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PIANO NAZIONALE
DI RIPRESA E RESILIENZA



Centro Nazionale di Ricerca in HPC,
Big Data and Quantum Computing

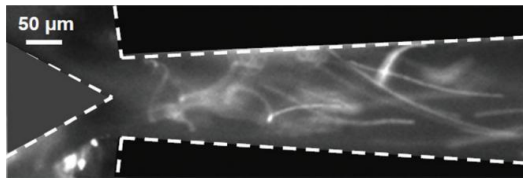
TURBO 3D: Complex flows and Lagrangian observables.

Victor de J. Valadão
University of Turin

Perugia - 27th May 2025.



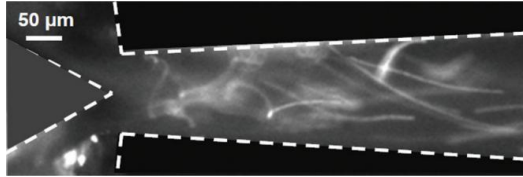
Brief Review on 3D turbulence



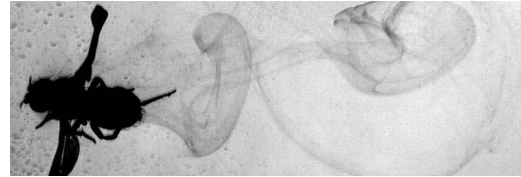
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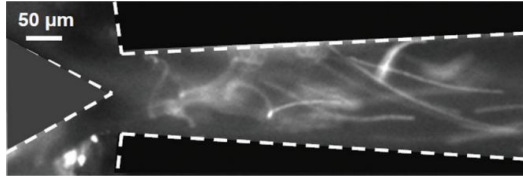


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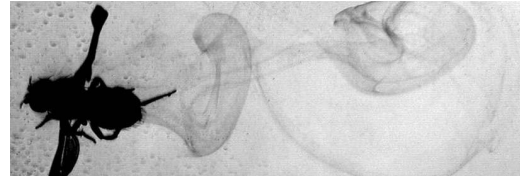


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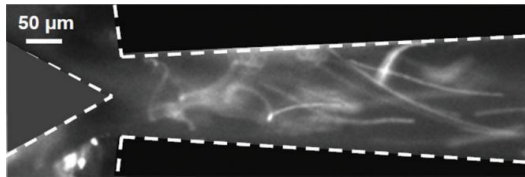
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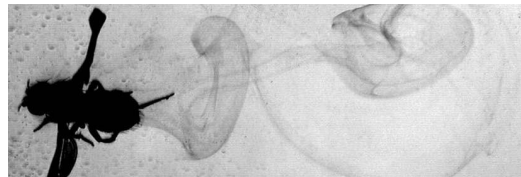
NASA-LaRC @ nasa.gov



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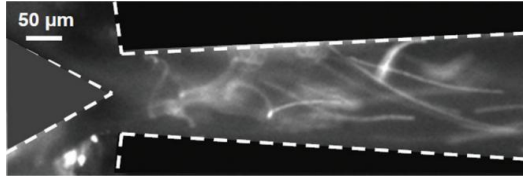


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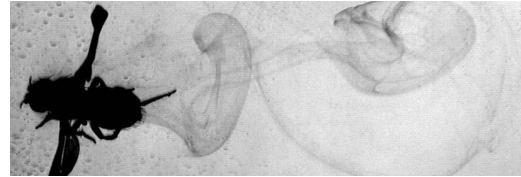


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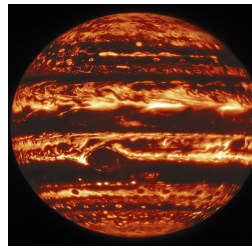
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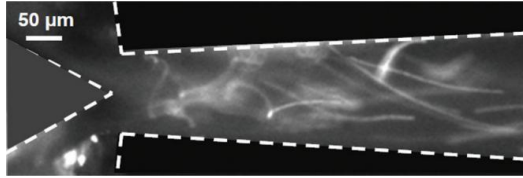


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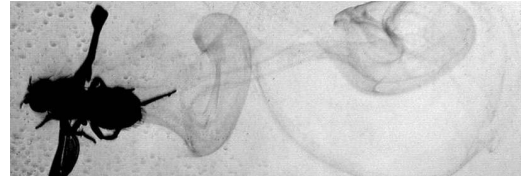


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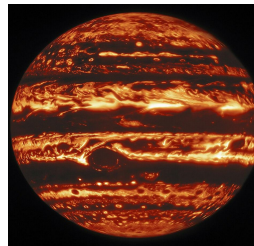
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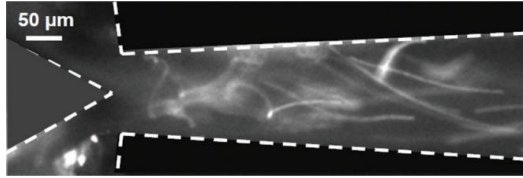
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$$\partial_t v_i + v_j \partial_j v_i - \nu \partial^2 v_i = f_i - \partial_i P$$

