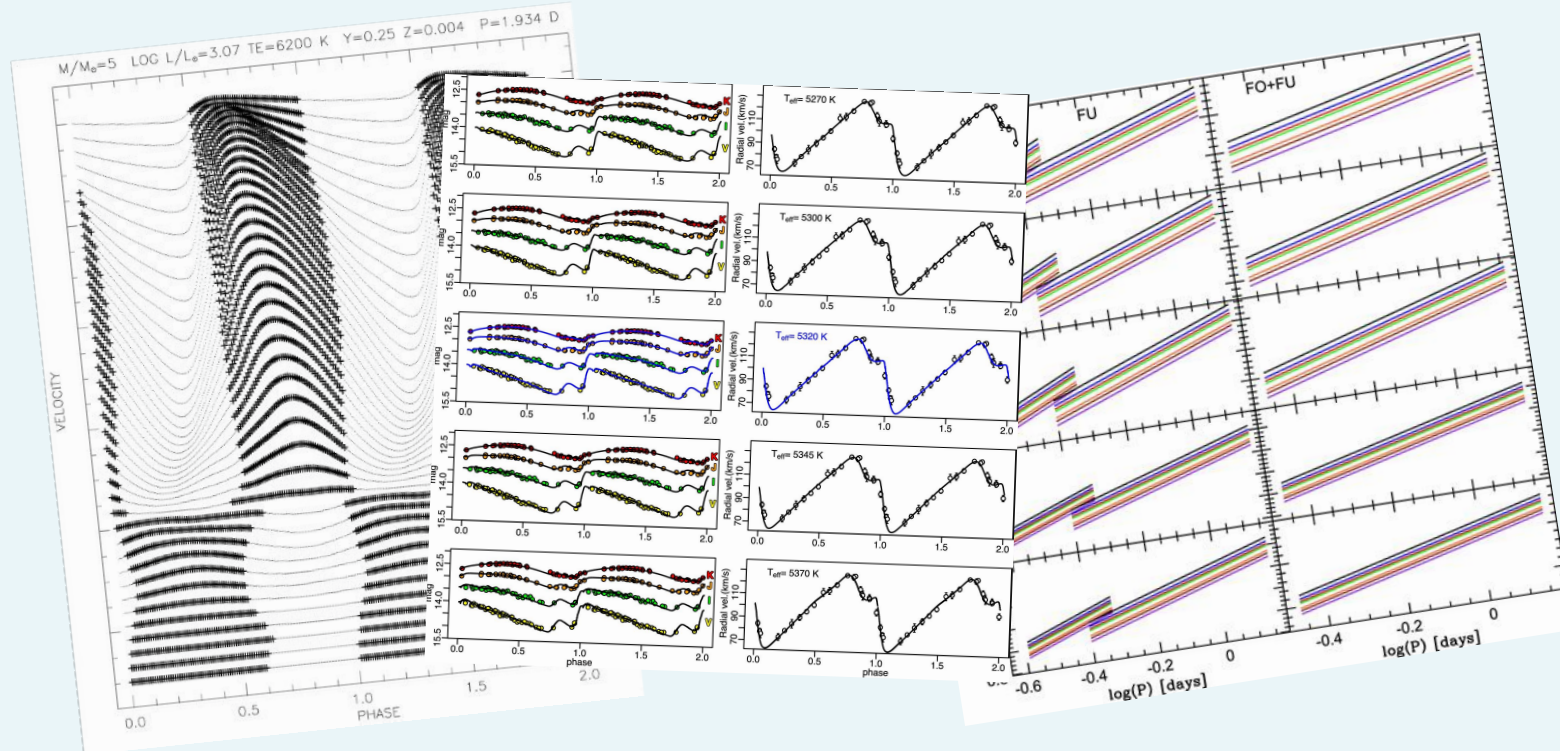




Hands-on session: Pulsation models



Giulia De Somma, Marcella Marconi
INAF-Osservatorio Astronomico di Capodimonte

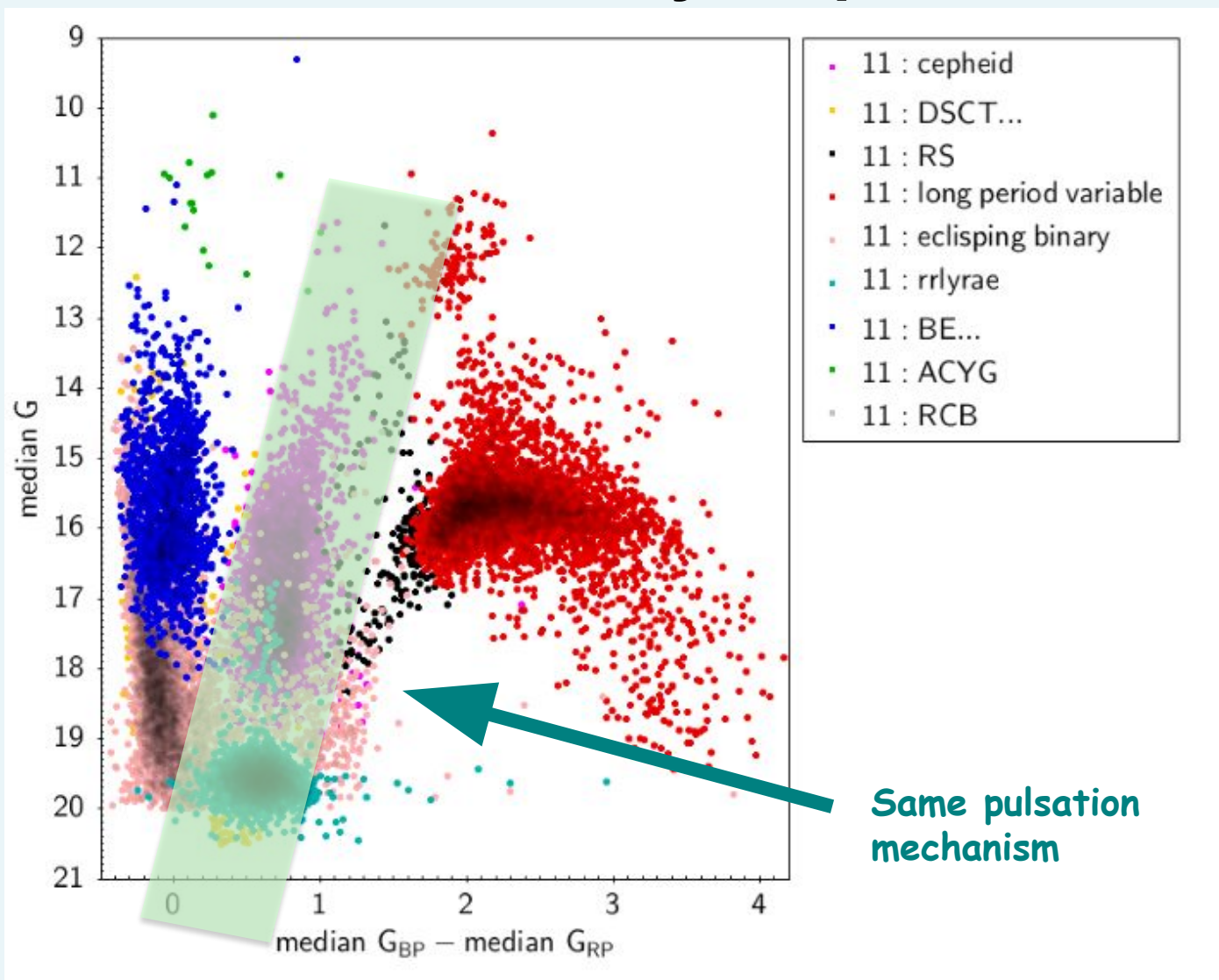


Outline

- Radial pulsation models: physics and state of the art
- Recent results for RR Lyrae and Cepheids
- How to apply model results



Radially pulsating stars: the instability strip





Driving mechanisms

In the classical instability pulsation mechanisms expected to be connected with the position in the HR diagram, i.e. related to L , T_e (R)



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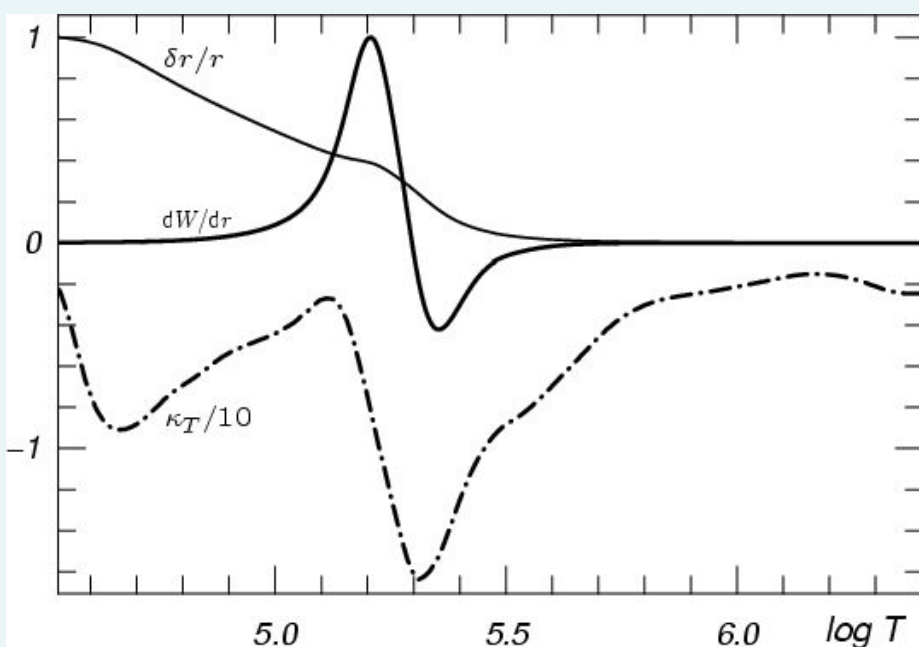
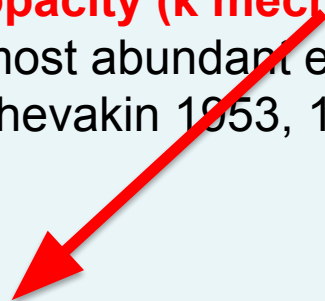
Valve effect: variation of **opacity (κ mechanism)** and $\Gamma_3 - 1$ (**γ mechanism**) in the ionization regions of the most abundant elements of stellar envelopes: H, He and He^+ (Eddington 1918, 1926, Zhevakin 1953, 1954)



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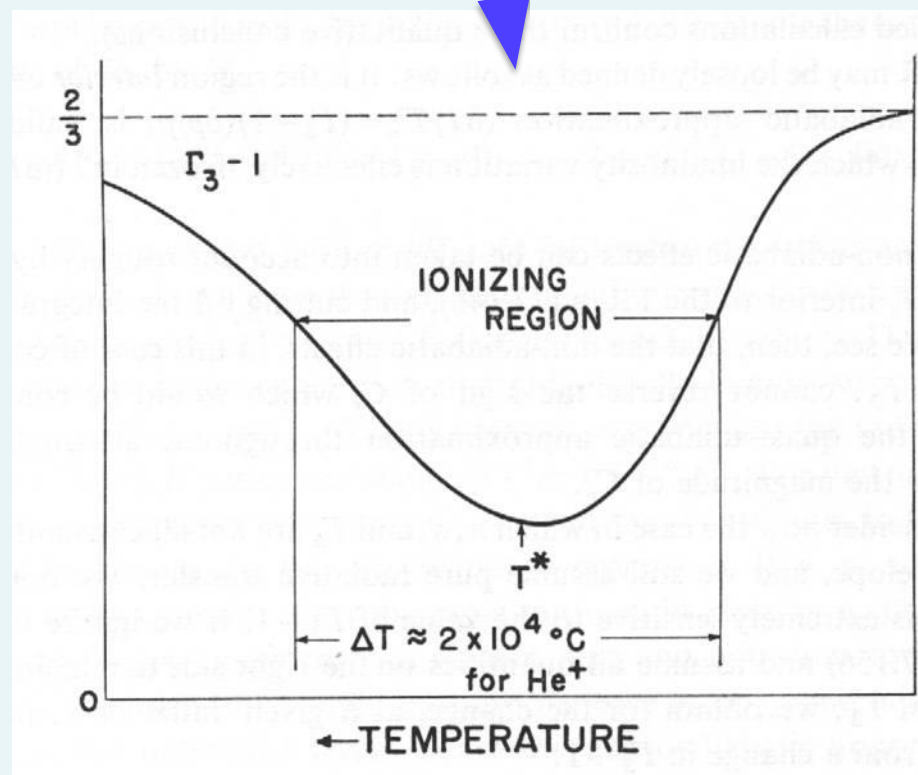
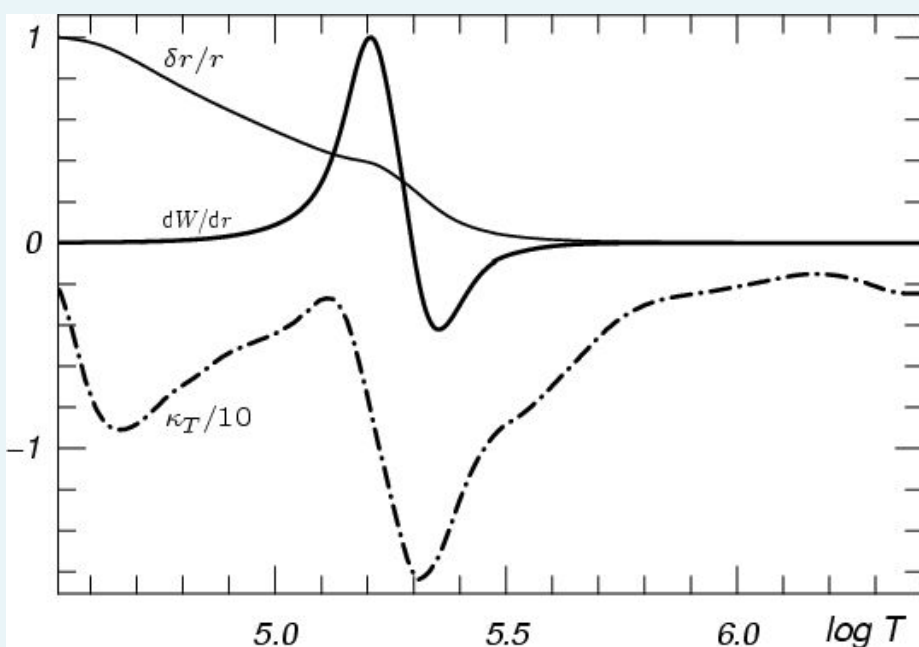




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The pulsation mechanisms

k mechanism

Opacity variations in the H, HeI, HeII ionization regions:

$$(k\rho=1/\lambda)$$

$$k \propto \rho^n T^{-s}$$

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k mechanism

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stellar interior $\Rightarrow$ positive **n** and **s**: opacity decreases during contraction of the stellar envelope producing heat loss

ionization regions $\Rightarrow$ **s** becomes large and negative: small temperature variations cause an increase of κ during contraction

$\Rightarrow$ radiation trapping $\Rightarrow$ energy excess $\Rightarrow$ **pulsation work**

The pulsation mechanisms

γ mechanism

The adiabatic exponent $\Gamma_3 - 1 = (d \log T / d \log \rho)$ decreases in the ionization regions.

Most of the released energy during the phases of contraction goes into ionization:

The pulsation mechanisms

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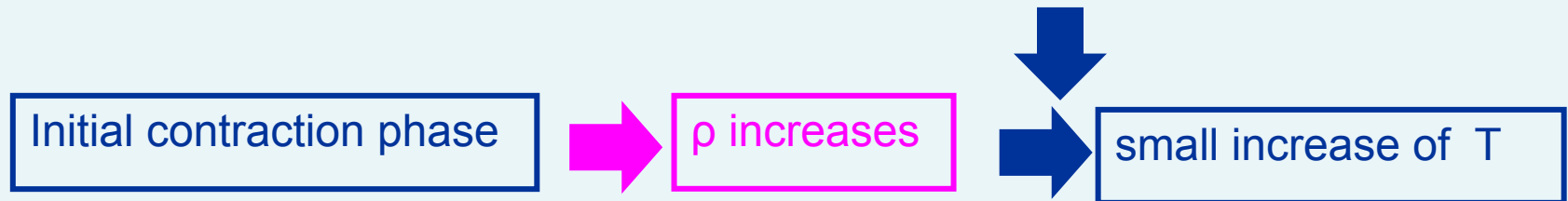


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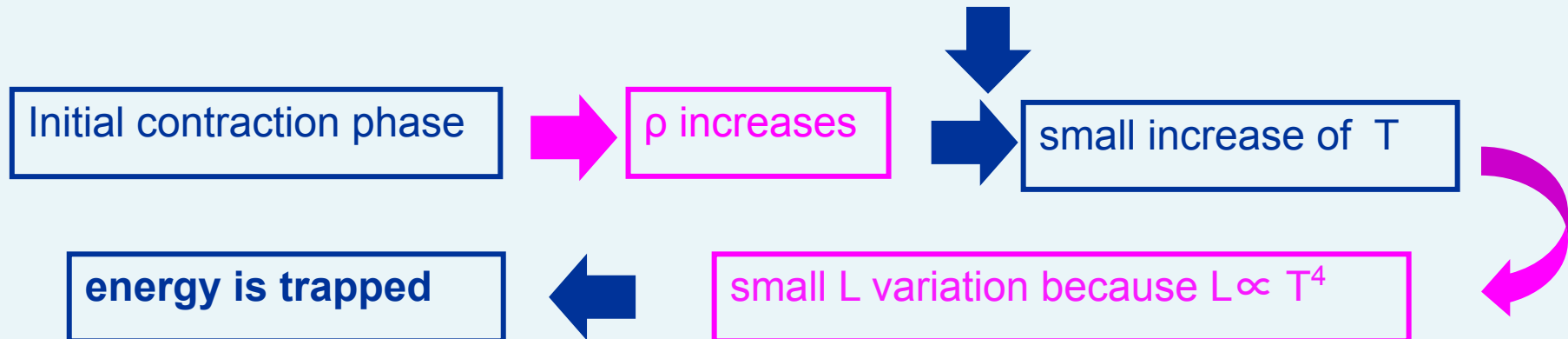


