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From Few to Many Maps: A Fast Map-Level Emulator for Extreme Augmentation of Small Datasets

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Scientific Rationale

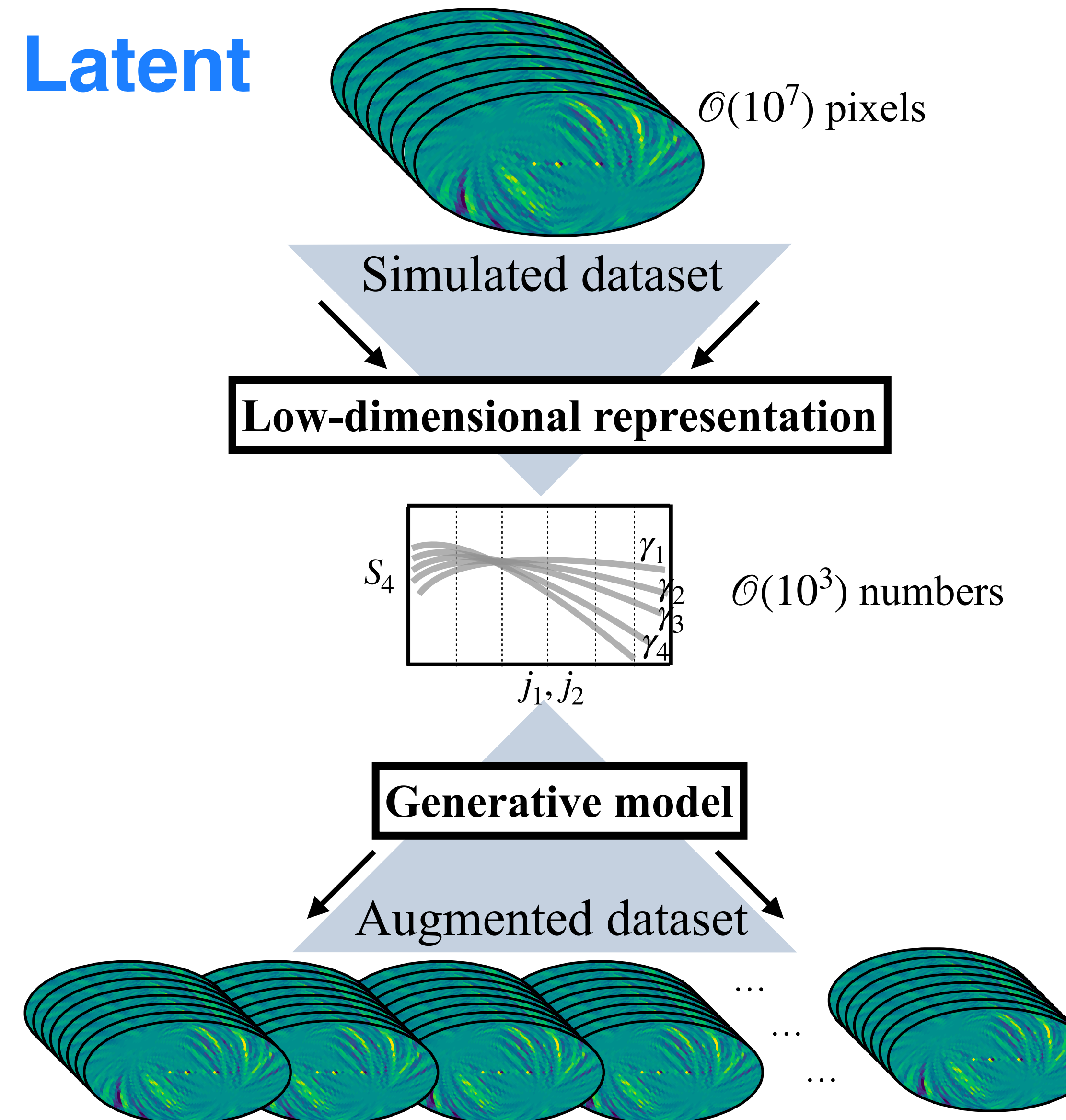
Massive Monte Carlos of end-to-end TOD simulations of CMB systematic effects are highly computationally expensive

- Typical **simulation campaign** $\mathcal{O}(10^3)$ maps costs $\mathcal{O}(100)$ **million CPU hrs** (e.g. *Planck*)
 - Limited number of simulations \rightarrow high sample variance in empirical covariance matrices \rightarrow non-optimal error bars, biased inverse covariance
- For **future surveys** (e.g. *LiteBIRD*, SO, CMB-S4):
 - *High-accuracy inference* needs $\mathcal{O}(10^{4-5})$ simulations [*Beck+ '22*]
 - *Simulation-Based Inference* needs $\mathcal{O}(10^{5-6})$ simulations [*Wolz+ '23*]
 - *Instrument design*: repeat for many different systematics effects, different noise models to find optimal configuration

$\left. \begin{array}{l} \mathcal{O}(10^{4-5}) \text{ simulations} \\ \mathcal{O}(10^{5-6}) \text{ simulations} \end{array} \right\} \mathcal{O}(10^{4-6}) \text{ millions CPU hrs!?$
- Computational cost might make these simply unfeasible \rightarrow **finding a solution is urgent!**

Solution: Generative Modelling from Latent Representation

- A good *Generative Model (or Emulator)* produces new *synthetic* samples which:
 1. reproduce true data features
 2. are representative of the true underlying data distribution
- Direct emulation often fails due to high dimensionality and/or not enough training data
- Train instead on a low-dimensional *latent representation*!
- But GANs, VAEs, diffusion models still need massive and expensive training sets...**catch 22!**



Scattering Covariance solves the small training set problem

- Very powerful as latent representation for generative models
- Interpretable *Non-Gaussian* summary statistic inspired by CNNs [Mallat'12, Bruna&Mallat'13, Cheng+ '23]
- (Iterative) **convolution** of field I with **fixed** wavelet kernels ψ_λ at oriented scale $\lambda = (j, \gamma) + \text{nonlinearity}$ (modulus)
- Extended also to **Cross-Scattering** Covariance between 2 maps

Convolution separates field I into individual scales

$$\begin{aligned}
 S_1^{\lambda_1} &= \left\langle \left| I \star \psi_{\lambda_1} \right| \right\rangle \\
 S_2^{\lambda_1} &= \left\langle \left| I \star \psi_{\lambda_1} \right|^2 \right\rangle \\
 S_3^{\lambda_1, \lambda_2} &= \text{Cov} \left[I \star \psi_{\lambda_1}, \left| I \star \psi_{\lambda_2} \right| \star \psi_{\lambda_1} \right] \\
 S_4^{\lambda_1, \lambda_2, \lambda_3} &= \text{Cov} \left[\left| I \star \psi_{\lambda_3} \right| \star \psi_{\lambda_1}, \left| I \star \psi_{\lambda_2} \right| \star \psi_{\lambda_1} \right]
 \end{aligned}$$

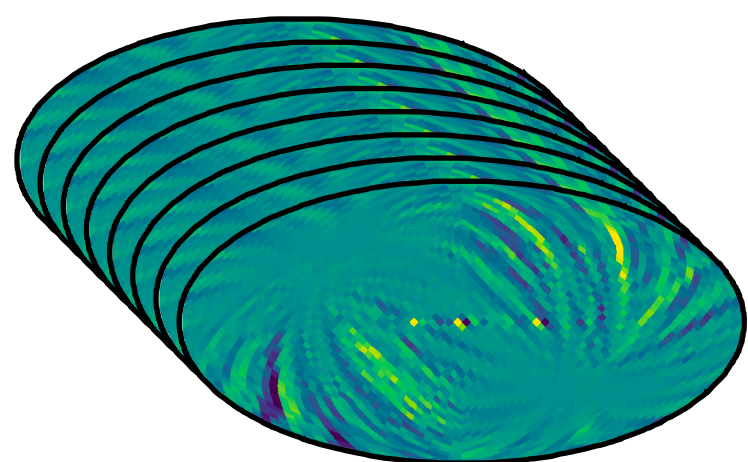
Interaction among different scales through non-linearity

Extreme Augmentation Algorithm

[Bruna & Mallat '19, Allys+ '20, Price+ '23,
Cheng+ '23, Häggbom+ '24]

1. Simulation

Simulate small
end-to-end *input*
dataset $\{\tilde{x}_i\}$



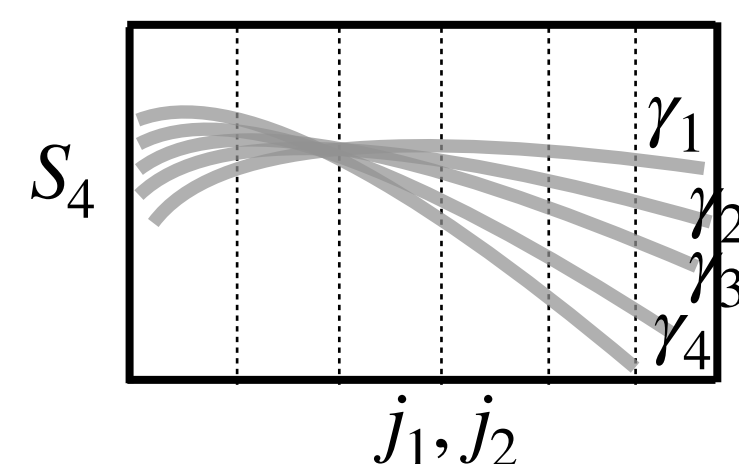
2. Latent vector of targets

Latent representation:

