UNIVERSITY *of the* WESTERN CAPE

# PRoybing the Cosmological Principle with weak lensing shear

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[arXiv:2411.08560], JCAP02(2025)016

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# Introduction

# Motivation

- Homogeneity and isotropy on large scales is foundational to modern cosmology
- Some dark energy, modified gravity models lead to large-scale anisotropies
- Fundamentally, this assumption must be tested
- Renewed interest in anisotropic cosmologies (e.g. SN Ia measurements, CMB dipole)
  $\implies$ Can use weak lensing to probe anisotropies
- Formalism developed by Pitrou, Pereira, & Uzan to estimate B-mode shear generated by anisotropies:
  [arXiv:1503.01125], [arXiv:1503.01127]
- Incorporate non-linear corrections and tomography into results of Pitrou et al. and quantify possible SNR for Euclid

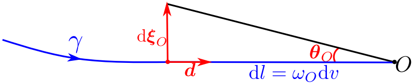
# Lensing Formalism

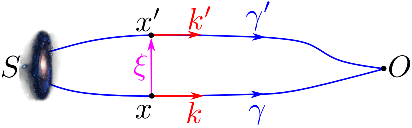
# Lensing distortions

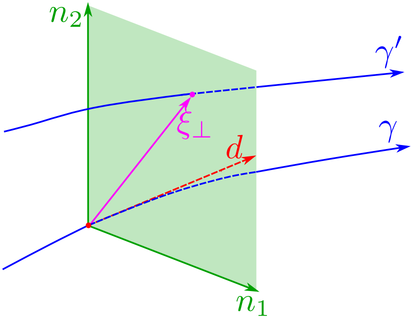
## Jacobi matrix

- Observed angular size $\longmapsto$ physical separation

$$\xi^A\big|_S \propto \mathcal{D}^A{}_B\,\theta^B\big|_O$$



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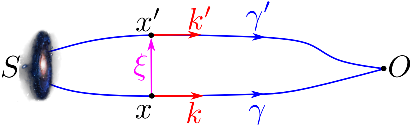
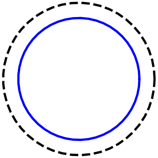


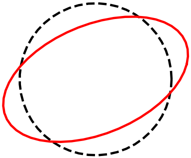

$$\boldsymbol{\mathcal{D}} \approx \bar{D}_A \left[ \overbrace{\begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix}}^{\text{Convergence}} + \underbrace{\begin{pmatrix} 0 & -\psi \\ \psi & 0 \end{pmatrix}}_{\text{Rotation}} + \overbrace{\begin{pmatrix} -\gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_1 \end{pmatrix}}^{\text{Shear}} \right]$$

- Weak lensing: $\kappa$, $\gamma \ll 1$ and $\dot{\psi} = \mathcal{O}(\gamma^2)$
  $\implies$ Ignore rotation (usually)

Convergence:



Shear:



# E-modes, B-modes, and multipoles

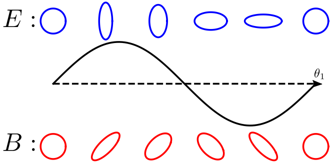
- Expand $\kappa$ in spherical harmonics

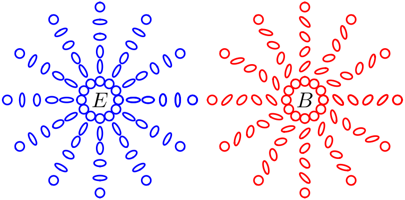
$$\kappa = \sum_{\ell,m} \kappa_{\ell m} Y_{\ell m}$$

- Expand $\gamma$ in spin-weighted spherical harmonics

$$\gamma^{\pm} = \gamma_1 \pm i\gamma_2 = \sum_{\ell,m} (E_{\ell m} \pm iB_{\ell m}) Y_{\ell m}^{\pm 2}$$

- $E$ =even parity, $B$ =odd parity
- No B-modes on large scales (FLRW)*



# Anisotropic Spacetime

# Bianchi-I universes

## Metric

$$\mathrm{d}s^2 = a^2(-\mathrm{d}\eta^2 + \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j)$$

- $a$ = scale factor, $\gamma_{ij}$ = spatial metric
- Hubble rate: $\mathcal{H} \equiv \frac{a'}{a}$
- Spatial shear: $\sigma_{ij} \equiv \frac{1}{2}\gamma'_{ij}$