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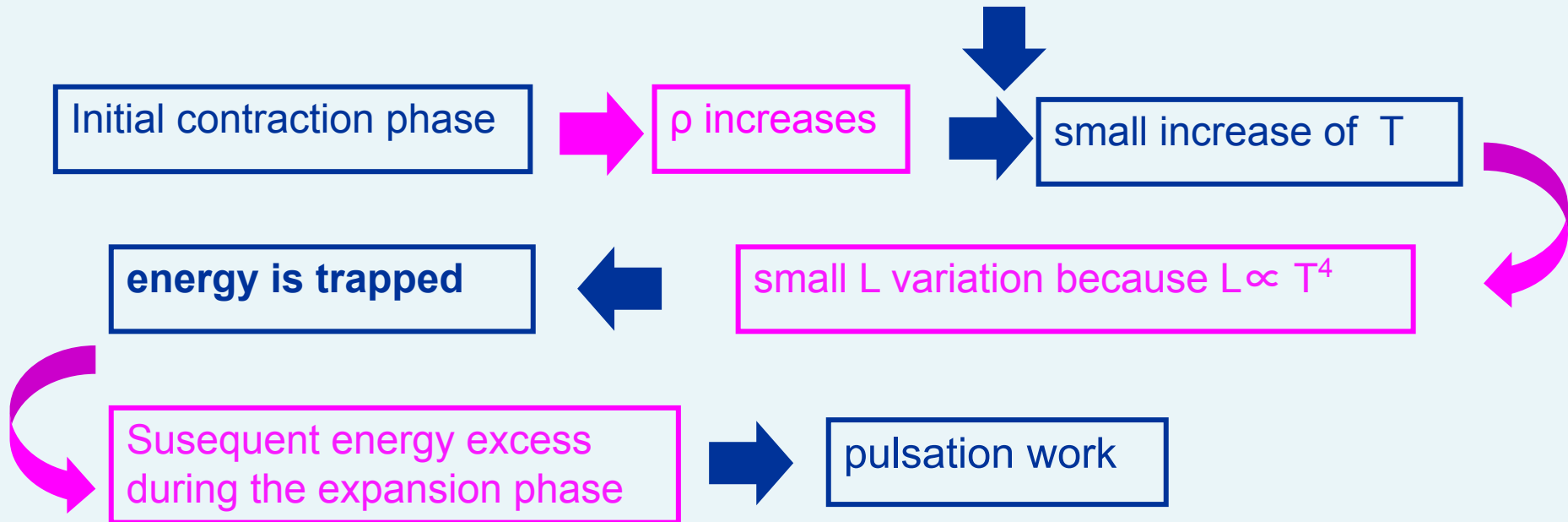


The pulsation mechanisms

γ mechanism

The adiabatic exponent $\Gamma_3 - 1 = (d \log T / d \log p)$ decreases in the ionization regions.

Most of the released energy during the phases of contraction goes into ionization:



The pulsation mechanisms

γ mechanism : effect of T variations on L variations

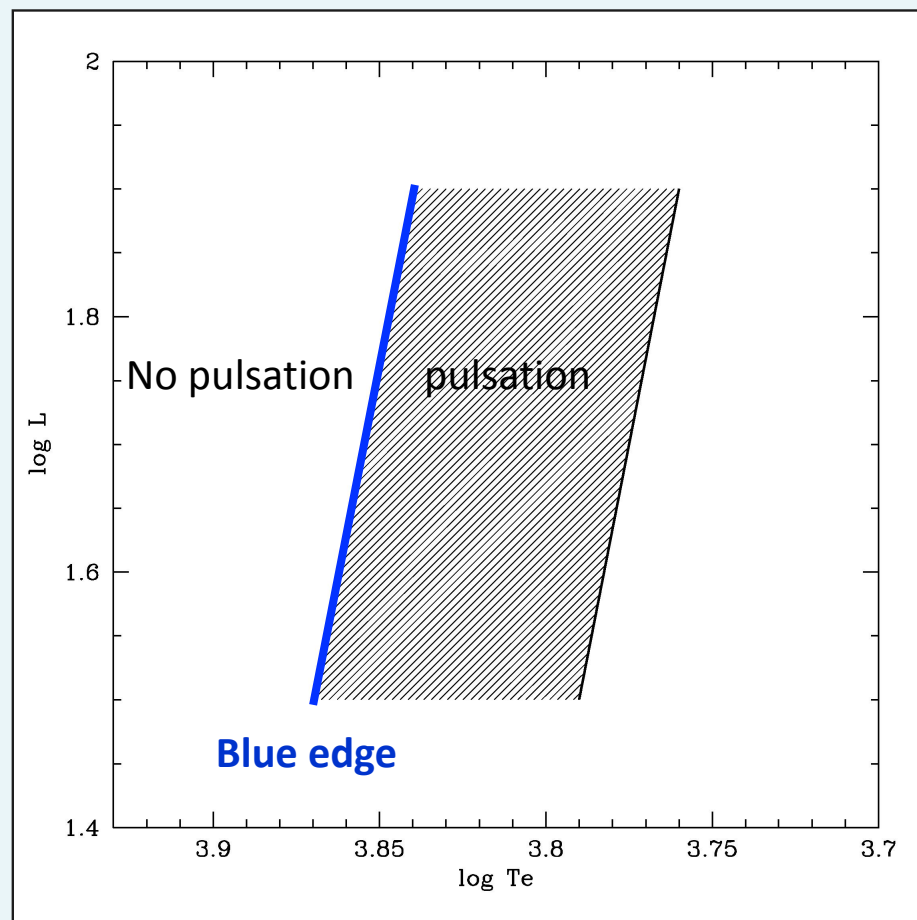
k mechanism: effect of k variations on L variations

Both the k and γ mechanisms are efficient in driving the pulsation but the phenomenon is started by a *stochastic fluctuation* of the external layer properties (e.g. contraction)



The physical basis of the instability strip: why the **blue** edge?

Only when the ionization regions are deep enough the mass involved in the pulsation driving mechanisms prevails $\Rightarrow$ Pulsation $\Rightarrow$ Blue Edge





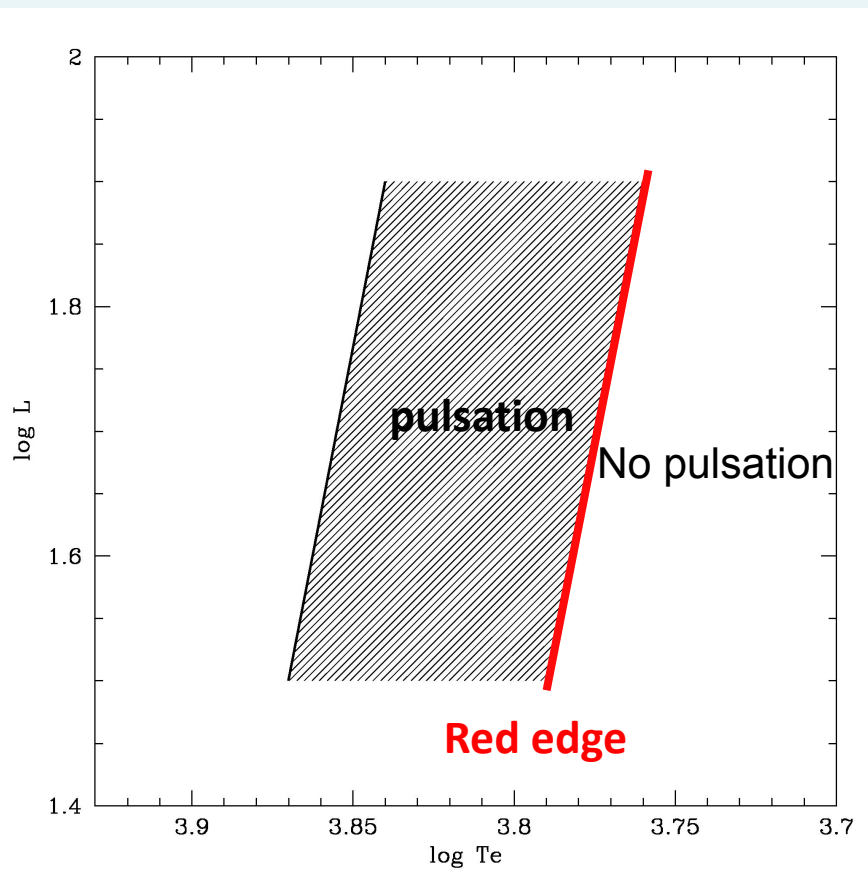
The physical basis of the instability strip: why the **red** edge?

Convection becomes more efficient
at lower effective temperatures

⇒ k and γ gradients are reduced

⇒ **quenching of pulsation**

(Baker & Kippenhahn 1965)





The physical basis of the instability strip: why the **red** edge?

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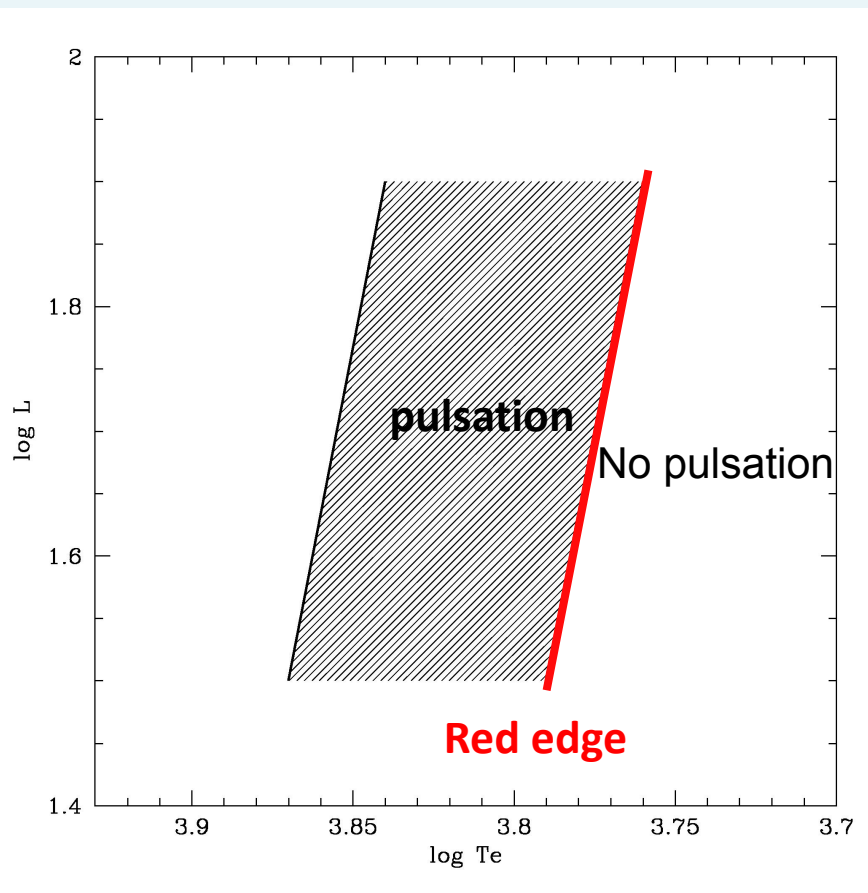
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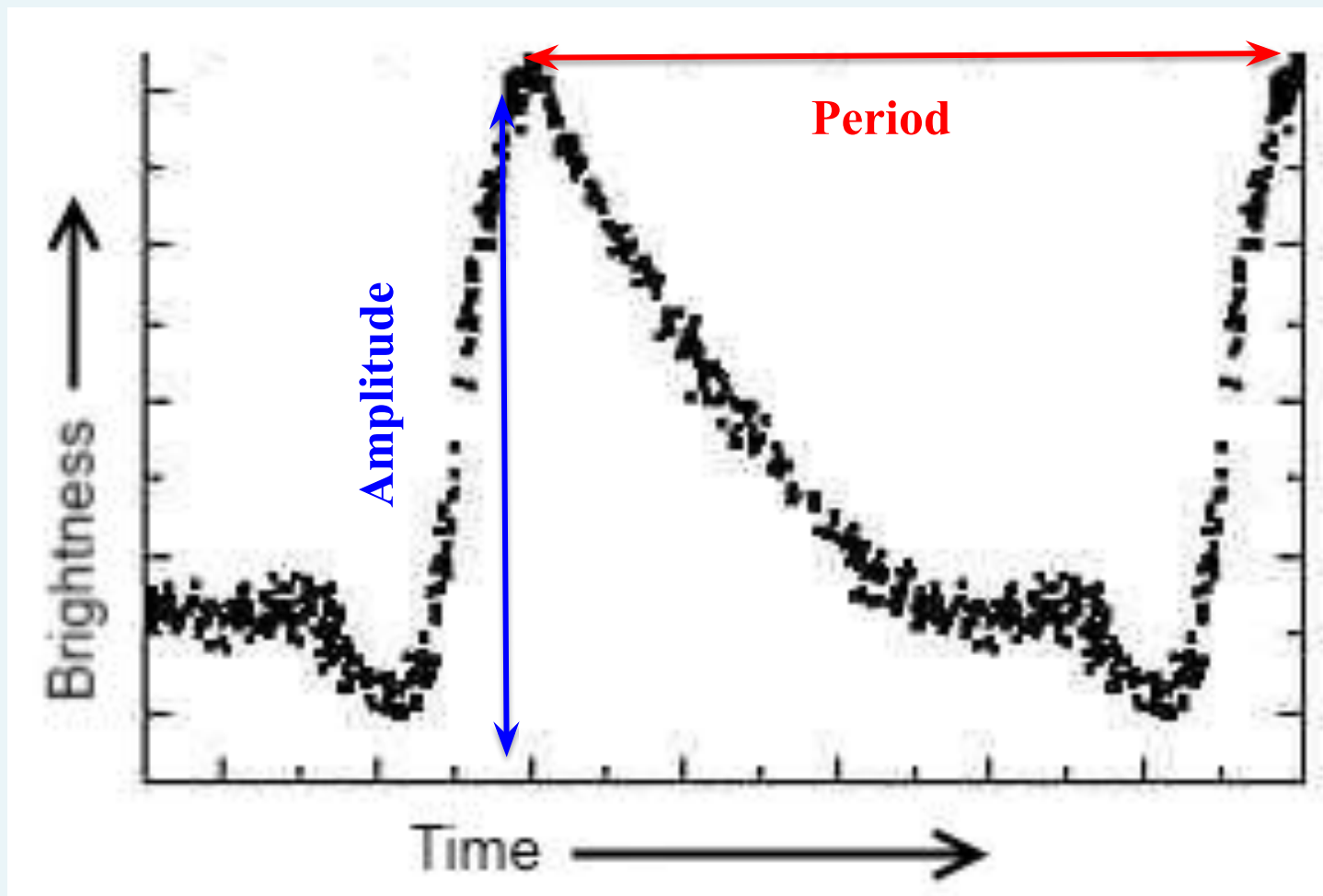
When the quenching effect due to
convection prevails, pulsation is no
more efficient

⇒ **Red edge** of the instability strip





Radially pulsating stars: the variation properties



The period and the amplitude of the oscillation are very solid quantities: they are independent of distance and reddening.



Radially pulsating stars: the period-density relation

In particular, the period of the oscillation can be connected to the intrinsic properties of the pulsating star.

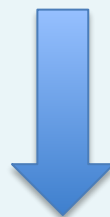
$P\sqrt{\rho} = \text{constant} + \text{Stefan-Boltzmann} \rightarrow$ **Period** is a function of **mass, luminosity, effective temperature (and chemical composition)**



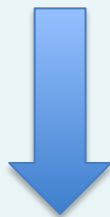
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PLCM relation



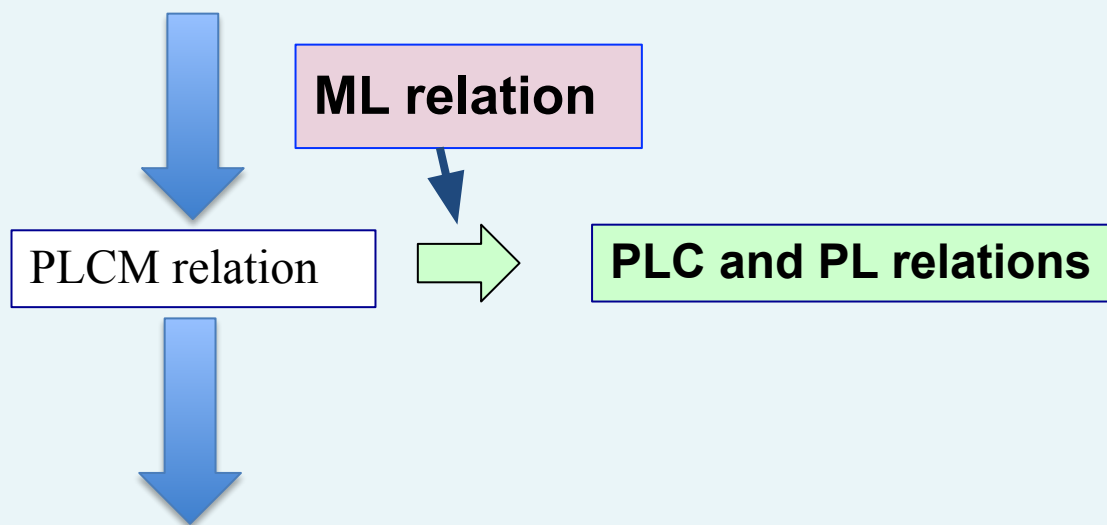
This makes pulsating stars standard candles and stellar population tracers



