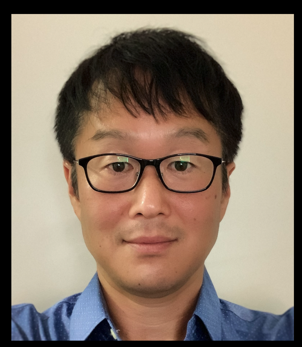

Decoding and Forecasting Non-linear Variability Patterns in X-ray Binaries Using Koopman Operator Theory

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Abstract: The variability observed in stellar-mass BH and NS binaries is highly complex and non-linear, exhibiting QPOs, stochastic and aperiodic components. Short-term variability, such as rapid flares, is commonly attributed to magnetic reconnection, plasmoid ejection and MHD turbulence, while long-term variability reflects changes in accretion states, jets, coronae, and their interactions. Decoding these non-linear variabilities can help us understand the dynamics of accretion disks, coronae, and relativistic jets, and potentially connect X-ray observations to GRMHD simulations. However, conventional Fourier-based methods, while powerful for detecting periodic signals in the frequency domain, have limitations in capturing the underlying non-linear evolution and state transitions. I will introduce a new time-domain framework for accreting compact objects based on Koopman operator theory (KOT) and its modern, data-driven implementation, extended dynamic mode decomposition (EDMD). In this approach, the non-linear variability is embedded in a higher-dimensional linear space, where the Koopman operator decomposes light curve data into independently evolving dynamical modes with well-defined eigenvalues and eigenfunctions. KOT/EDMD is a well-developed yet actively evolving data-driven framework for analyzing and modeling complex nonlinear dynamical systems, with a growing range of real-world applications across science and engineering. I will present promising pilot-study applications of KOT/EDMD to X-ray lightcurve data of several BH and NS binaries, demonstrating its potential to provide new diagnostics for determining the physical processes that drive X-ray variability and to predict future state transitions. By applying these modern time-domain analysis methods to multi-wavelength light-curve data, we may improve the interpretability and predictability of the many tones of accretion.

(1) Motivation

- Variability in accreting objects
  - Short-term: flares from magnetic reconnection and plasmoid ejection. MHD turbulence
  - Long-term: accretion state changes from the dynamics of accretion disks, coronae and relativistic jets

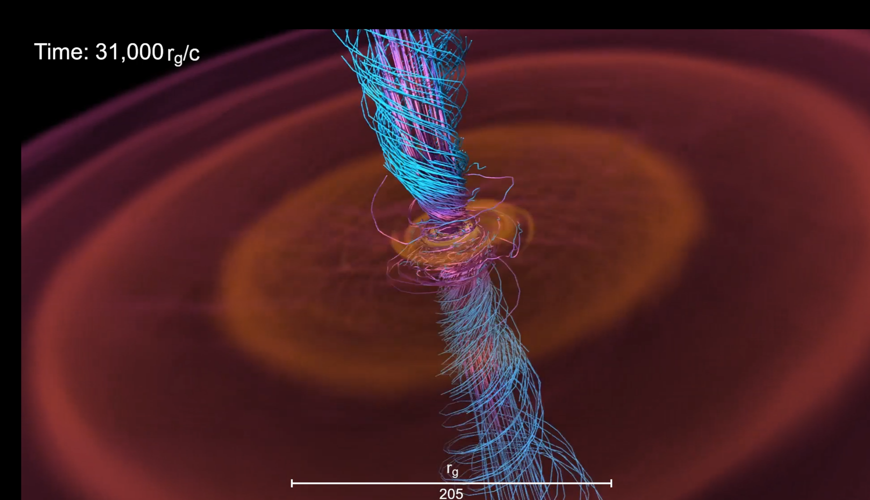

Fig. 1: Snapshots from GRMHD simulations for a BH binary (produced by Lizhong Zhang[1])

- Two approaches to studying variability from accreting objects
  - Frequency-domain methods cannot capture the non-linear dynamics of accreting objects => Prediction is limited to stationary periodic variations
  - Time-domain methods are suited to capturing chaotic and unstable state changes.