$$z_i = \Phi(\tilde{x}_i)$$

Scattering covariance

$$\Phi = \{S_1, S_2, S_3, S_4\}$$



3. Emulation

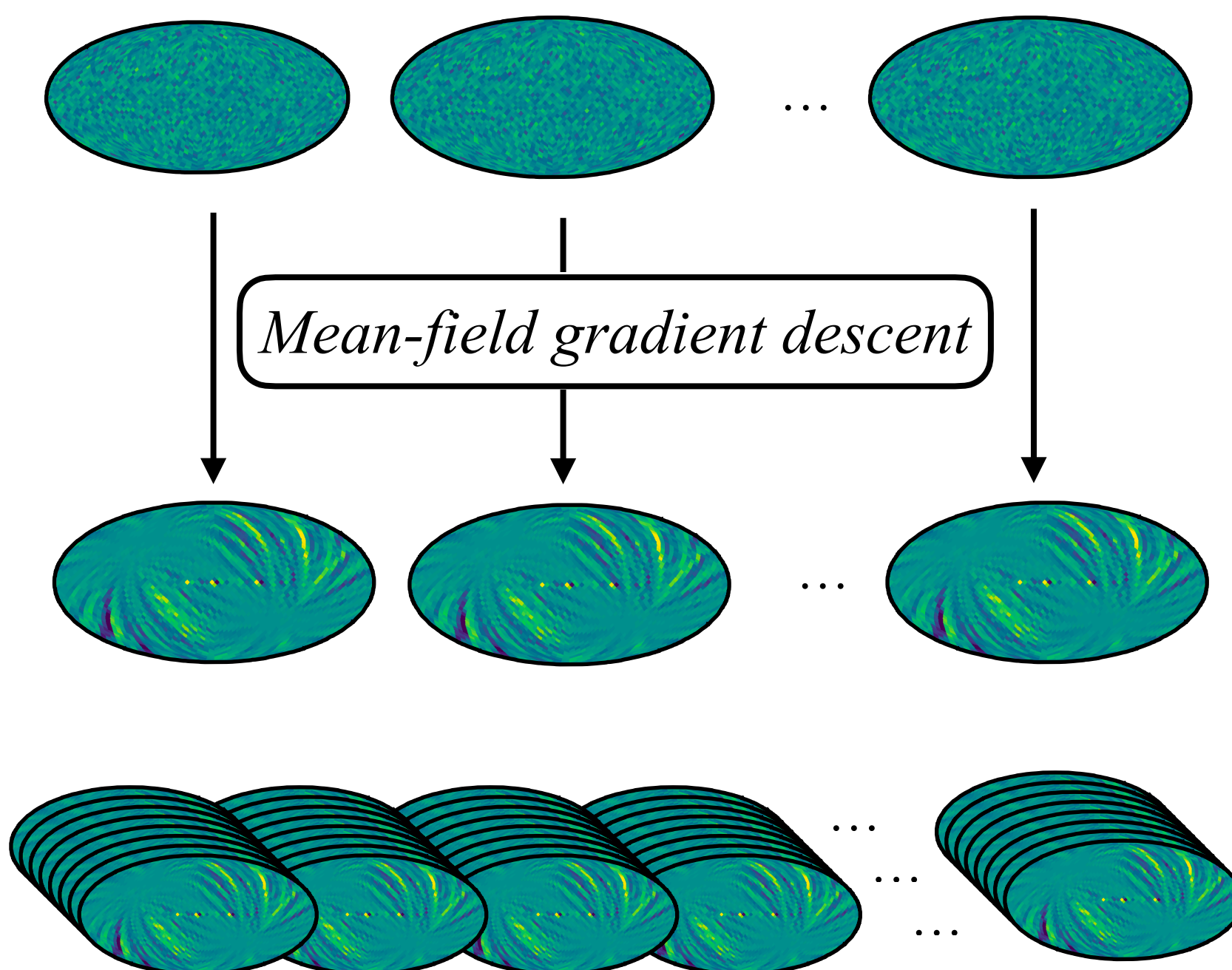
A. Take batch of *white noise* samples $\{x_j\}$

B. Minimize *loss* w.r.t. latent target in
pixel space:

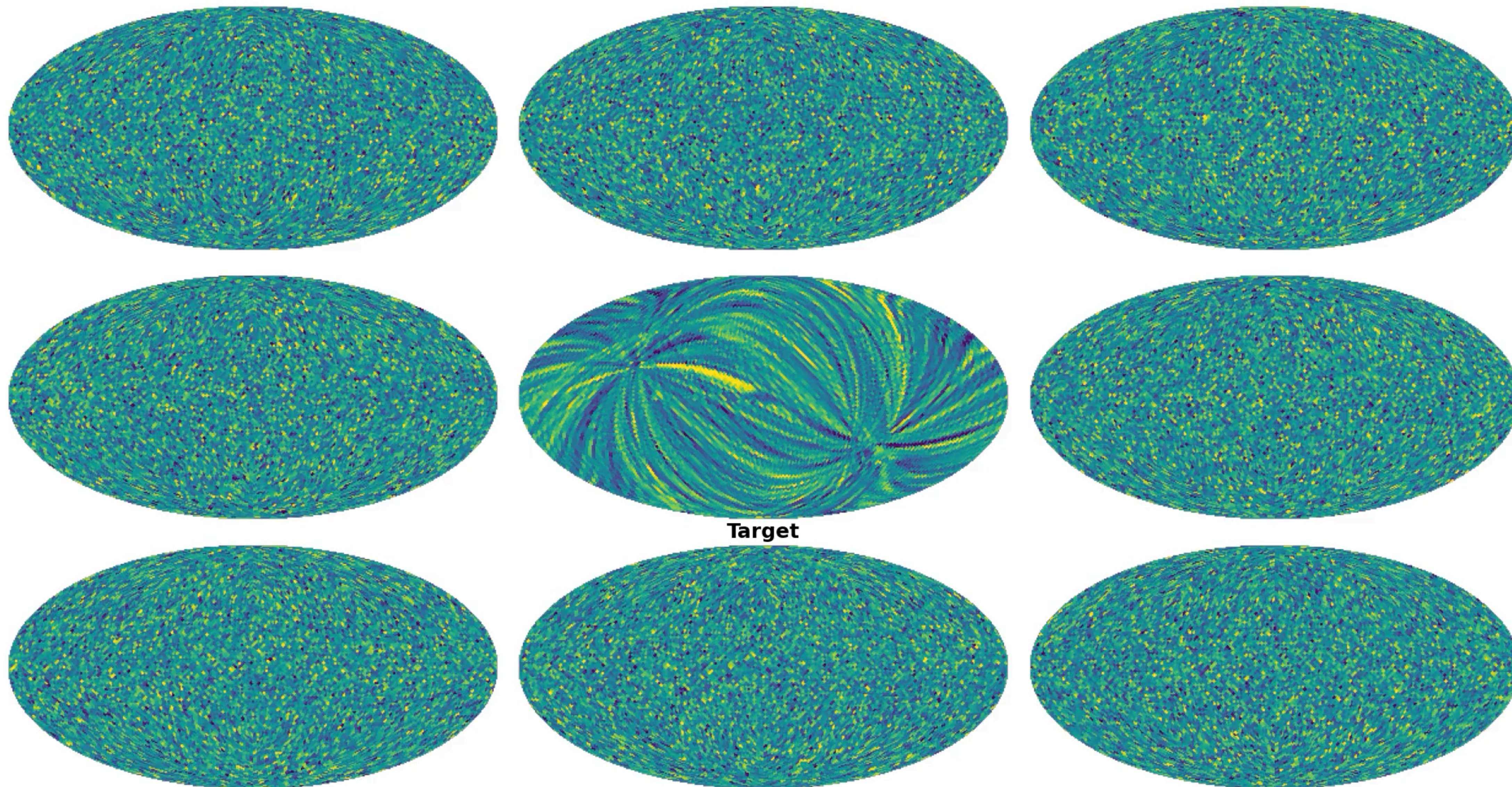
$$\mathcal{L} = \|\text{Ave}_j \Phi(x_j) - z_i\|_2^2$$