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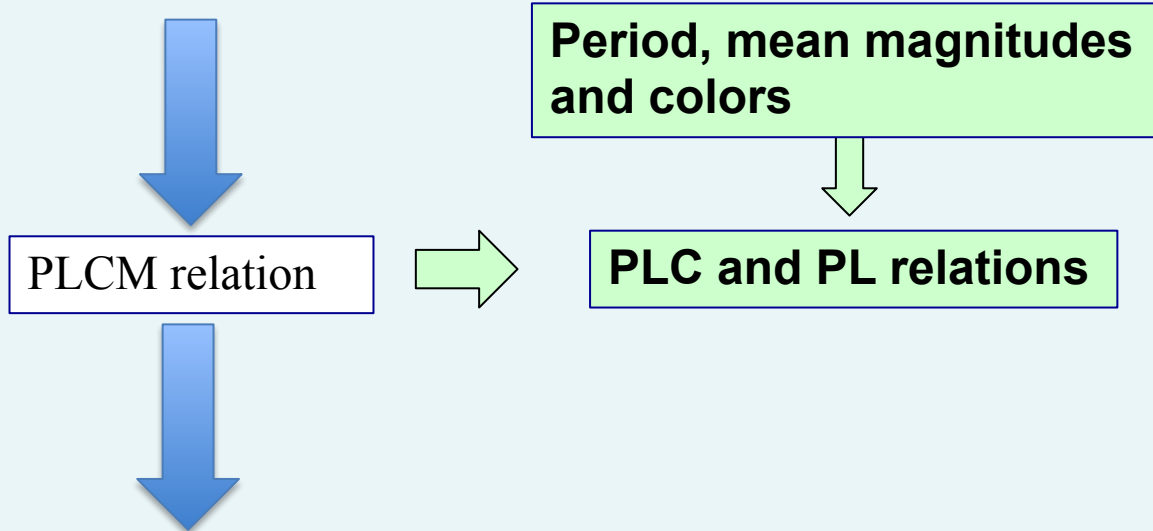
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The theory of stellar pulsation

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi\rho r^2}$$

Mass conservation equation

$$\frac{\partial^2 r}{\partial t^2} = -4\pi r^2 \frac{\partial P}{\partial m} - \frac{GM}{r^2}$$

Momentum equation

$$\frac{\partial E}{\partial t} - \frac{P}{\rho^2} \frac{\partial \rho}{\partial t} = \varepsilon - \frac{\partial H}{\partial m}$$

Energy equation

$$H = 4\pi r^2 F$$

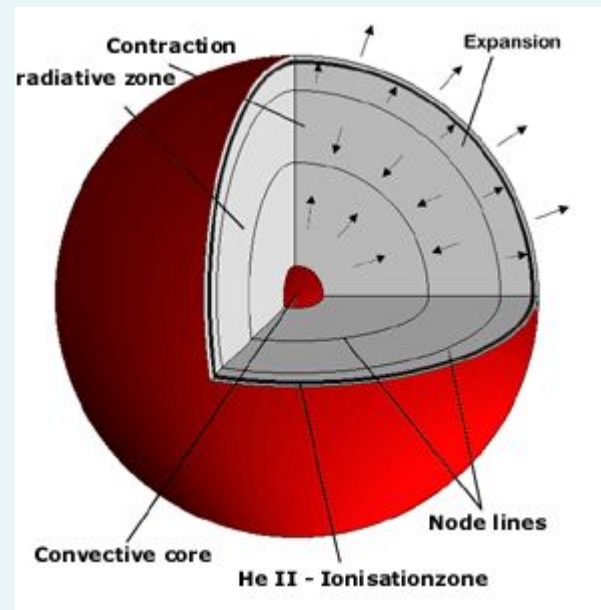
+ heat transfer equation

The lagrangian description is usually preferred ← better physical interpretation of equations



Radially pulsating envelope

- **Spherical symmetry:** the star varies its volume and luminosity on the pulsation time scale but the shape remains spherical.
- **No rotation, no magnetic fields.**



- The core is excluded (nuclear reactions evolve on much longer time scales, pulsation mechanisms excited in the envelopes)
- The envelope is divided in ~ 150 -250 mass zones.



State of the art of modeling:

Linear non adiabatic convective 1D models

Nonlinear convective 1 D models

2D/3D simulations



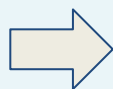
Linear non adiabatic pulsation models

Linear non adiabatic hydrodynamical models means that non adiabatic effects related to the driving mechanisms are taken into account but the hydrodynamic equations describing the pulsation phenomenon are linearized.



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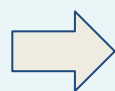


Periods

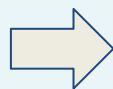


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Periods



Instability strip



Amplitudes



Nonlinear convective pulsation models

Nonlinear convective hydrodynamical 1D models means that the equations describing the pulsation phenomenon are NOT linearized and that not only the periods and the instability strip edges can be estimated but also the pulsation amplitudes (full amplitude variation of all the relevant quantities along the pulsation cycle)

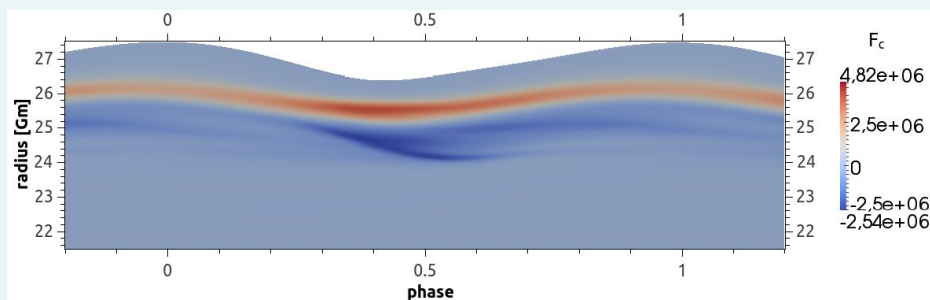
⇒ **Periods, amplitudes, lightcurves, blue and red edges**

Several authors developed nonlinear convective pulsation models of Cepheids and RR Lyrae (e.g. *Gehmeyr et al. 1990, Bono & Stellingwerf 1994, Yecko et al. 1998; Kolláth et al. 2002; Bono, Marconi Stellingwerf 1999, Szabo et al. 2000, 2004, Smolec & Moskalik 2008, Paxton et al. 2019*)

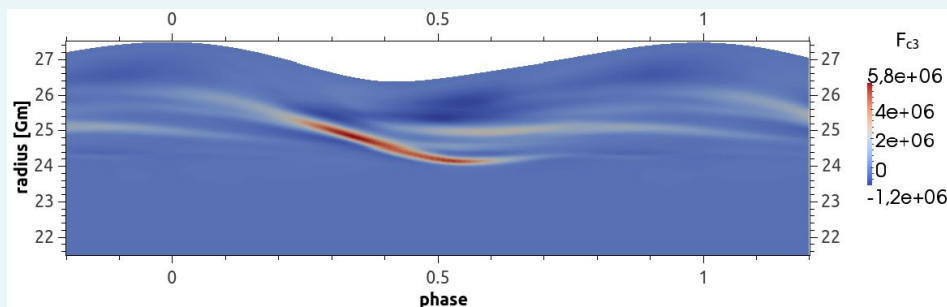


2D and 3D hydrodynamical simulations

Multidimensional hydrodynamical simulations means that the 1D assumption is released and that the pulsation convection coupling is treated in 2D or 3D (e.g. *Mundprecht et al. 2013, 2015 MNRAS* → 2D approach to pulsation, But see also Geroux & Deupree 2015, Deupree 2021)



More realistic simulations of the convection–pulsation interaction in Cepheids



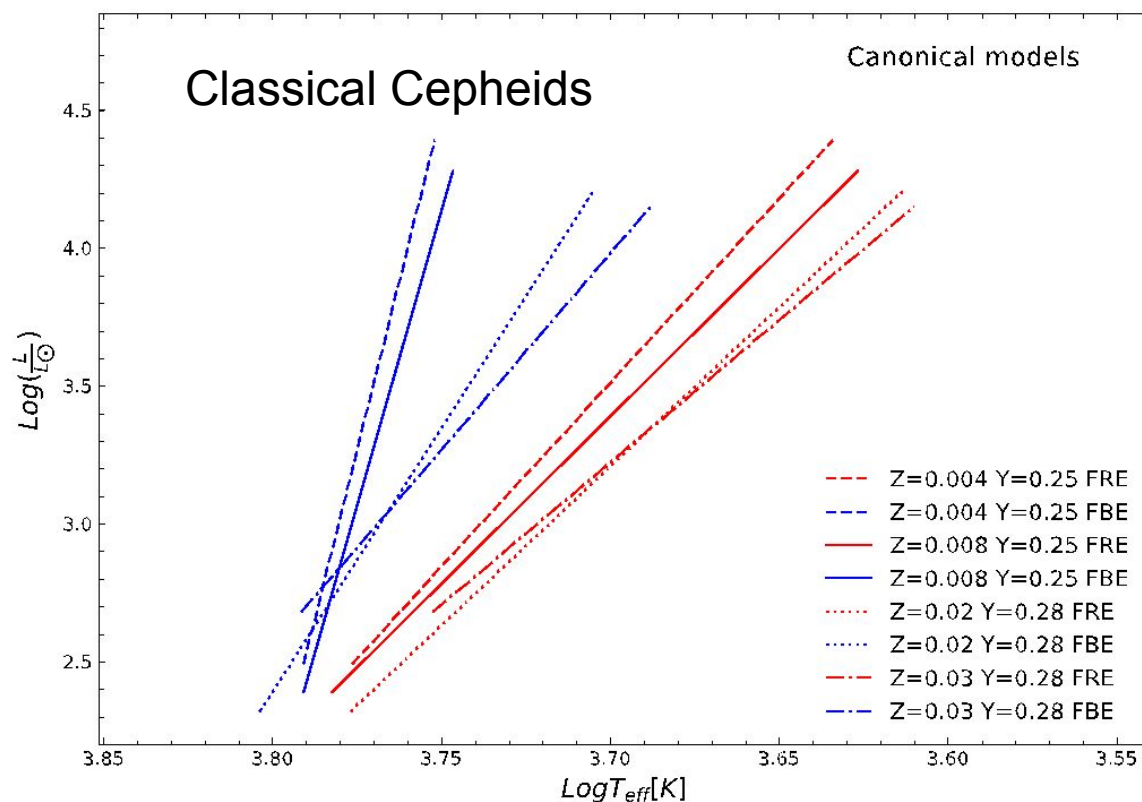
→ guidelines for developing descriptions of convection to be applied in 1D modelling.



1D nonlinear convective pulsation models



Typical predictions of 1D nonlinear convective pulsation models: the instability strip