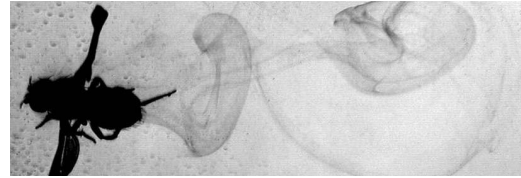
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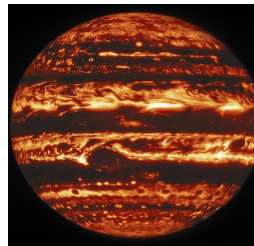
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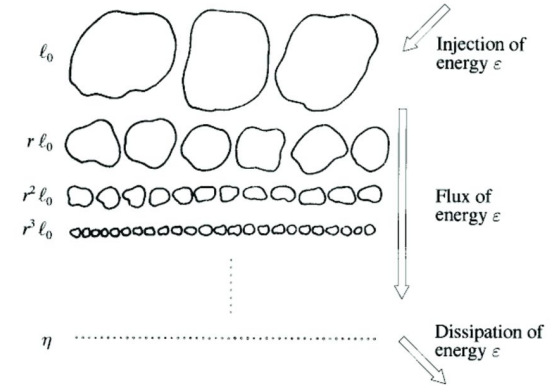
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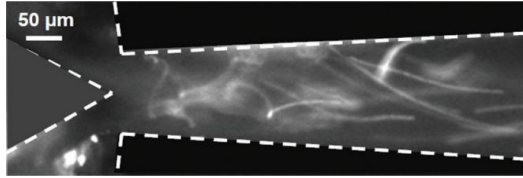
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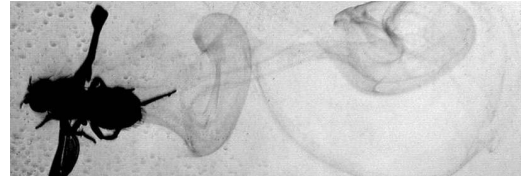


Andrey N. Kolmogorov

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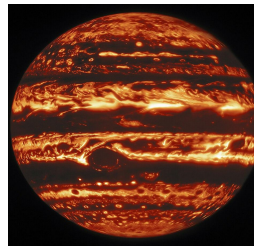
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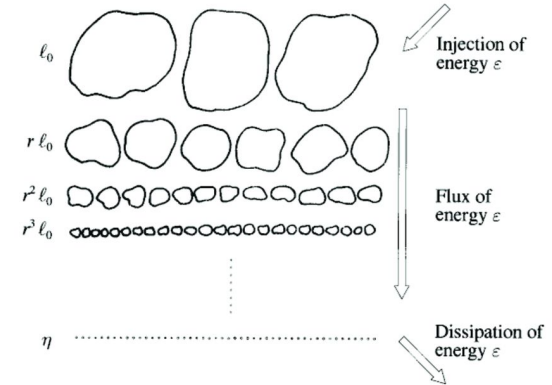
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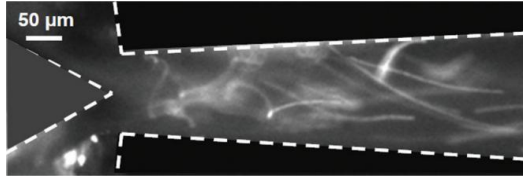
Andrey N. Kolmogorov

$$E(k) = \frac{\langle |\vec{v}_k|^2 \rangle}{2} \propto \varepsilon^{2/3} k^{-5/3}$$

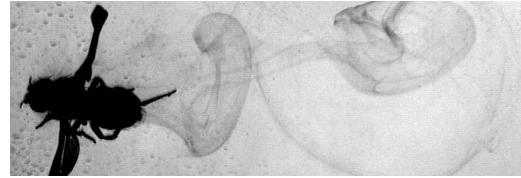
$$\ell_f^{-1} < k < \eta_\kappa^{-1}$$

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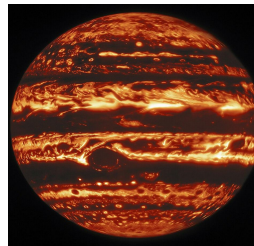
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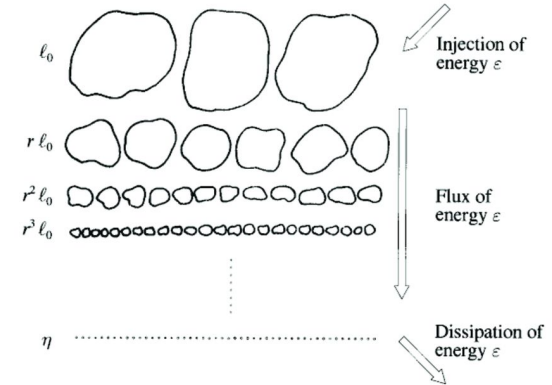
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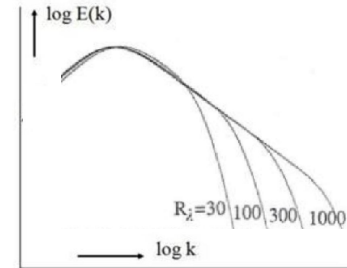


