

Multi-frequency Gaussian modeling of AGN jets with the extended Korean VLBI Network

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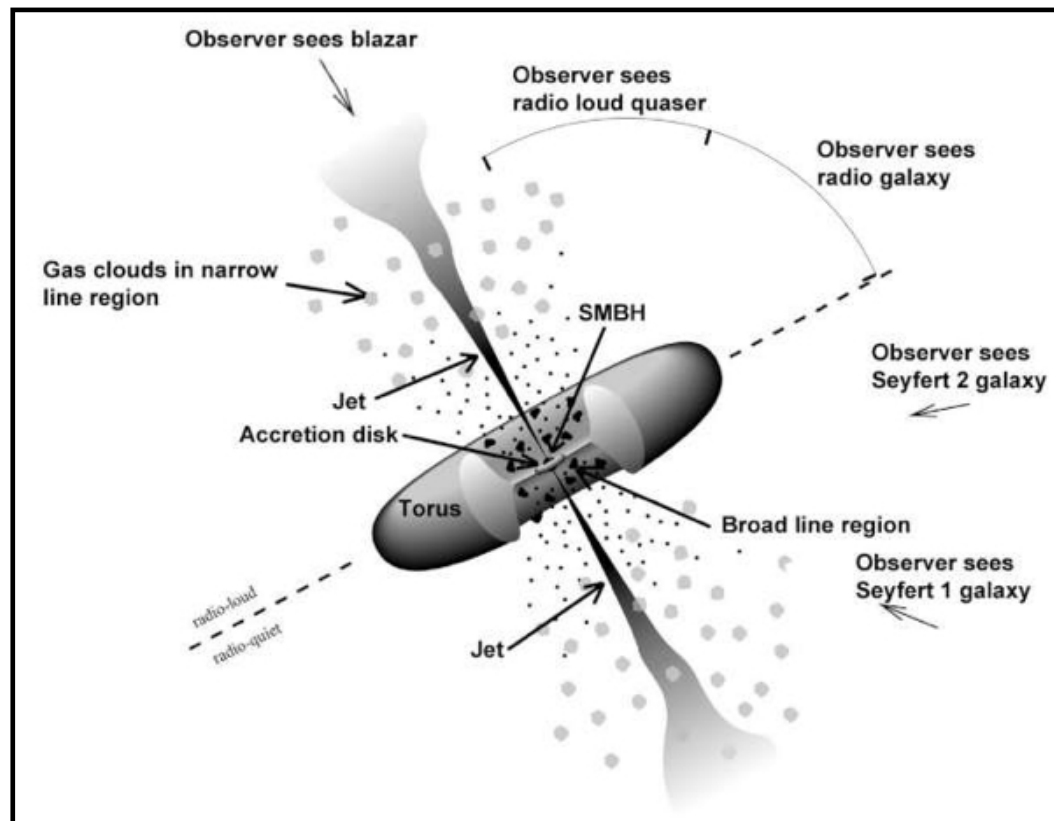
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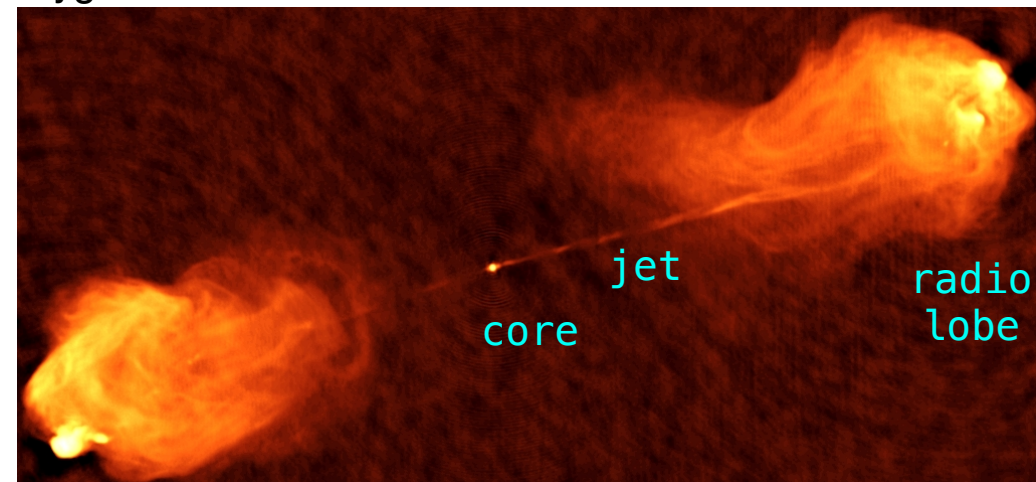


Jets from AGNs



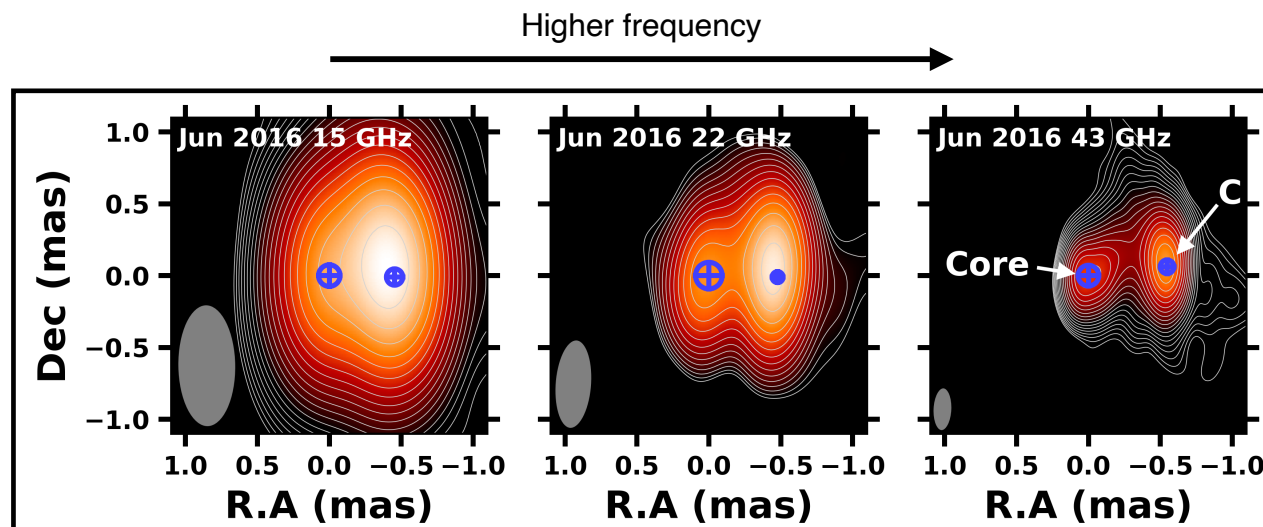
Mass accretion into the supermassive black hole and ejection of plasma population with relativistic speed

Cygnus A



Jets from AGNs

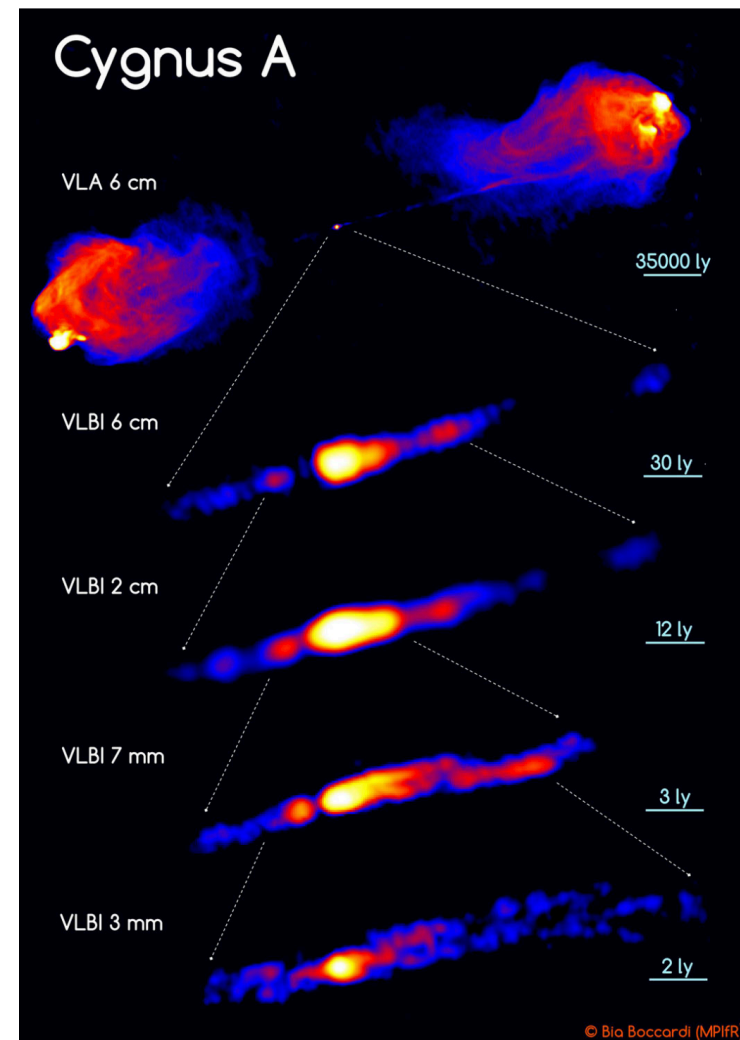
Jet are **resolved** in VLBI observations
 into several knot-features
 in **VLBI** observations
 &
 when operated **at high frequencies**



VLBA images of 3C 454.3 at 15–43 GHz

Jeong+2023

*mas: milli-arcsecond

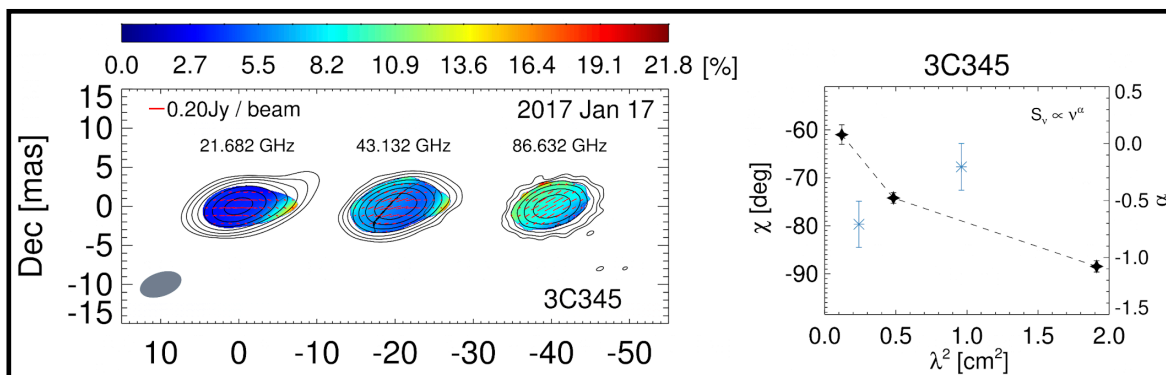


Boccardi+2017 (Rv)

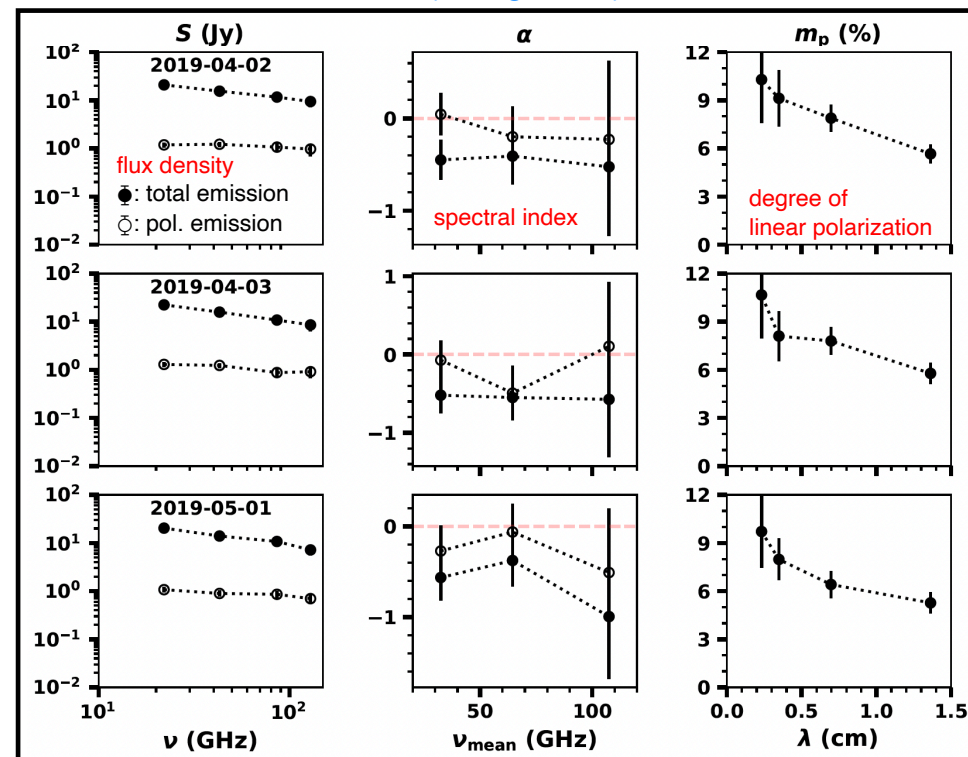
Jets from AGNs

Jet are **resolved** in VLBI observations
into several knot-features
in **VLBI** observations
&
when operated **at high frequencies**