De Somma et al. 2022 ApJS



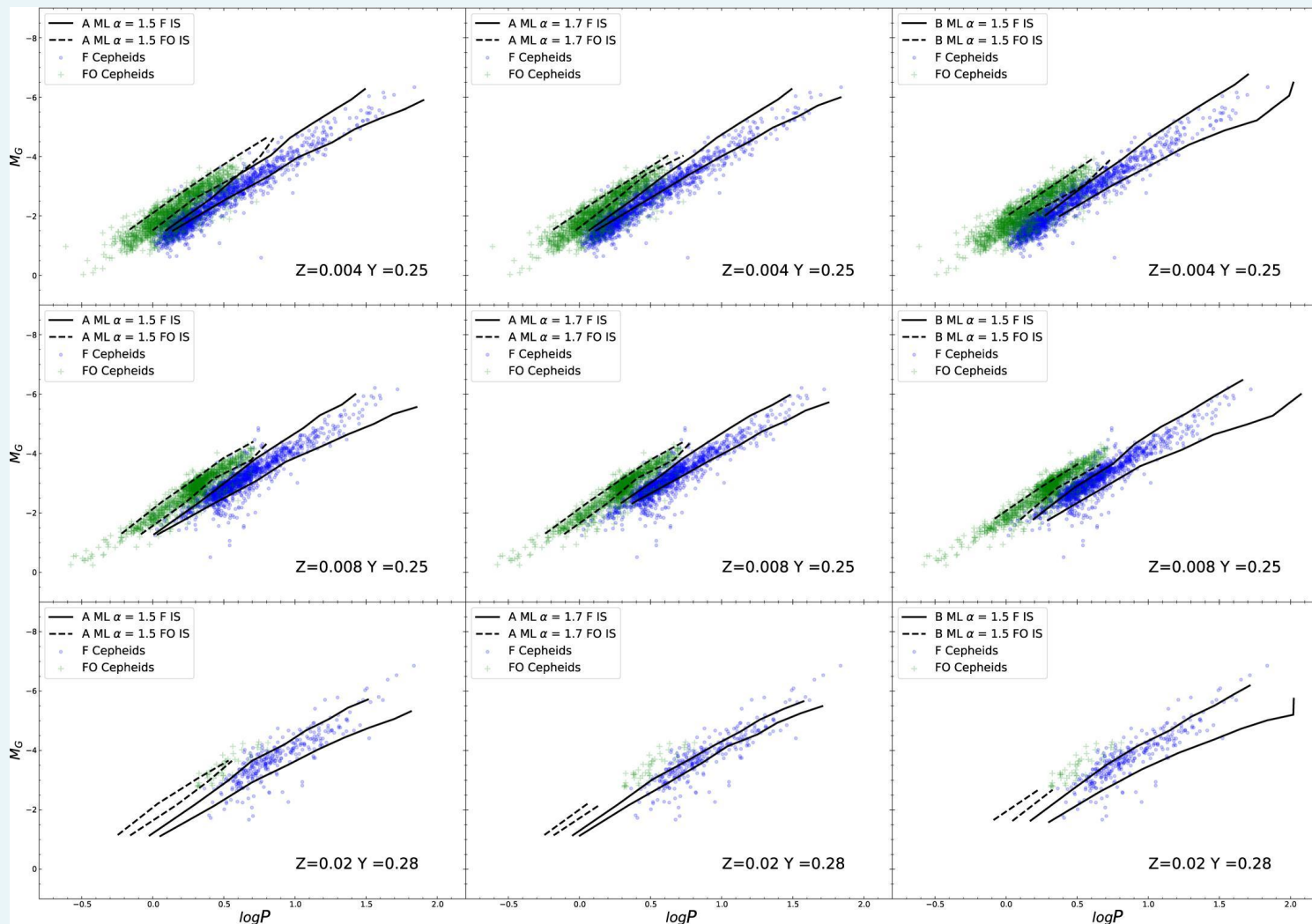
Comparisons with the observations

Canonical $I/H_p = 1.5$

Canonical $I/H_p = 1.7$

Non Canonical $I/H_p = 1.5$

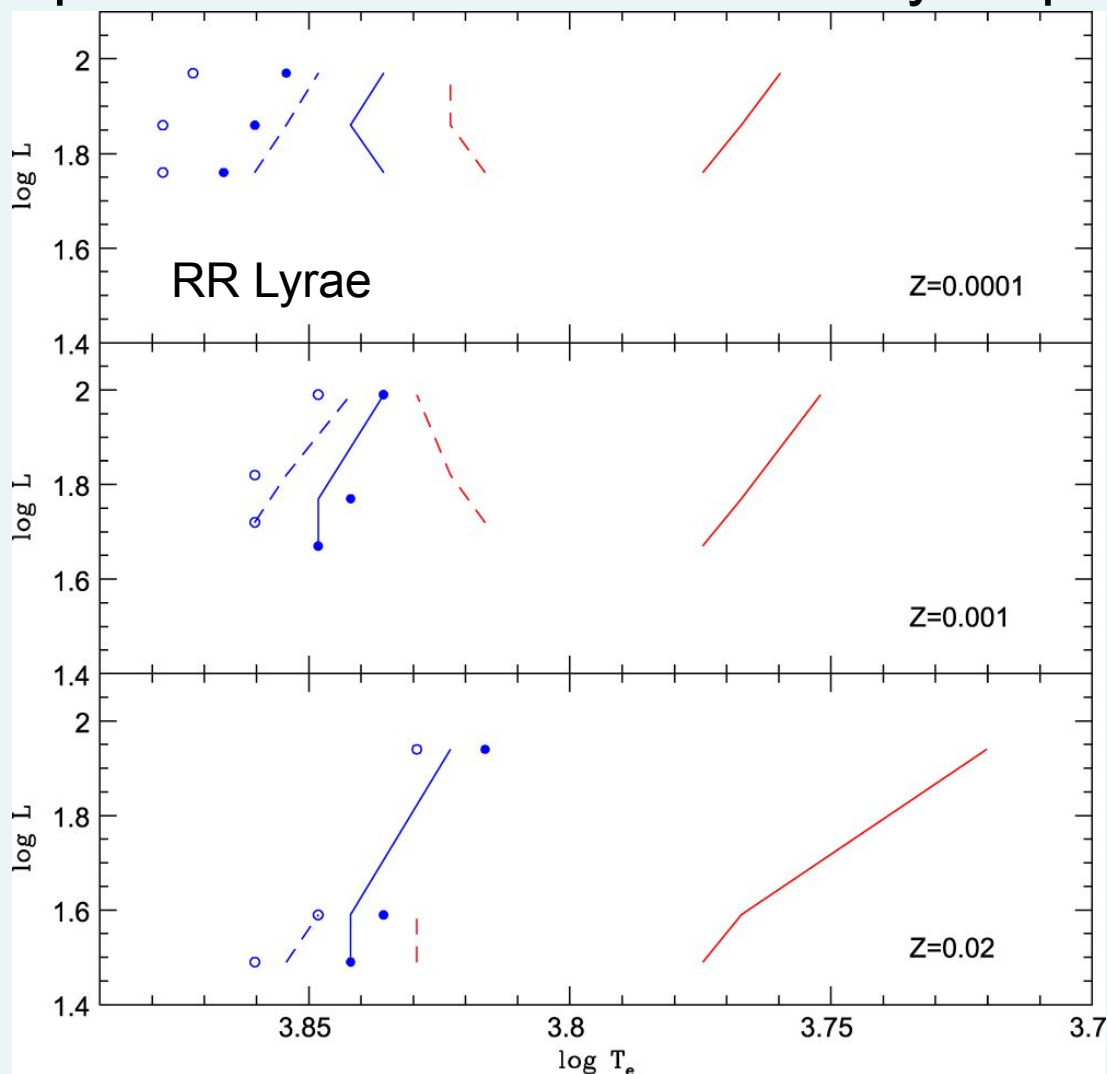
Z



De Somma et al. 2022 ApJS



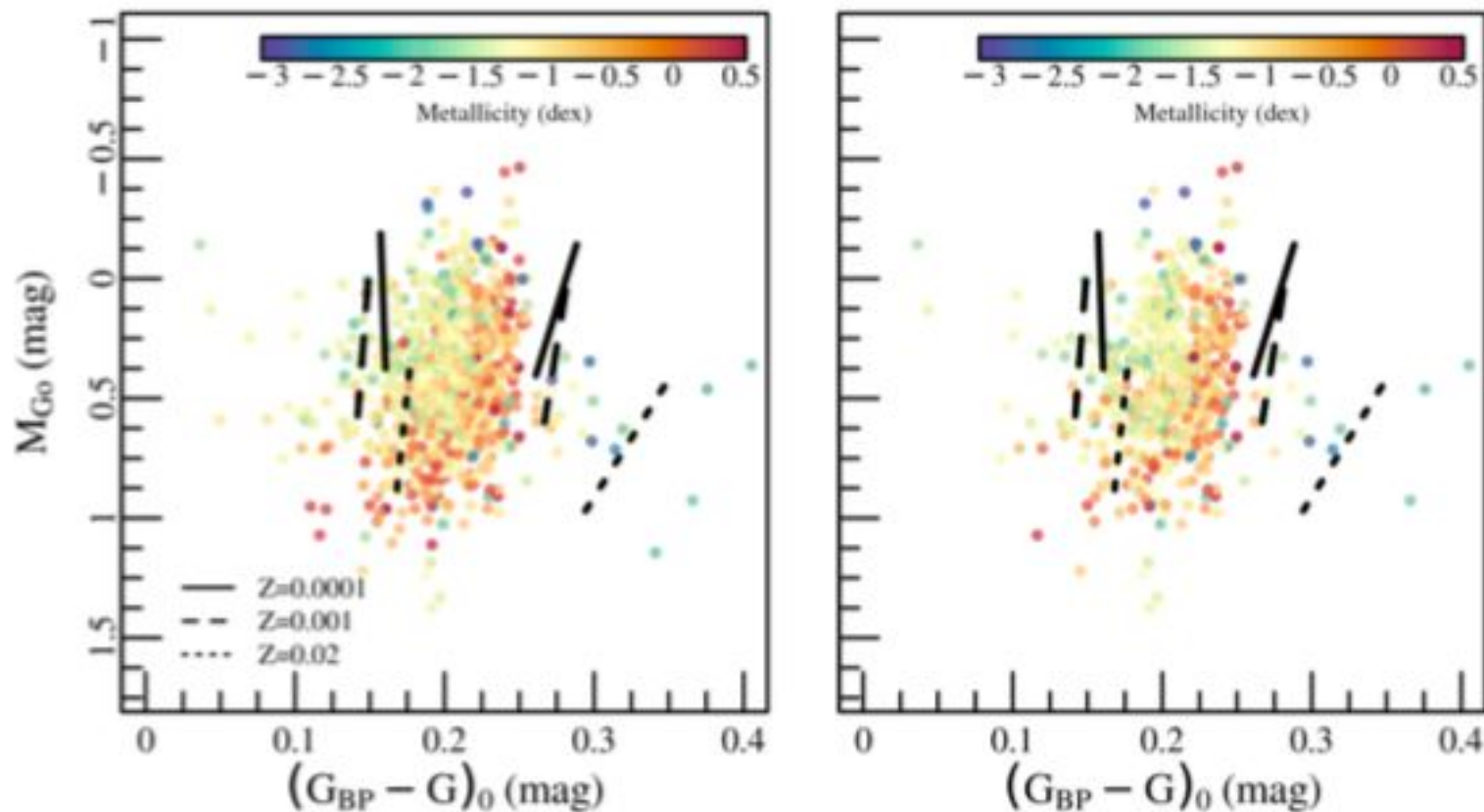
Typical predictions of 1D nonlinear convective pulsation models: the instability strip



Marconi et al. 2015 ApJS



Comparison with the observations

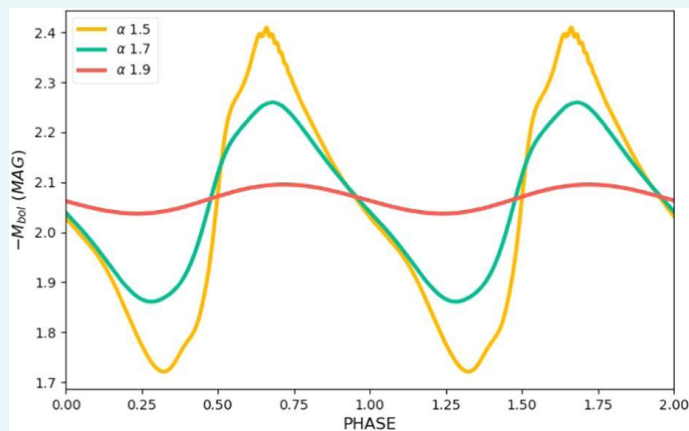


Clementini et al. 2023 A&A

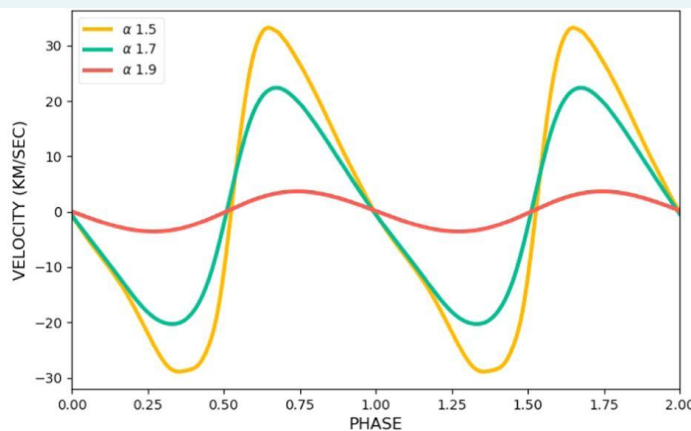
Marconi et al. 2015 ApJS



Typical predictions of 1D nonlinear convective pulsation models: the light curves



(a)



(b)

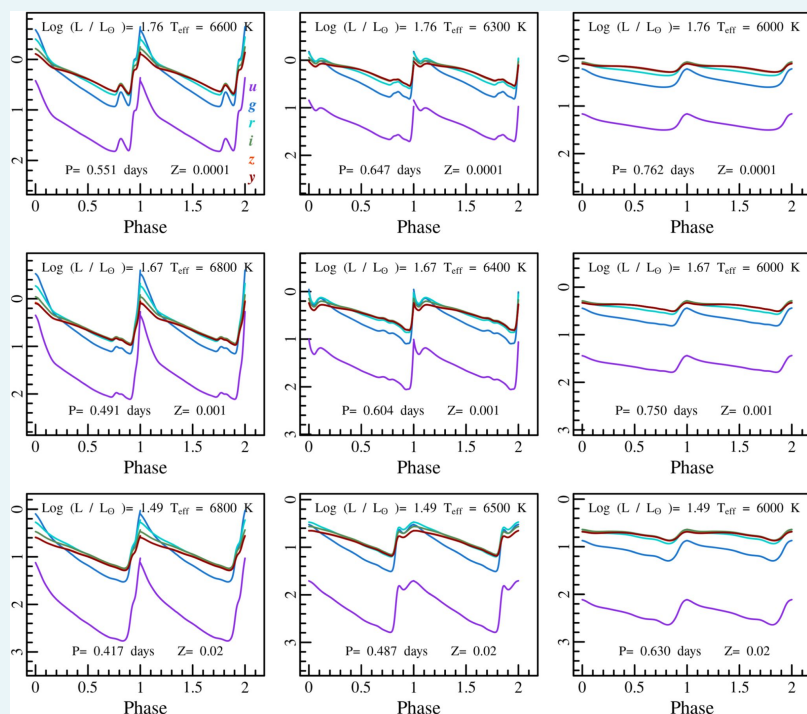
De Somma et al. 2020 ApJS



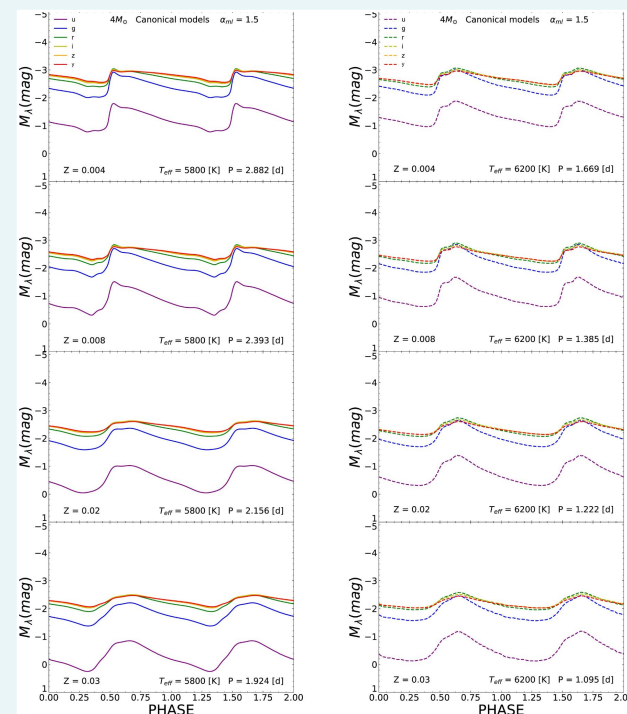
Typical predictions of 1D nonlinear convective pulsation models: the light curves

Bolometric light curves are transformed into various photometric systems through model atmospheres.

RR Lyrae



Cepheids

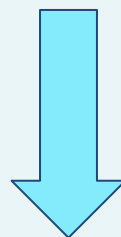




Pulsation amplitudes in the selected photometric filters.



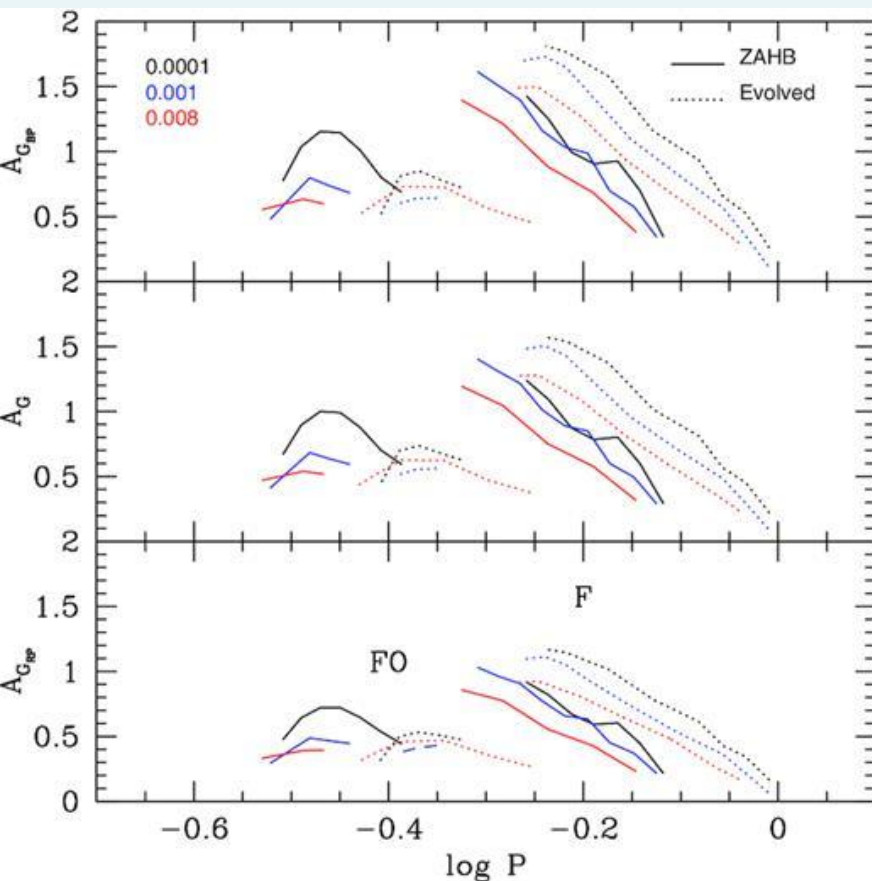
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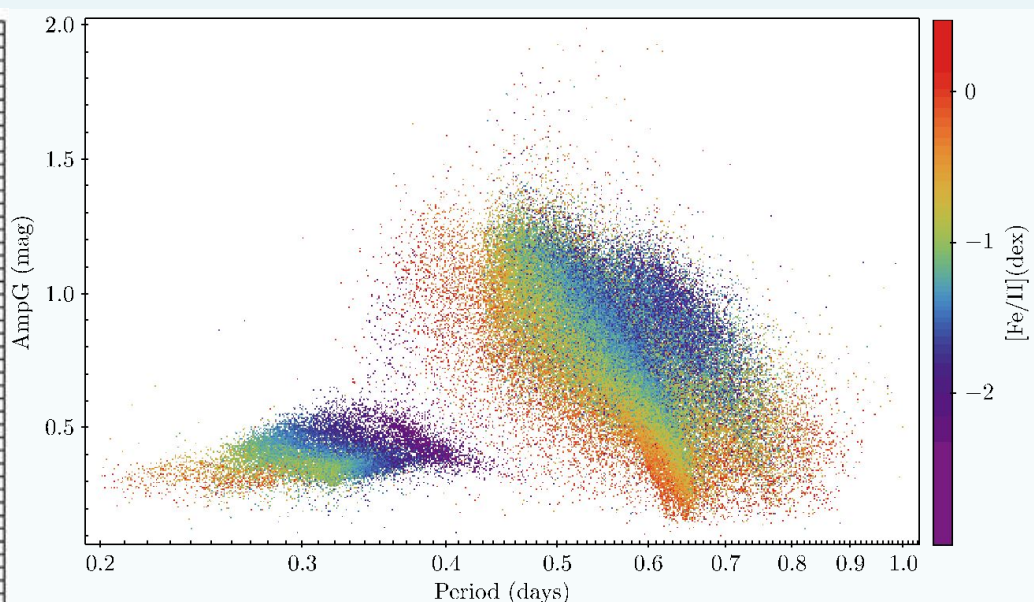
Theoretical Period-Amplitude diagrams



Theoretical versus observational Bailey diagrams



Marconi et al. 2021 MNRAS



Clementini et al. 2022 A&A



Mean magnitudes and colors in the selected photometric filters.



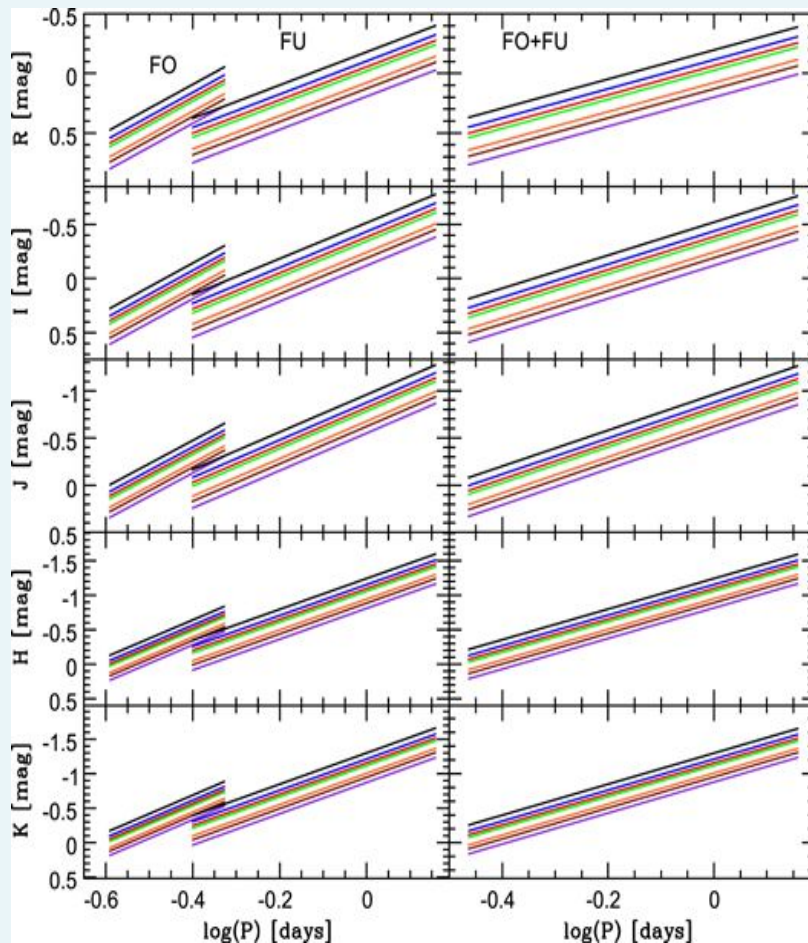
Mean magnitudes and colors in the selected photometric filters.