## Dark Energy

$$T^{de}_{\mu\nu} = (\rho_{de} + P_{de})u_\mu u_\nu + P_{de} g_{\mu\nu} + \Pi_{\mu\nu}$$

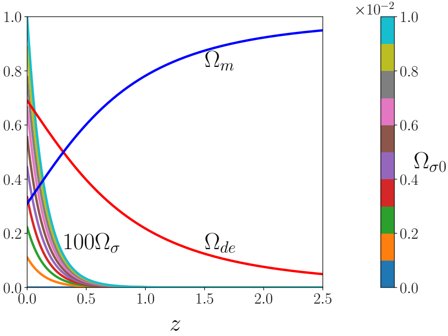
Anisotropic stress

- EoS: $P_{de} = -\rho_{de}$
- Anisotropic stress model: ${\Pi^i}_j \propto \Omega_{de} {W^i}_j$

## Evolution

$$\mathcal{H}^2 = \frac{1}{3}\kappa a^2 \rho + \frac{1}{6}\sigma^2$$

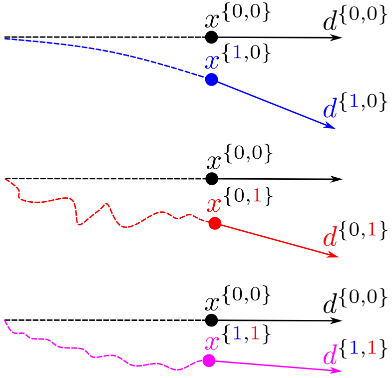
$$({\sigma^i}_j)' = -2\mathcal{H}{\sigma^i}_j + \kappa {\Pi^i}_j$$



# Weak shear limit and perturbation scheme

**Perturbed metric**

$$\mathrm{d}s^2 = a^2\Big[ -(1+2\Phi)\mathrm{d}\eta^2 + 2\bar{B}_i \mathrm{d}x^i \mathrm{d}\eta + (\gamma_{ij} + h_{ij})\mathrm{d}x^i \mathrm{d}x^j \Big]$$




**Perturbation scheme**

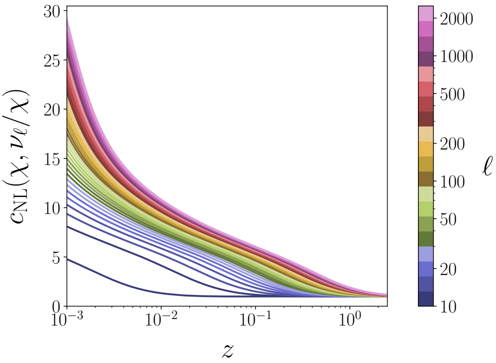
- Treat $\sigma_{ij}/\mathcal{H} \ll 1$ as perturbation along with scalar-vector-tensor (SVT) perturbations
- SVT: $\Phi$, $\bar{B}^i$, $h_{ij} = -2\Psi\left(\gamma_{ij} + \frac{\sigma_{ij}}{\mathcal{H}}\right) + 2E_{ij}$
- Two-fold perturbation $\{n, m\}$ for shear and SVT*:

$$X^{\{n,m\}} = \underbrace{X^{\{0,0\}}}_{\text{FLRW}} + \overbrace{\delta X^{\{0,1\}}}^{\text{SVT}} + \underbrace{\delta X^{\{1,0\}}}_{\text{Shear}} + \overbrace{\delta X^{\{1,1\}} + \ldots + \delta X^{\{n,m\}}}^{\textit{Shear}+\textit{SVT}}$$

# Results

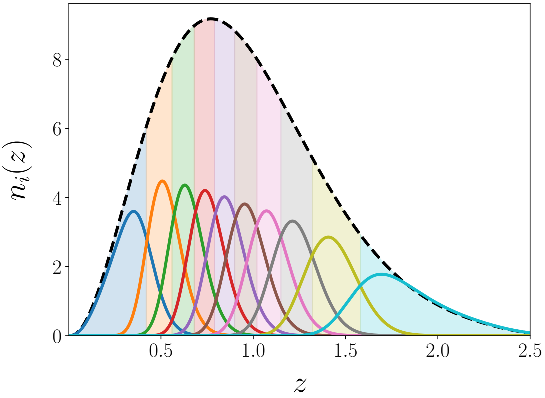
# Non-linear corrections

- Angular power spectra of the form $C_\ell \sim \ell^4 \int \frac{\mathrm{d}\chi}{\chi^2}\, P(k) T^2_\varphi(\chi, k)\big|_{k=\ell/\chi}$
- Calculate non-linear matter power spectrum $P_m^{NL}$ with HaloFit
- Rescale transfer function $T_\varphi \longmapsto c_{NL} T_\varphi$
- Non-linear factor $c_{NL} = \sqrt{P_m^{NL}/P_m^L}$
- Smooth interpolation from linear to non-linear regimes
- Better understand limits of validity for predictions



