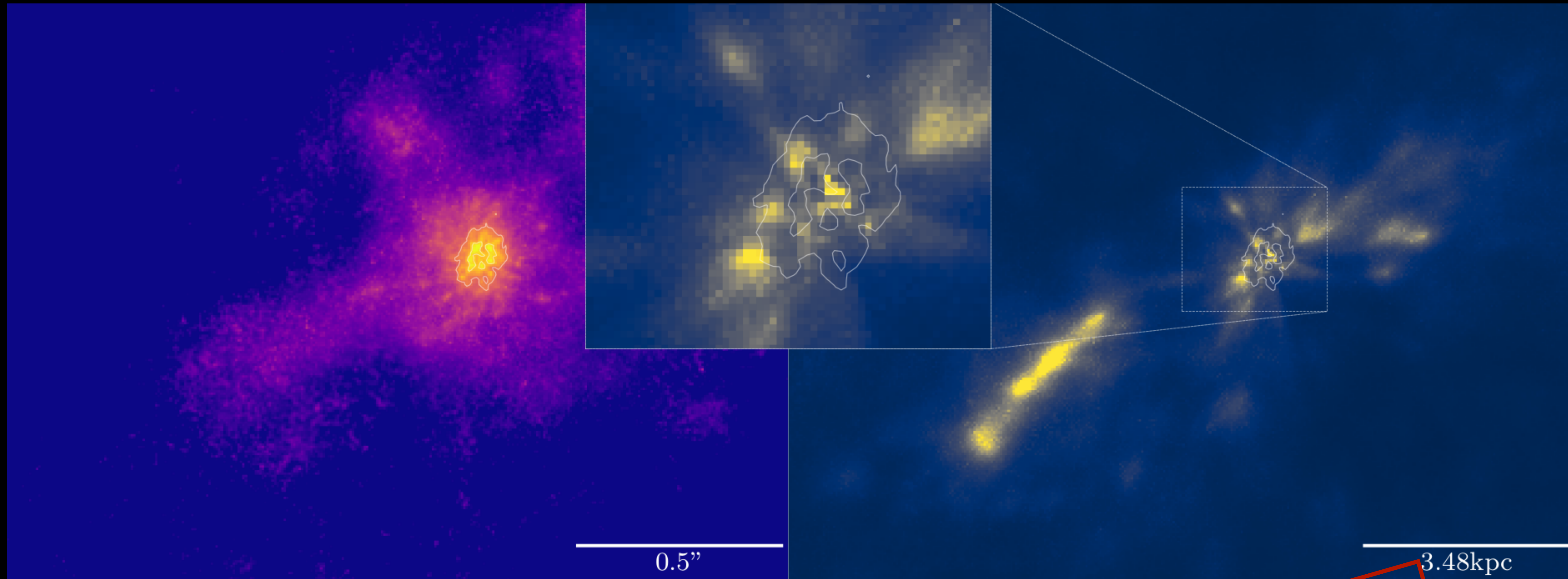
SPT 0418-47 ($z=4.22$)