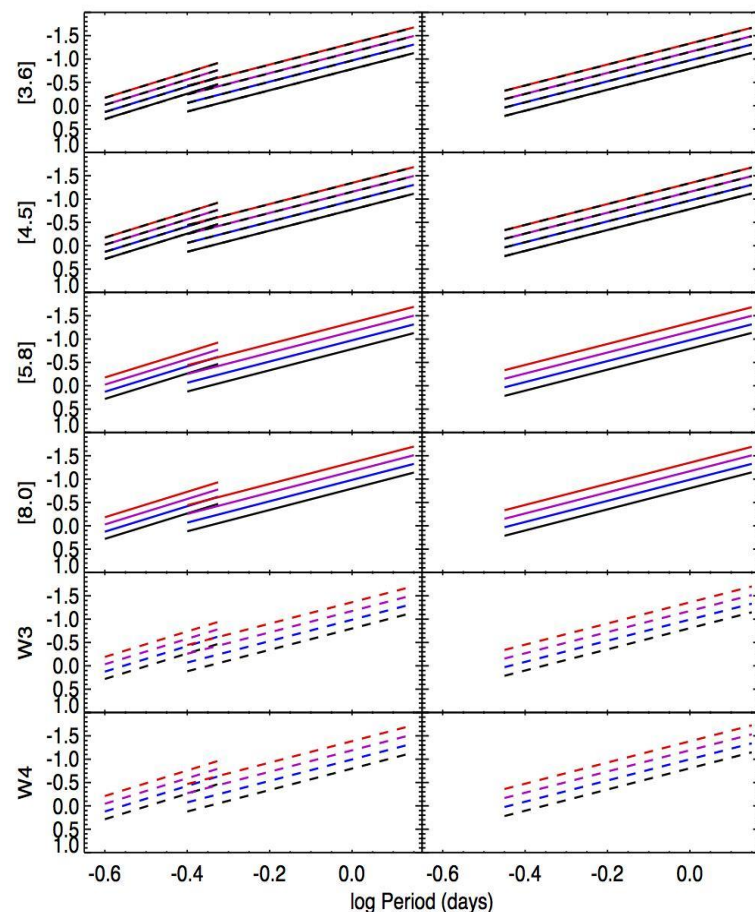
Multi-filter PL, PLC and PW relations



Multi-filter PL relations predicted by RR Lyrae models



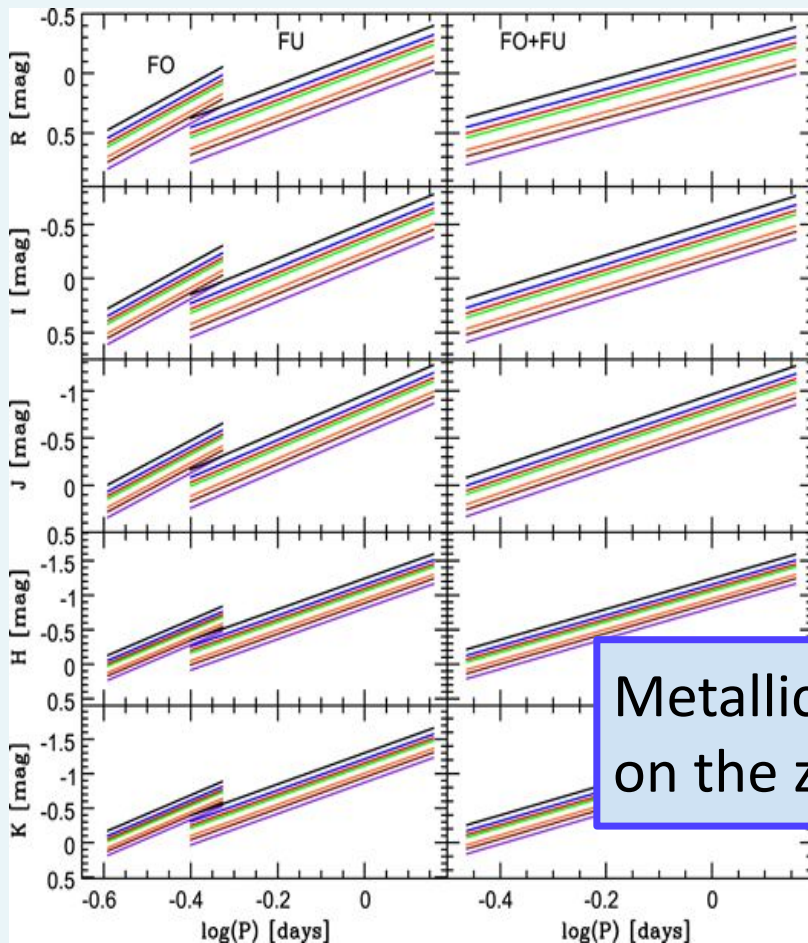
Marconi+ 2015 ApJ



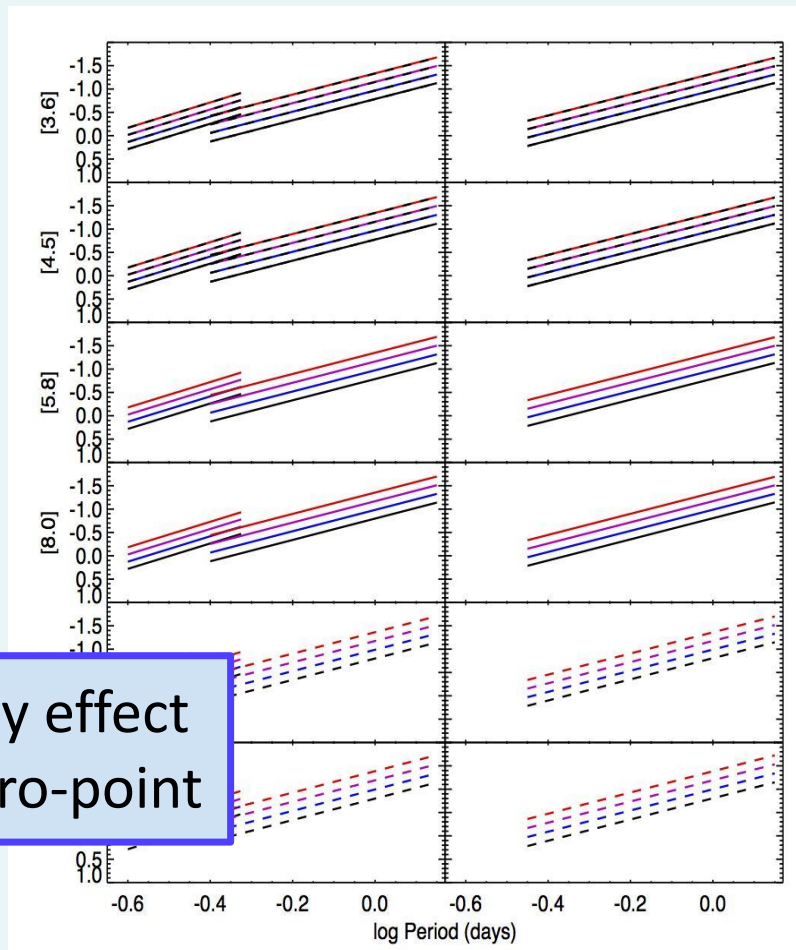
Neeley+ 2017 ApJ



Multi-filter PL relations predicted by RR Lyrae models



Marconi+ 2015 ApJ



Neeley+ 2017 ApJ

Metallicity effect
on the zero-point



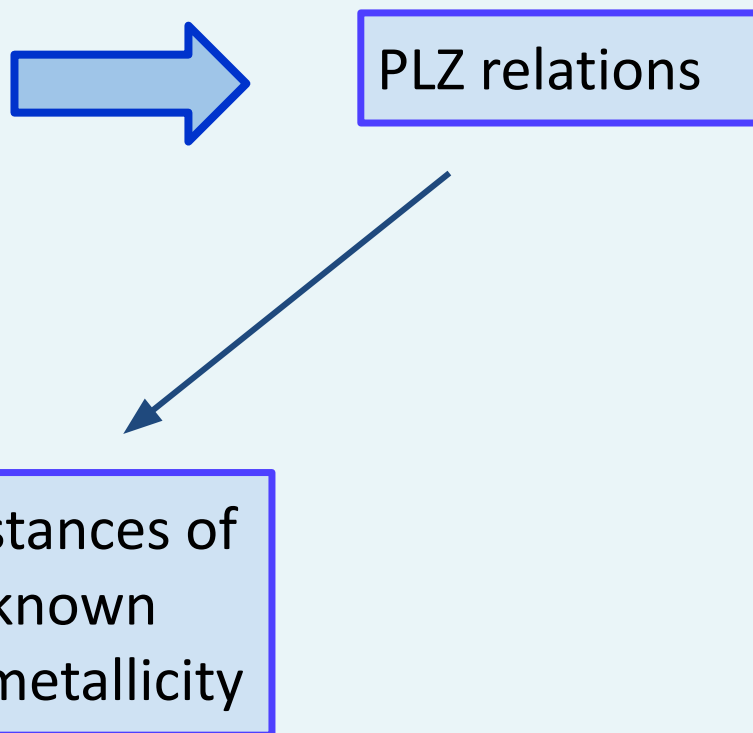
Multi-filter PLZ relations predicted by RR Lyrae models



PLZ relations

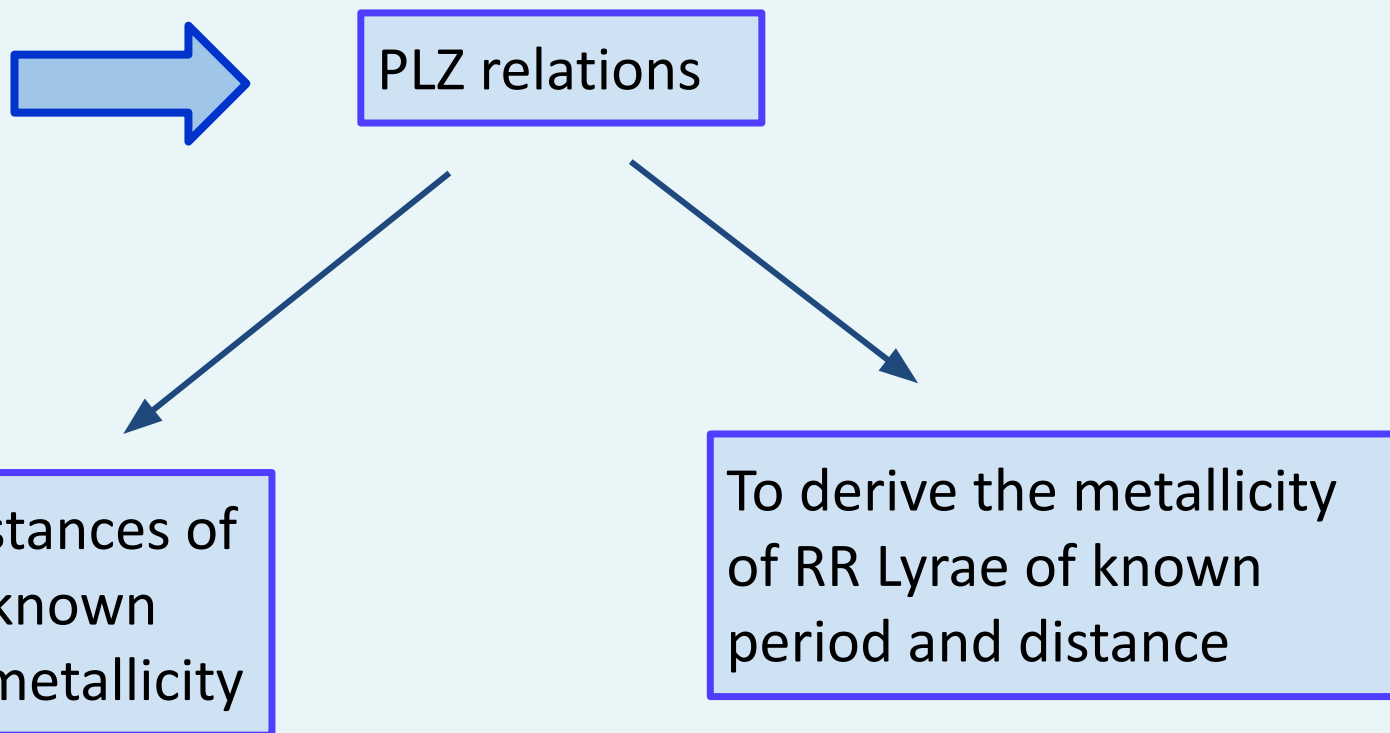


Multi-filter PLZ relations predicted by RR Lyrae models



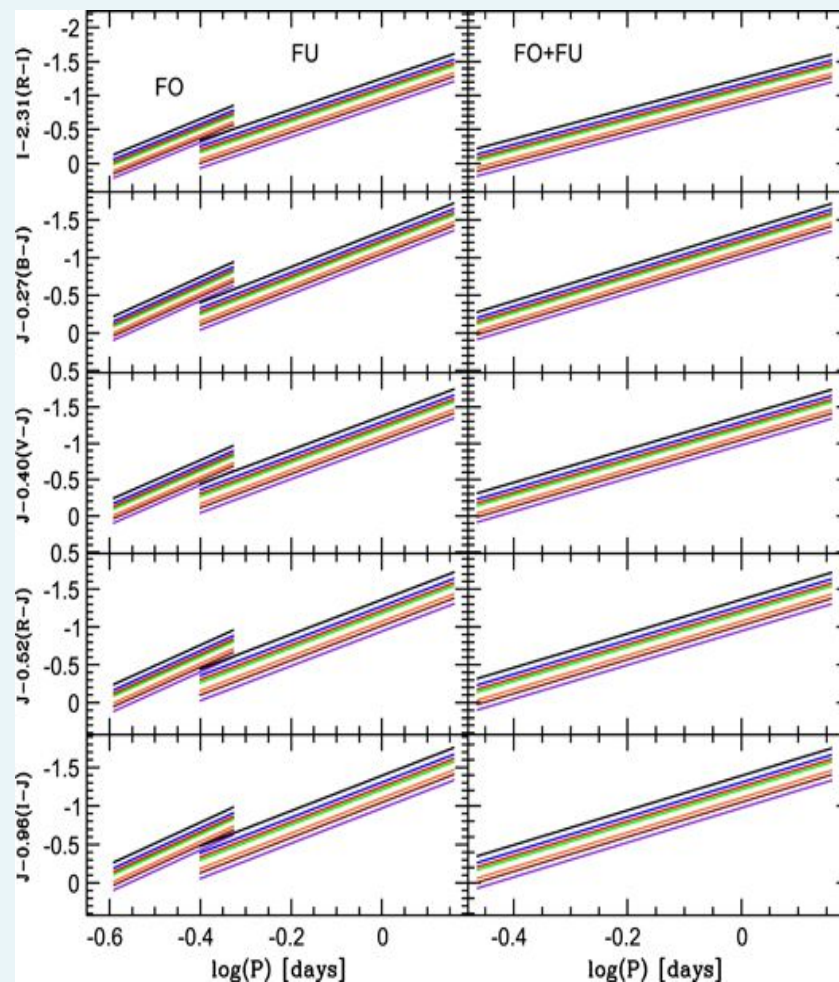
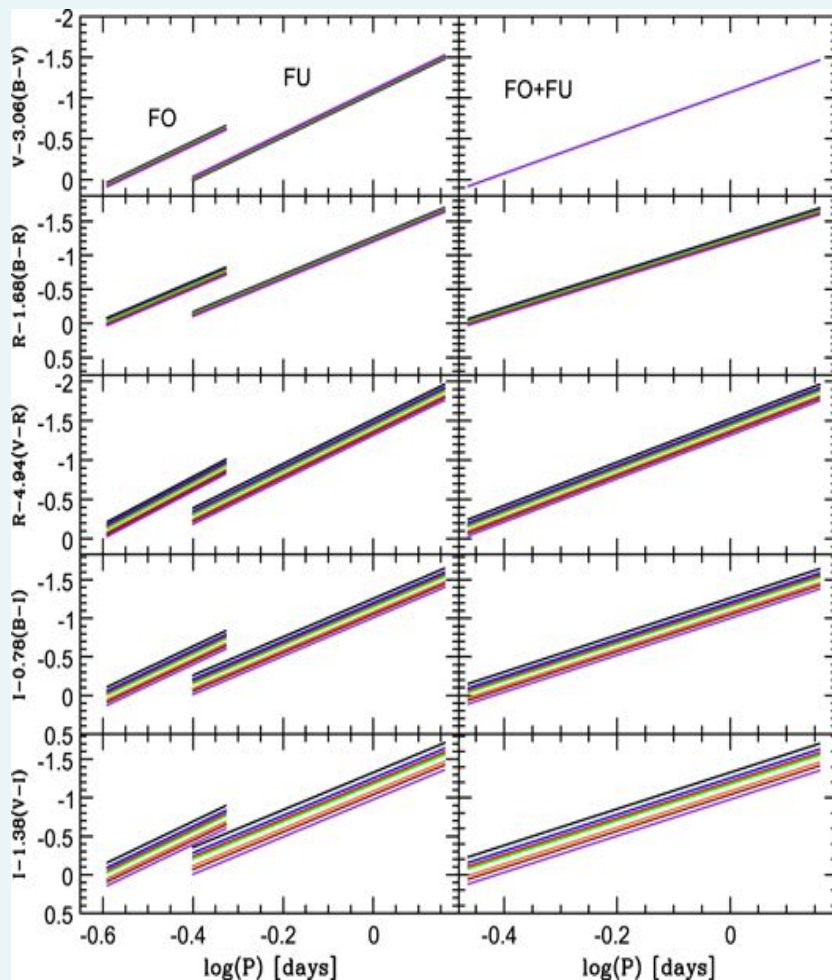