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Pseudospectral approach

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- Solves NSE in the Fourier space;
- Linearity is easily solvable;
- Non-linear part is calculated in the physical space;
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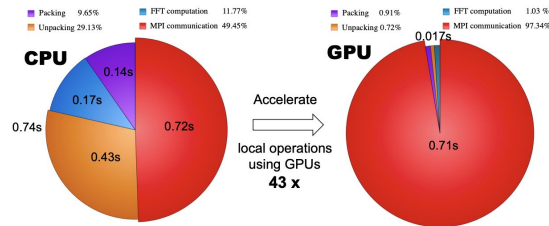
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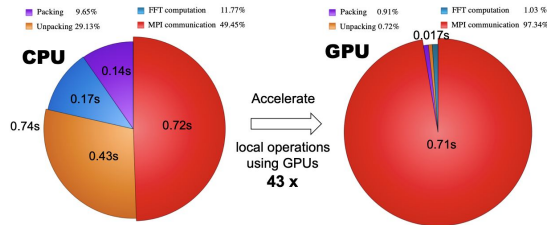
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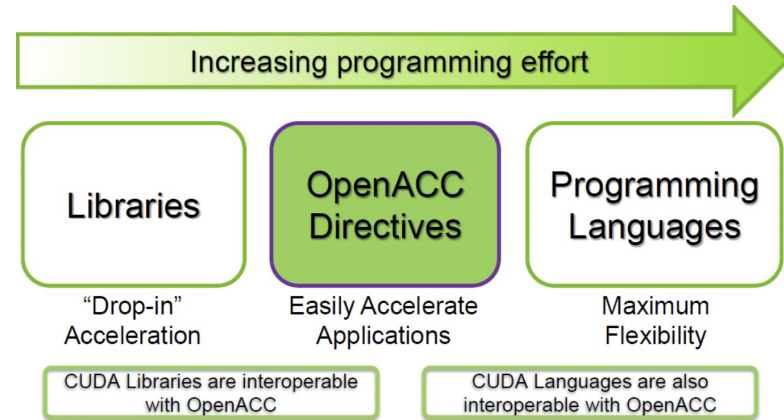
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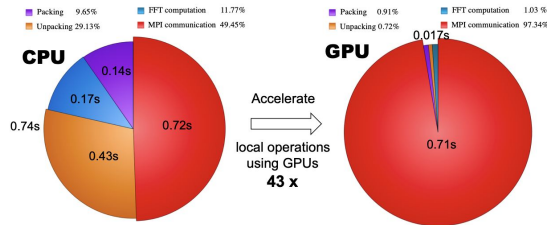
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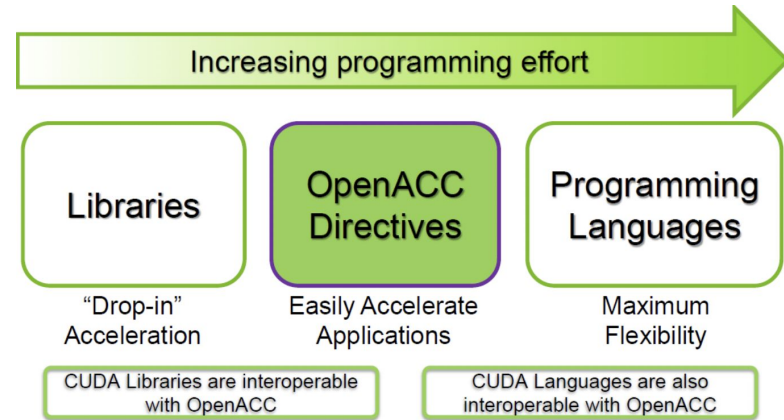
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```

54  !$acc parallel loop collapse(2) present(z, u, v, fxx, fky)
55  do j=1,NY
56  do i=1,NX2P1
57    z(1,i,j)=fxx(i)*u(2,i,j)+fky(j)*v(2,i,j)
58    z(2,i,j)=-fxx(i)*u(1,i,j)-fky(j)*v(1,i,j)
59  end do
60  end do
61  !$acc end parallel loop
    