Mult-frequency observations
provide valuable information of jets
(e.g., spectral index, Faraday rotation measure)

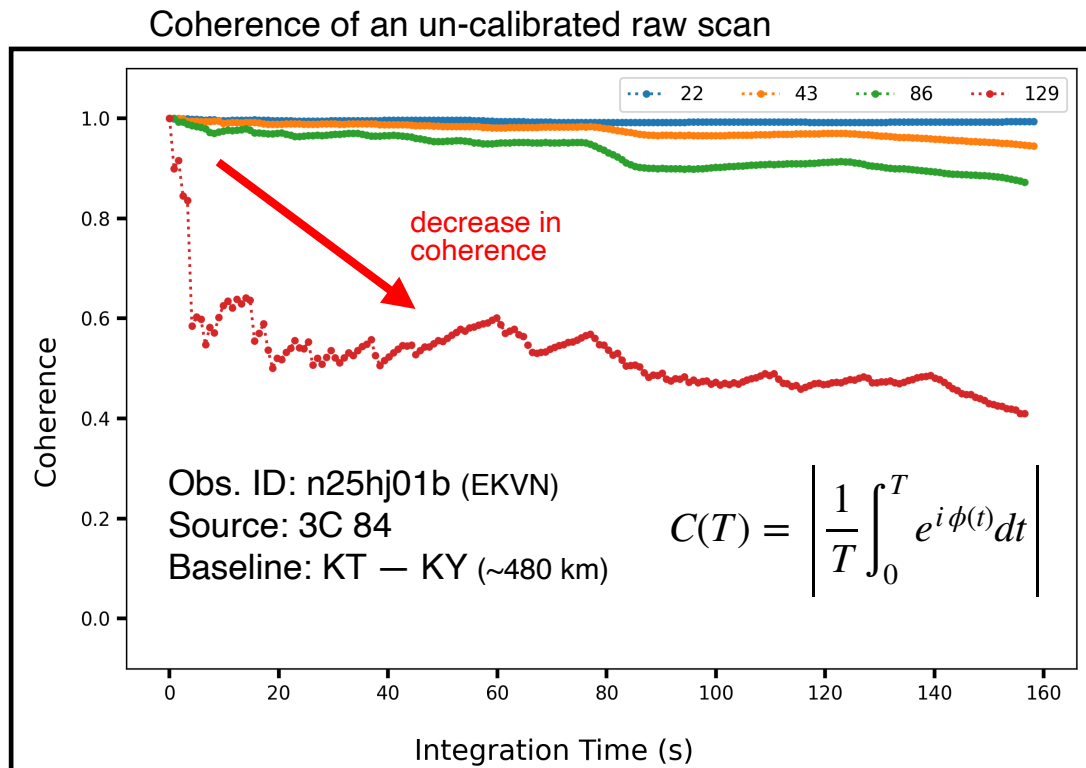


Multi-frequency monitoring of KVN single-dish ([MOGABA; Kang+2015](#))
on 3C 454.3 at 22—129 GHz ([Jeong+2025](#))

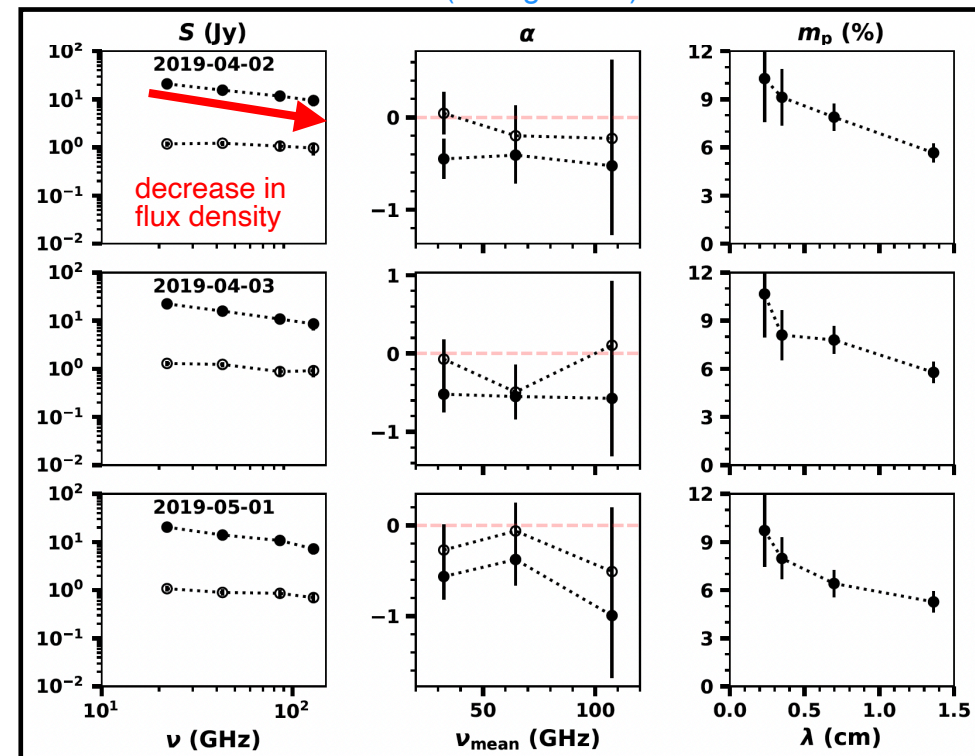


Why multi-frequency?

High-frequency observations are challenging due to **atmospheric fluctuations**, **decreasing coherence**, and low brightness of AGN jets



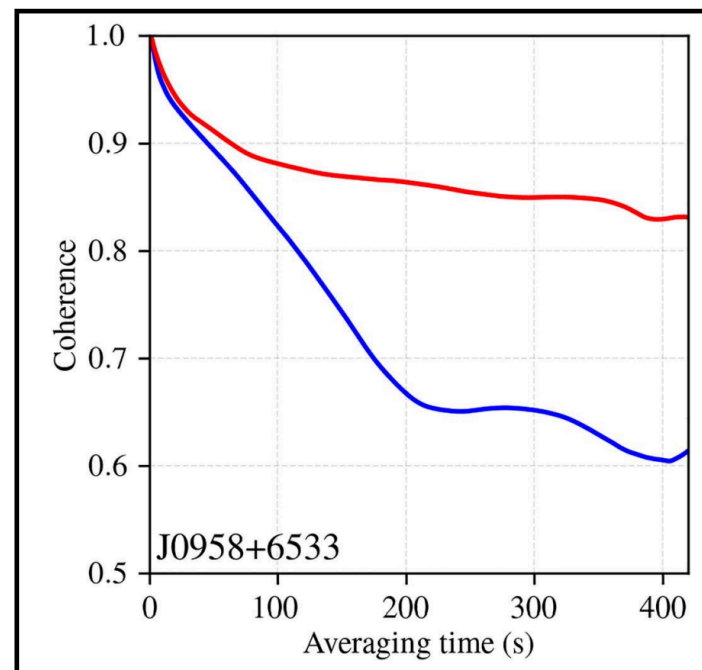
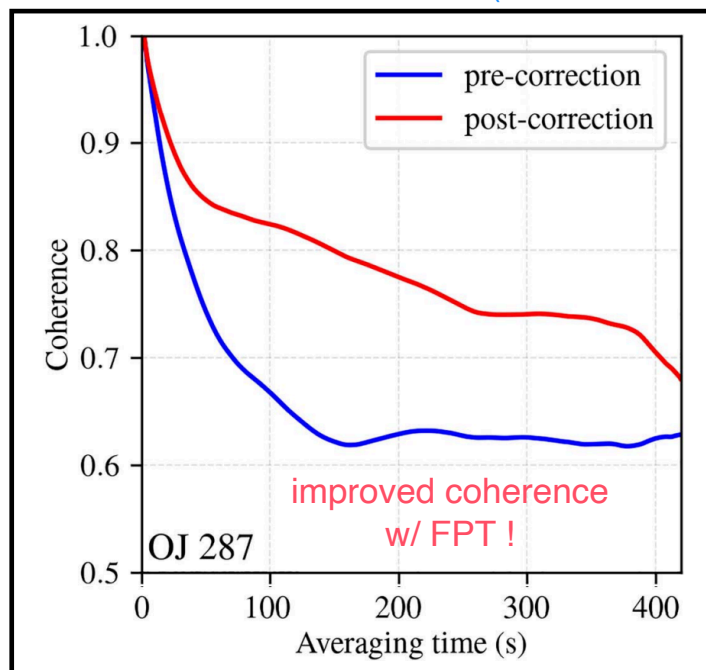
Multi-frequency monitoring of KVN single-dish (MOGABA; Kang+2015) on 3C 454.3 at 22—129 GHz (Jeong+2025)



Why multi-frequency?

To improve coherence at high frequencies,
truly simultaneous multi-frequency capability is required
(i.e., **frequency phase transfer; FPT**; [Rioja+2015](#))

Phase coherence at 215 GHz ([Issaoun+2025](#))

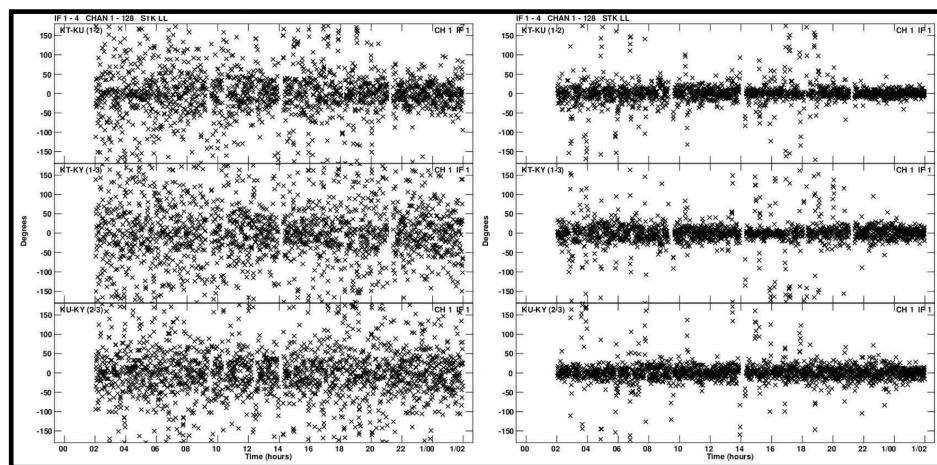


Extended Korean VLBI Network (EKVN)

KVN telescopes are equipped with simultaneous multi-frequency receiver
(typically, 22 – 129 GHz)

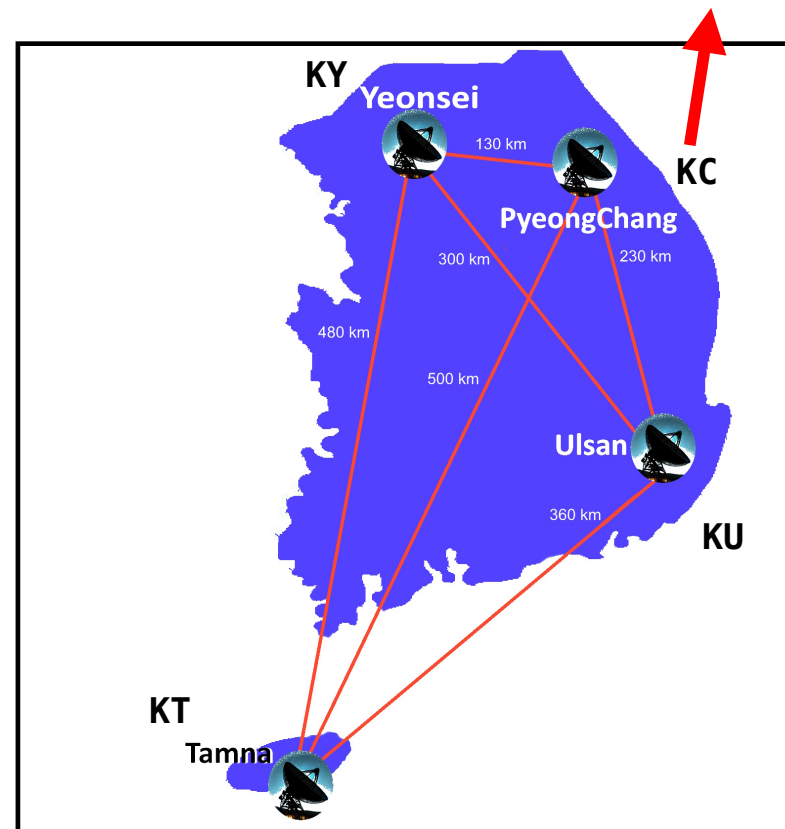
w/o FPT

w/ FPT



phase residuals, [Hodgson+2016](#)