preliminary

LensCharm 2.0

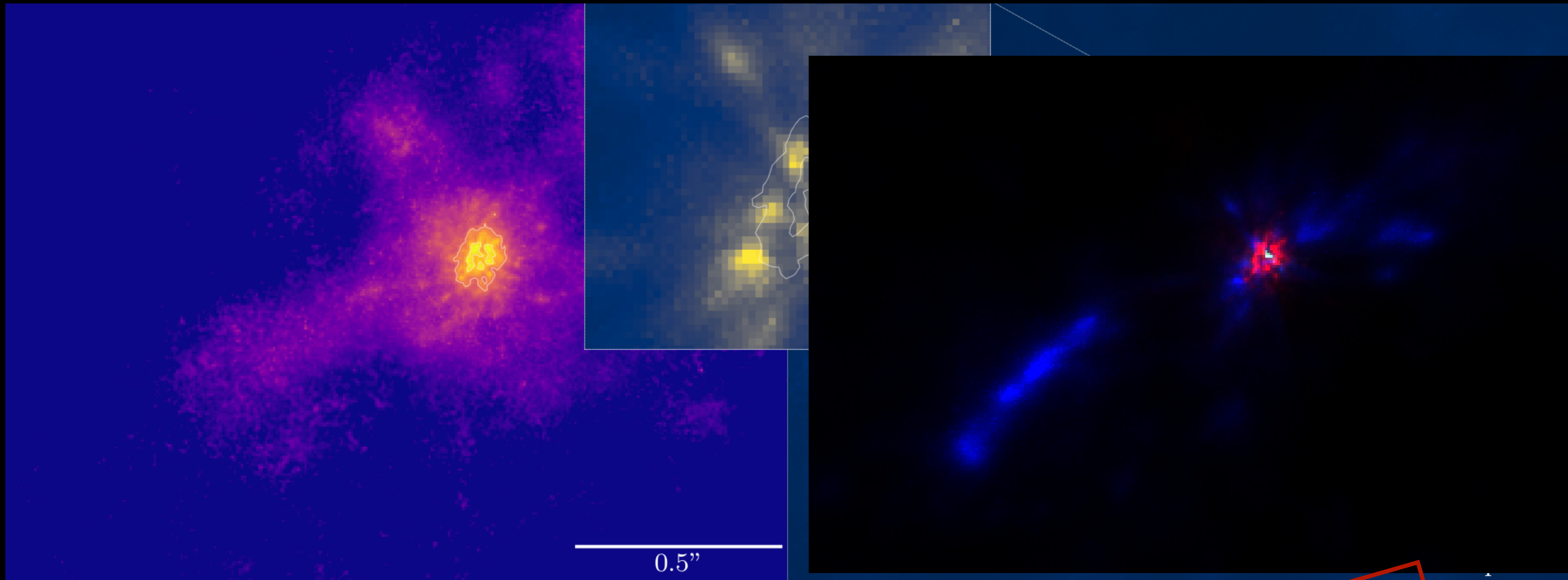
SPT 0418-47 ($z=4.22$)



preliminary

LensCharm 2.0

SPT 0418-47 ($z=4.22$)