```



Turbo 2D

$$\partial_t \omega + v_i \partial_i \omega + (-1)^n \nu \partial^{2n} \omega + (-1)^m \mu \partial^{2m} \omega = F$$

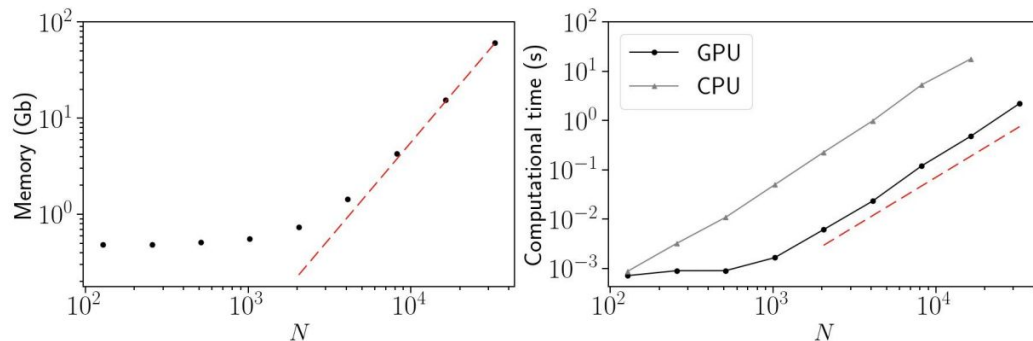
$$v_i = \epsilon_{ij} \partial_j \psi \qquad \hat{\omega}(k, t) = k^\alpha \hat{\psi}(k, t)$$

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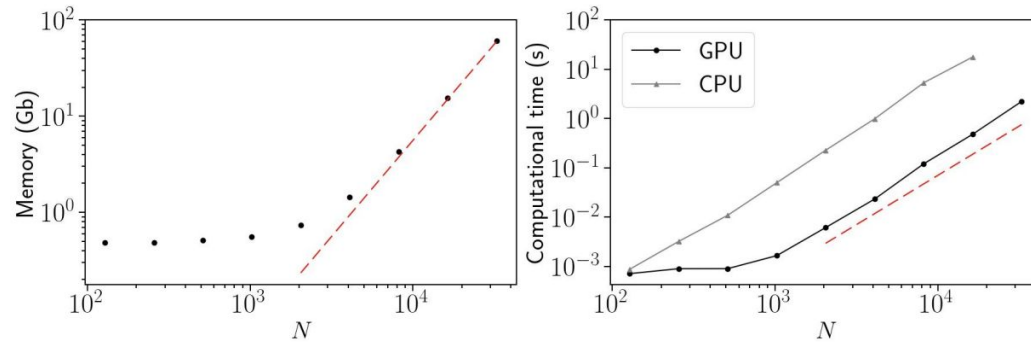


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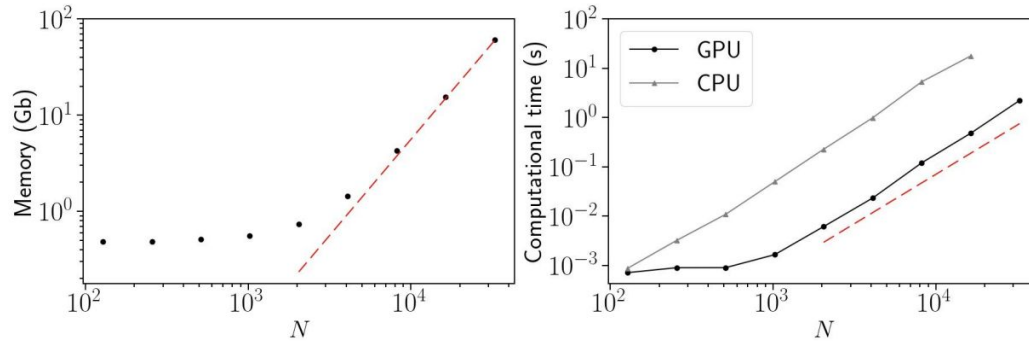
Up to $N^2 = (32768)^2$

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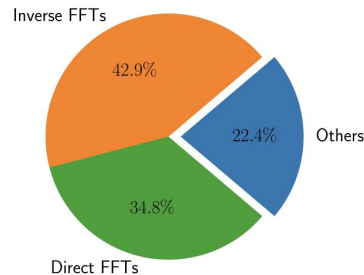
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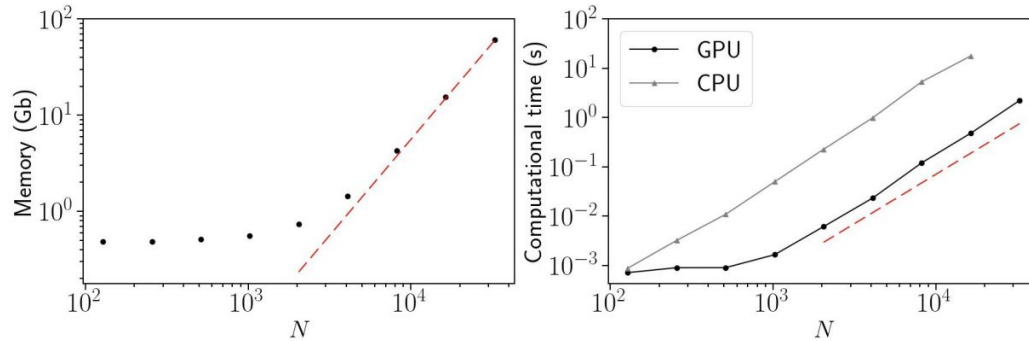


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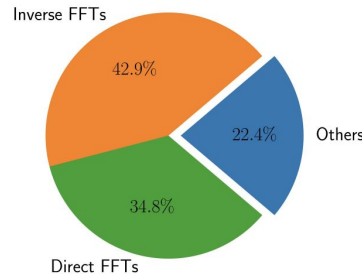
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Functionalities:

- Simulation of Generalized active Transport Equations;
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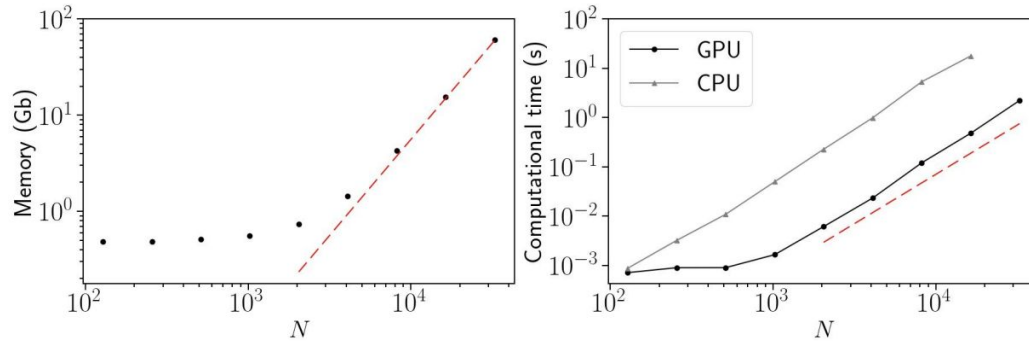