Multi-filter PLZ relations predicted by RR Lyrae models





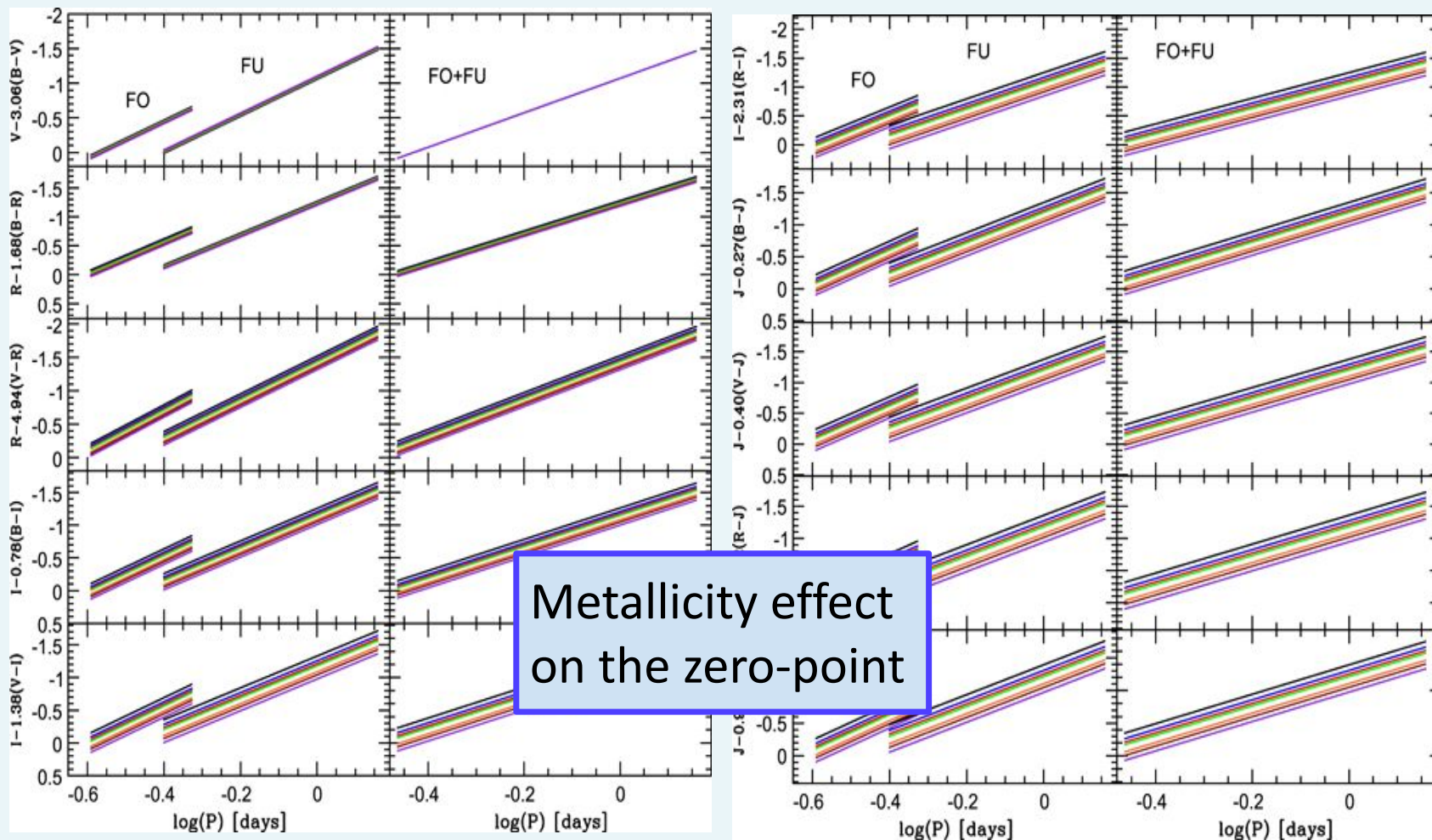
Multi-filter PW relations predicted by RR Lyrae models



Marconi+ 2015 ApJ



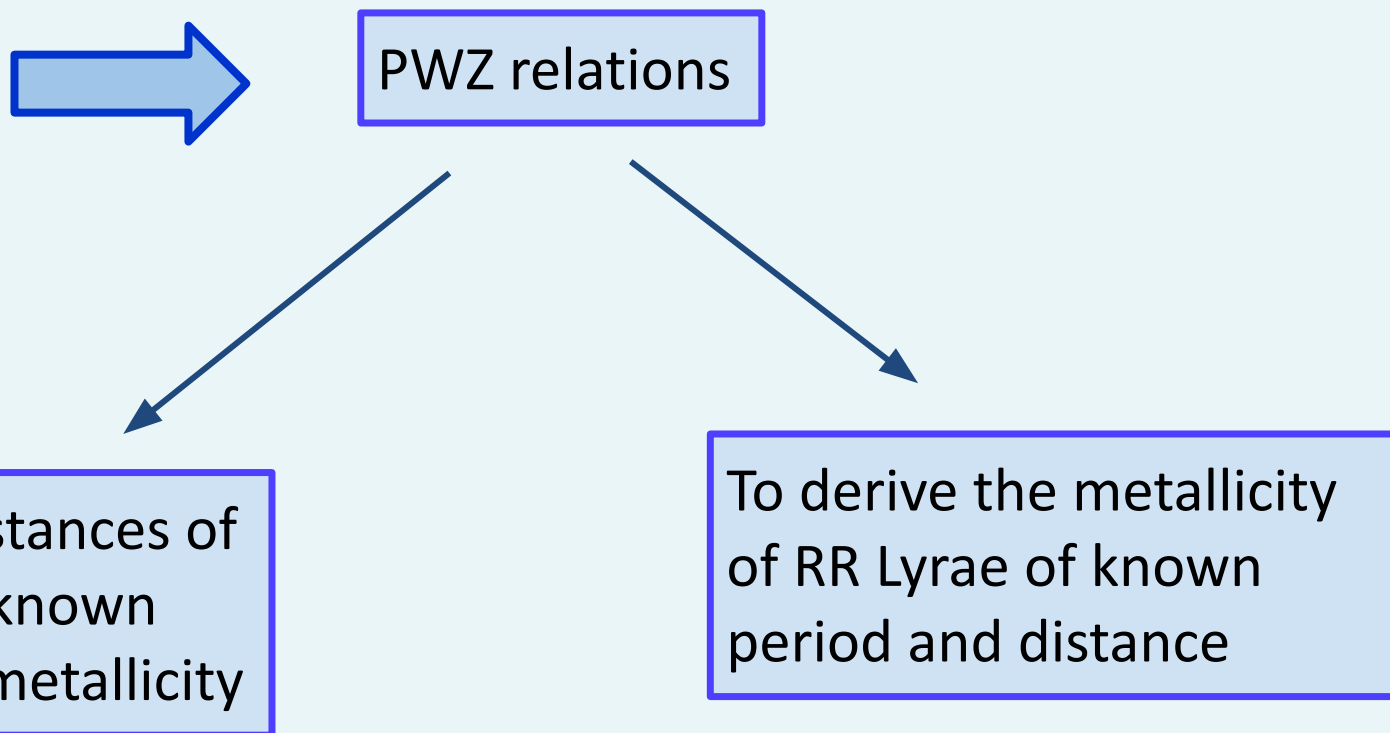
Multi-filter PW relations predicted by RR Lyrae models



Marconi+ 2015 ApJ

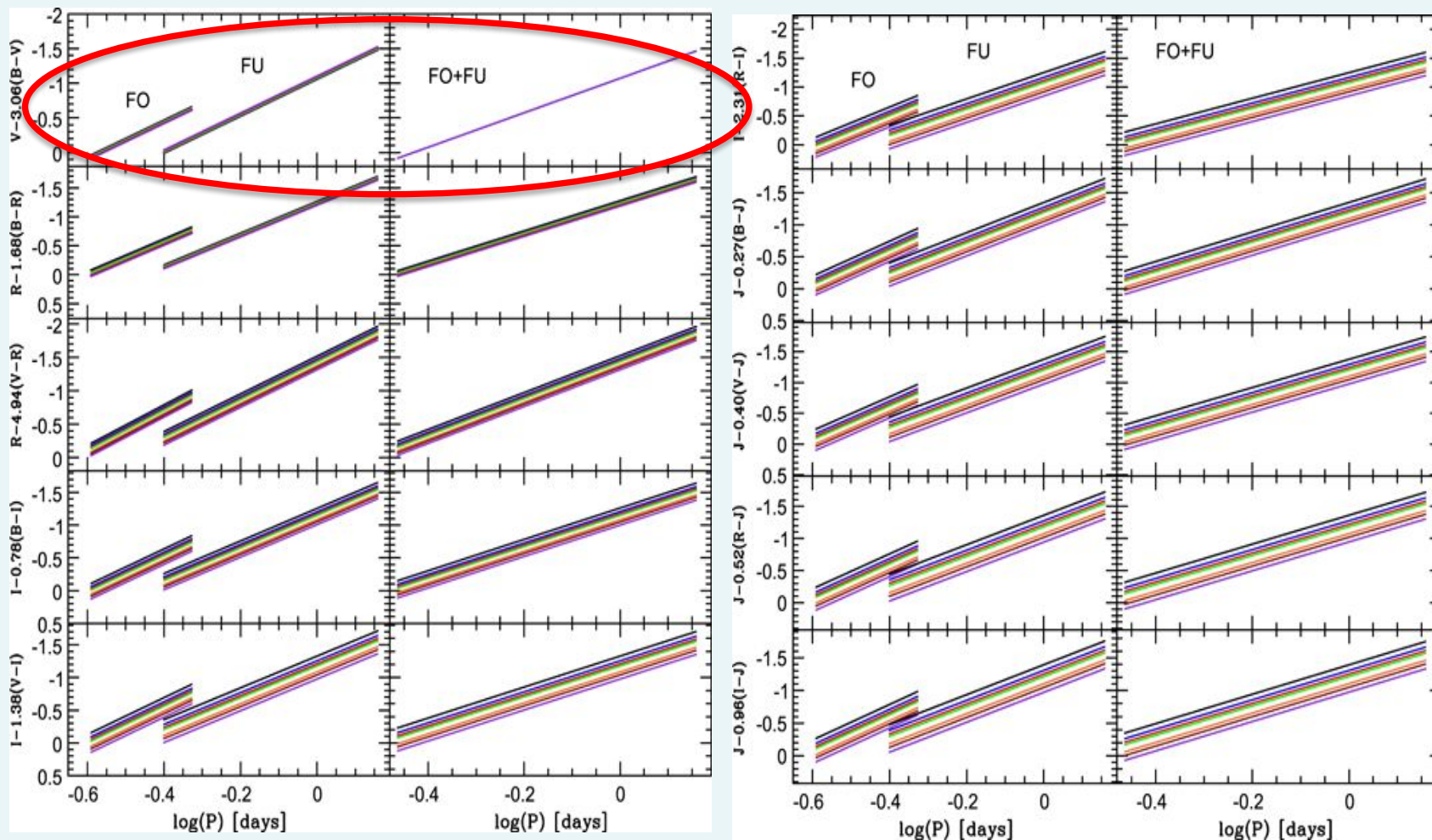


Multi-filter PWZ relations predicted by RR Lyrae models





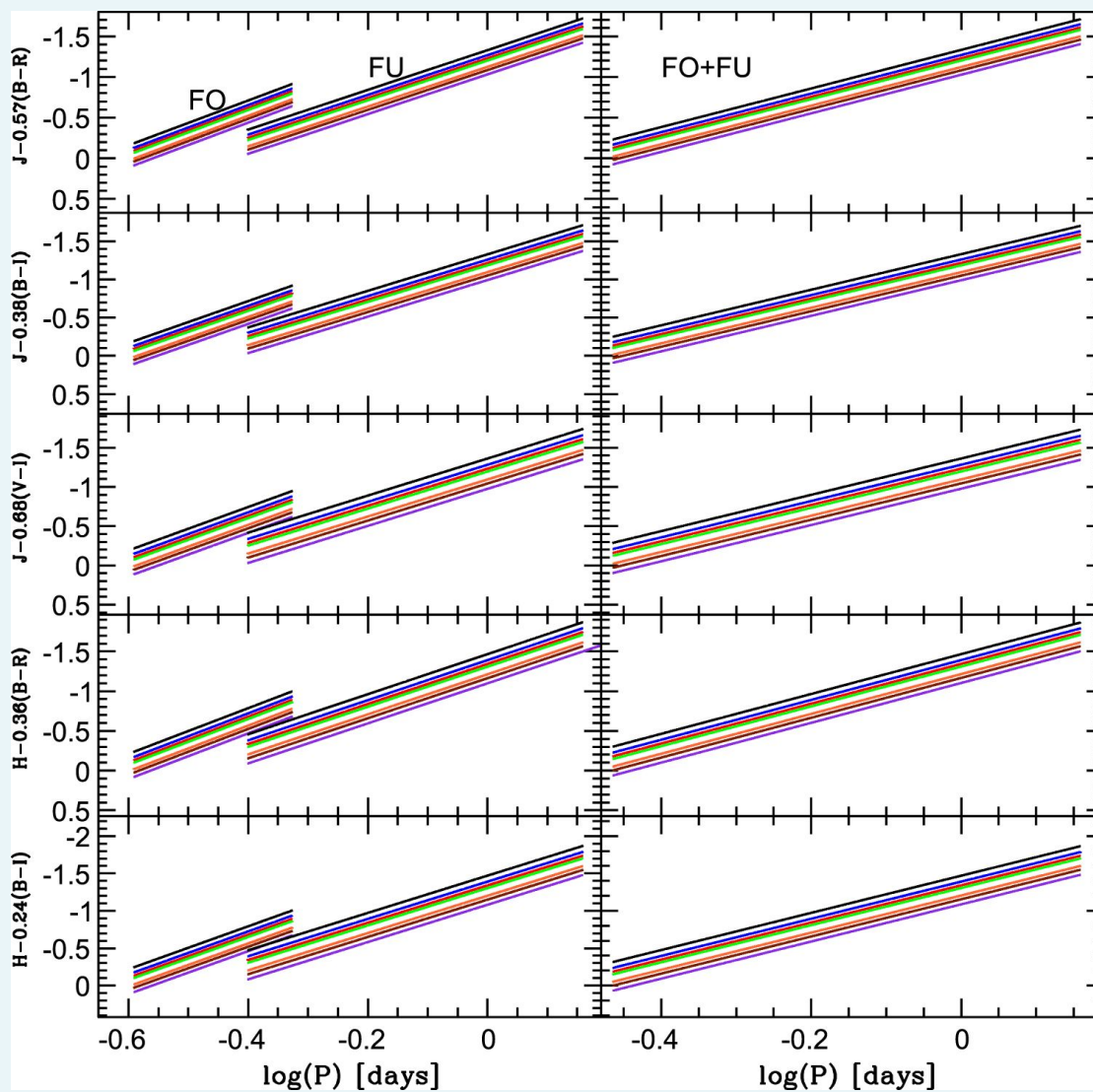
Multi-filter PW relations predicted by RR Lyrae models



Marconi+ 2015 ApJ



3 band PW relations predicted by RR Lyrae models



Marconi+ 2015 ApJ



Table 8
Same as Table 7, but for Triple-band PWZ Relations

Mode ^a	Band ^b	ξ^c	a^d	b^c	c^f	σ_a^g	σ_b^g	σ_c^g	rms ^h
FU	V, B-I	1.34	-1.01	-2.54	0.08	0.01	0.03	0.01	0.05
FO	V, B-I	1.34	-1.48	-2.74	0.08	0.03	0.07	0.01	0.03
Glob	V, B-I	1.34	-0.99	-2.42	0.08	0.01	0.02	0.01	0.05
FU	V, B-R	2.02	-1.17	-2.72	0.02	0.01	0.04	0.01	0.07
FO	V, B-R	2.02	-1.58	-2.78	0.04	0.04	0.08	0.01	0.04
Glob	V, B-R	2.02	-1.15	-2.58	0.03	0.01	0.03	0.01	0.07
FU	J, B-R	0.57	-0.99	-2.44	0.13	0.01	0.02	0.01	0.03
FO	J, B-R	0.57	-1.51	-2.75	0.12	0.03	0.05	0.01	0.02
Glob	J, B-R	0.57	-0.99	-2.37	0.13	0.01	0.02	0.01	0.04
FU	J, B-I	0.38	-0.95	-2.39	0.14	0.01	0.02	0.01	0.03
FO	J, B-I	0.38	-1.48	-2.74	0.13	0.02	0.05	0.01	0.02
Glob	J, B-I	0.38	-0.95	-2.33	0.15	0.01	0.02	0.01	0.04
FU	J, V-I	0.68	-0.93	-2.36	0.17	0.01	0.02	0.01	0.03
FO	J, V-I	0.68	-1.48	-2.76	0.14	0.02	0.04	0.01	0.02
Glob	J, V-I	0.68	-0.93	-2.31	0.17	0.01	0.02	0.01	0.03
FU	H, B-R	0.35	-1.05	-2.52	0.16	0.01	0.02	0.01	0.03
FO	H, B-R	0.35	-1.58	-2.87	0.14	0.02	0.04	0.01	0.02
Glob	H, B-R	0.35	-1.06	-2.51	0.16	0.01	0.01	0.01	0.03
FU	H, B-I	0.24	-1.03	-2.49	0.17	0.01	0.02	0.01	0.03
FO	H, B-I	0.24	-1.57	-2.87	0.14	0.02	0.03	0.01	0.02
Glob	H, B-I	0.24	-1.04	-2.49	0.16	0.01	0.01	0.01	0.03
FU	H, V-I	0.42	-1.02	-2.48	0.18	0.01	0.02	0.01	0.03
FO	H, V-I	0.42	-1.57	-2.88	0.15	0.02	0.03	0.01	0.01
Glob	H, V-I	0.42	-1.03	-2.48	0.18	0.01	0.01	0.01	0.03
FU	K, B-R	0.23	-1.02	-2.46	0.16	0.01	0.02	0.01	0.03
FO	K, B-R	0.23	-1.55	-2.84	0.14	0.02	0.04	0.01	0.02
Glob	K, B-R	0.23	-1.02	-2.45	0.16	0.01	0.01	0.01	0.03
FU	K, B-I	0.16	-1.00	-2.44	0.17	0.01	0.02	0.01	0.03
FO	K, B-I	0.16	-1.54	-2.83	0.15	0.02	0.04	0.01	0.02
Glob	K, B-I	0.16	-1.01	-2.43	0.17	0.01	0.01	0.01	0.03
FU	K, V-I	0.28	-0.99	-2.43	0.18	0.01	0.02	0.01	0.03
FO	K, V-I	0.28	-1.54	-2.84	0.15	0.02	0.03	0.01	0.02
Glob	K, V-I	0.28	-1.00	-2.42	0.18	0.01	0.01	0.01	0.03

Note.

- ^a Pulsation mode.
^b Photometric bands.
^c Coefficient of the color term.
^d Zero-point of the relation.
^e Coefficient of the logarithmic
^f Coefficient of the metallicity term.
^g Errors on coefficients.
^h Standard deviation.

Color term
of the PW

Marconi+ 2015 ApJ



Classical Cepheid Results



The Updated Dataset of CC Pulsation models

- **Wide range of masses and effective temperatures**
- **3 assumptions about the Mass-Luminosity (ML) relation**
- **3 values for the efficiency of superadiabatic convection**
- **4 chemical compositions**

The ML relation and the efficiency of superadiabatic convection were varied simultaneously for the first time!