preliminary

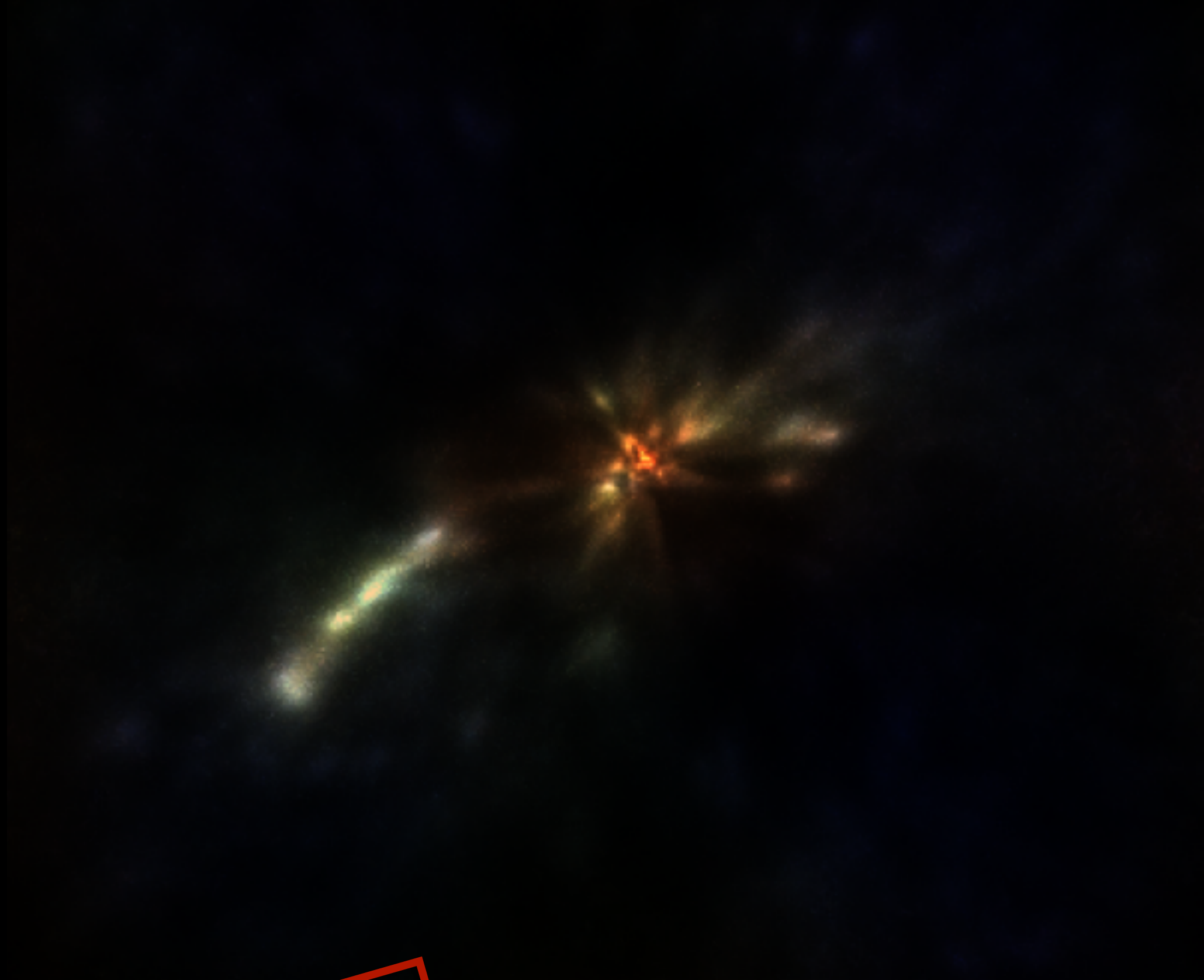
LensCharm 2.0

SPT 0418-47 ($z=4.22$)

preliminary

LensCharm 2.0

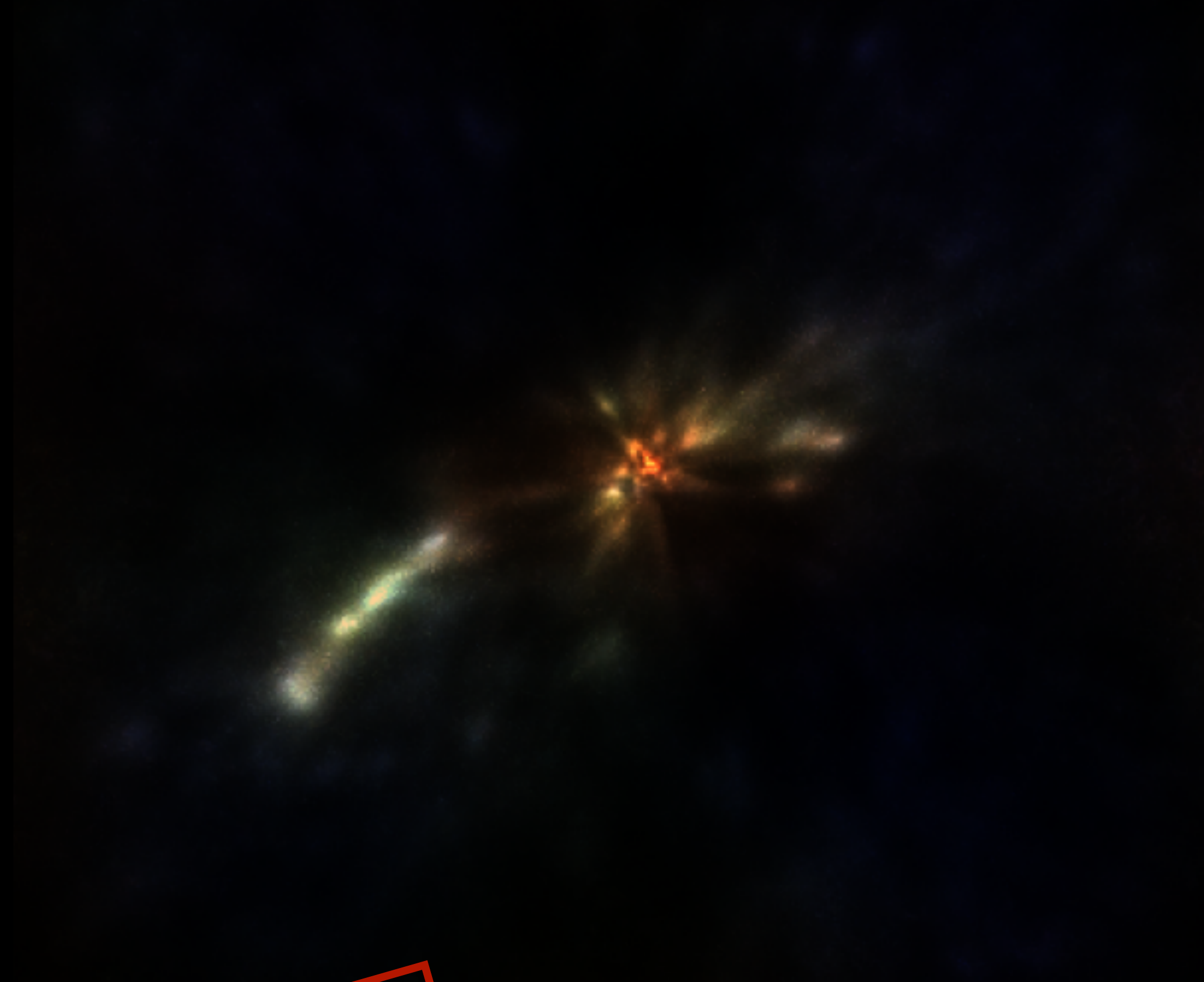
SPT 0418-47 ($z=4.22$)