Frequency domain:
- Fourier transform of timing data
- e.g., power density spectrum
- Periodicities, QPOs and red noise

Time domain:
- Directly analyze lightcurve data
- e.g., Bayesian block
- Flares and state changes

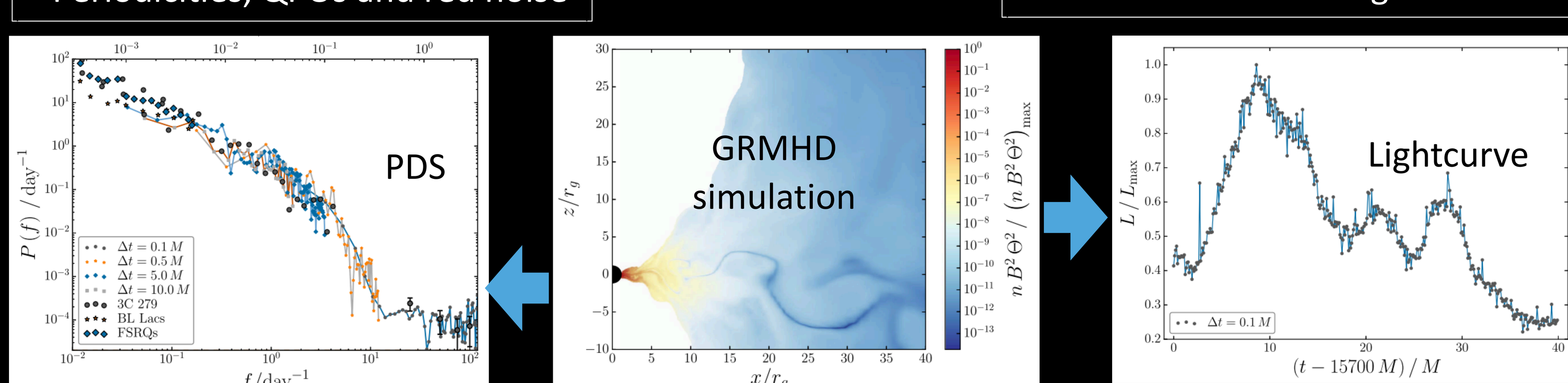

Fig. 2: Model PDS and lightcurve plots produced from GRMHD simulation (Riordan et al. 2017[2])

(2) Koopman and EDMD algorithm

- Koopman (1931): "Hamiltonian Systems and Transformations in Hilbert Space"
  - Transforms non-linear systems into linear ones in infinite-dimensional Hilbert space
- Extended dynamic mode decomposition (EDMD)
  - Modern-era approximation of the Koopman operator as a finite-dimensional matrix
  - Broad and successful applications in engineering and science (Brunton et al. 2022[3])

(1) Stack time-shifted pairs $\{x_t, x_{t+1}\}$ from lightcurve data into time-delay vectors $\vec{x}_t$ and $\vec{x}_{t+1}$.
(2) Lift $\vec{x}_t$ and $\vec{x}_{t+1}$ to higher-dimension observable space by a set of dictionary functions: $\vec{y}_t = \vec{g}(\vec{x}_t)$ and compile $\vec{y}_t, \vec{y}_{t+1}$ into data matrices $\mathbf{Y}$ and $\mathbf{Y}'$ respectively.
(3) Derive Koopman operator $\mathcal{K}$ from training data $\vec{y}_t$ by minimizing $\|\mathbf{Y}' - \mathbf{Y}\mathcal{K}\|$
(4) Decompose $\mathcal{K}$ into a linear combination of eigenvalues, eigenfunctions, and spatial modes.
(5) Evolve system $\vec{y}_{t+n} = \vec{y}_t\mathcal{K}^n$ to forecast n steps into the future
(6) Convert back to the original space and forecast lightcurve: $\vec{x}_{t+n} = \vec{g}^{-1}(\vec{y}_{t+n})$