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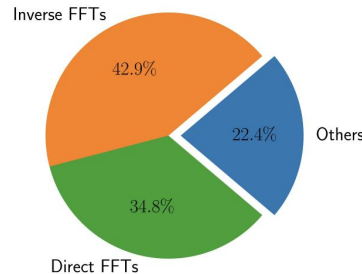
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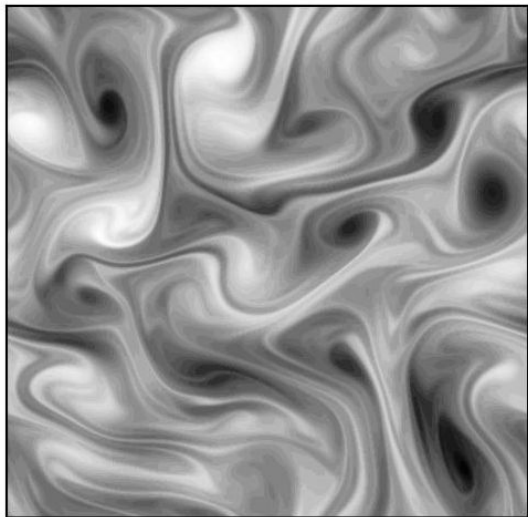
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Papers

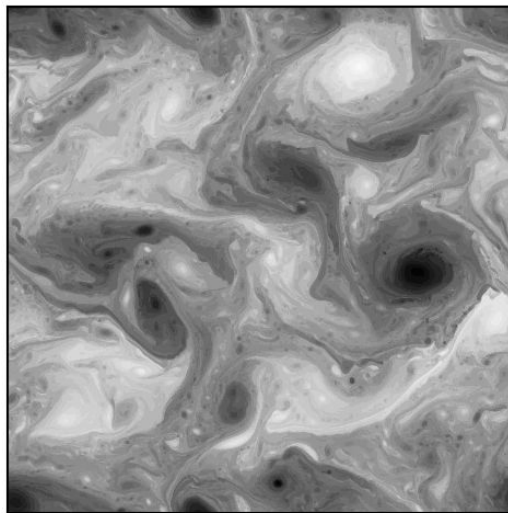
1. V. J. Valadão, T. Ceccotti, G. Boffetta, and S. Musacchio, Phys. Rev. Fluids 9, 094601. 2024.
2. Valadão V. J., Boffetta G., De Lillo F., Musacchio S., and Ciraletti-Esposito M.; Journal of Turbulence, 1-10. 2025.
3. V.J. Valadão, F. De Lillo, S. Musacchio, G. Boffetta. ArXiv preprints: arXiv:2504.07914. 2025 (Under Review at Journal of Fluid. Mech.).



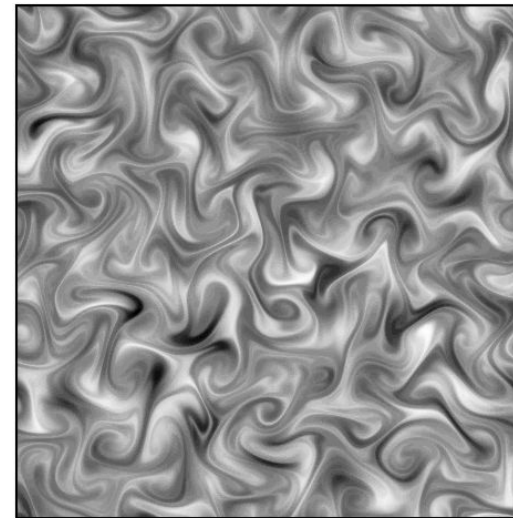
Some nice images



Navier-Stokes Turbulence

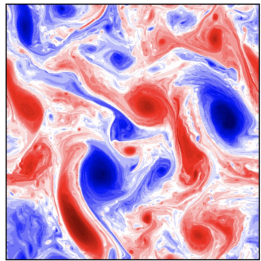


Surface Quasi-geostrophic
Turbulence

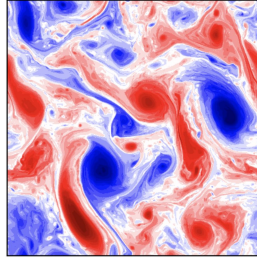


Passive-scalar advection

Eulerian x Lagrangian Predictability



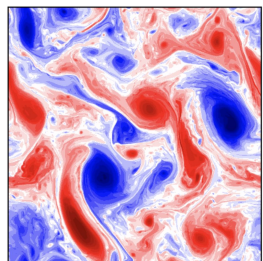
θ'