C. Get batch of *emulated samples*

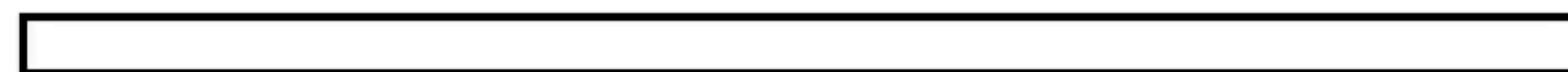
Repeat steps A-C to get many emulations



Gradient descent on a batch of 8 maps

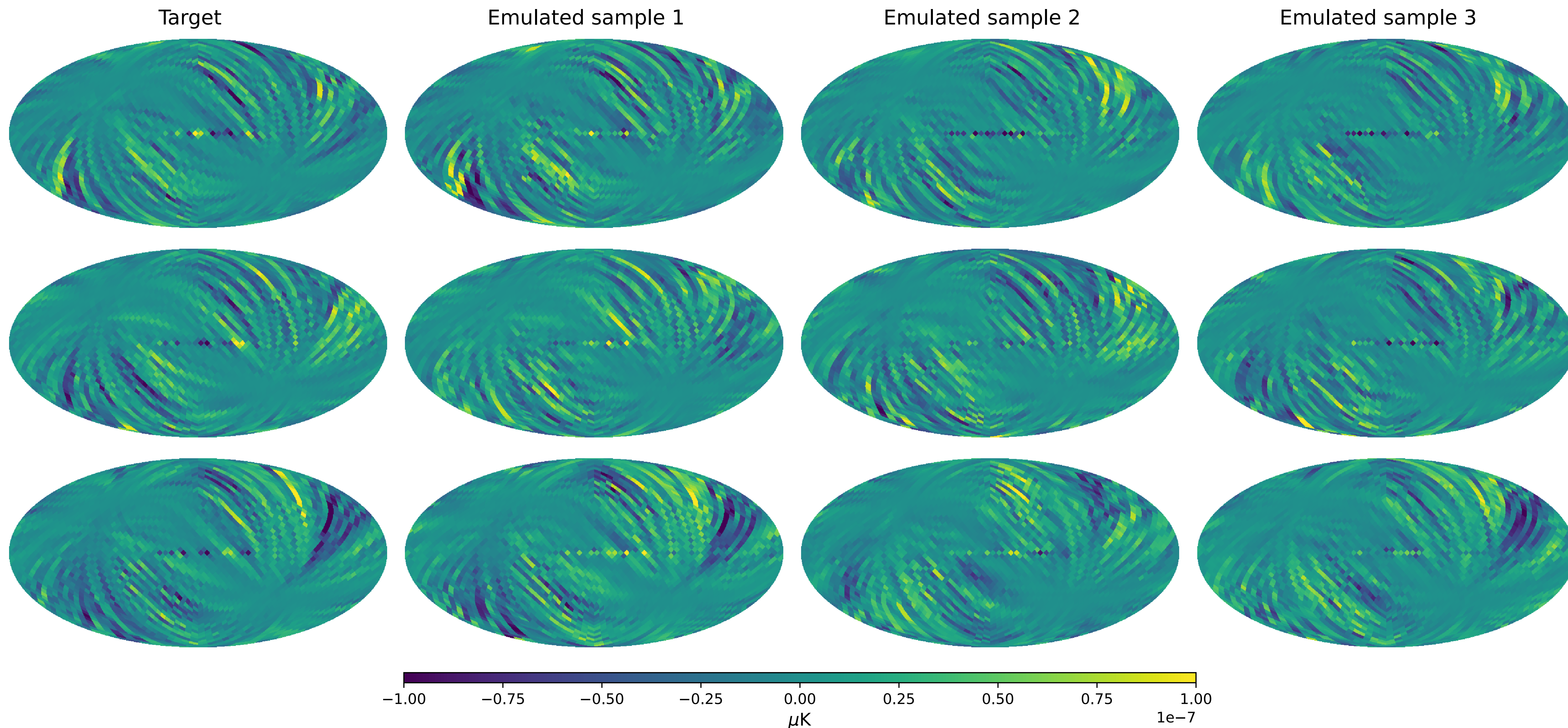


Target



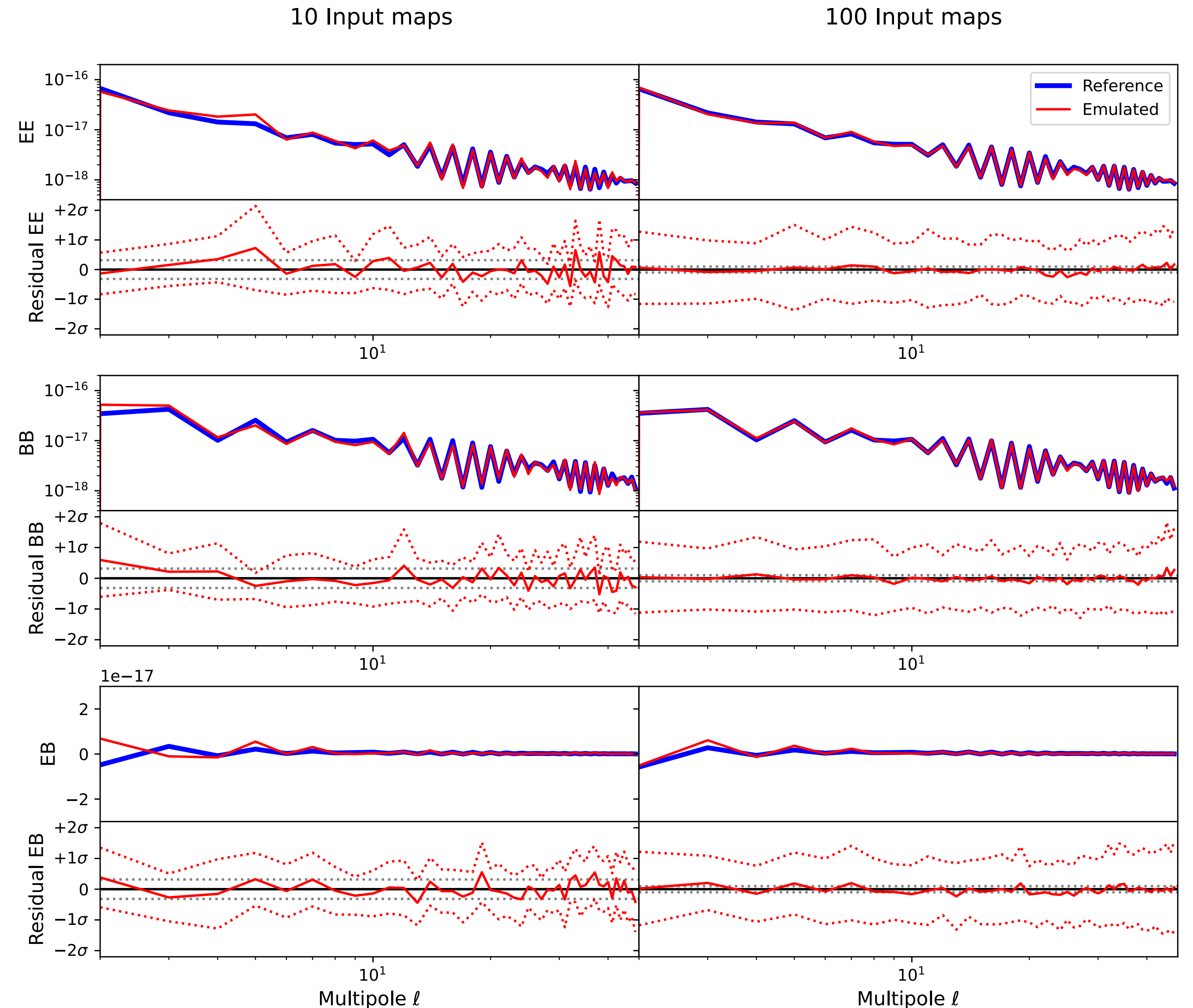
0 / 999
number of steps

Main Results - Application to *Planck*-like scanning with additive Gaussian random *gain* miscalibration systematic

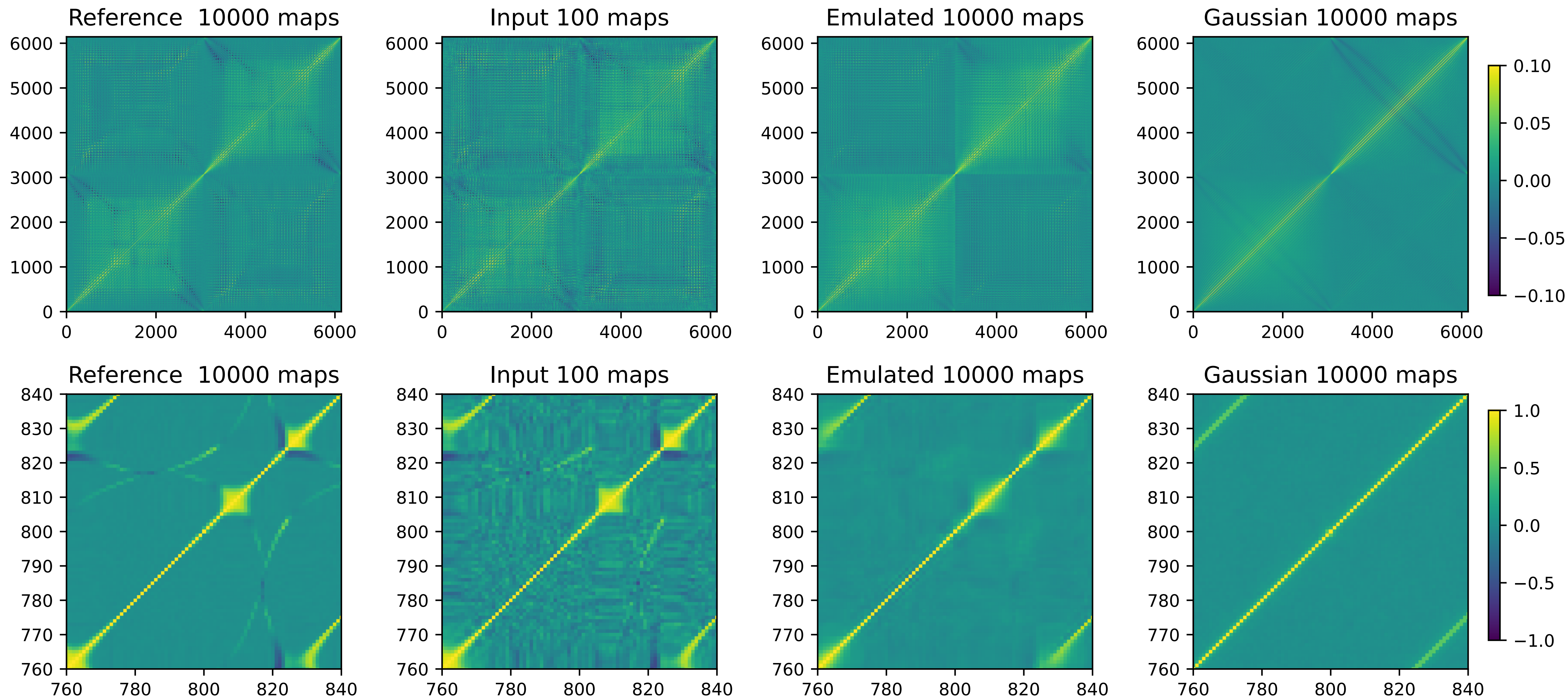


Main Results: Validation on Power Spectra

- **Blue:** Validation set (10,000 sims)
- **Red:** Emulated set (10,000 emulations)
- Gray band: std of validation set
- Residual **solid red:** $\frac{\langle C_\ell^{\text{emu}} \rangle - \langle C_\ell^{\text{val}} \rangle}{\sigma_\ell^{\text{val}}}$
- **Dotted red:** $\frac{\langle C_\ell^{\text{emu}} \rangle - \langle C_\ell^{\text{val}} \rangle \pm \sigma_\ell^{\text{emu}}}{\sigma_\ell^{\text{val}}}$
- Dashed grey: error on the mean $\pm \frac{\sigma_{\text{val}}}{\sqrt{N_{\text{input}}}}$