A new station of the KVN at PyeongChang
(KPC / KC)



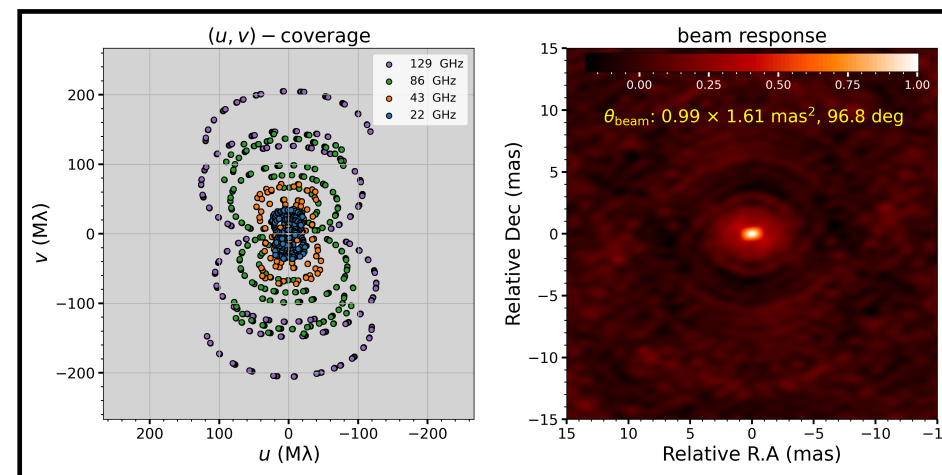
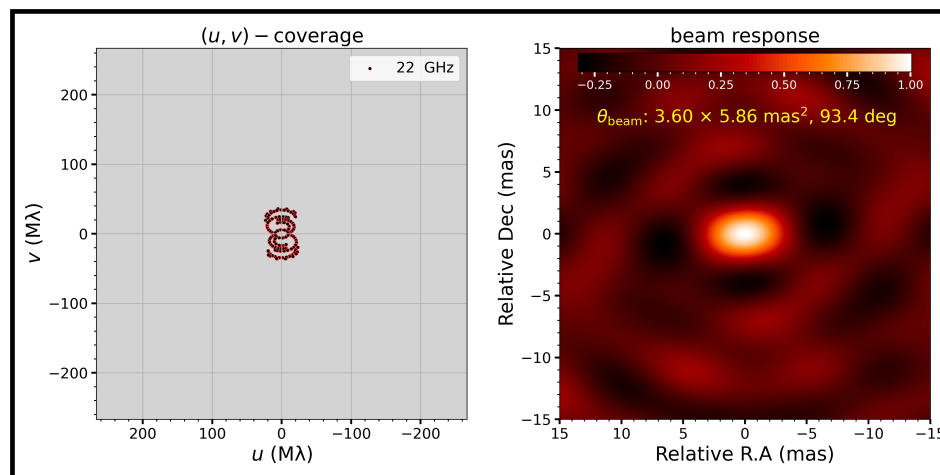
Multi-frequency synthesis (MFS)

Multi-frequency receiver enables the **MFS technique**

Through MFS, one can perform

1. **high angular resolution** analysis,
2. with the **increased sensitivity**
3. and the improved information through the combined **dense (u,v)-coverage**

* Data from the EKVN (3C 111, 2025-03-18)



Gaussian multi-frequency modeling

Using the MFS and the assumptions below,
**we developed a new modeling script,
Gaussian Multi-frequency VLBI Analyses (GaMVAs)*,**
for AGN jets

*
based on the python package,
DYNesty ([Speagle 2020](#))

1. Jet morphology can be described
by multiple 2D circular Gaussian model components
2. Common position of the Gaussian model components
across frequencies
 - valid for optically thin jets
 - may not be valid for optically thick core
(e.g., core-shift effect ≤ 0.1 mas, at 22–43 GHz; [Pushkarev+2019](#), [Chamani+2023](#), [Benke+2025](#))
 - nevertheless, may be valid for those in the EKVN images
($B_{\min,129} \sim 0.6$ mas)

Gaussian multi-frequency modeling // complex visibility of a multi-frequency Gaussian

1. Complex visibility in a baseline : $\mathcal{V}(u, v) = \iint I(l, m) e^{-2\pi i(ul+vm)} dl dm$

→ circular Gaussian : $\mathcal{V}_m(u, v; \theta') = S e^{-2\pi a^2(u^2+v^2)} e^{2\pi i(ul_0+vm_0)}$
(single frequency)

$$\theta' = (S, a, l_0, m_0)$$

S : flux density [Jy]

a : angular size [mas]

l_0 : right ascension offset [mas]

m_0 : declination offset [mas]

2. Synchrotron self-absorption :
(Turler+1999)

$$S(\nu; S_m, \nu_m, \alpha) = S_m \left(\frac{\nu}{\nu_m} \right)^{2.5} \times \frac{1 - \exp(-\tau_m(\nu/\nu_m)^{\alpha-2.5})}{1 - e^{-\tau_m}}$$

$$\theta = (S, a, l_0, m_0, \alpha, \nu_m)$$

3. Multi-frequency Gaussian :

$$\mathcal{V}_m(u_\nu, v_\nu, \nu; \theta) = S(\nu; S_m, \nu_m, \alpha) \times e^{-2\pi a^2(u_\nu^2+v_\nu^2)} e^{2\pi i(u_\nu l_0+v_\nu m_0)}$$

α : spectral index

ν_m : turnover frequency [GHz]

ν : observing frequency [GHz]

Gaussian multi-frequency modeling // antenna-gain-free closure relations

1. Observed visibility :

$$\widetilde{\mathcal{V}}_{12} \equiv \widetilde{A}_{12} e^{i\widetilde{\phi}_{12}} \approx A_{g1} A_{g2} A_{12} e^{i(\phi_{12} + \phi_{g1} - \phi_{g2})} + n_t$$

A_{ij} : visibility amplitude [Jy]

ϕ_{ij} : visibility phase [rad]

n_t : random thermal noise

$\mathcal{G}_1 := A_{g1} e^{i\phi_{g1}}$: complex gain of antenna 1

2. Closure amplitude :

$$\begin{aligned} \mathcal{A}_{1234} &= \frac{|\widetilde{\mathcal{V}}_{12}| |\widetilde{\mathcal{V}}_{34}|}{|\widetilde{\mathcal{V}}_{13}| |\widetilde{\mathcal{V}}_{24}|} \\ &\approx \frac{|A_{g1} A_{g2}| |A_{g3} A_{g4}|}{|A_{g1} A_{g3}| |A_{g2} A_{g4}|} \frac{|A_{12}| |A_{34}|}{|A_{13}| |A_{24}|} = \frac{A_{12} A_{34}}{A_{13} A_{24}} \end{aligned}$$

3. Closure phase :

$$\begin{aligned} \psi_{123} &= \widetilde{\phi}_{12} + \widetilde{\phi}_{23} + \widetilde{\phi}_{31} \\ &\approx (\phi_{12} + \phi_{g1} - \phi_{g2}) \\ &\quad + (\phi_{23} + \phi_{g2} - \phi_{g3}) \\ &\quad + (\phi_{31} + \phi_{g3} - \phi_{g1}) = \phi_{12} + \phi_{23} + \phi_{31} \end{aligned}$$

Gaussian multi-frequency modeling // objective function, optimal number of models

1. Modeling script aims to minimize **Bayesian information criteria (BIC)**

$$\text{BIC} = k \ln(n) - 2 \ln(\mathcal{L})$$

k : degree of freedom

n : the number of data points

2. Likelihood \mathcal{L} with independent normal Gaussian noise is given by

$$\mathcal{L} = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp \left(-\frac{1}{2} \left(\frac{\text{model}_i - \text{data}_i}{\sigma_i} \right)^2 \right) \quad \sigma_i : \text{uncertainty of } i^{\text{th}} \text{ data}$$

then,

$$\text{BIC} = k \ln(n) + \sum_{i=1}^n \left[\left(\frac{\text{model}_i - \text{data}_i}{\sigma_i} \right)^2 + \ln(2\pi\sigma_i^2) \right]$$

Observation // n25hj01a

Obs. date: 2025 02-10 06:00 – 02-11 06:00 (24h)

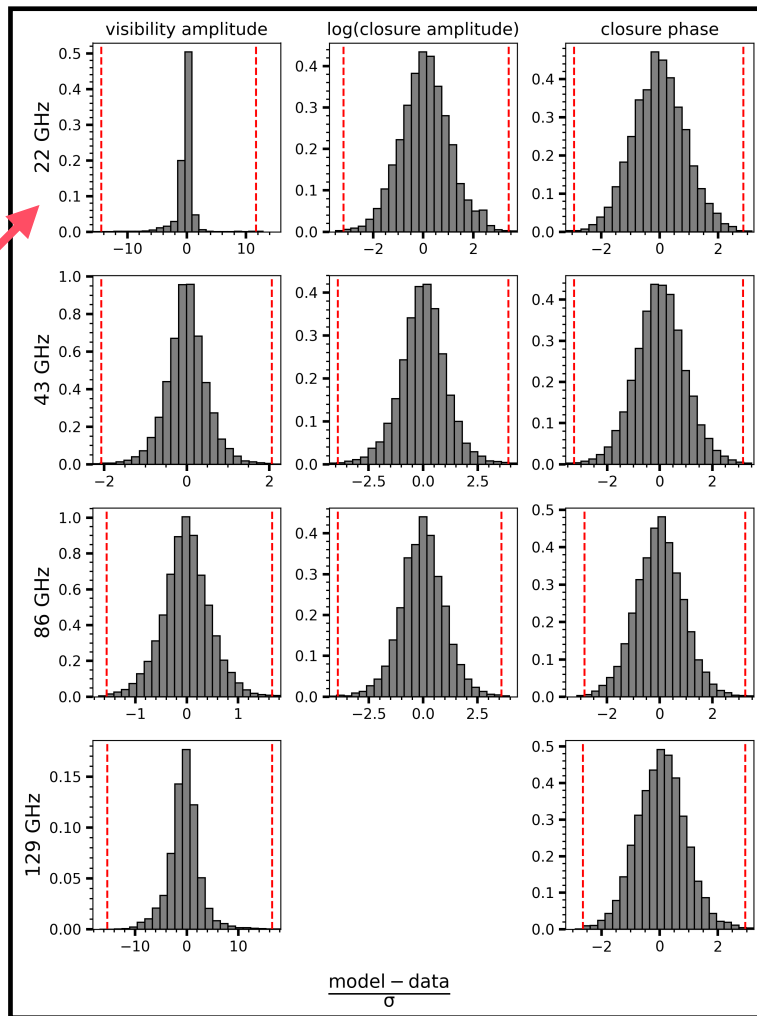
Bands: 22 / 43/ 86 / 129 GHz

Sources: 21 AGNs

0235+164	1226+023 (3C 273)
0316+413 (3C 84)	1228+126 (M 87)
0415+379 (3C 111)	1253-055 (3C 279)
0420-014	1510-089
0430+052 (3C 120)	1553+113
0446+112	1641+399 (3C 345)
0735+178	1652+398 (Mrk 501)
0851+202 (OJ 287)	1749+096
0954+658	1807+698 (3C 371)
1156+295 (Ton 599)	2200+420 (BL Lac)
	2251+158 (3C 454.3)

Broad distribution
in vis. amplitude
caused by technical
issue at 22 GHz
during the observation