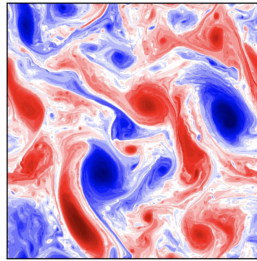
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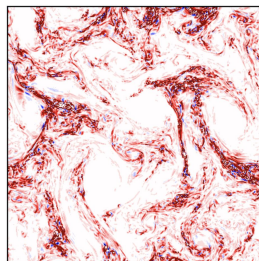


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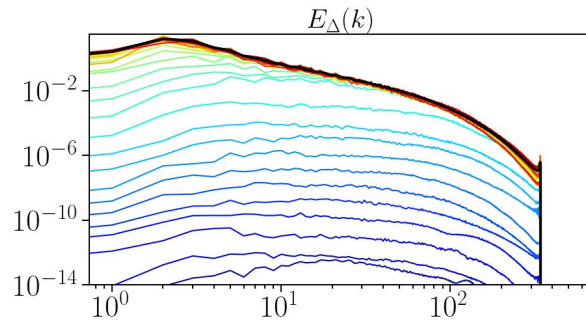
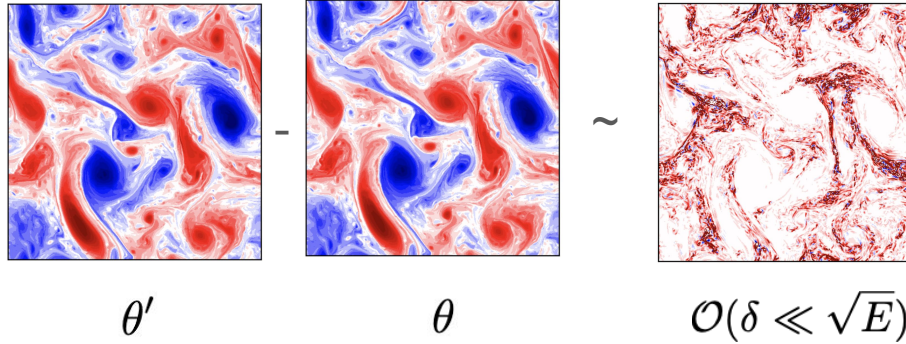
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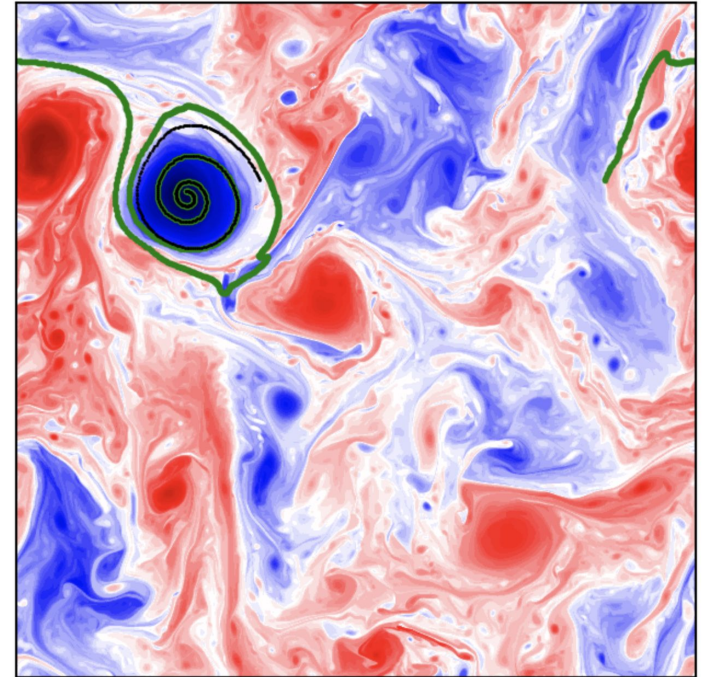
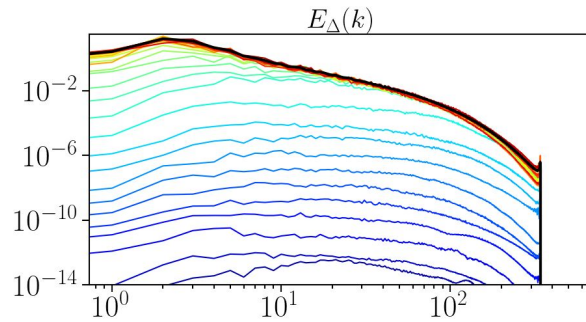
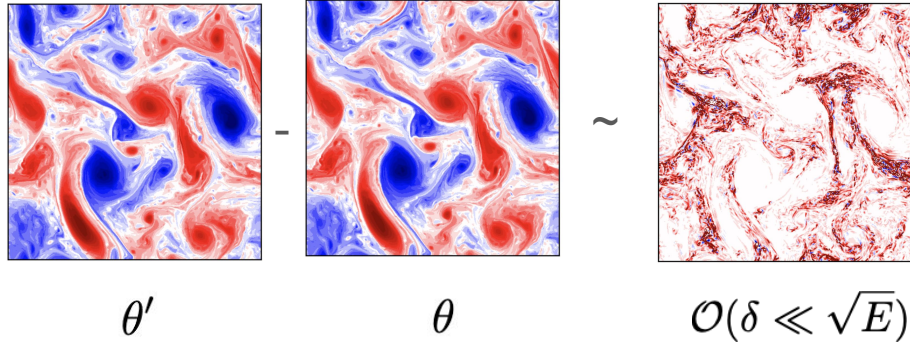


$\mathcal{O}(\delta \ll \sqrt{E})$

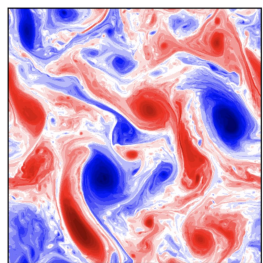
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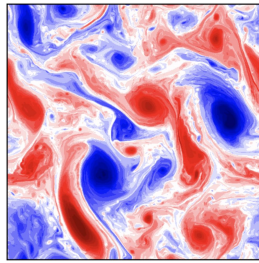
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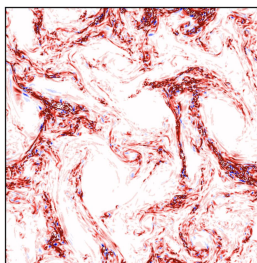
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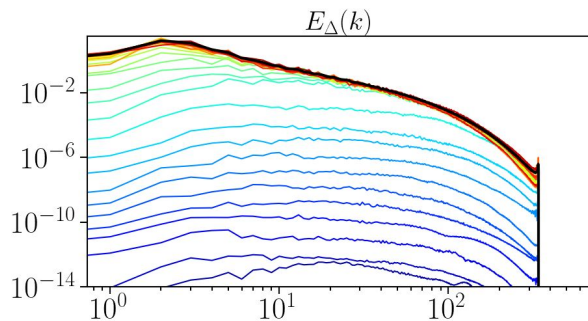
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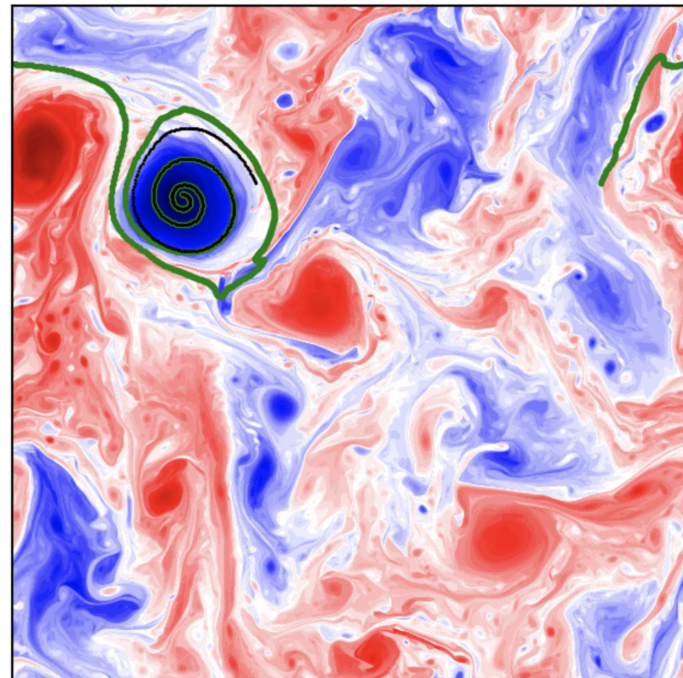
θ



$\mathcal{O}(\delta \ll \sqrt{E})$