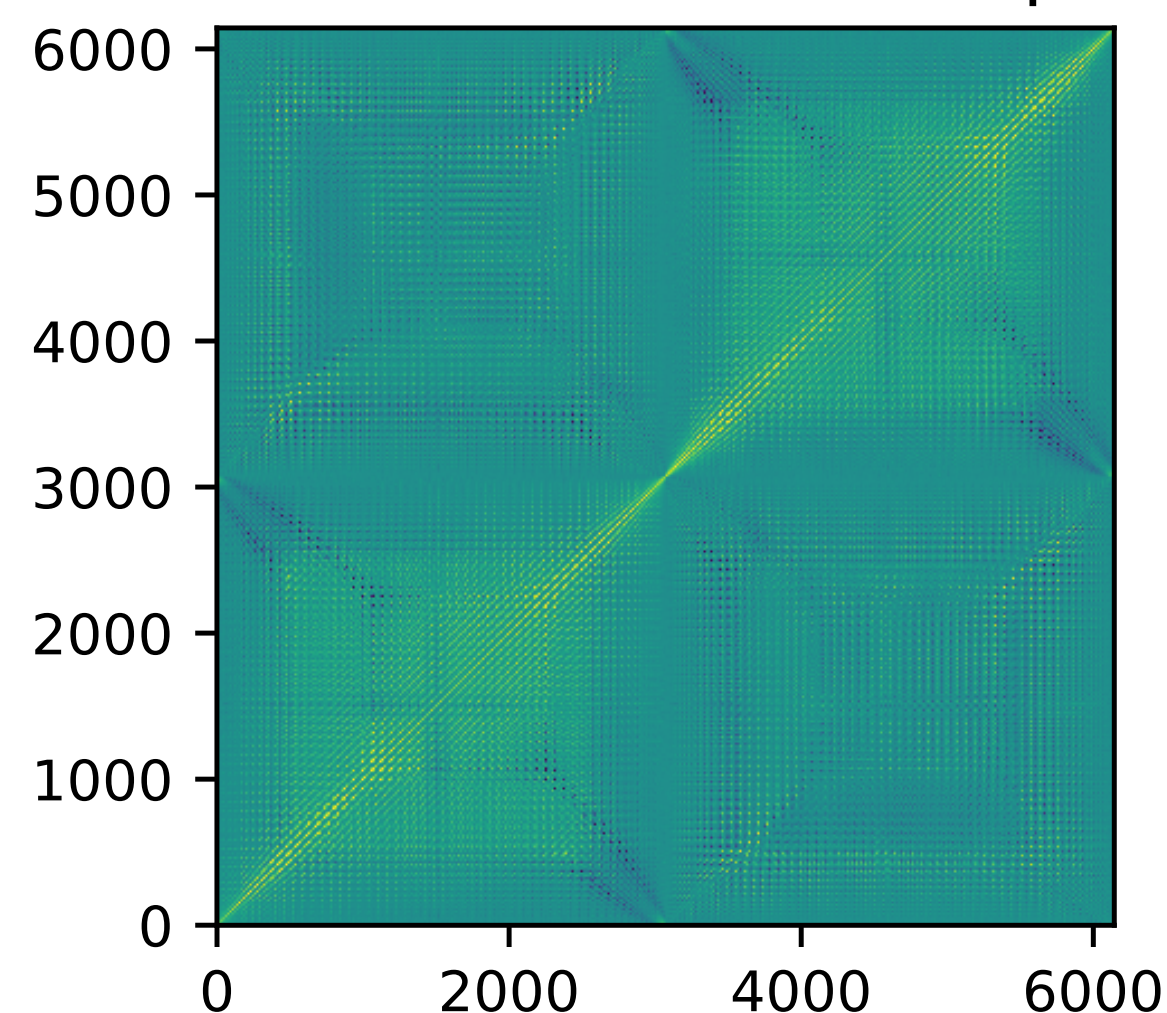


Main results - Pixel-Pixel Correlation Matrices from 100 input maps

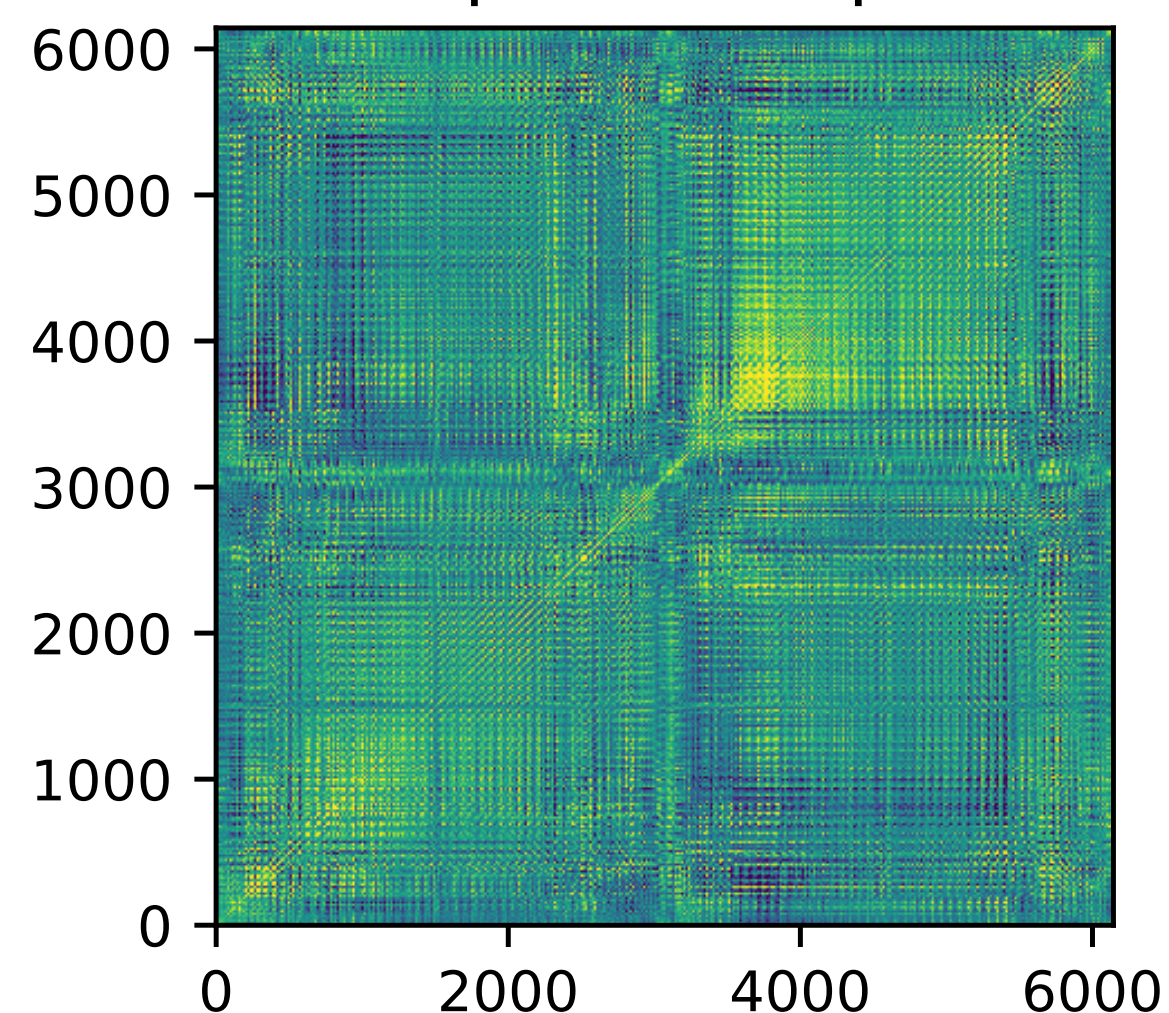


Main results - Pixel-Pixel Correlation Matrices from 100 input maps