* red vertical lines:
99% level



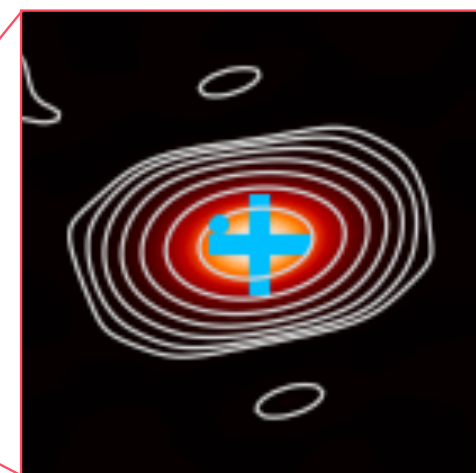
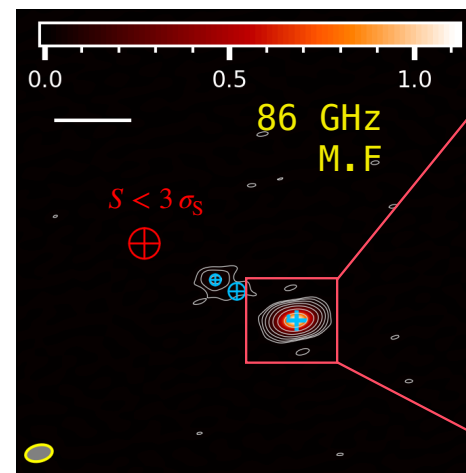
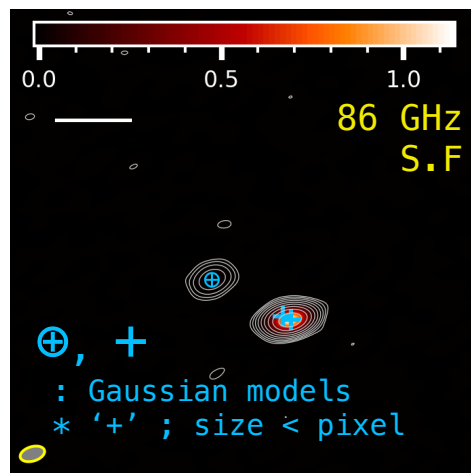
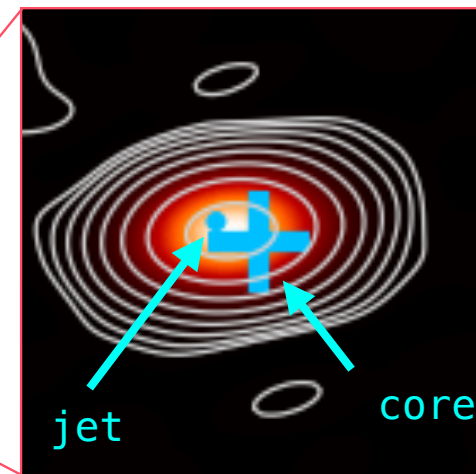
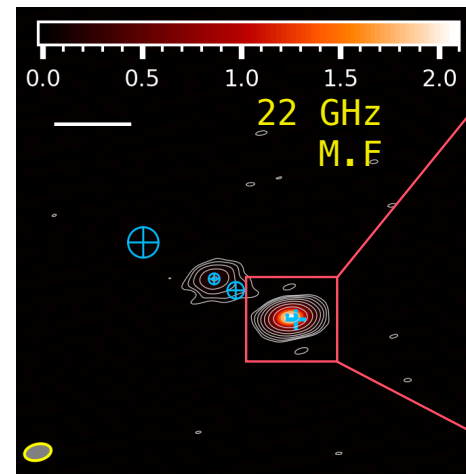
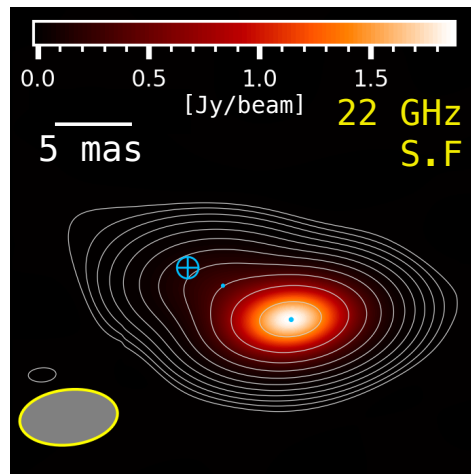
Normalized residual distribution
obtained from all modeling results,
except for 3C 84

Modeling results

Representative sources: **3C 111** & 3C 345

$$\nu_{\text{m,core}} = 40.4 \pm 2.4 \text{ GHz}$$

$$\alpha = -0.58 \pm 0.05$$



*
residual noise maps
are obtained from
DFT of
residual visibility

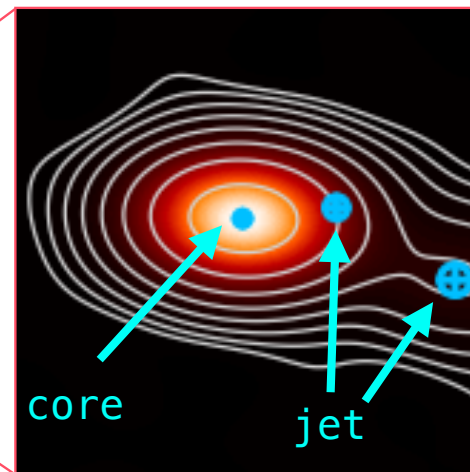
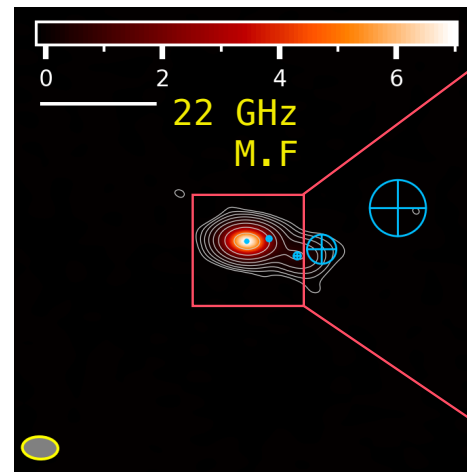
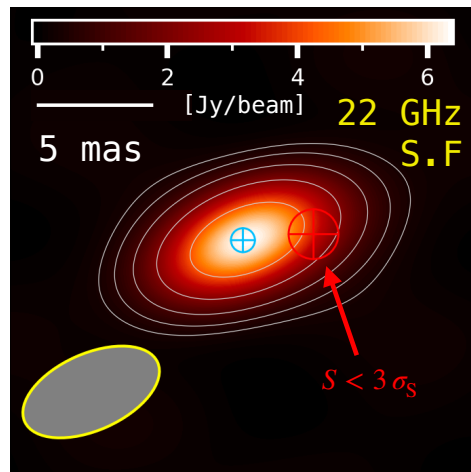
Modeling results

Representative sources: 3C 111 & 3C 345

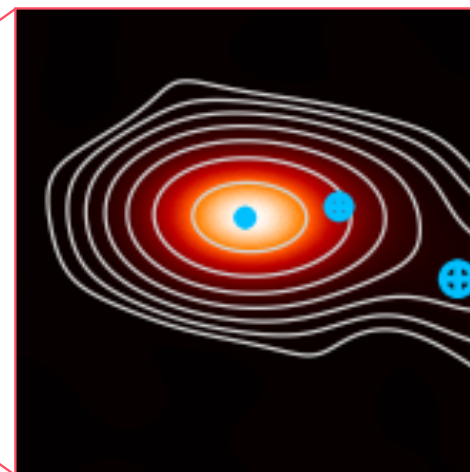
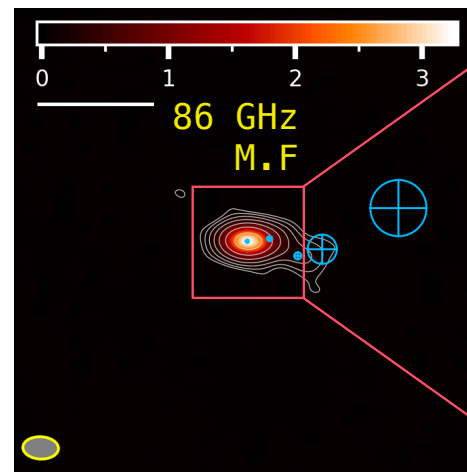
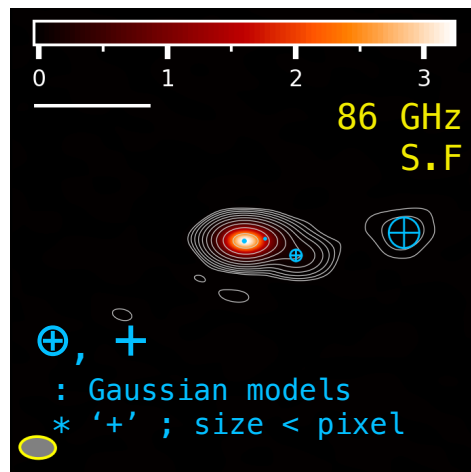
$$\nu_{m,core} = 23.2 \pm 0.4 \text{ GHz}$$

$$\alpha = -0.73 \pm 0.01$$

**
for this source,
there are only records
of **three stations**
at 22 GHz
(technical issue at KPC)

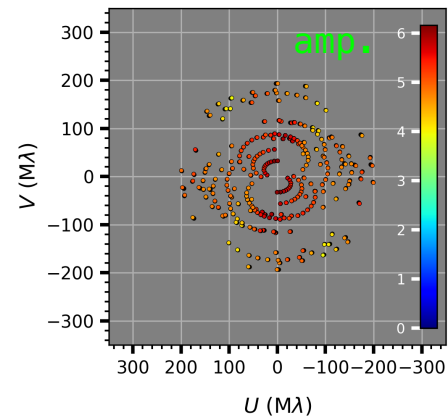
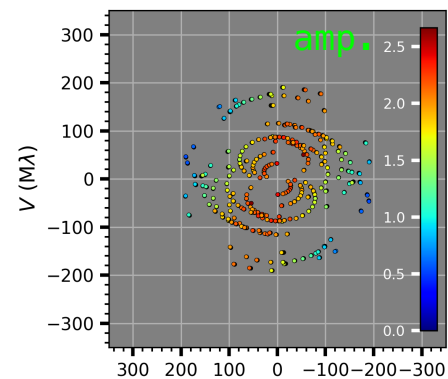


*
residual noise maps
are obtained from
DFT of
residual visibility

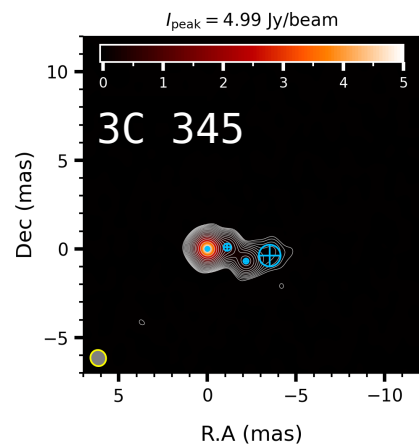
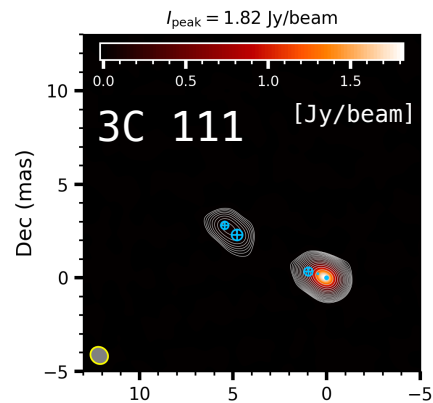


Compare with uv-radius-limited VLBA at 43 GHz

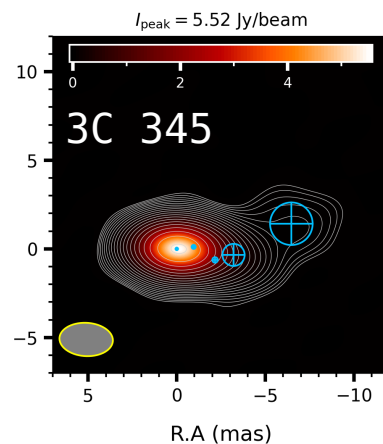
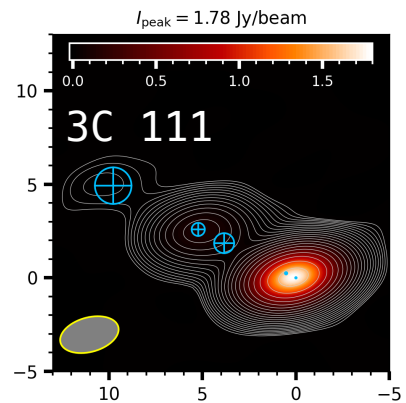
(u, v)-coverage
($\leq 200 \text{ M}\lambda$, VLBA)