$$\dot{\mathbf{v}}_p = \frac{\mathbf{u}(\mathbf{x}_p(t), t) - \mathbf{v}_p(t)}{\tau_s}, \quad \epsilon = \frac{\rho_0}{\rho_p}, \quad \tau_s = \frac{2R_p^2}{9\nu\epsilon}$$





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- Requires a MULTI-GPU approach (memory bottleneck);
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Observations

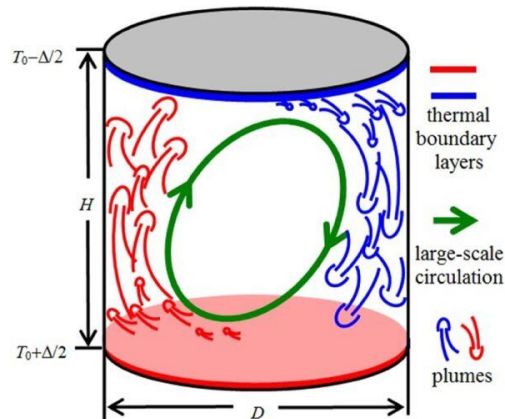
- Fluid equations with increasing complexity demands less spatial resolution:
 - Rotational / Thermal coupling;
 - Multiphase / phase transition flow;
 - Eletromagnetic coupling?;

Turbo 3D

Problems

- Requires a MULTI-GPU approach (memory bottleneck);
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Complex fluids in single GPU



Observations

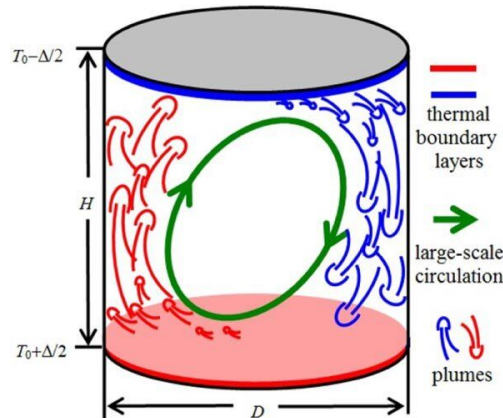
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Homogeneous Rayleigh-Bénard (HRB)

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} + 2\boldsymbol{\Omega} \times \mathbf{v} - \nu \nabla^2 \mathbf{v} = -\nabla P - g \frac{\delta \rho}{\rho_0} \mathbf{e}_z$$