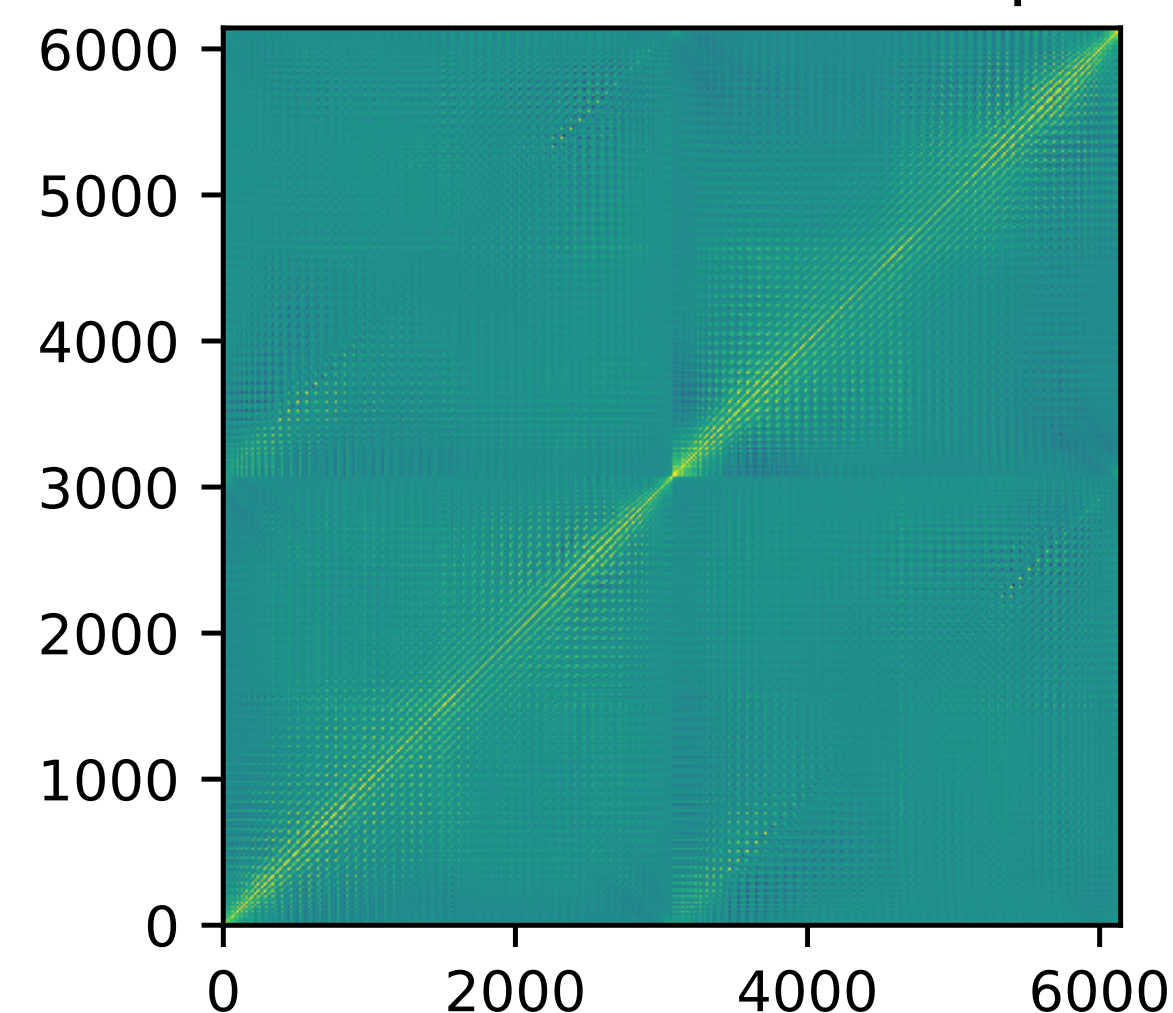
Reference 10000 maps



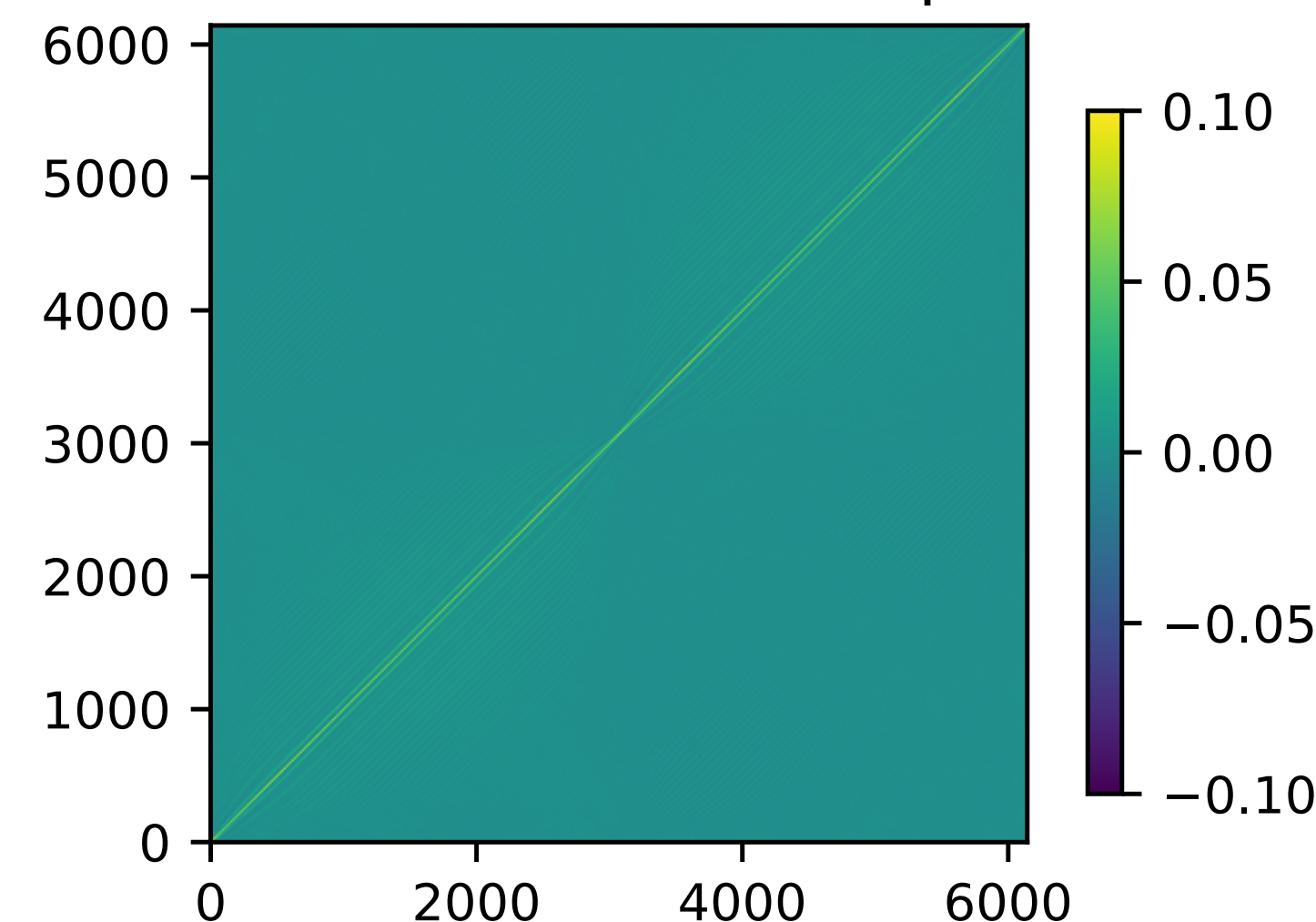
Input 10 maps



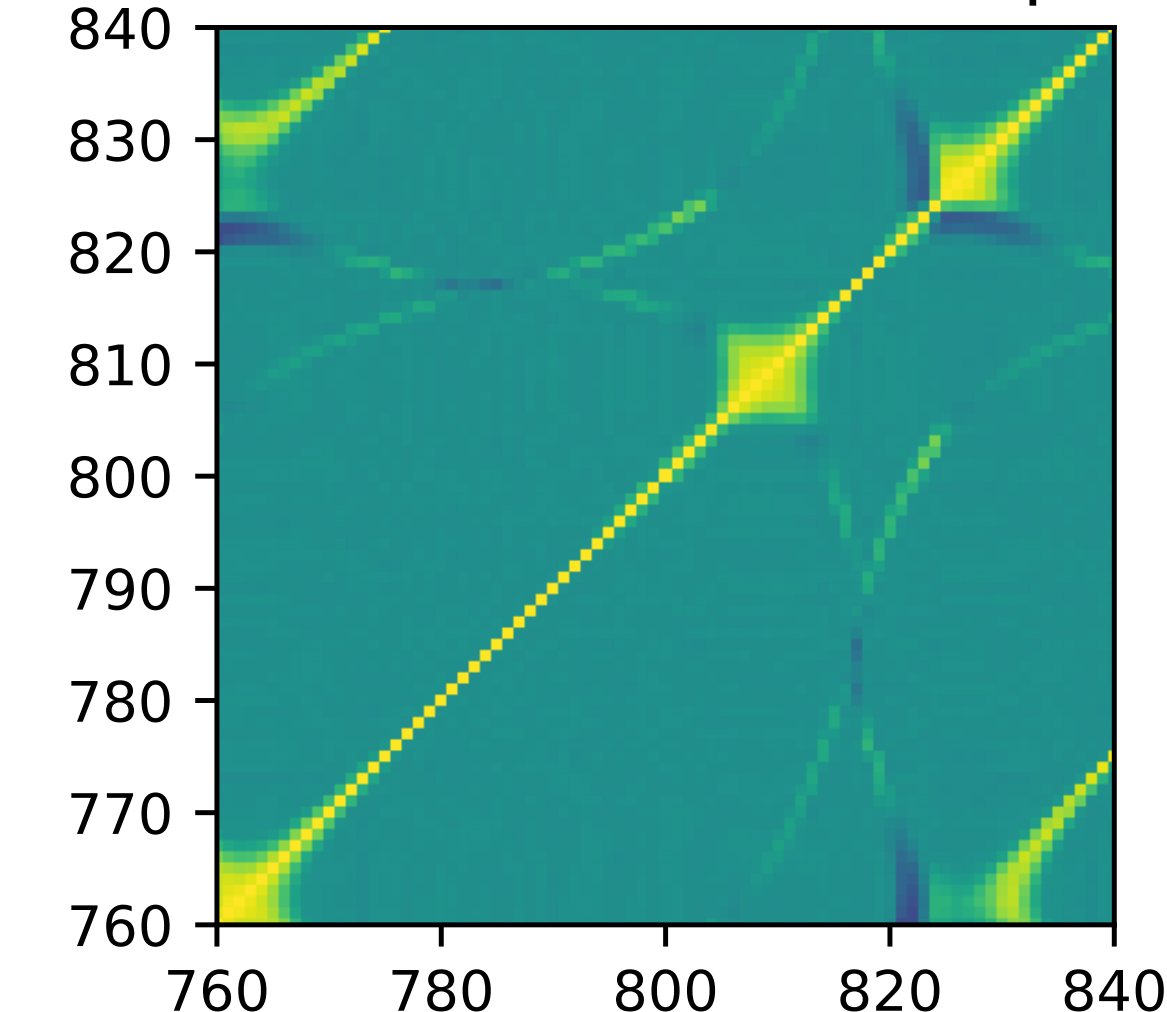