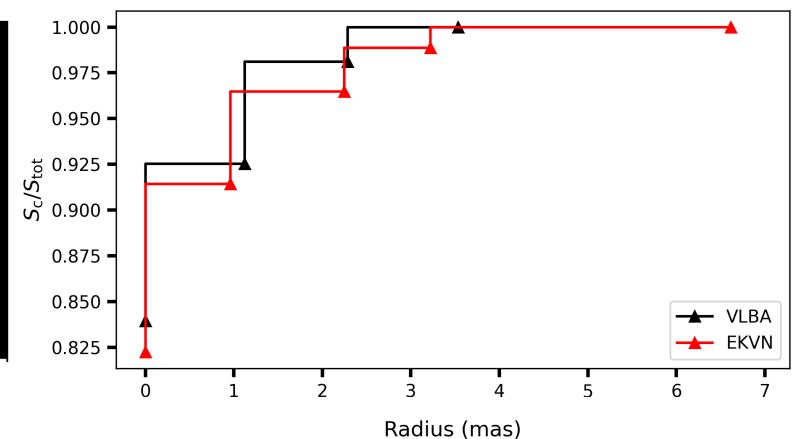
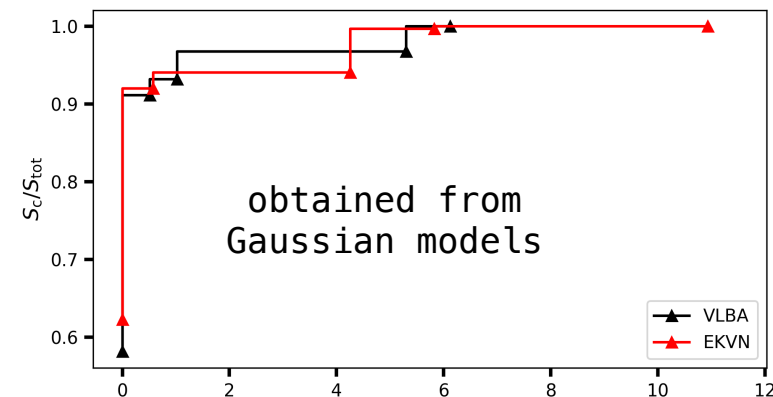
VLBA
(2025-02-16)



EKVN
(2025-02-10)

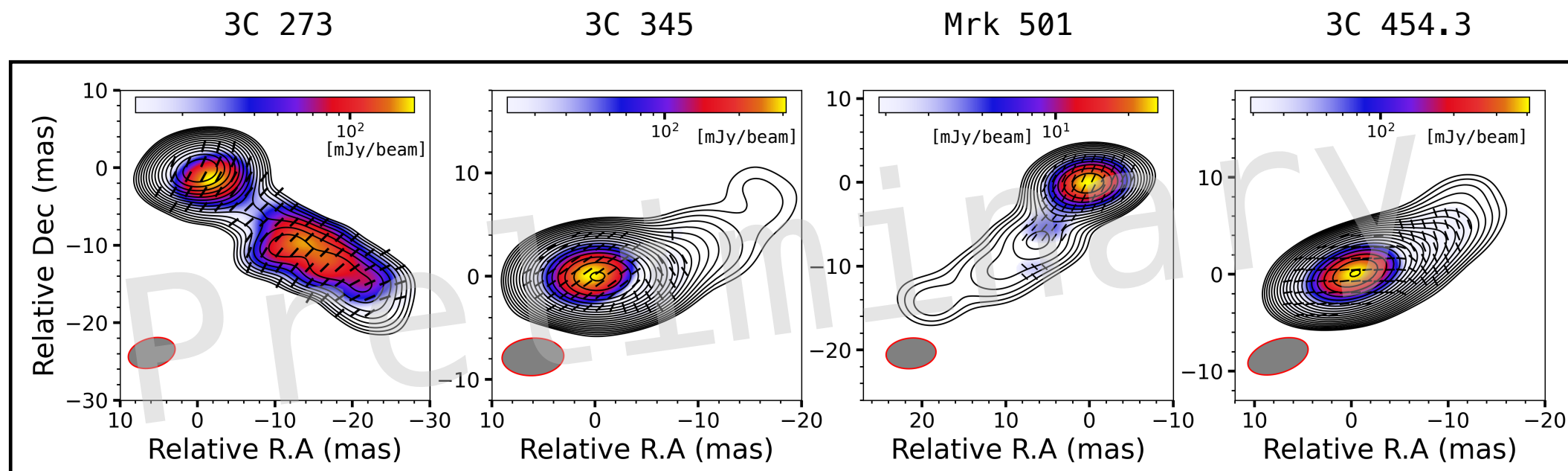


normalized cumulative
flux density
(43 GHz)



Reconstruction on extended jet (22 GHz, n25hj01b, 2025-03-18)

The short baseline length of the KC station enables detection on **large-scale jets** & its polarization



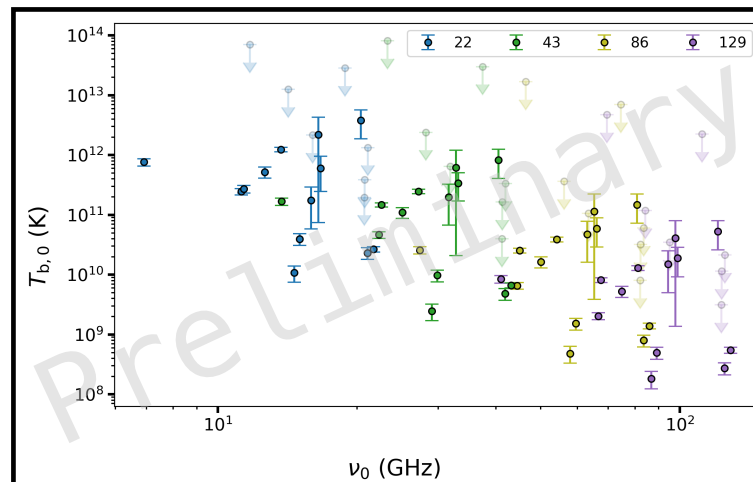
*NOTE: obtained from CLEAN images

Further scientific analysis?

Physical evolution along the jet

e.g.,

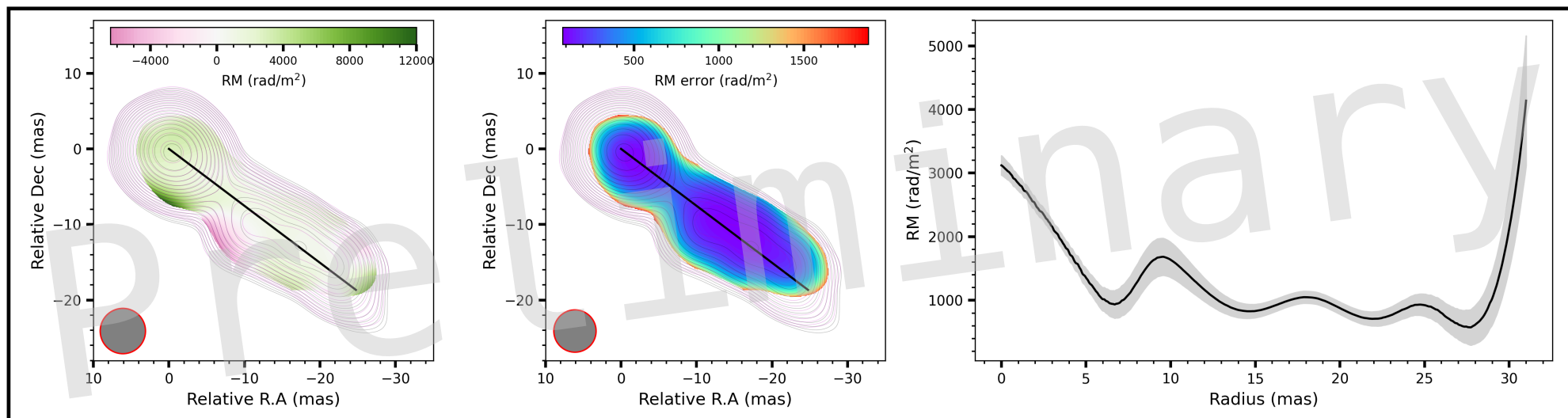
brightness temperature,
spectral index,
Faraday rotation measure,
...



observed
brightness temperature
of cores
(source rest frame)

$$* \nu_0 = \nu_{\text{obs}} / (1 + z)$$

Faraday rotation measure of 3C 273 along jet (19 – 22 GHz, n25hj02b, 2025-03-18)



*NOTE: obtained from CLEAN images

Summary

- Truly simultaneous multi-frequency receiver is crucial for increasing high-frequency coherence
- MFS technique enables us to use higher angular resolution and sensitivity, resolving inner jet morphology within the EKVN resolution
- Our modeling scheme using the GaMVAs yields acceptable statistics and comparable modeling results with uv-limited VLBA

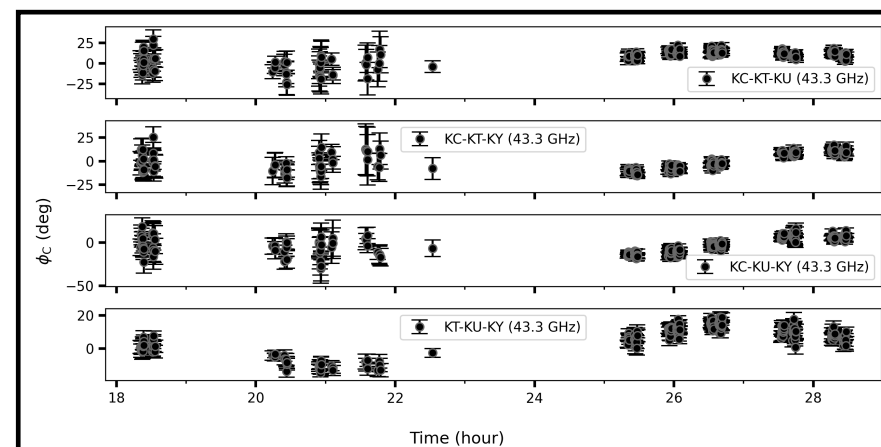
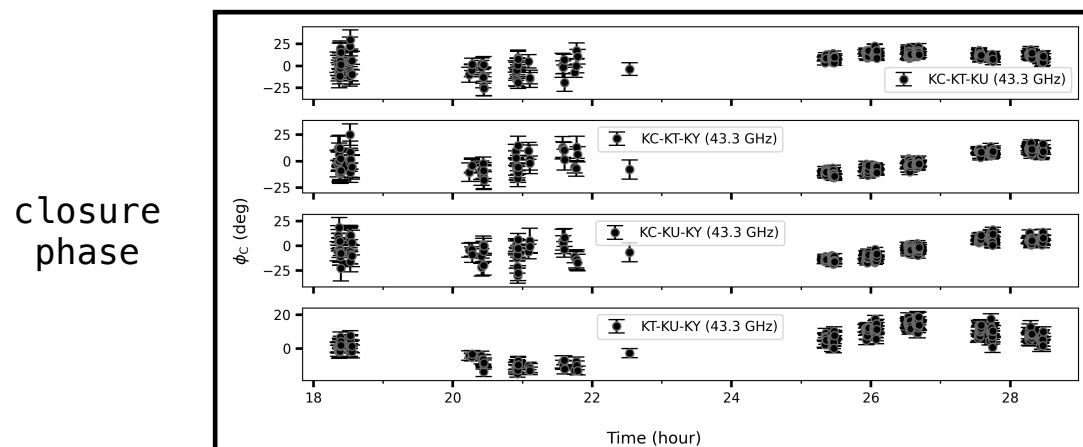
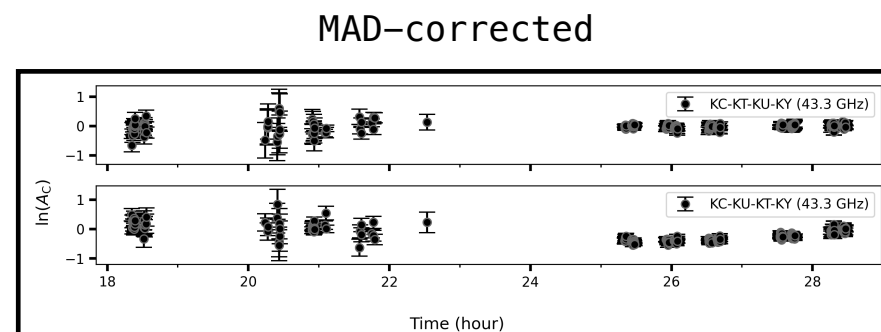
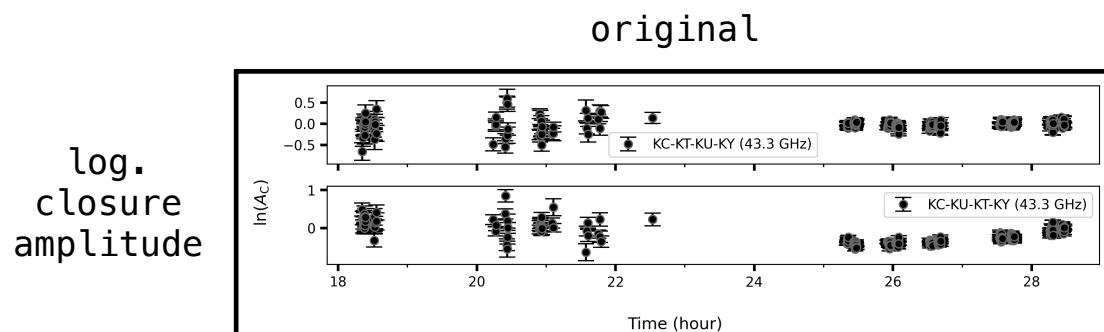
Thank you