Low metallicity

$Z=0.004$ $Y=0.25$

$Z=0.008$ $Y=0.25$

Solar metallicity

$Z=0.02$ $Y=0.28$

$Z=0.03$ $Y=0.28$

Oversolar metallicity

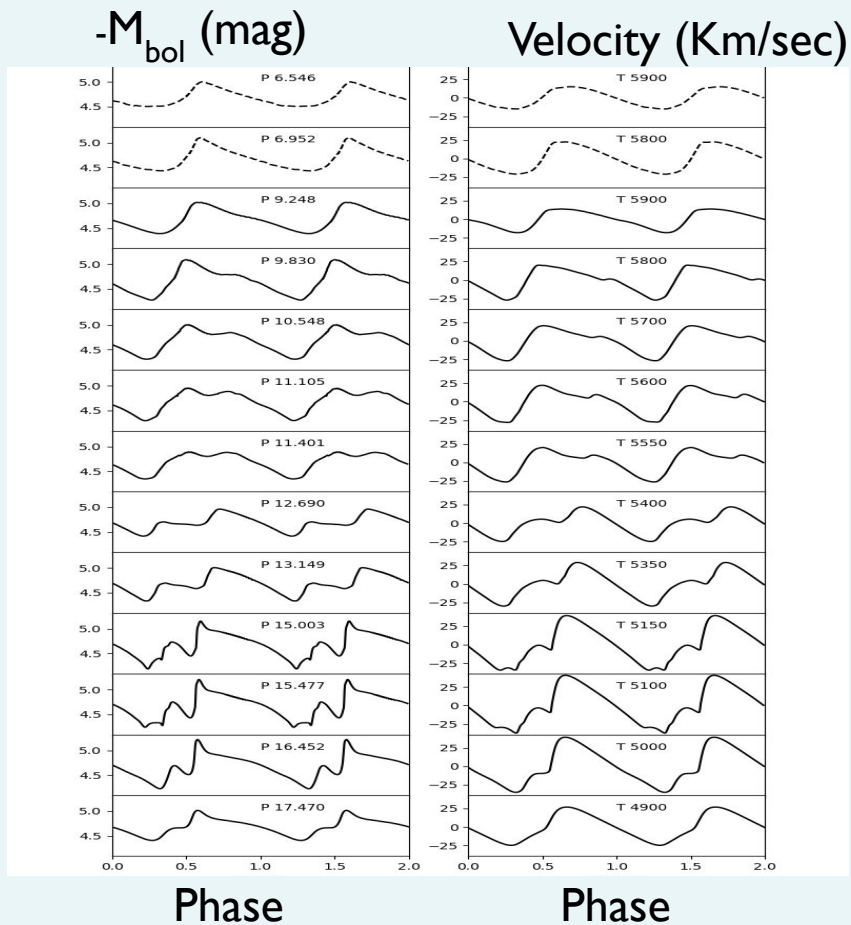
**More than
2000 models!!**



The Atlas of light and Radial Velocity Curves

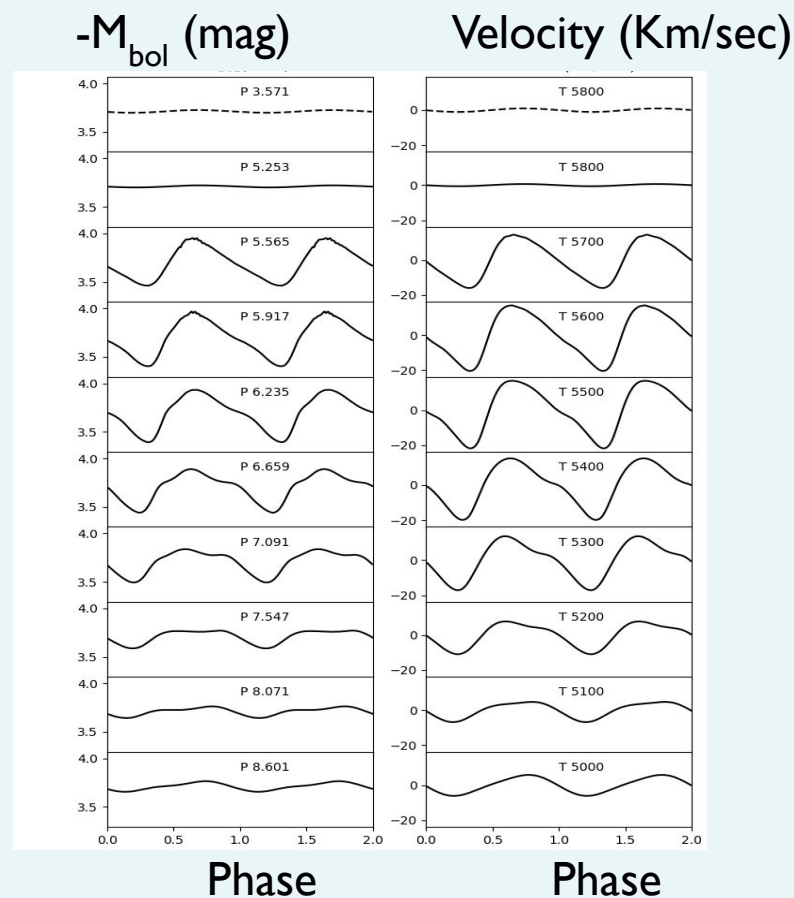
$Z=0.004$ $Y=0.25$

$7 M_{\odot}$



$Z=0.03$ $Y=0.28$

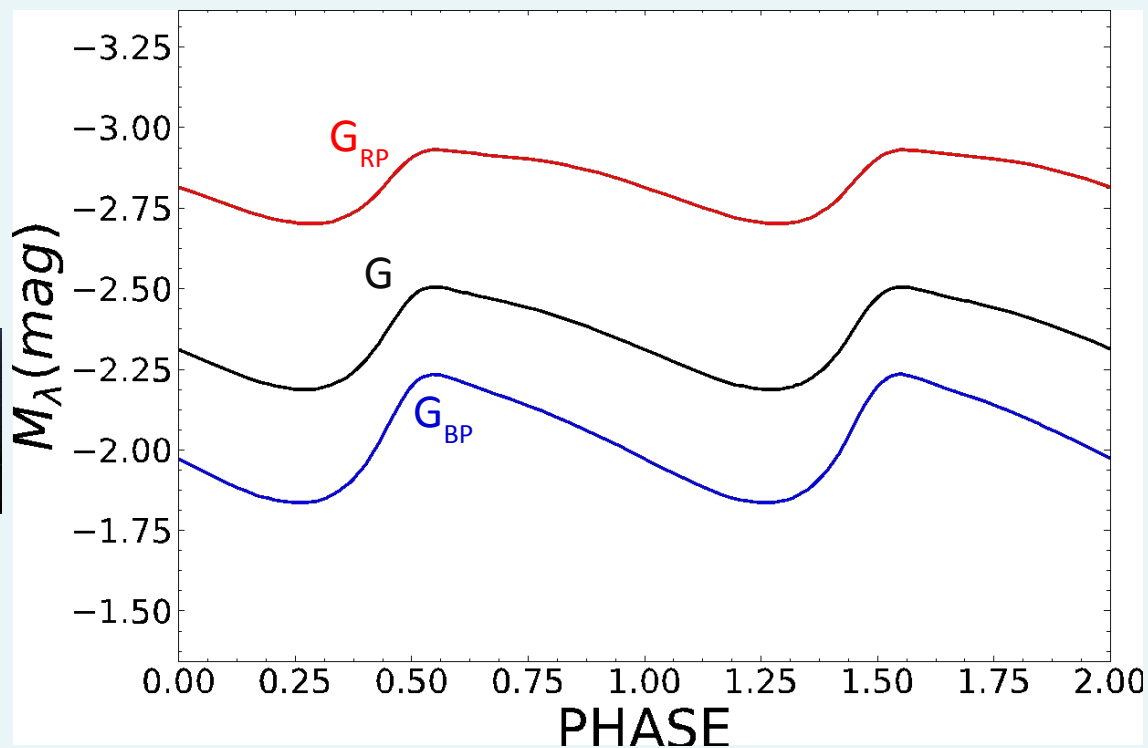
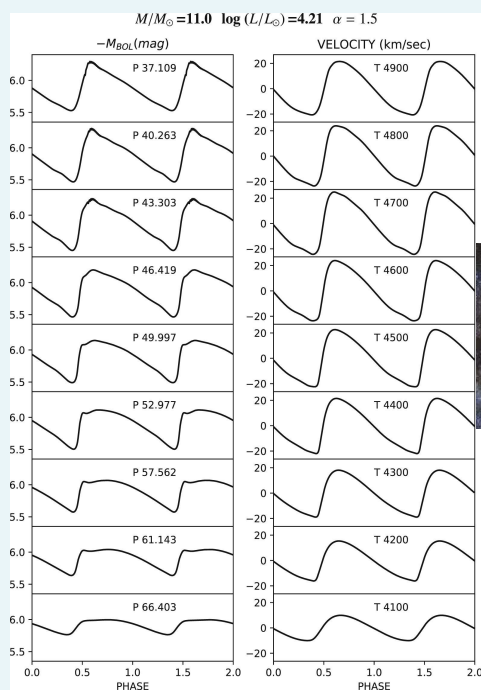
$6 M_{\odot}$





The First Theoretical Atlas of Gaia Light Curves

Using bolometric corrections (*Chen, Girardi et al. 2019 A&A*),
bolometric light curves were converted into the Gaia photometric system.



De Somma et al., 2020, ApJS

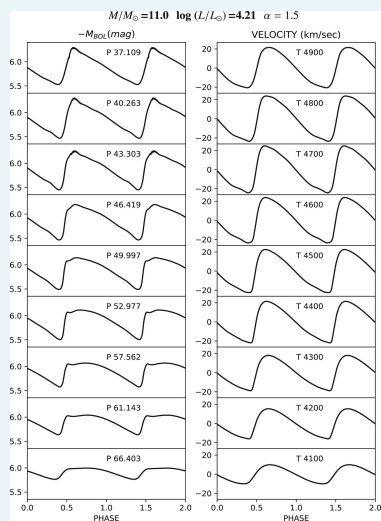
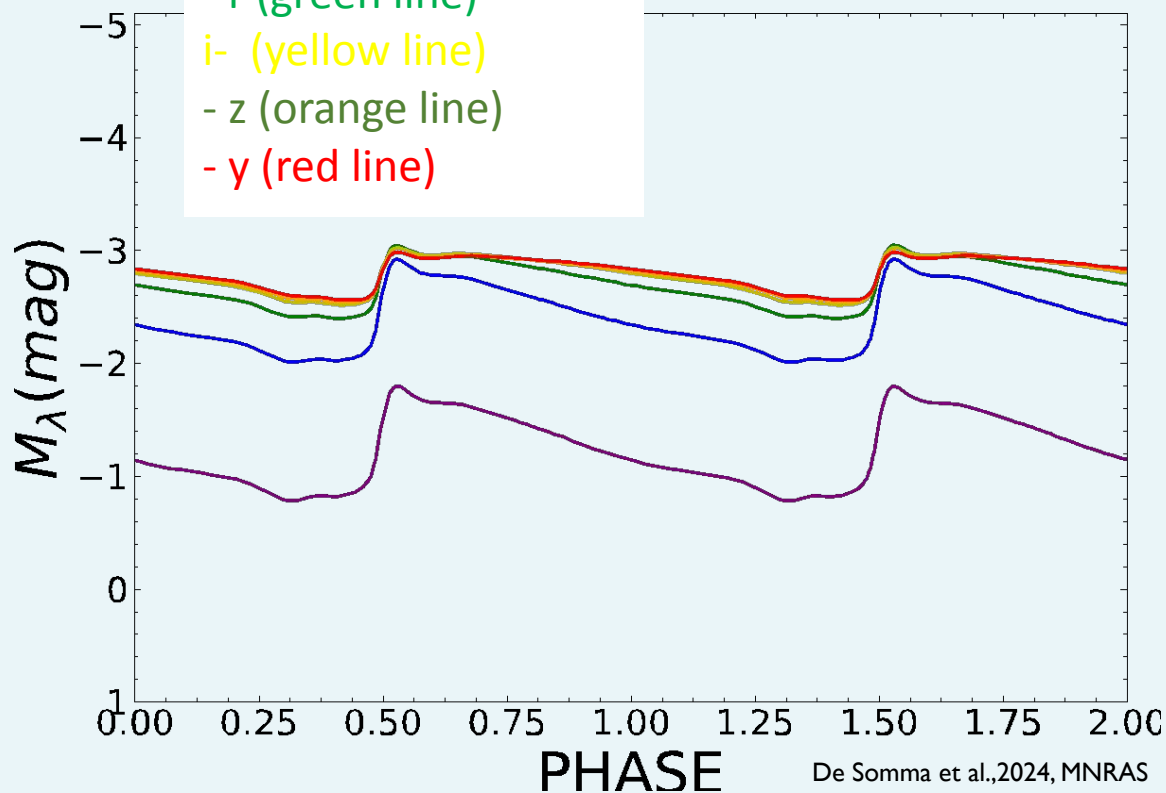


The First Theoretical Atlas of Rubin-LSST Light Curves

Using bolometric corrections (*Chen, Girardi et al. 2019 A&A*),
bolometric light curves were converted into the Gaia photometric system.

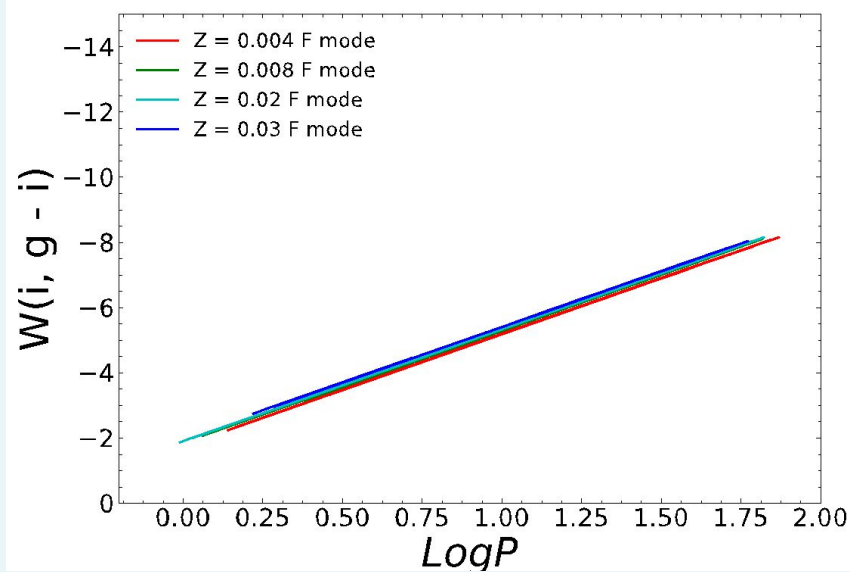
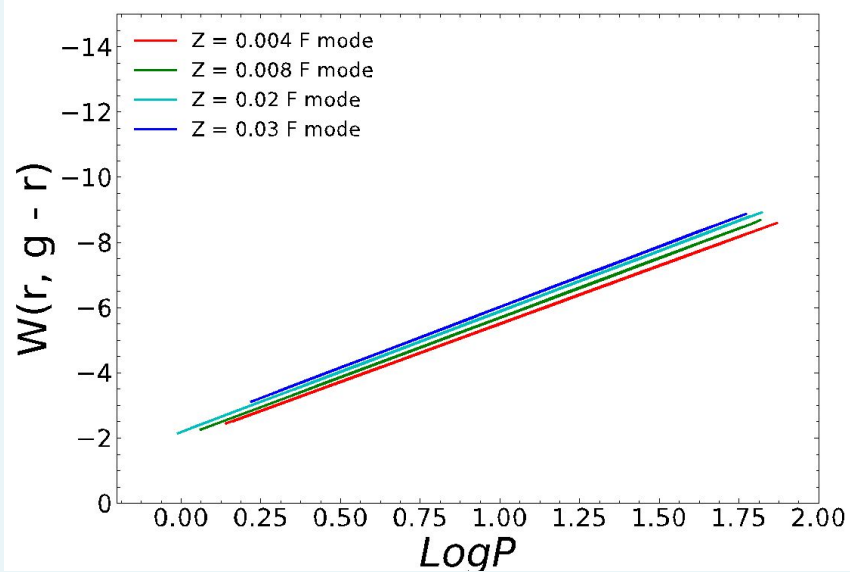
Filters:

- u (purple line)
- g (blue line)
- r (green line)
- i- (yellow line)
- z (orange line)
- y (red line)





The Dependence of CC PW relation on Metallicity



There is a metallicity effect and it depends on the color!!