Emulated 10000 maps



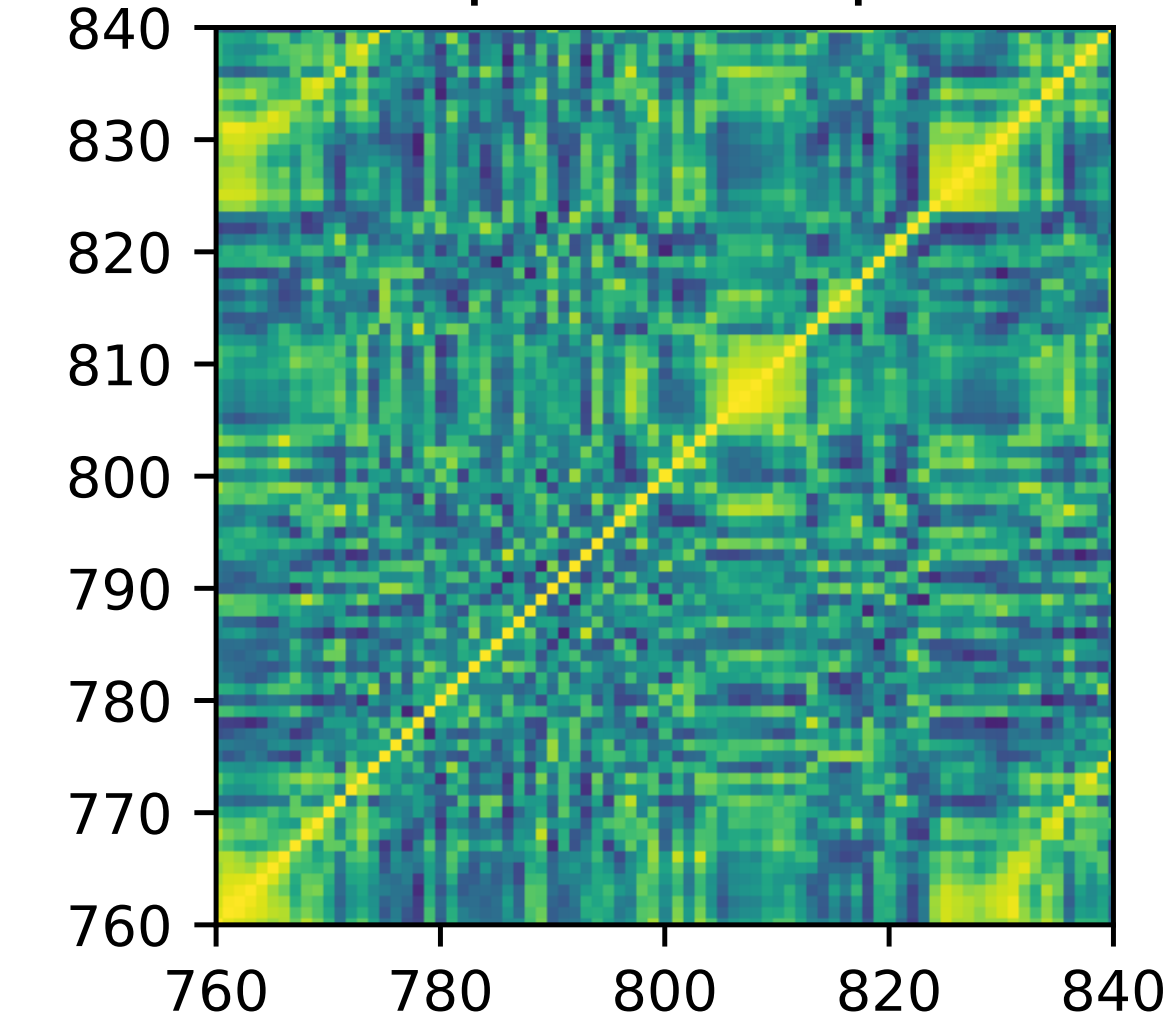
Gaussian 10000 maps



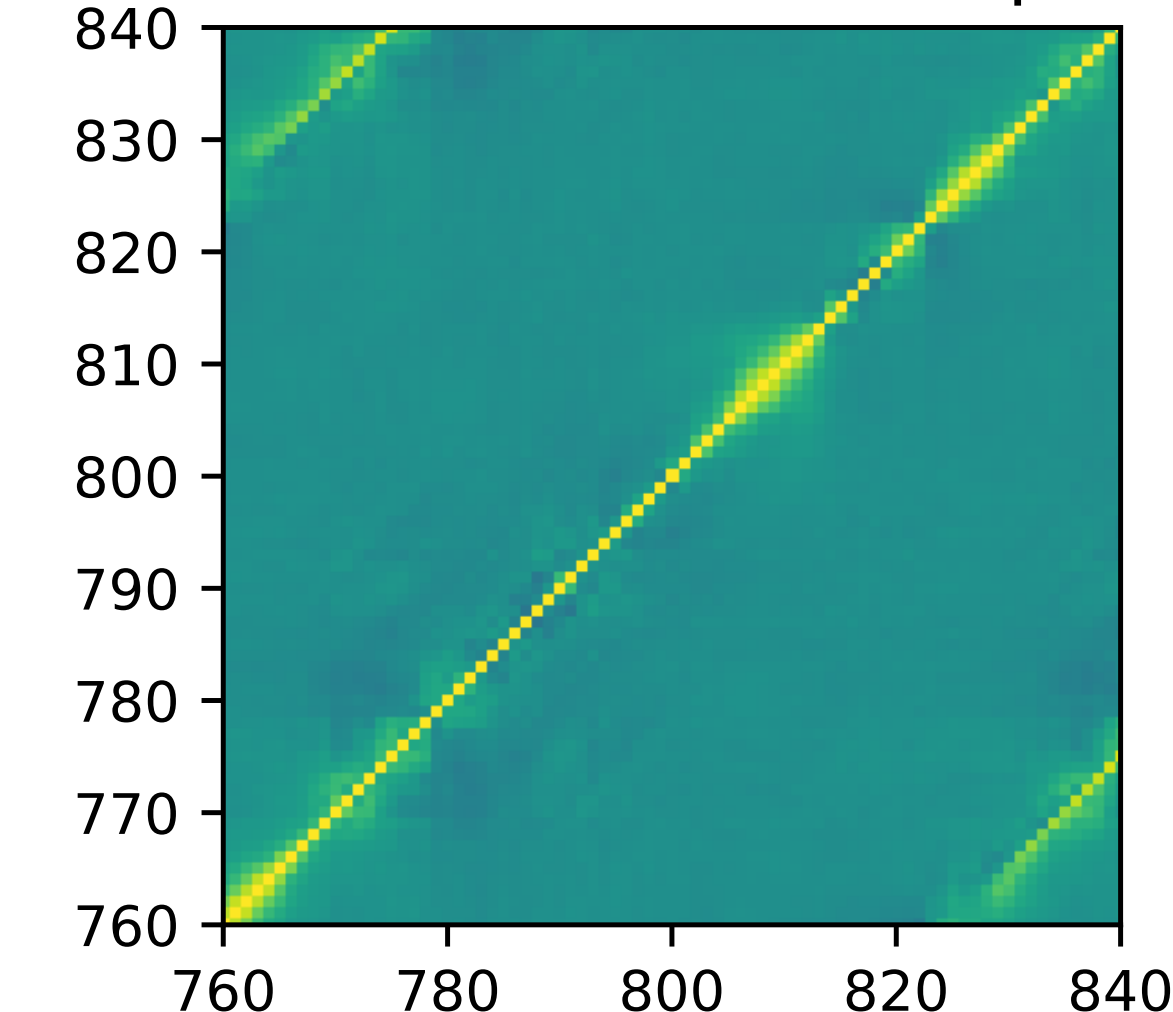
Reference 10000 maps



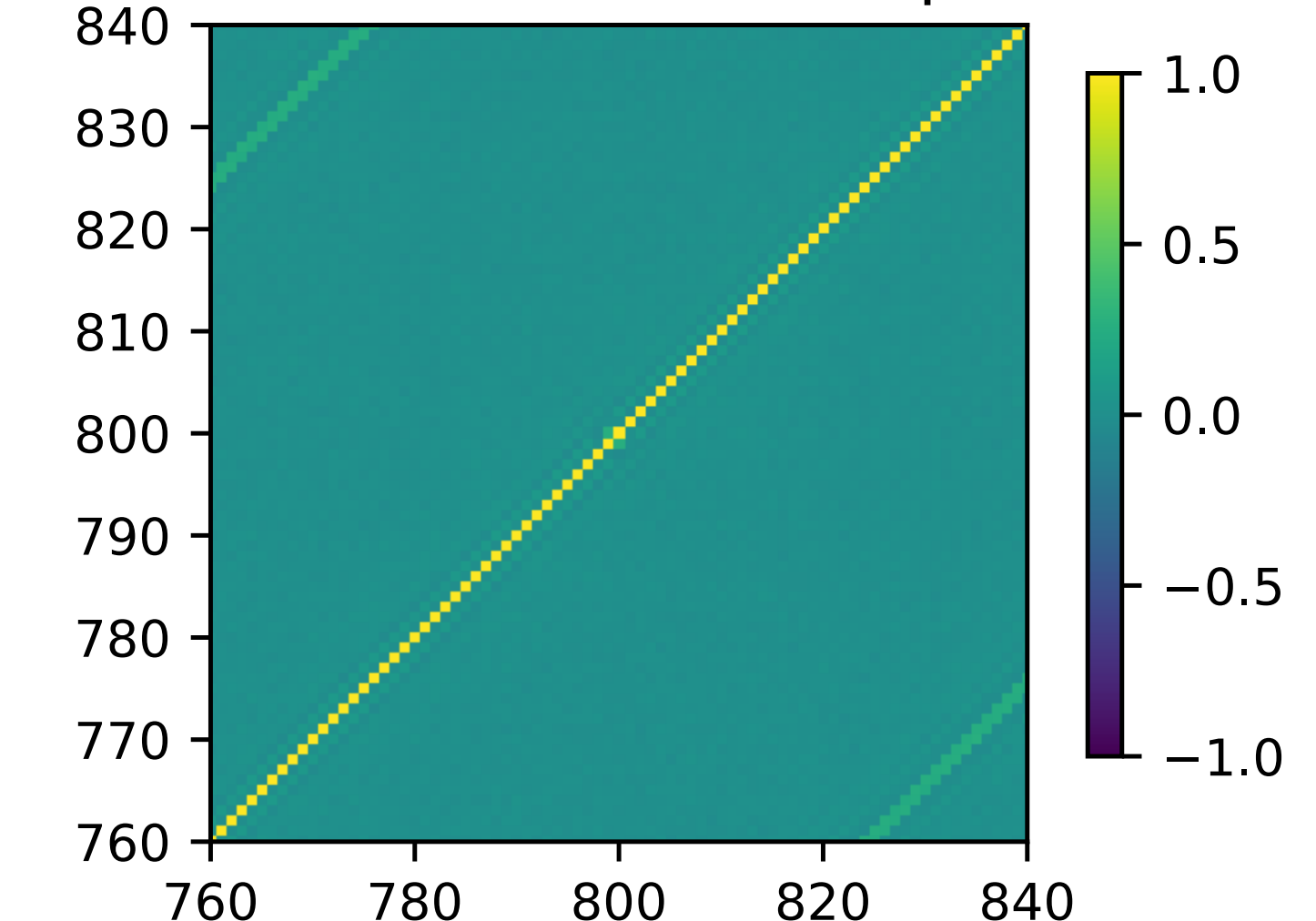
Input 10 maps