Supplementary

Systematics // median absolute deviation (MAD), Mrk 501 @ 43 GHz (supplementary)

assumption: normally-distributed closure relations (SNR > 3)

$$\text{mad}(Y) = 1.483 \text{ med}(|Y|) = 1.483 \text{ med}\left(\frac{|X - X_{\text{med}}|}{\sigma}\right) = 1$$

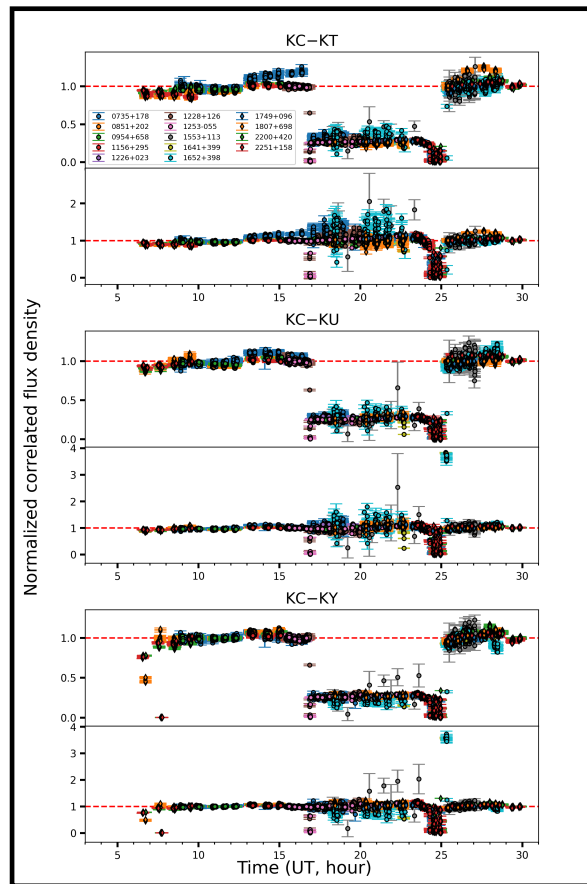


Observation // n25hj01a, technical issue (supplementary)

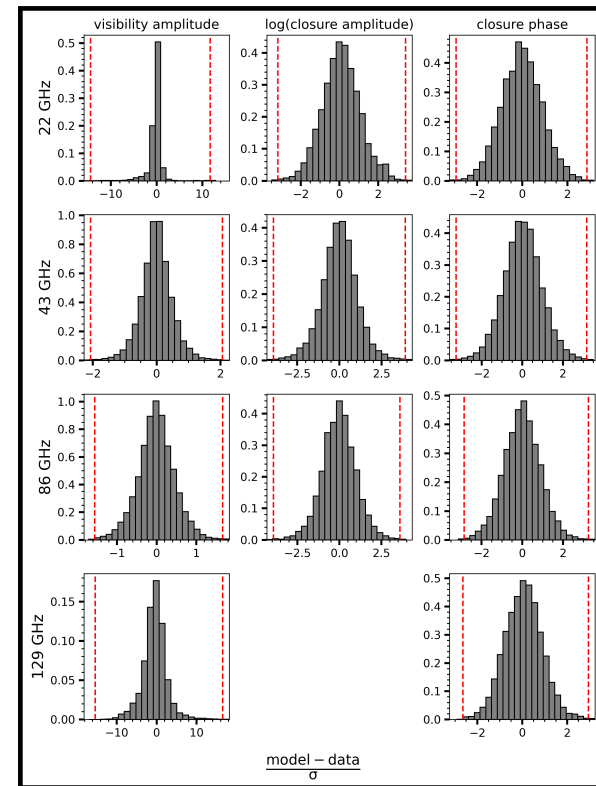
probably induced by clock synchronization

KC	KT	KU	KY
PPSGAP @ K-band (+ Q-band, unknown) 00-16:53:18 — 01-00:57:42	PPSGAP @ K-band 01-00:19:47 — 01-01:18:44	clear	late start due to freq. setup 00:06:52:00 —

For KC-baselines,
no-fringe @ K-band
weaker fringe @ Q-band



Normalized vis. amplitude
of KC-baselines
upper: w/o correction
lower: w/ correction

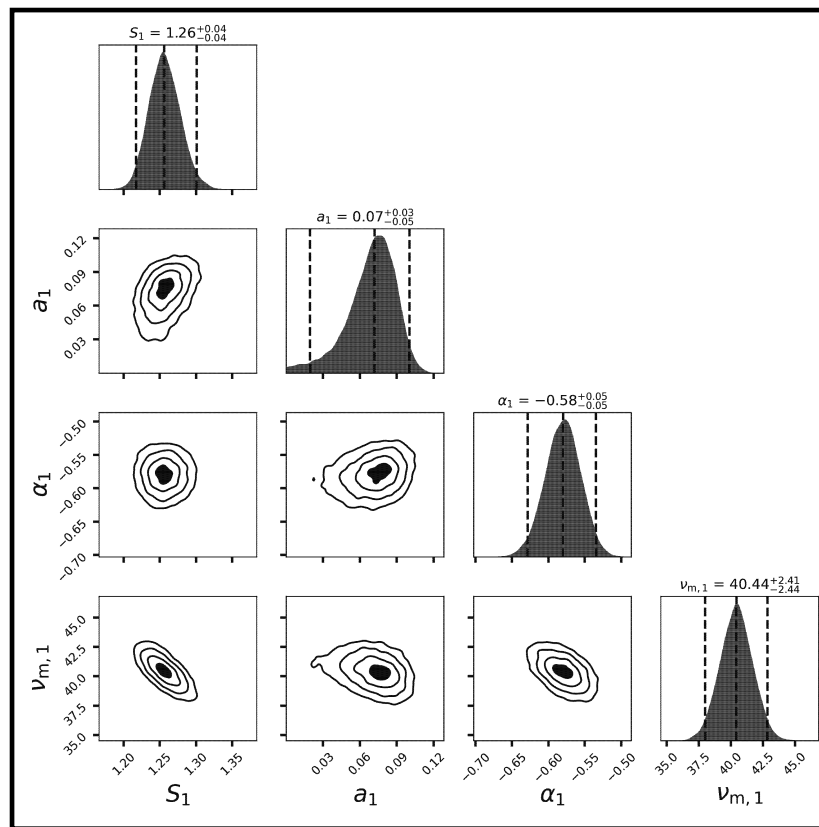


Broader normalized residual
in logarithmic closure amplitude
→ affected by the systematics

Error estimation from posterior distributions // (supplementary)

* NOTE:
first model
is fixed to (0,0)
in the map

3C 111
(core comp.)



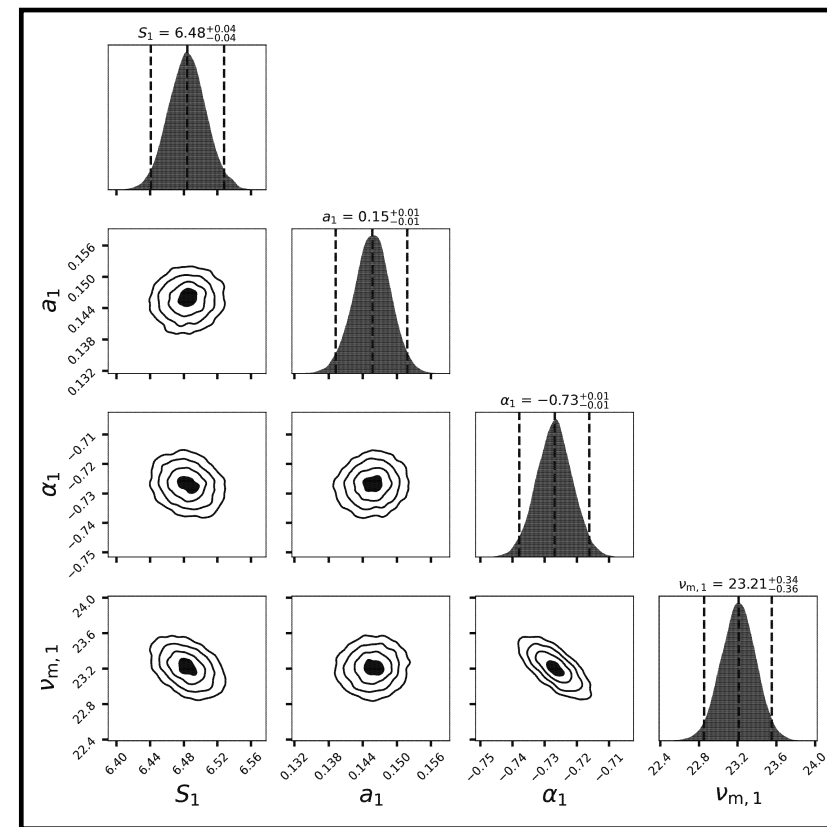
flux density
(Jy)

size
(mas)

spectral
index
($S \propto \nu^\alpha$)

turnover
freq.
(GHz)

3C 345
(core comp.)



flux density
(Jy)

size
(mas)

spectral
index
($S \propto \nu^\alpha$)

turnover
freq.
(GHz)

Pros and cons // (supplementary)

Pros

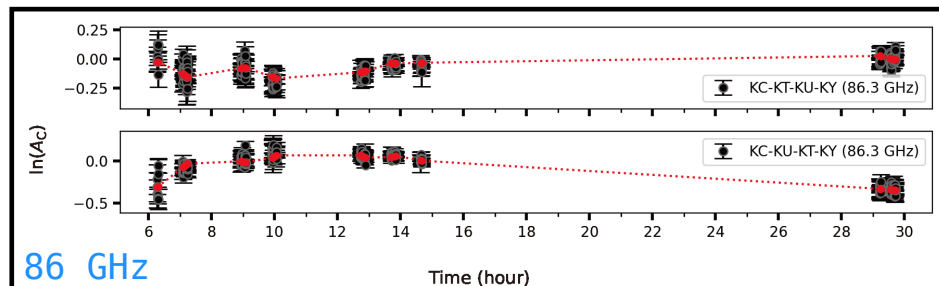
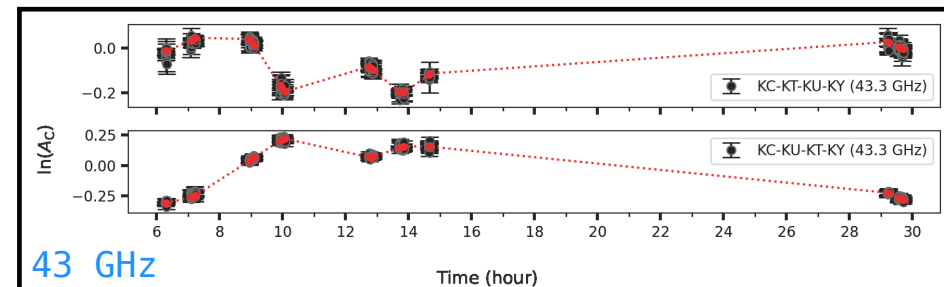
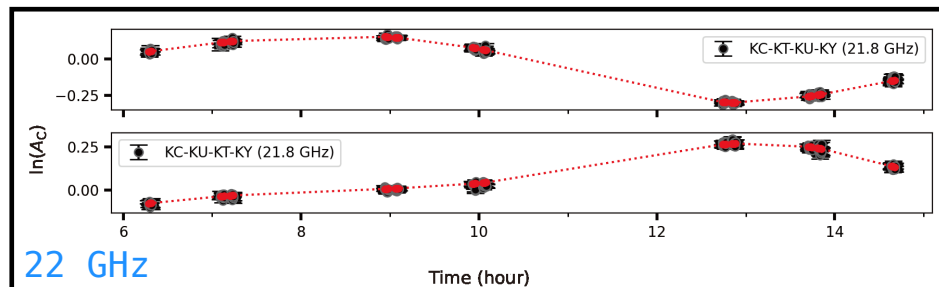
- Dense sampling on (u,v) -plane through the MFS technique
- Antenna-gain-independent closure quantities during modeling, which are not supported in DIFMAP
- Direct error estimation from posterior distribution, provided by the Bayesian modeling
- Characteristic physical parameters from Gaussian modeling