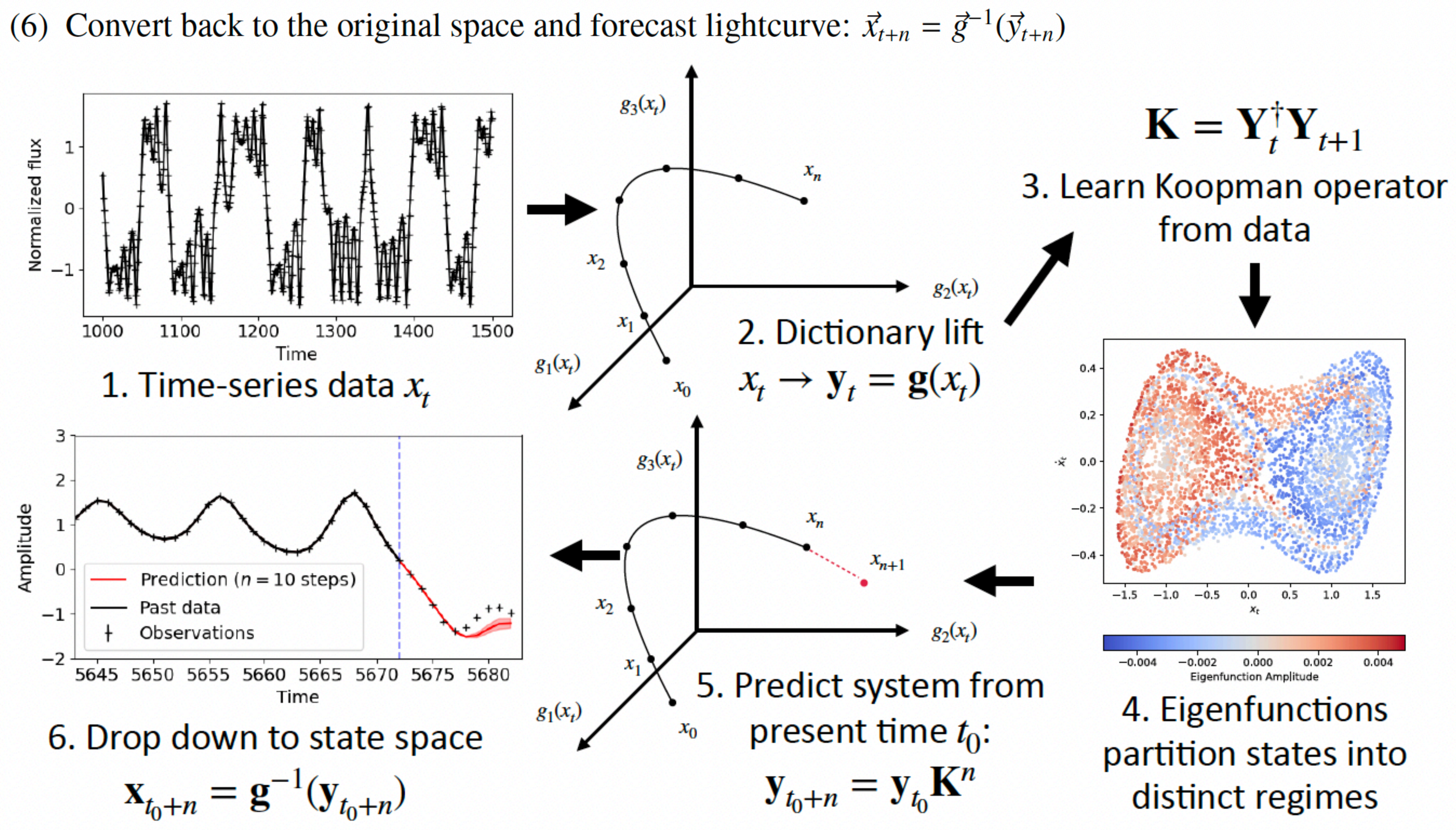

Fig. 3: Flowchart of the EDMD procedures and products

(3) Simulation

- Forced Duffing oscillator is a well-known chaotic system.

$$\ddot{x} = -\delta\dot{x} - \alpha x - \beta x^3 + \gamma\cos\Omega t$$

- 4U 1705-44 exhibits dynamics similar to Duffing oscillator (Phillipson et al. 2018[4])
- Apply EDMD:
  - Flux prediction: Lift, evolve, drop.
  - Eigenfunction predictive partitioning: Lift, decompose, identify "slow" eigenvalue, plot eigenfunction.
- Enables *prediction* and *interpretation*