Emulated 10000 maps



Gaussian 10000 maps

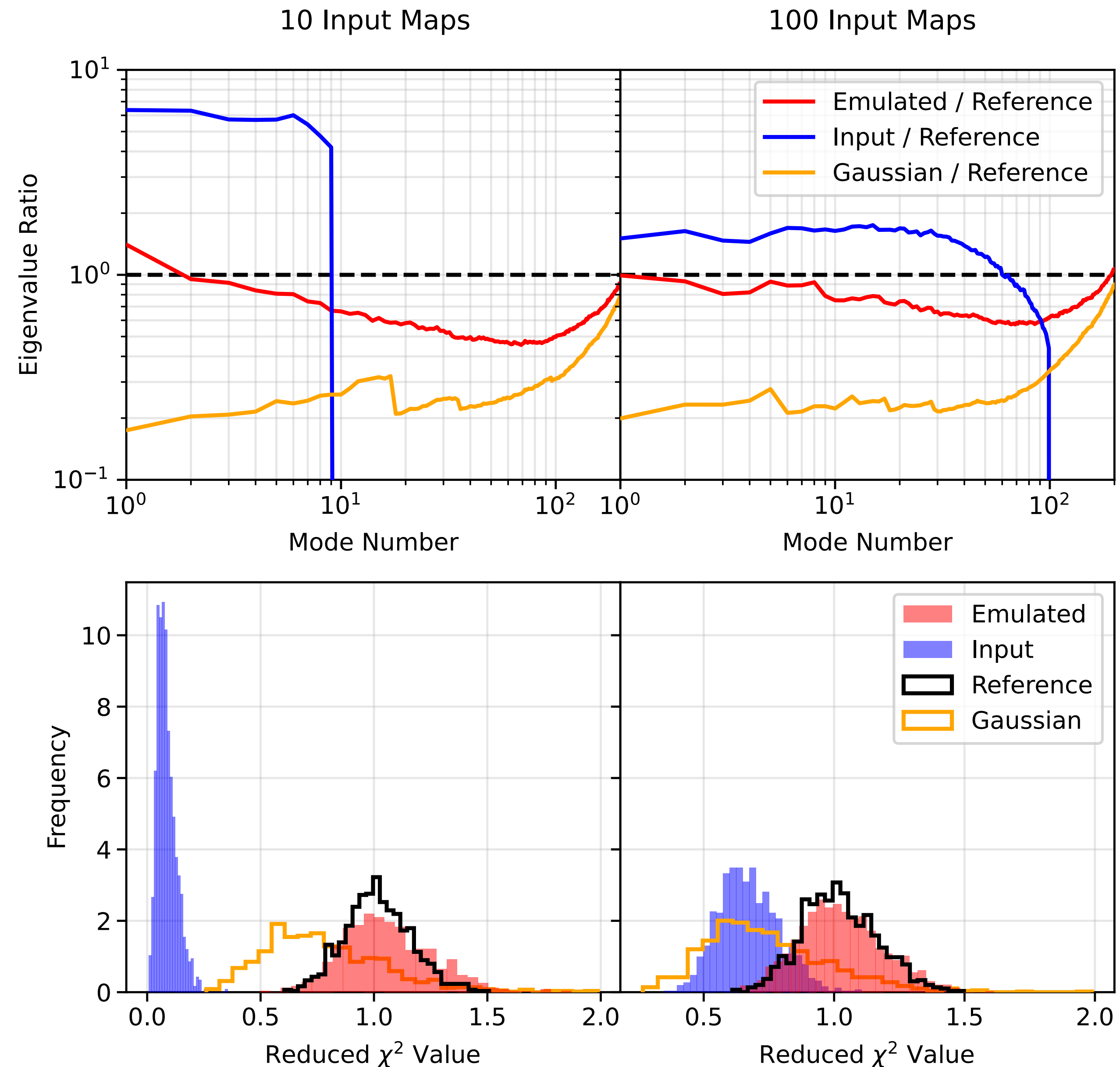


Main results - Eigenvalues and χ^2

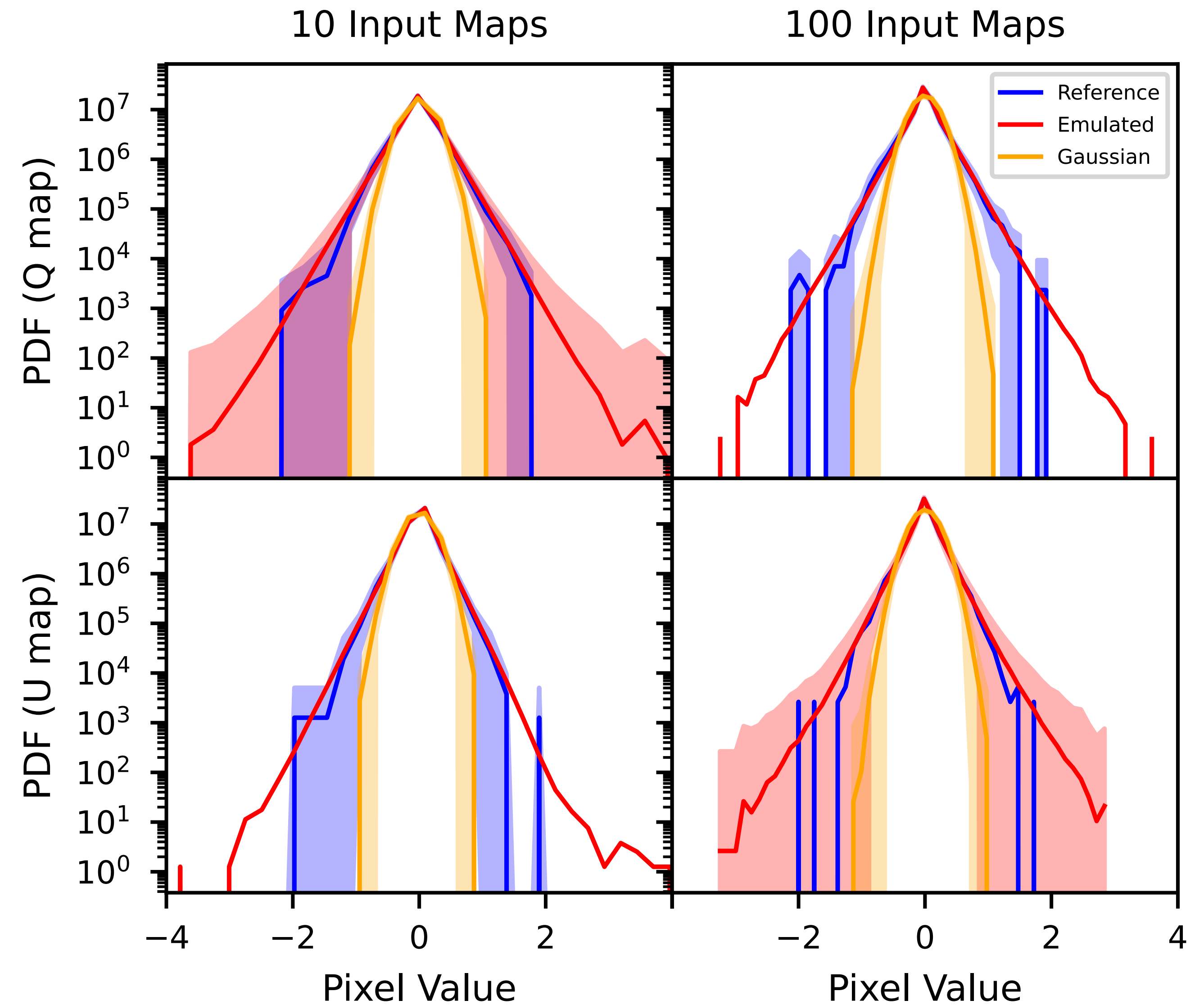
- Reduced χ^2 histogram:

$$\chi^2 = \mathbf{m}^T \mathbf{C}^{-1} \mathbf{m} / d$$

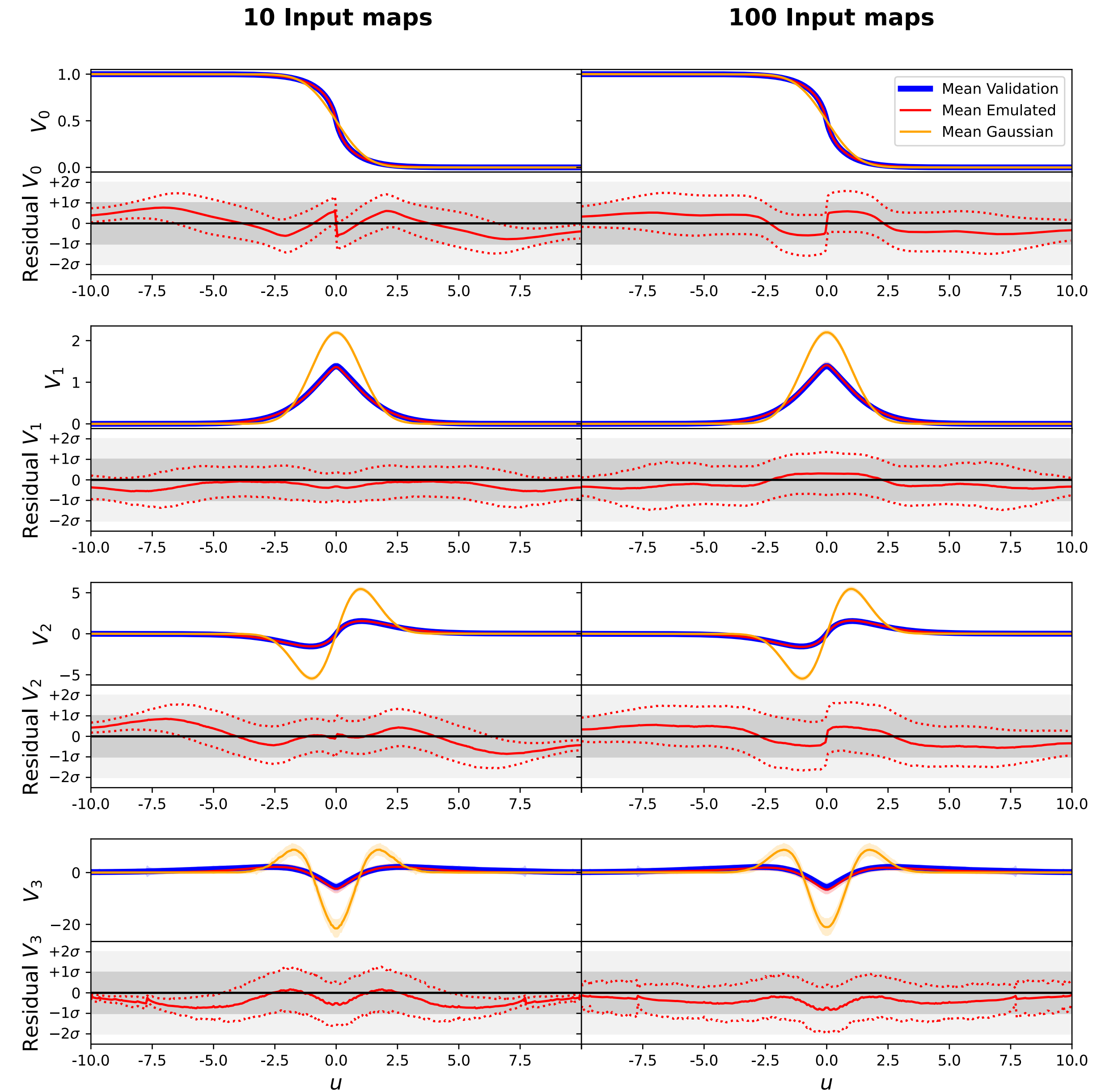
- \mathbf{m} : mean subtracted validation set maps
- \mathbf{C} either reference, input or emulated pixel covariance
- Compare also to **naive Gaussian realizations** from isotropic power spectrum



Main results - Validation on PDFs of maps

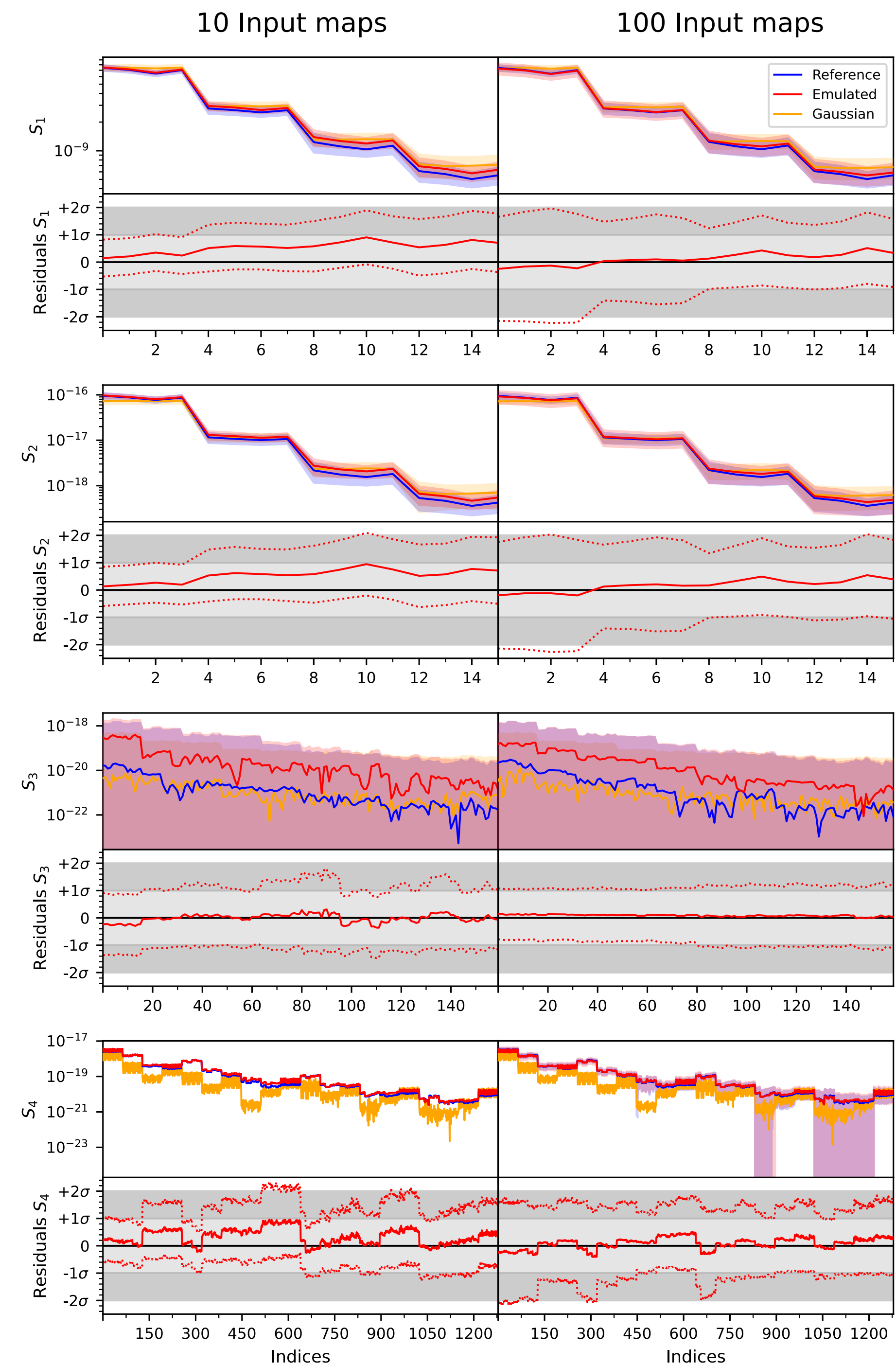


Main results - Minkowski Functionals



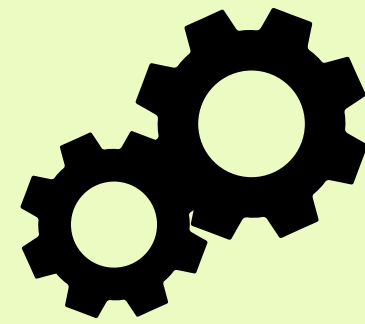
Main results - Scattering Coefficients

- Kernel 3x3
- 4 orientations
- 4 scales ($J_{\max} = 4$)
- Dyadic
- Total of 1474 parameters in scattering coefficients:
 - $S_0 : 2$
 - $S_1 : 4 \times 4 = 16$
 - $S_2 : 4 \times 4 = 16$
 - $S_3 : 10 \times 4 \times 4 = 160$
 - $S_4 : 20 \times 4 \times 4 \times 4 = 1280$



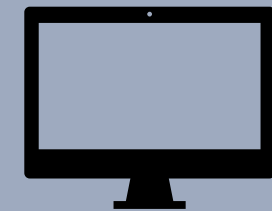
Next Steps and Expected Results

Computational benchmark



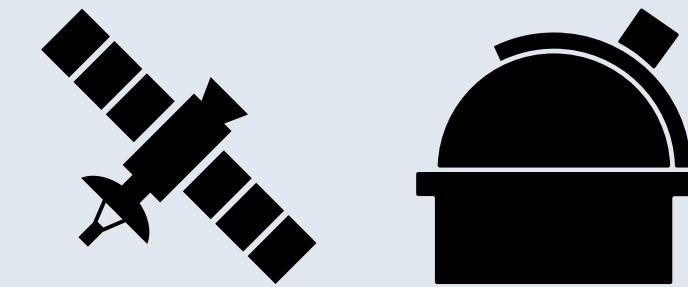
- Emulator is $\sim 10^4$ times faster than going through TOD simulation and runs
- Extremely efficient at low-resolution ($N_{\text{side}} \lesssim 256$)
- Still very fast but too memory-hungry for high resolution ($N_{\text{side}} \gtrsim 512$) \rightarrow we're working on that

Software



- HEALPIXML:
Python+TensorFlow/Torch software for scattering transform:
 - Runs on single/multiple GPUs
 - Available on **GitHub**
- CMBSCAT Emulator + Jupyter notebook demo available on **GitHub**
- Paper out [arxiv:2503.11643](https://arxiv.org/abs/2503.11643) submitted to A&A, extremely positive report

Future Applications



- Applications:
 - Killing sample variance in covariance matrices
 - *Simulation-based inference*
 - *Denoising* of complex instrumental systematics
 - ...
- Completion rate 85-90% exceeding expectations