Cons

- Lack of deeper consideration to the core-shift effect
- Inflexibility in spectrum model (e.g., free-free absorption or synchrotron aging)
- Insufficient criteria for the goodness of the model fitting
- Expensive computational cost

Modeling results // closure relations (supplementary)

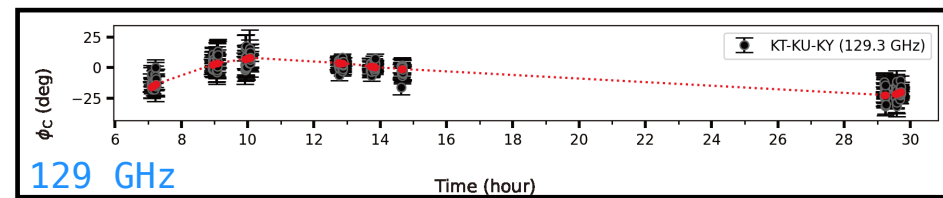
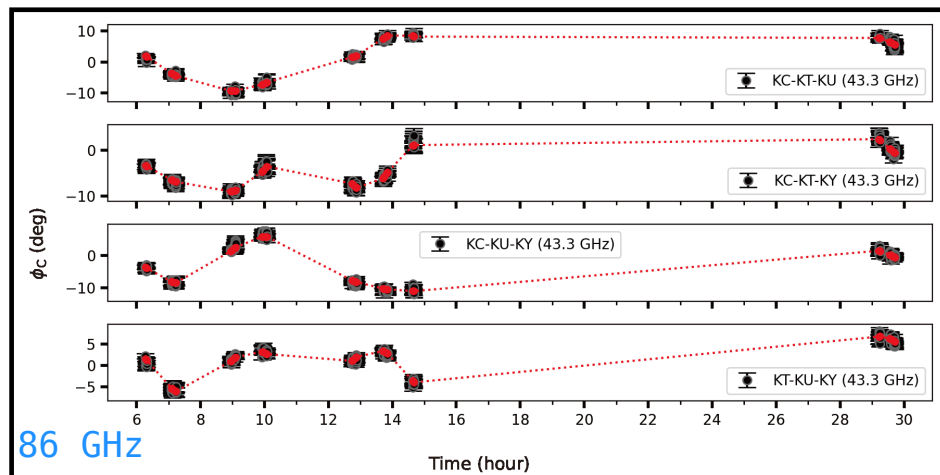
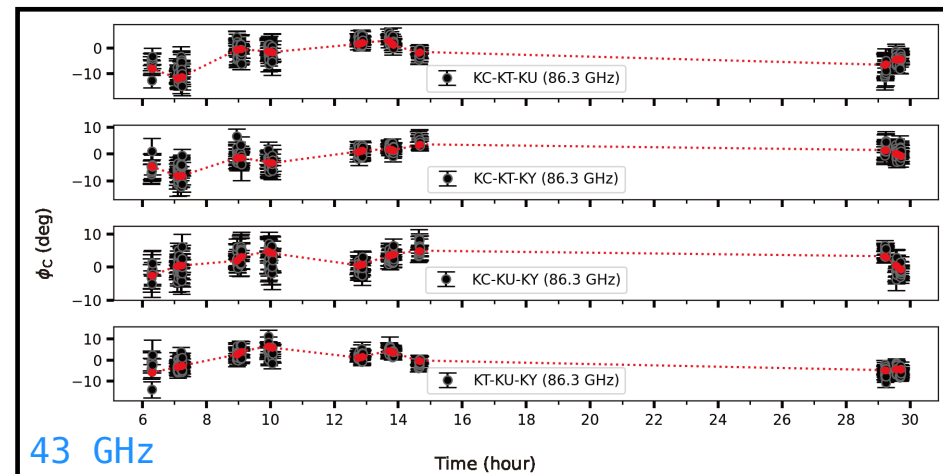
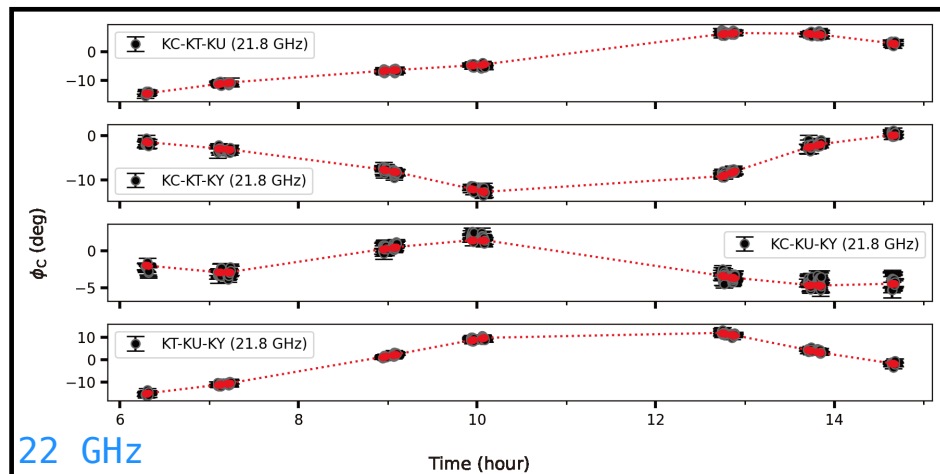
Source: 3C 111 (logarithmic closure amplitude)



129 GHz: None of Cl. amp
(only 3 stations)

Modeling results // closure relations (supplementary)

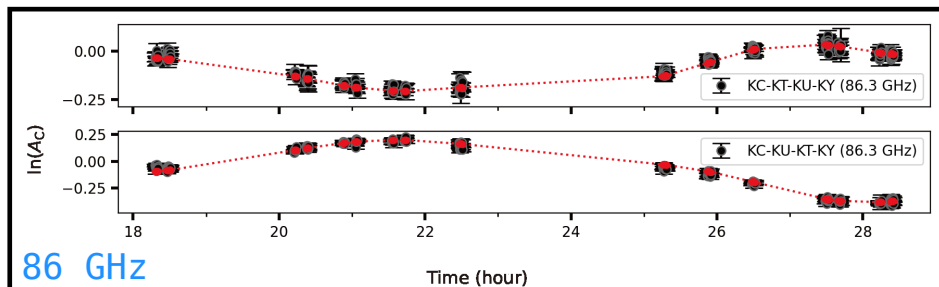
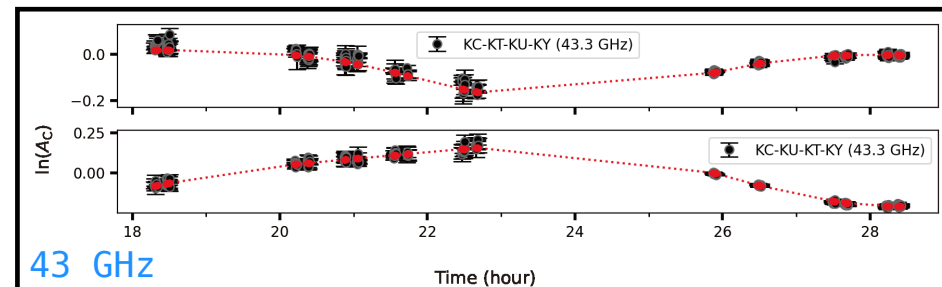
Source: 3C 111 (closure phase)



Modeling results // closure relations (supplementary)

Source: 3C 345 (logarithmic closure amplitude)

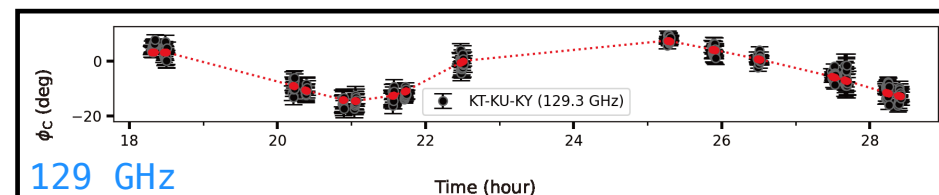
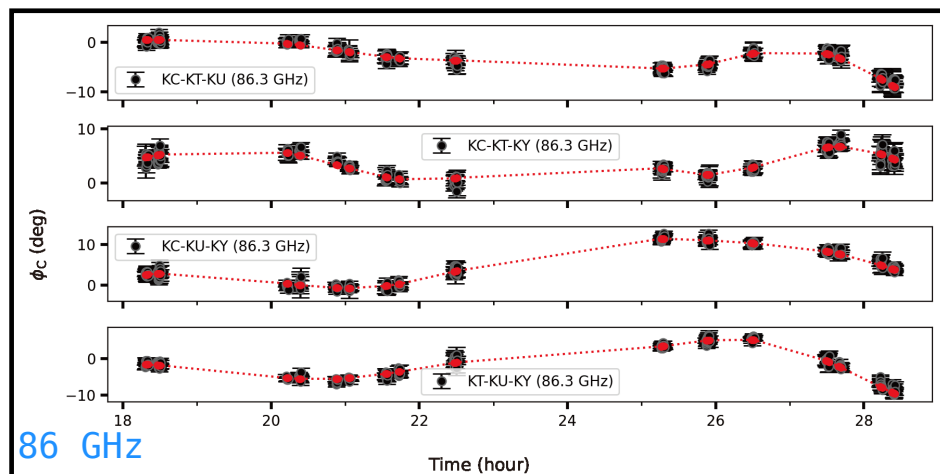
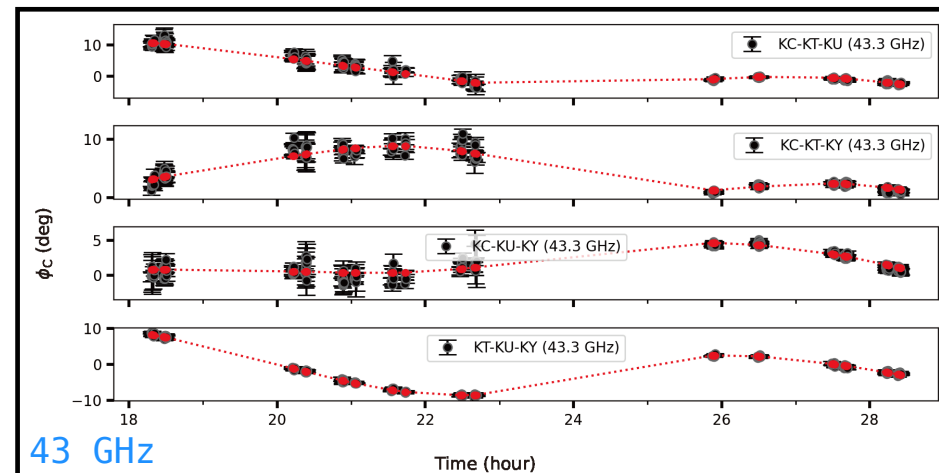
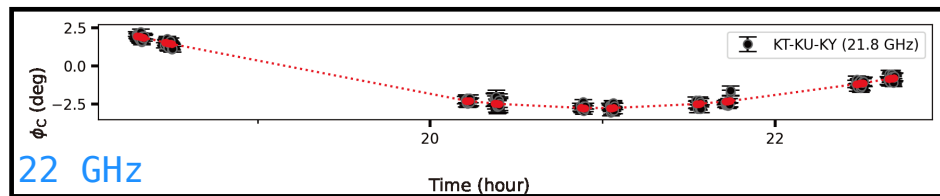
22 GHz: None of Cl. amp
(only 3 stations)