The Metal-Dependent PW relation on Metallicity

$$W = G - 1.9 (G_{BP} - G_{RP}) = a + b \log P + c [Fe/H]$$

α_{ml}	ML	a	b	c	σ_a	σ_b	σ_c	σ	R^2
F									
1.5	A	-6.018	-3.314	-0.189	0.009	0.016	0.021	0.118	0.993
1.7	A	-6.072	-3.379	-0.129	0.010	0.016	0.021	0.090	0.996
1.9	A	-6.170	-3.472	-0.245	0.023	0.018	0.040	0.072	0.998
1.5	B	-5.853	-3.234	-0.190	0.011	0.016	0.022	0.139	0.991
1.7	B	-5.871	-3.262	-0.260	0.012	0.015	0.023	0.118	0.995
1.9	B	-5.968	-3.370	-0.189	0.026	0.017	0.047	0.092	0.997
1.5	C	-5.694	-3.270	-0.105	0.012	0.017	0.023	0.141	0.991
1.7	C	-5.722	-3.274	-0.140	0.012	0.015	0.022	0.116	0.994
1.9	C	-5.800	-3.327	-0.167	0.023	0.016	0.043	0.094	0.997
FO									
1.5	A	-6.676	-3.450	-0.221	0.051	0.048	0.059	0.145	0.985
1.7	A	-6.818	-3.627	-0.243	0.040	0.034	0.049	0.073	0.996
1.9	A	-6.933	-3.688	-0.349	0.045	0.030	0.052	0.034	0.999
1.5	B	-6.634	-3.566	-0.304	0.063	0.063	0.062	0.097	0.988
1.7	B	-6.616	-3.533	-0.303	0.095	0.083	0.095	0.103	0.987
1.9	B	-6.719	-3.627	-0.304	0.066	0.050	0.068	0.030	0.998
1.5	C	-6.473	-3.510	-0.235	0.043	0.051	0.038	0.038	0.996
1.7	C	-6.486	-3.506	-0.261	0.049	0.056	0.051	0.030	0.998

De Somma et al. 2022, ApJS



The Metal-Dependent PW relation on Metallicity

$$W = G - 1.9 (G_{BP} - G_{RP}) = a + b \log P + c [Fe/H]$$

α_{ml}	ML	a	b	c	σ_a	σ_b	σ_c	σ	R^2
F									
1.5	A	-6.018	-3.314	-0.189	0.009	0.016	0.021	0.118	0.993
1.7	A	-6.072	-3.379	-0.129					
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1.5	C	-5.694	-3.270	-0.105					
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1.9	C	-5.800	-3.327	-0.167					
FO									
1.5	A	-6.676	-3.450	-0.221					
1.7	A	-6.818	-3.627	-0.243					
1.9	A	-6.933	-3.688	-0.349					
1.5	B	-6.634	-3.566	-0.304					
1.7	B	-6.616	-3.533	-0.303	0.095	0.083	0.095	0.103	0.987
1.9	B	-6.719	-3.627	-0.304	0.066	0.050	0.068	0.030	0.998
1.5	C	-6.473	-3.510	-0.235	0.043	0.051	0.038	0.038	0.996
1.7	C	-6.486	-3.506	-0.261	0.049	0.056	0.051	0.030	0.998

Metal-dependent PW relations point towards a metallicity effect on the zero point varying from **~-0.1 dex to ~-0.2 dex for the F-mode relations** and from **~-0.1 dex to ~-0.3 dex for the FO-mode relations.**

De Somma et al. 2022, ApJS

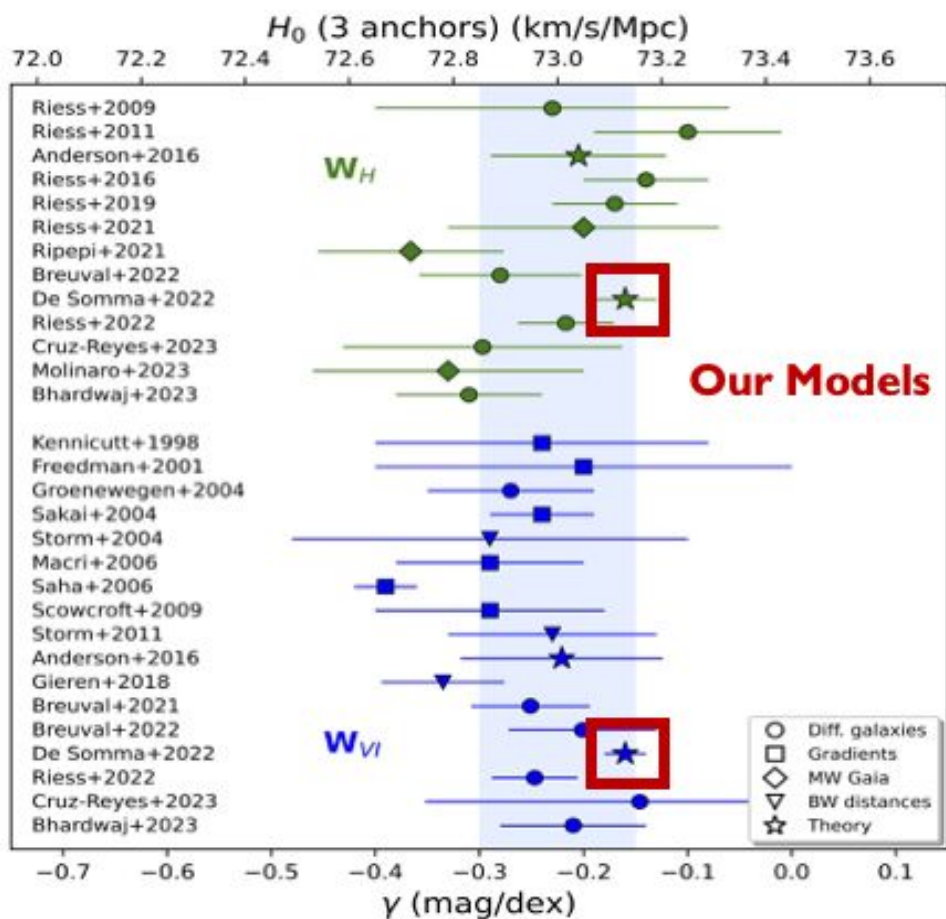
The first theoretical PWZ relation in the Gaia filters

At fixed period and color, models predict that more metallic Cepheids are brighter → effect on the distance scale □

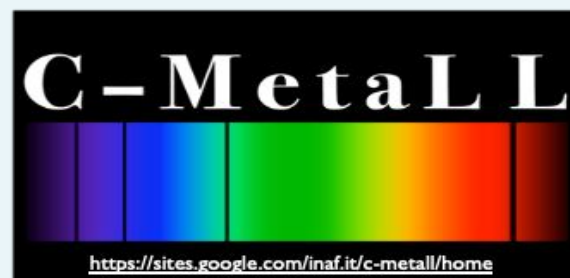
Accurate corrections for metallicity are crucial for reducing uncertainties and making the anchors consistent in the distance ladder.



Metallicity Effects on PL and PW Relations



In 2020 we started the C-MetaLL (Cepheid Metallicity in the Leavitt Law) project (PI:V. Riipepi)



Aim of the project : Measure accurate PLZ/PWZ relations based on homogeneous spectroscopy and photometry

Theoretical and empirical metallicity correction results are in good agreement!!



Application: Derivation of Individual Theoretical Parallaxes

Steps

1. **Upload** the Gaia table (*GaiaDR3_table.csv*) and familiarize yourself with the meaning of each column.
2. **Apply** the theoretical PW/PWZ relations (selecting the assumptions on *alfa* and the *ML relation*) to the stars in the Gaia table to compute:

$$\text{distance modulus: } \mu = W_{obs} - W_{th}$$

Where

$$W_{th} = a + b \log P_{obs} \text{ (neglecting metallicity dependence)}$$
$$W_{th} = a + b(\log P_{obs} - 1) + c [Fe/H] \text{ (including metallicity dependence)}$$

for each individual Cepheid

3. **Derive** theoretical distances and parallaxes:

$$d(\text{pc}) = 10^{(\mu+5)/5}$$
$$\varpi_{th}(\text{mas}) = \frac{1000}{d}$$

4. **Compare** with Gaia astrometric parallax (column 8 in Gaia table).
5. **Evaluate** the effects of the model inputs on the derived distance scale of:
 - changing α (mixing-length parameter) (column 1 in PW PWZ tables)
 - changing ML relation (A/B/C) (column 2 in PW PWZ)
 - including or excluding metallicity ([Fe/H]) (i.e. using PWZ or PW)



Application: Derivation of Individual Theoretical Parallaxes

Steps

1. **Upload** the Gaia table (*GaiaDR3_table.csv*) and familiarize yourself with the meaning of each column.

	GaiaEDR3_sourceid	RA	Dec	Mode	Period	G	GBP-GRP	plx_corr	plx_err	[Fe/H]	[Fe/H]_err
1	3345254098763568768	92.592625	14.6445	DCEP_10	4.83807	9.535	1.389	0.3342	0.0157	0.01	0.11
2	3373097817690243840	98.829917	21.296694	DCEP_10	1.512365	11.289	0.959	0.3492	0.0257	-0.18	0.15
3	3157125975624005632	103.556375	7.941417	DCEP_10	0.777405	11.179	0.971	0.3998	0.0147	-0.27	0.15
4	2936274153063501824	107.132042	-14.907444	DCEP_10	6.38771	9.242	1.492	0.3713	0.0141	-0.11	0.11
5	3045301790490616960	107.295	-12.285889	DCEP_10	2.412282	10.298	1.301	0.324	0.0213	-0.11	0.11
6	3054600330268450560	111.102167	-7.855444	DCEP_10	2.071284	10.164	0.979	0.3262	0.017	-0.04	0.11
7	5718760258978008320	116.049667	-17.081	DCEP_10	4.7013	10.093	1.11	0.2199	0.0171	-0.18	0.1
8	4256464464696335616	278.445167	-4.809139	DCEP_10	3.102301	10.052	1.985	0.5138	0.0153	0.2	0.11
9	4155550943909053696	279.218125	-9.118	DCEP_10	2.589889	10.651	2.041	0.5071	0.014	0.16	0.11
10	4154758024243120640	279.766792	-10.822556	DCEP_10	3.05774	9.97	1.827	0.5749	0.0199	0.04	0.11
11	4319599865468760832	290.792667	13.856667	DCEP_10	2.972999	10.156	2.12	0.587	0.0155	0.03	0.15
12	5848500161483878400	222.626196	-67.497632	DCEP_10	3.06522	7.144	1.218	1.1389	0.0191	0.14	0.12
13	5334449269746243328	171.304037	-61.369148	DCEP_10	3.21197	8.398	0.912	0.4975	0.0161	0.06	0.12
14	5334591003669216000	178.389681	-62.852195	DCEP_10	3.99786	9.745	1.328	0.3296	0.0114	0.19	0.12
15	429635993926502912	2.464122	61.514037	DCEP_10	3.651514	10.459	1.799	0.3959	0.0122	-0.02	0.12
16	429182376660681984	3.166667	60.226417	DCEP_10	2.8417	9.923	1.327	0.3605	0.0125	-0.1	0.14
17	5877460679352962048	221.674867	-61.461959	DCEP_10	2.3981	7.327	1.045	1.0429	0.0223	-0.01	0.12
18	511226491219235840	26.799652	61.422489	DCEP_10	3.223247	9.897	1.729	0.4825	0.0171	0.09	0.12
19	5311598634949142400	139.319632	-53.084847	DCEP_10	2.482007	10.403	1.156	0.2656	0.011	0.13	0.12
20	468646563398354176	61.611833	56.382611	DCEP_10	2.99108	10.786	1.532	0.3256	0.0142	-0.45	0.11
21	514736269771300224	33.881083	63.517778	DCEP_10	3.29896	10.338	1.48	0.3226	0.0129	-0.09	0.15
22	3356119713188276096	98.477542	14.471379	DCEP_10	3.13574	10.406	1.309	0.3242	0.0191	-0.01	0.12
23	5614274807174596864	115.457168	-25.041774	DCEP_10	2.6259	10.369	1.173	0.33	0.0125	-0.15	0.12
24	3398383973788673024	86.418876	18.656884	DCEP_10	2.102324	7.951	0.972	0.8611	0.0233	0.12	0.12
25	4156512638614879104	279.165006	-8.184834	DCEP_10	3.090811	9.644	1.627	0.5265	0.0177	0.09	0.12
26	5337634589276921856	167.685752	-60.750272	DCEP_10	5.725573	8.859	1.256	0.3913	0.0121	0.19	0.12



Application: Derivation of Individual Theoretical Parallaxes

2. **Apply** the theoretical PW/PWZ relations (selecting the assumptions on *alfa* and the *ML relation*) to the stars in the Gaia table to compute:

PWZ

$$W = a + b \log P + c [Fe/H]$$

	Alfa	ML	a	b	c	sa	sb	sc	sigma	R^2
1	1,5	A	-6,018	-3,314	-0,189	0,009	0,016	0,021	0,118	0,993
2	1,7	A	-6,072	-3,379	-0,129	0,01	0,016	0,021	0,09	0,996
3	1,9	A	-6,17	-3,472	-0,245	0,023	0,018	0,04	0,072	0,998
4	1,5	B	-5,853	-3,234	-0,19	0,011	0,016	0,022	0,139	0,991
5	1,7	B	-5,871	-3,262	-0,26	0,012	0,015	0,023	0,118	0,995
6	1,9	B	-5,968	-3,37	-0,189	0,026	0,017	0,047	0,092	0,997
7	1,5	C	-5,694	-3,27	-0,105	0,012	0,017	0,023	0,141	0,991
8	1,7	C	-5,722	-3,274	-0,14	0,012	0,015	0,022	0,116	0,994
9	1,9	C	-5,8	-3,327	-0,167	0,023	0,016	0,043	0,094	0,997

PW for Z=0.02 and Y=0.28

$$W = a + b \log P$$

	col1	col2	col3	col4	col5	col6	col7	col8	col9	col10
	Z	Y	Alfa	ML	a	b	sa	sb	sigma	R^2
1	0.02	0.28	1.5	A	-2.698	-3.300	0.035	0.029	0.110	0.995
2	0.02	0.28	1.7	A	-2.677	-3.370	0.028	0.025	0.070	0.998
3	0.02	0.28	1.9	A	-2.582	-3.605	0.014	0.034	0.013	1.000
4	0.02	0.28	1.5	B	-2.636	-3.205	0.041	0.029	0.132	0.993
5	0.02	0.28	1.7	B	-2.641	-3.218	0.039	0.028	0.110	0.996
6	0.02	0.28	1.9	B	-2.439	-3.585	0.028	0.069	0.020	0.999
7	0.02	0.28	1.5	C	-2.476	-3.225	0.049	0.034	0.142	0.992
8	0.02	0.28	1.7	C	-2.479	-3.249	0.036	0.026	0.092	0.997
9	0.02	0.28	1.9	C	-2.342	-3.493	0.028	0.045	0.026	0.999



2. **Apply** the theoretical PW/PWZ relations (selecting the assumptions on *alfa* and the *ML relation*) to the stars in the Gaia table to compute:

$$\text{distance modulus: } \mu = W_{obs} - W_{th}$$

Where

$$W_{obs} = G - 1.9 (G_{BP} - G_{RP})$$

$$W_{th} = a + b \log P_{obs} \text{ (neglecting metallicity dependence)}$$
$$W_{th} = a + b (\log P_{obs} - 1) + c [Fe/H] \text{ (including metallicity dependence)}$$

for each individual Cepheid



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3. **Derive** theoretical distances and parallaxes:

$$d(\text{pc}) = 10^{(\mu+5)/5}$$
$$\varpi_{th}(\text{mas}) = \frac{1000}{d}$$

4. **Compare** with Gaia astrometric parallax (column 8 in Gaia table).

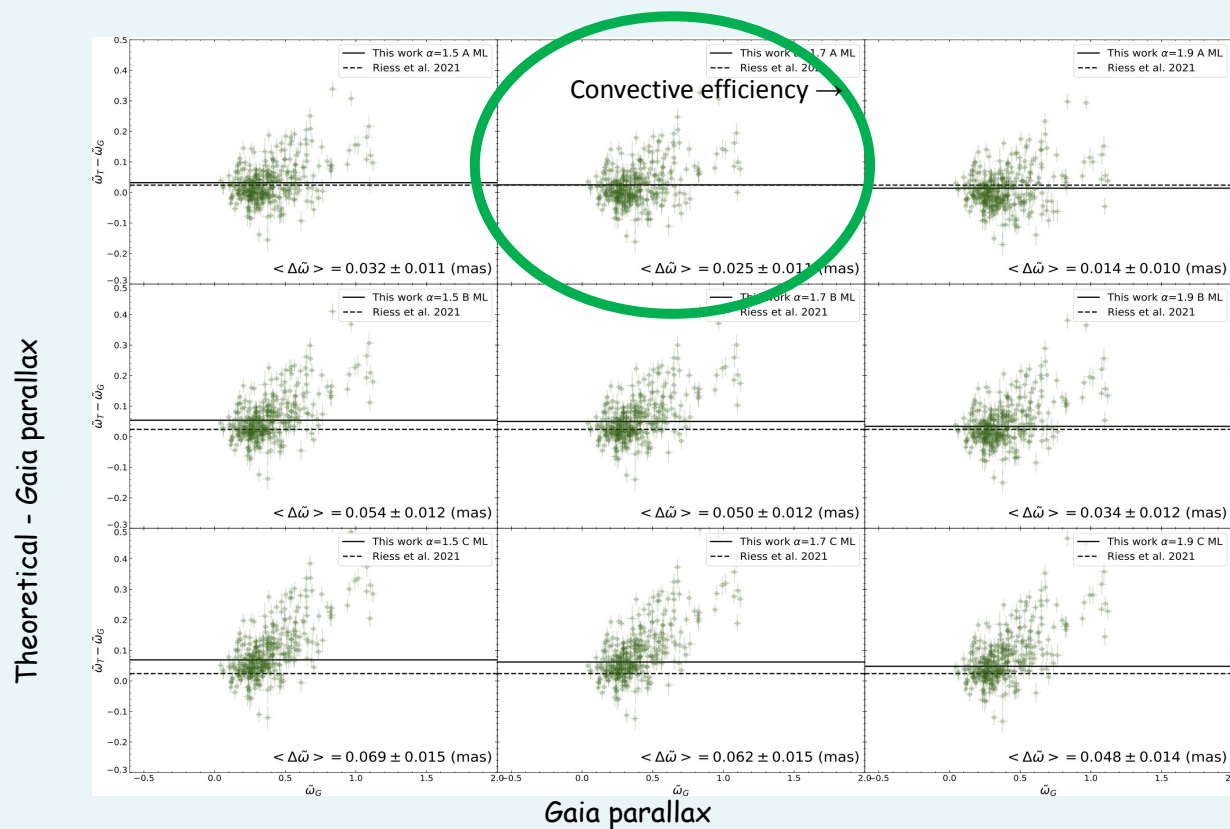
4. **Evaluate** the effects of the model inputs on the derived distance scale of:

- changing α (mixing-length parameter) (column 1 in PW PWZ tables)
 - changing ML relation (A/B/C) (column 2 in PW PWZ)
- including or excluding metallicity ([Fe/H]) (i.e. using PWZ or PW)



Theoretical versus Gaia Parallaxes for Galactic Cepheids: the Effect of the ML Relation

Theoretical parallaxes were obtained by applying the PWZ: $W = a + b \log P + c [\text{Fe}/\text{H}]$



$$\frac{\Delta \pi}{\pi} \sim \frac{\Delta H_0}{H_0}$$

- Riess et al. 2021
- De Somma et al. 2022

Best Match:
Can ML relation
 $\alpha = 1.7$

De Somma et al. 2022, ApJS

Brighter/fainter ML relation implies a shorter/longer distance scale → increase/decrease of H_0