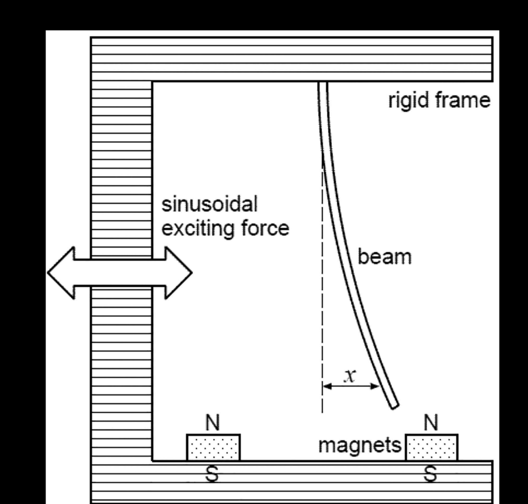

Fig 4. Schematic view of a forced duffing oscillator system

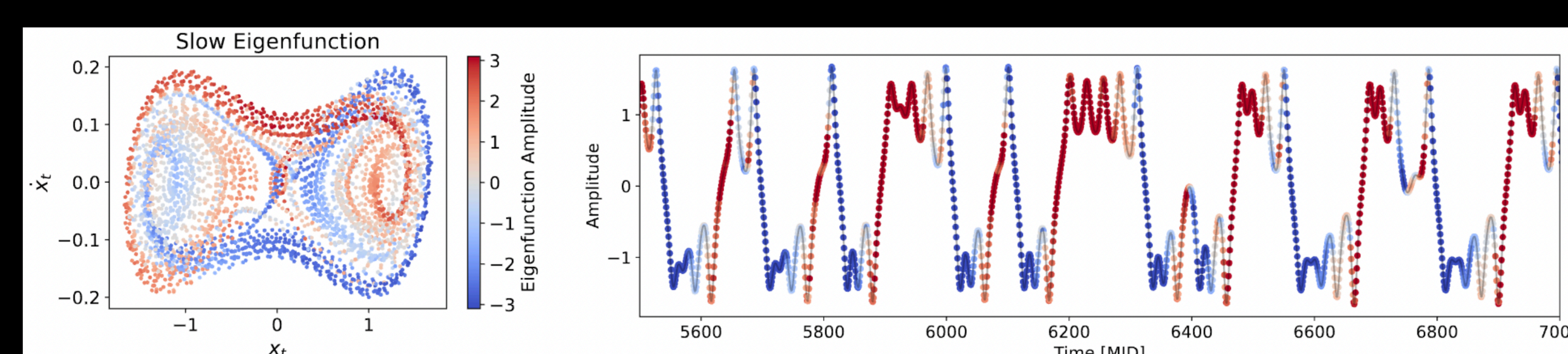

Fig 5. "Slow" eigenfunction value, partitioning states predictively

(4) Preliminary results

- We obtained X-ray and gamma-ray lightcurve data from RXTE, MAXI and Fermi-LAT.
  - 4U 1705-44 (NS-LMXB), Cyg X-3 (microquasar) and blazars
- 4U 1705-44 as the best case for EDMD decomposition and predictability
  - Accumulated ~30 years for RXTE and MAXI lightcurve data
- "Slow" eigenfunction *predictively partitions* the two stable basins