# Euclid tomography

- Lensing projects/flattens observables
- Tomography regains some projected info.
- Reduce noise and error
- Euclid:
  - 10 equi-populated bins $10^{-3} \leq z \leq 2.5$
  - Weight underlying distribution $n(z)$ with photometric error $p_{\mathrm{ph}}(z_{\mathrm{p}}|z)$



# Angular power spectra

**Order** $\{1, 0\}$

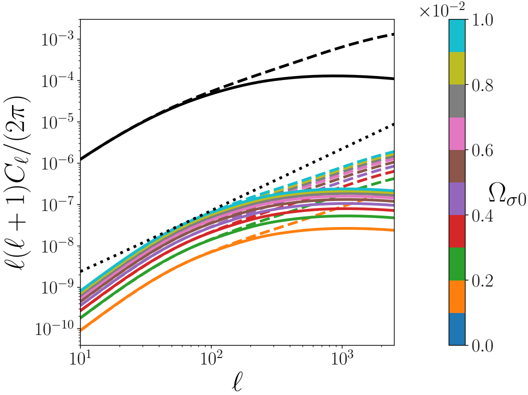
- $\gamma \sim \frac{\sigma}{\mathcal{H}}$
- Fully deterministic
  $\implies$ no power spectrum

**Order** $\{0, 1\}$

- First order in scalars
  $\implies$ only E-modes
- Auto: $C_\ell^{EE} \sim \varphi^2$
- CLASS: $P(k)$, $T_\varphi(\eta, k)$, HaloFit

**Order** $\{1, 1\}$

- Post-Born couples $\sigma$ and scalars
  $\implies$ Non-zero B-modes!
- Auto: $C_\ell^{BB} \sim \left(\frac{\sigma}{\mathcal{H}}\right)^2 \varphi^2$
- Cross: $\left\langle E_{\ell m} B^*_{\ell \pm 1 m'} \right\rangle \sim \left(\frac{\sigma}{\mathcal{H}}\right) \varphi^2$
  $\implies$ off-diagonal (parity)



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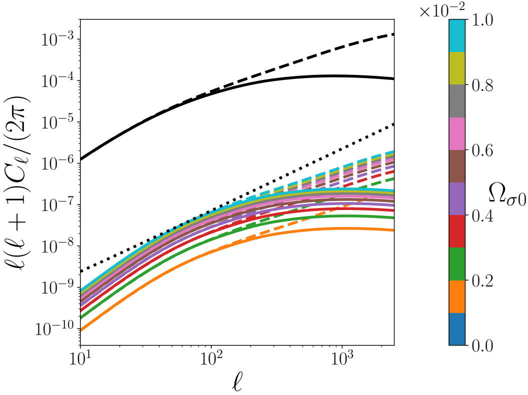



**$C_\ell^{BB}$ likely too small to detect :(**

$\implies$ **Use cross-correlation!**

# E-B cross-correlation

- Cross-corr. to BipoSH:

$$\langle EB \rangle \longmapsto {}^{EB}\mathcal{A}^{\cdots}_{\cdots}$$

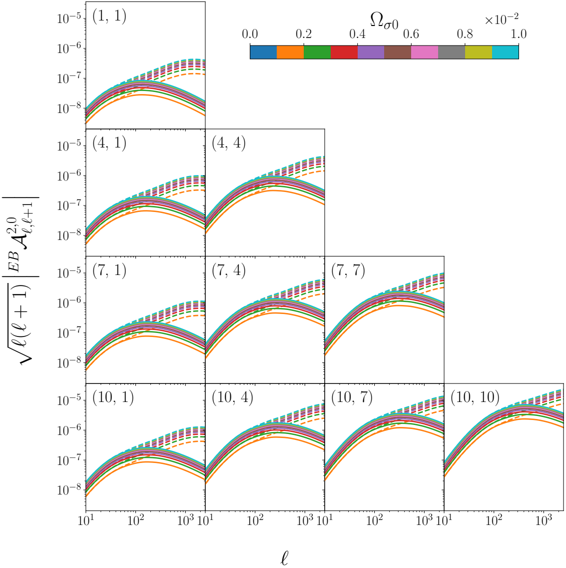
- $\mathcal{P}_{\ell M} \sim {}^{EB}\mathcal{A}^{2M}_{\ell,\ell\pm1}/\ell^{4.5}$ fully captures anisotropy
- Simple estimator $\hat{\mathcal{P}}_{\ell M}$ $\implies$ Compute SNR:

$$\left(\frac{S}{N}\right)^2 \sim \sum \left(\frac{\mathcal{P}_{\ell M}}{\Delta\mathcal{P}_{\ell M}}\right)^2$$

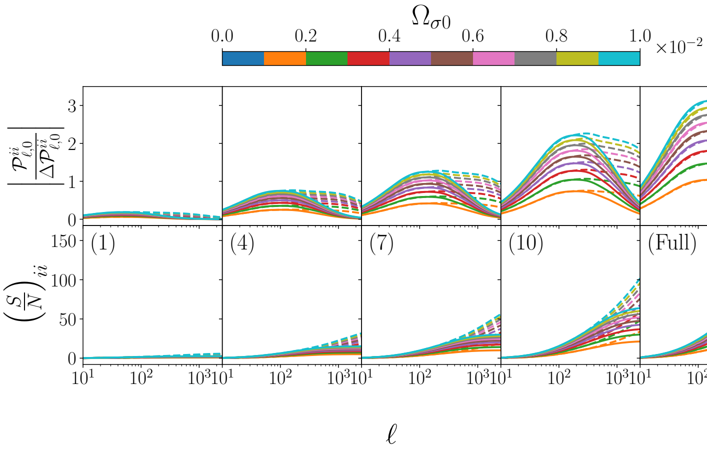
- SI variance:

$$(\Delta\mathcal{P}_{\ell M})^2 = \mathrm{Var}\left(\hat{\mathcal{P}}_{\ell M}\right)_{\mathrm{SI}}$$

- What about $\langle EE \rangle$? $\implies$ Lower SNR!



# Signal-to-noise



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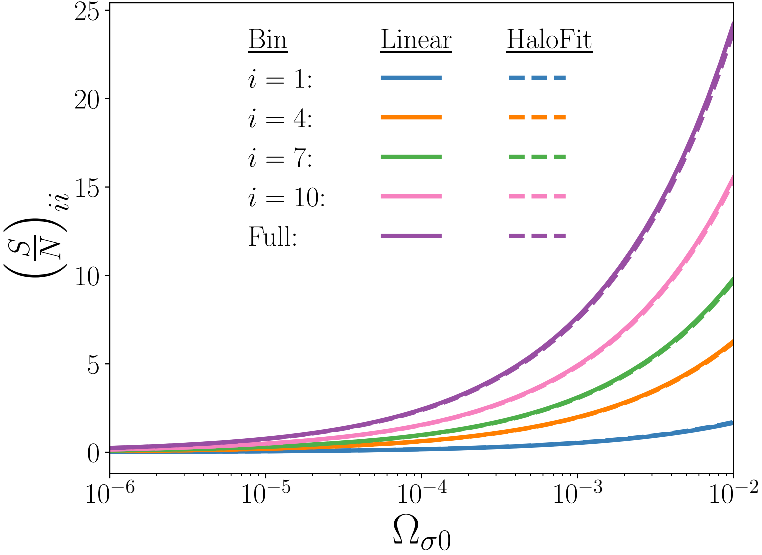

# Conclusion

# Concluding remarks

**Summary**

- Lightning review of lensing formalism
- Applied perturbation scheme and results Pitrou et al.
  $\implies$ incorporated tomography and non-linear corrections
- Use $E$-$B$ cross-correlations in order to constrain late-time anisotropic expansion
- Should construct appropriate estimators for cross-correlations

**Outlook**

- Weak lensing is of immense importance to upcoming surveys
- Currently looking at cross-correlating CMB $\kappa$ with cosmic shear $B$-mode
- Hope to place constraints on $\sigma/\mathcal{H}$ at the percent level

# Questions?