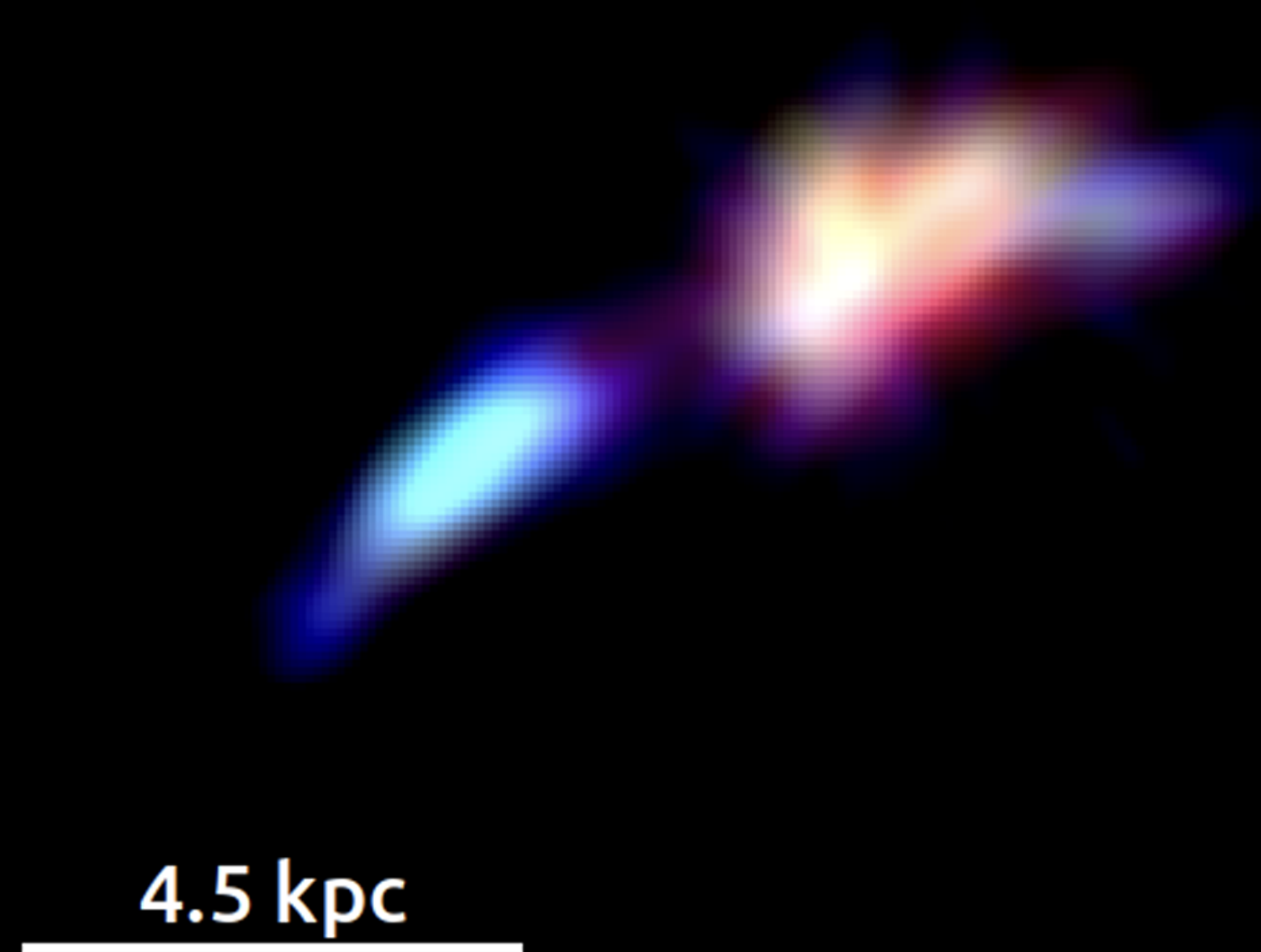
preliminary

LensCharm 2.0

SPT 0418-47 ($z=4.22$)



preliminary



4.5 kpc

Cathey et al.,
10.3847/1538-4357/ad33c9