$$\partial_t \delta \rho + (\mathbf{v} \cdot \nabla) \delta \rho - \kappa \nabla^2 \delta \rho = \gamma \mathbf{v} \cdot \mathbf{e}_z$$



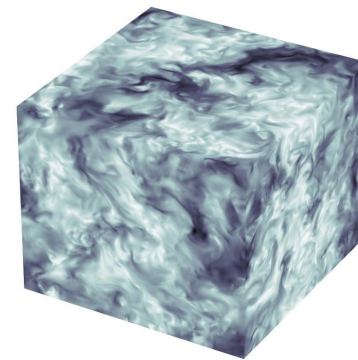
Finanziato
dall'Unione europea
NextGenerationEU



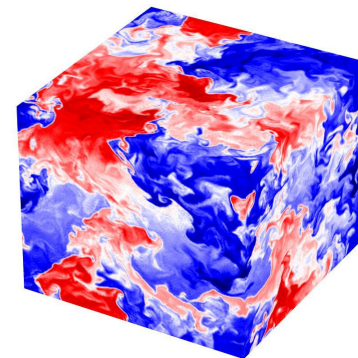
Ministero
dell'Università
e della Ricerca

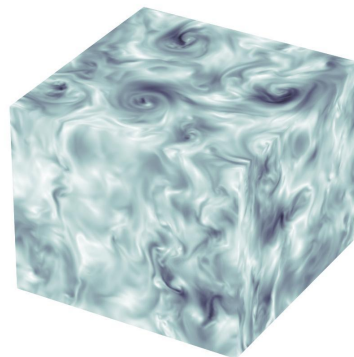


Italiadomani
PIANO NAZIONALE
DI RIPRESA E RESILIENZA

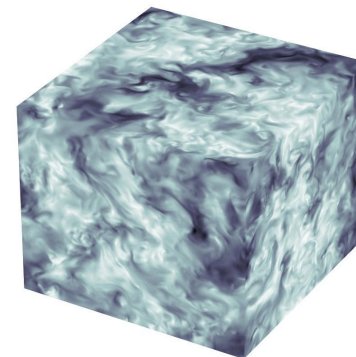


HRB

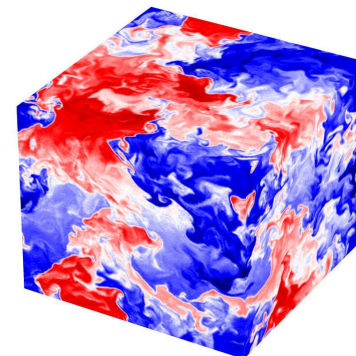
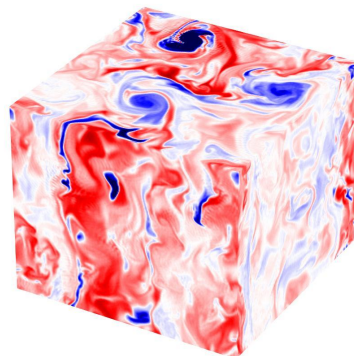


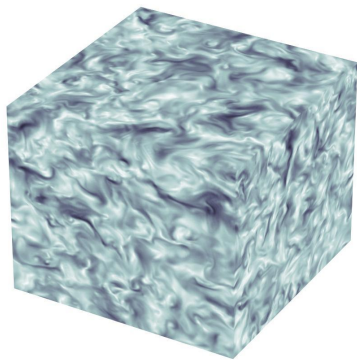


Rotating

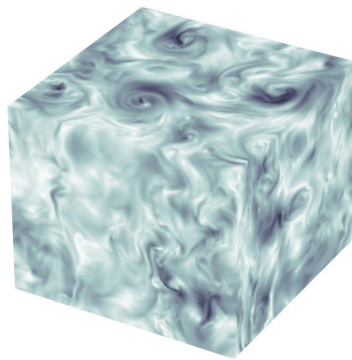


HRB

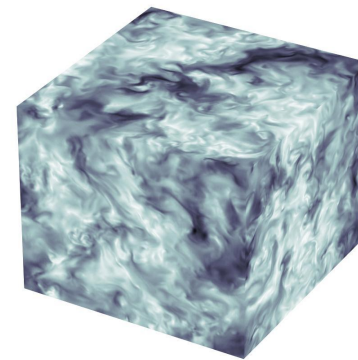




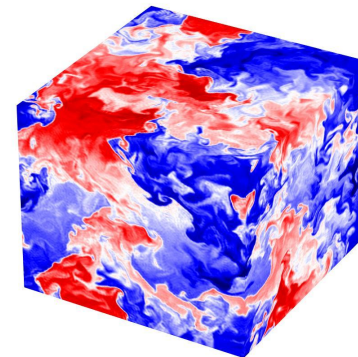
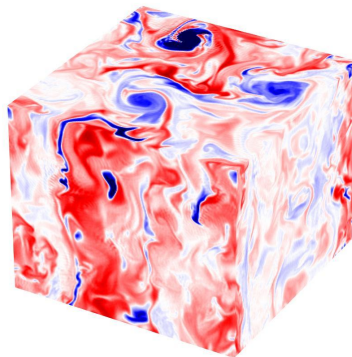
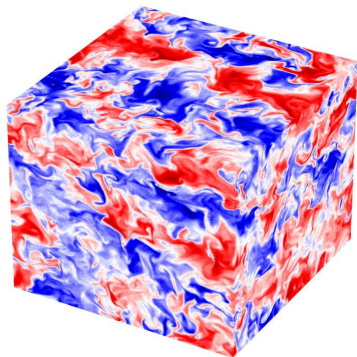
Homo & Iso Turb



Rotating



HRB



In collaboration with:



Stefano Musacchio



Filippo De Lillo



Guido Boffetta

Thank you for your attention!