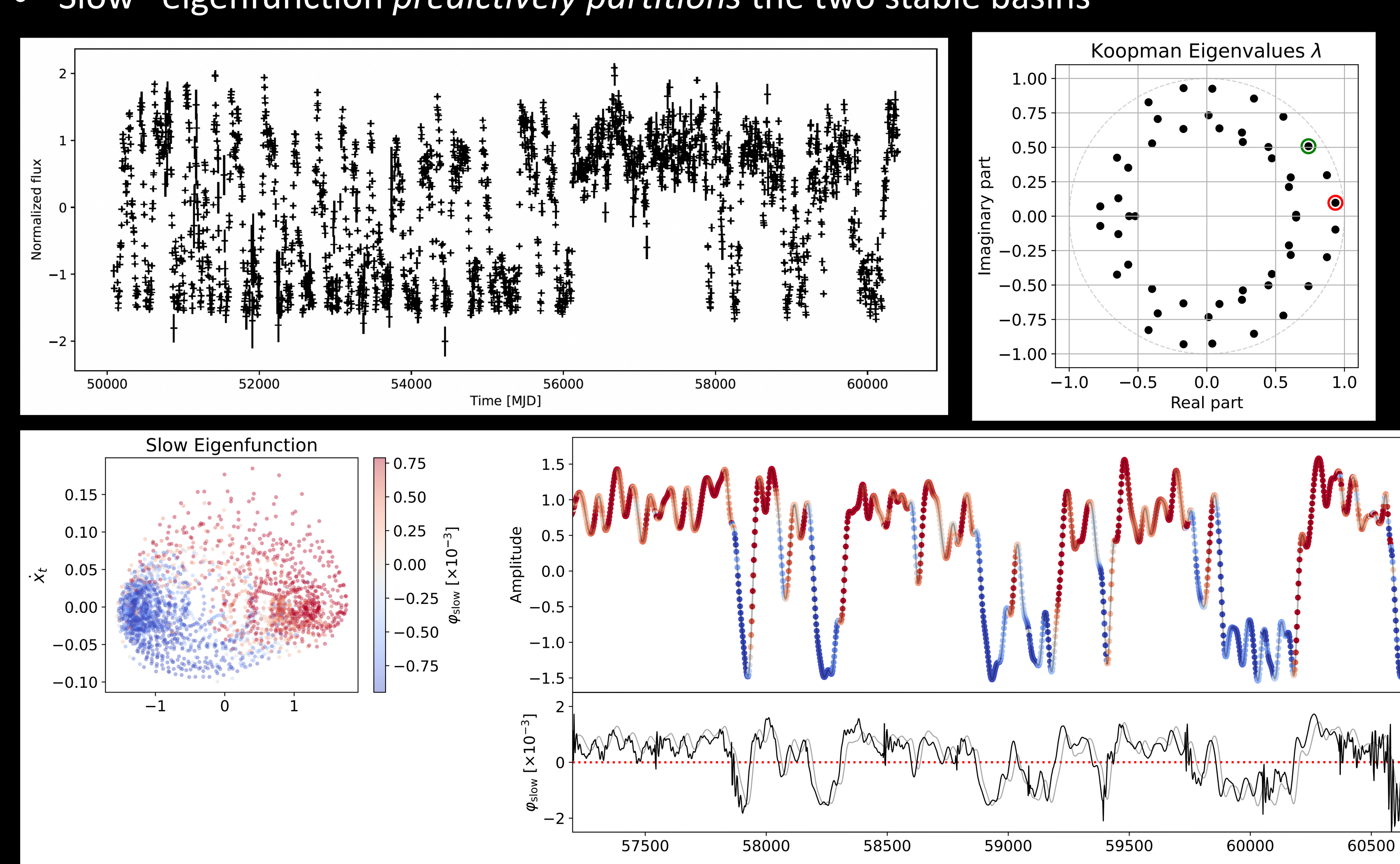

Fig 6. 4U 1705-44 X-ray lightcurves over 30 years (top) and "slow" eigenfunction value (bottom)

(5) Summary and future work

- Our pilot study is promising: the interpretability and predictability of accretion state changes may be feasible with the KOP/EDMD methods (Miao et al. 2026[5])
- Future work
  - Collect continuous and high-statistics lightcurves in various wavelength bands
  - Implement more noise-robust and physically motivated dictionary choices.
  - Physically interpret the eigenstates (which physical processes are associated with different eigenmodes).
  - Apply optimized KOT/EDMD methods to GRMHD simulation products

References: [1] lizhongzhang.com, [2] Riordan et al. (ApJ 834, 81, 2017), [3] Brunton et al. (SIAM Review, 64, 229, 2022), [4] Phillipson et al. (MNRAS, 477, 5220, 2018), [5] Miao et al. (ApJ in press, arXiv: 2609.01734)

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