129 GHz: None of Cl. amp
(only 3 stations)

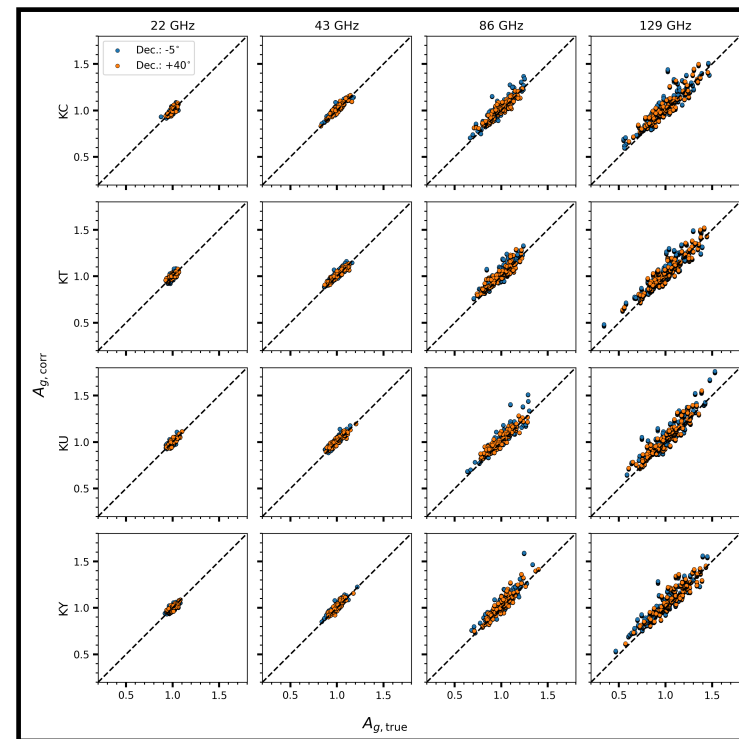
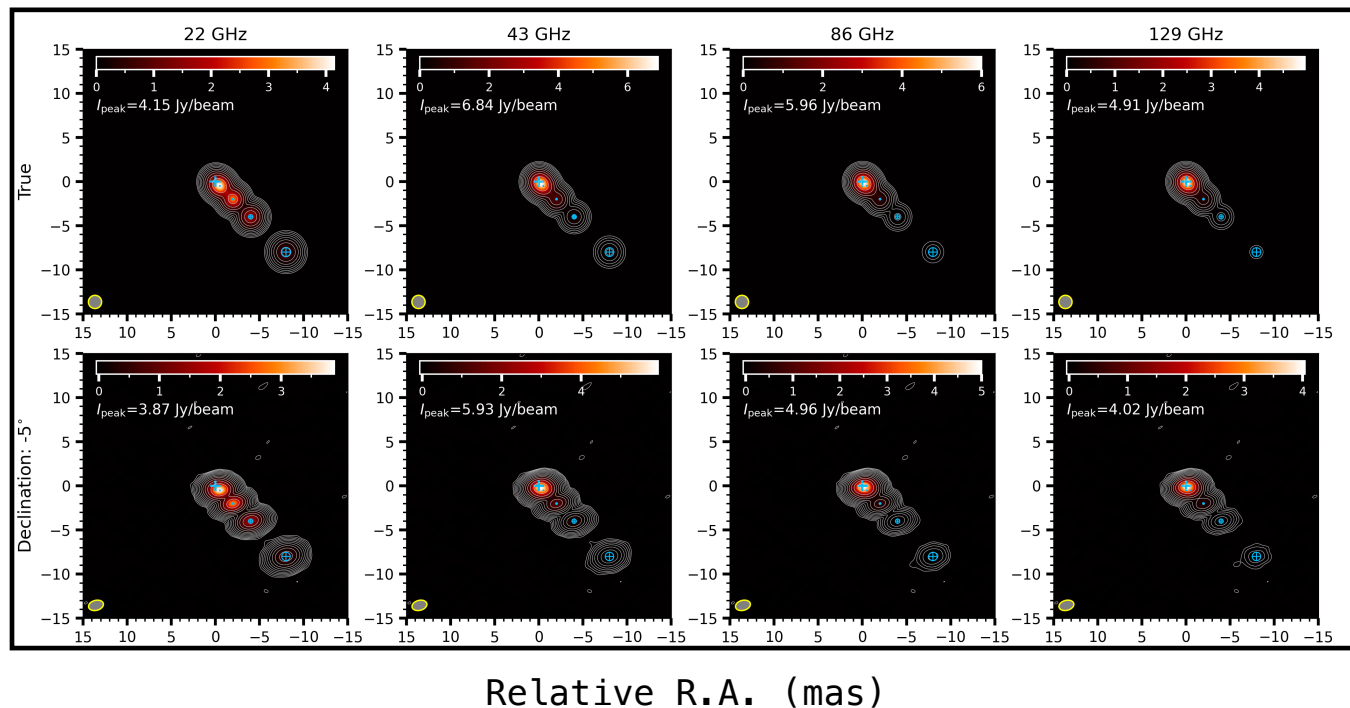
Modeling results // closure relations (supplementary)

Source: 3C 345 (closure phase)



Simulation // gain calibration (supplementary)

100 simulations with random gain offset,
on two declinations -5° & 40°

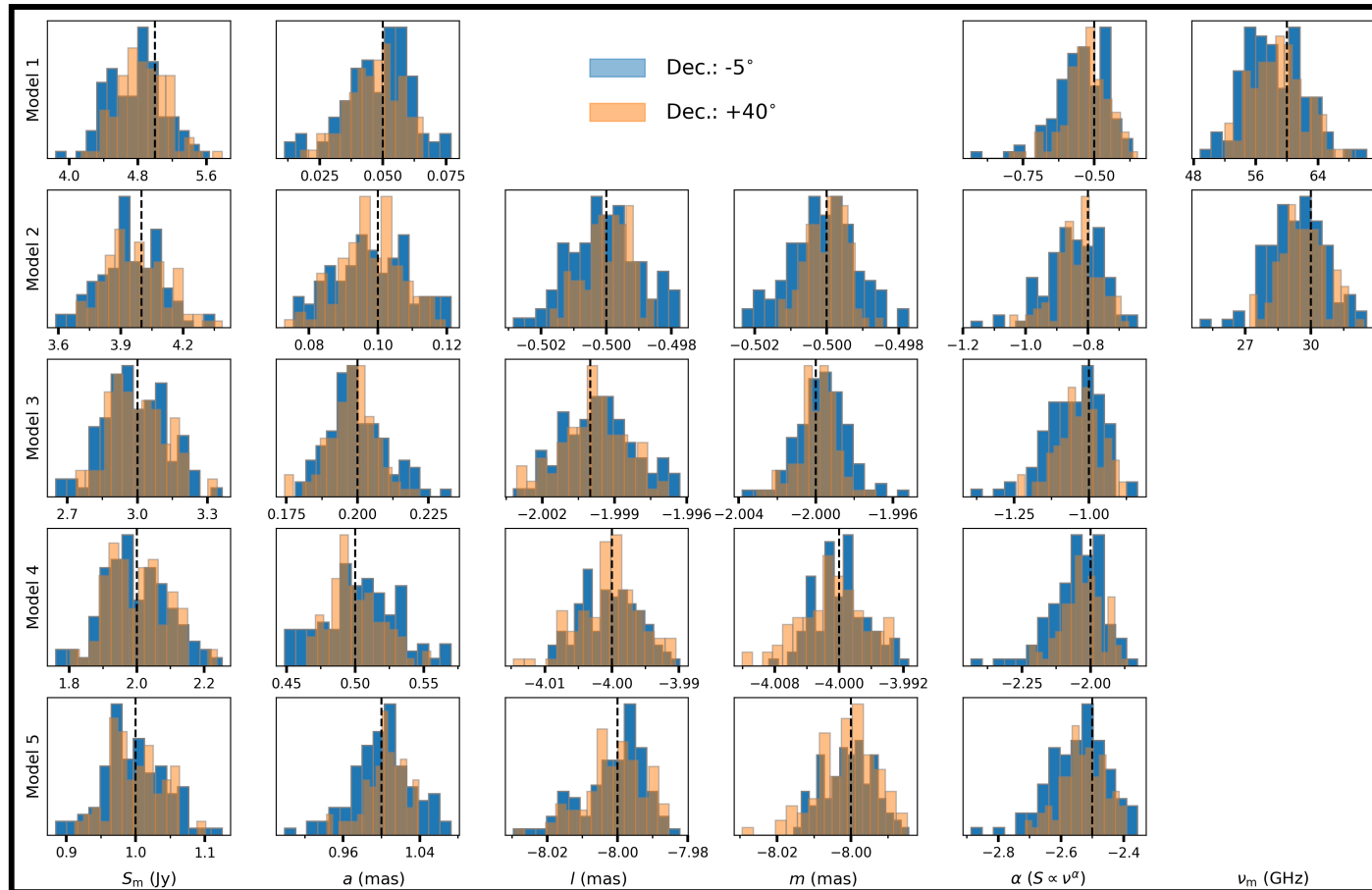


gain calibration results

-5° deg
 $+40^{\circ}$ deg

Simulation // posterior distributions (supplementary)

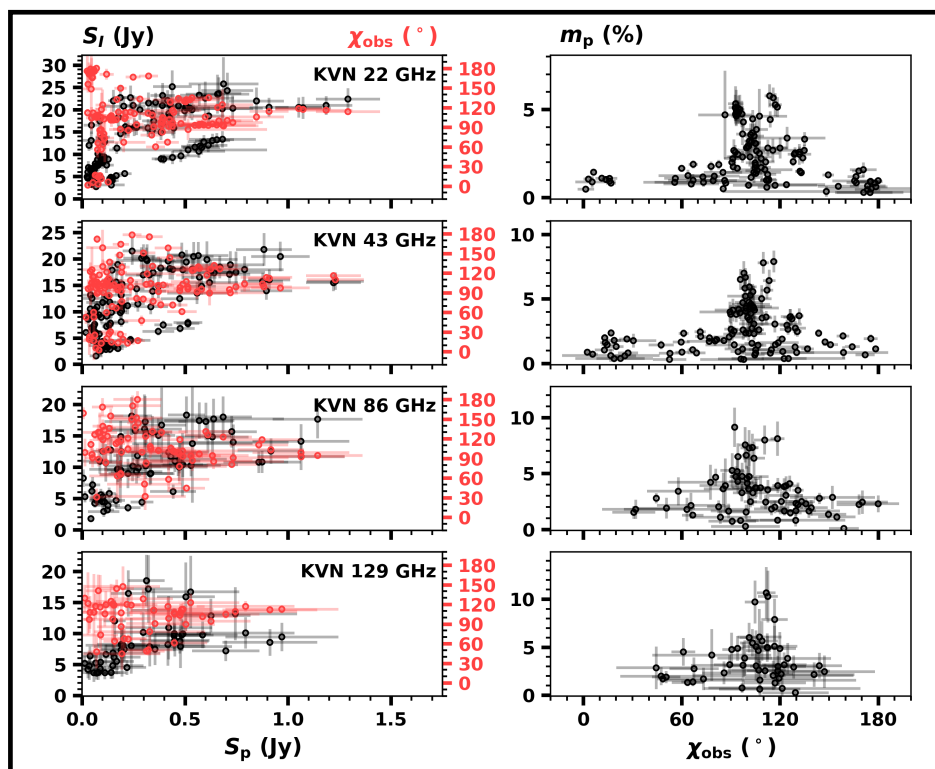
100 simulations with random gain offset,
on two declinations -5° & 40°



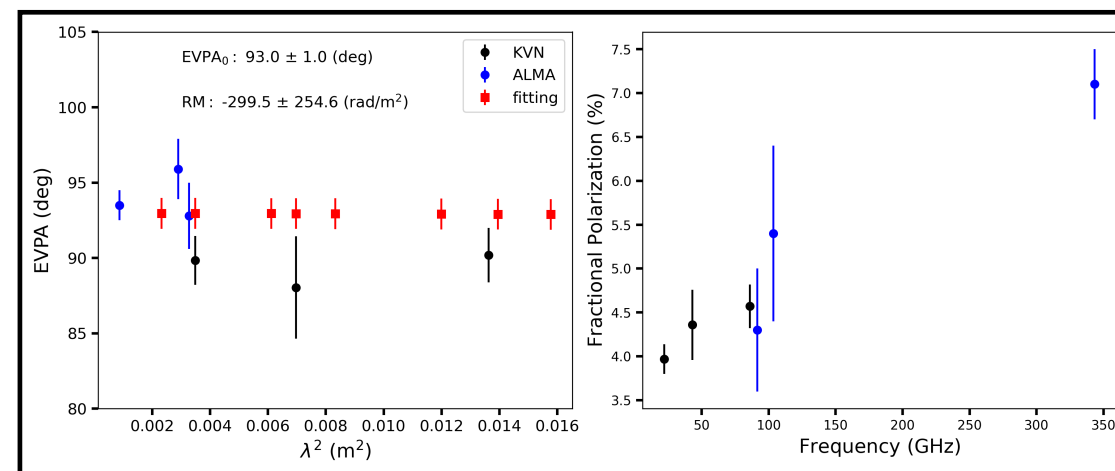
Absolute EVPA calibration // n25hj01b, 2025-03-18 (supplementary)

Aligned EVPA of 3C 454.3

Polarimetry of 3C 454.3
obtained from a decadal monitoring of the KVN (Jeong+2025)



→ EVPA of this tends to be aligned at 90 – 100 deg,
when highly polarized



Polarimetry of 3C 454.3 in March 2025,
obtained from KVN (22 – 86 GHz) and ALMA (91 – 340 GHz)

* NOTE:
errors on the KVN data does not involve
systematic uncertainties