

BARYOGENESIS FROM PRIMORDIAL MAGNETIC FIELDS

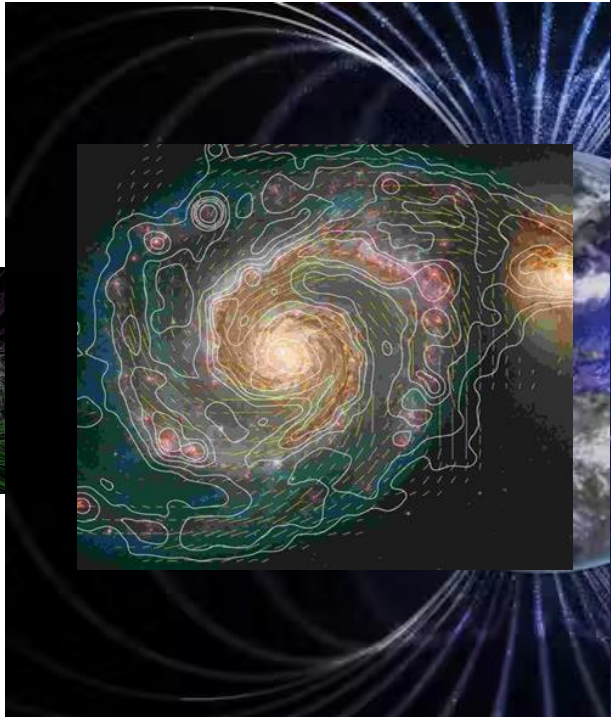
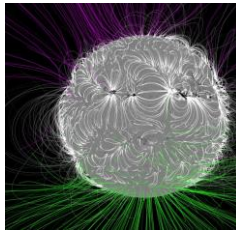
THÉO BOYER 3rd (last 😞) Year PhD student
Supervisor Andrii Neronov

About me

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PhD at APC 2022-2025

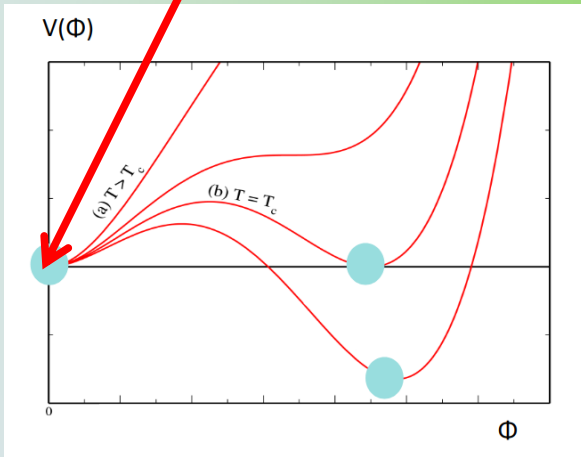
Introduction



Electroweak phase transition

Step 1 : Universe is stuck in the $\Phi=0$ ground state

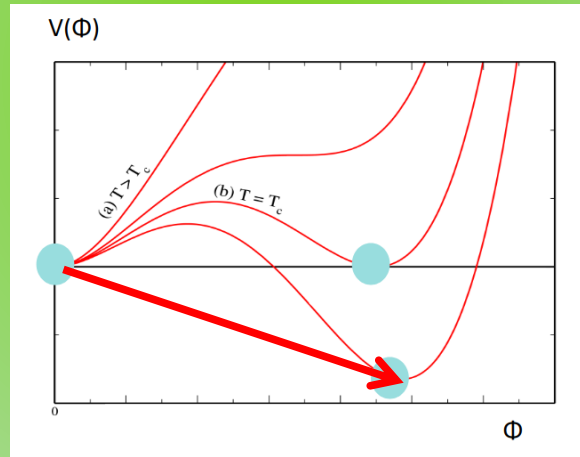
$SU(2) \times U(1)_y$



$T_{EWPT} \sim 160 \text{ GeV}$

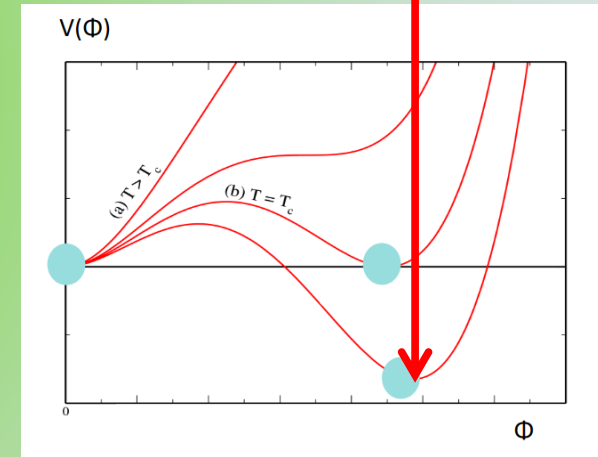
Step 2 : Universe is tunneling through the barrier to reach the true vacuum (on a 3-sphere actually)

$T \approx T_{EWPT}$



Step 3 : Universe has transitioned totally into the true vacuum

$U(1)_{em}$



PMF and Baryogenesis

Since the baryons and leptons are only interacting with SU(2) gauge fields through L-H particles, we can expect an anomaly conducting to a violation of B+L such that (['t Hooft \(1976\)](#)):

$$\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = N_g \frac{g^2}{16\pi^2} \text{Tr} \left[W_{\mu\nu} \widetilde{W}^{\mu\nu} \right] - N_g \frac{g'^2}{32\pi^2} Y_{\mu\nu} \widetilde{Y}^{\mu\nu}$$

We can simplify this expression by integrating it over time :

$$\Delta Q_B = \Delta Q_L = N_g \left(\Delta N_{\text{CS}}^L - \frac{g'^2}{16\pi^2} \Delta \mathcal{H}_Y \right)$$

→ **Baryogenesis from a decaying Hyperhelical magnetic field !**

PMF and Baryogenesis

Electroweak phase transition (EWPT) system of interacting particles ([Kamada & Long arXiv:1606.08891](#)) :

- 3 generation of quarks and leptons, 2 W^{+/-} bosons and 1 Higgs doublet + corresponding antiparticles
- Separation between Left-handed and Right-handed particles
- neutral particles do not intervene in this system

→ System of many kinetic equations

$$\frac{d\vec{\eta}}{dx} = \mathbf{M}\vec{\eta} + \vec{\mathcal{S}}$$

$x = M_0/T$
 $M_0 \sim 10^{17} \text{ GeV}$

\vec{n}_B/s

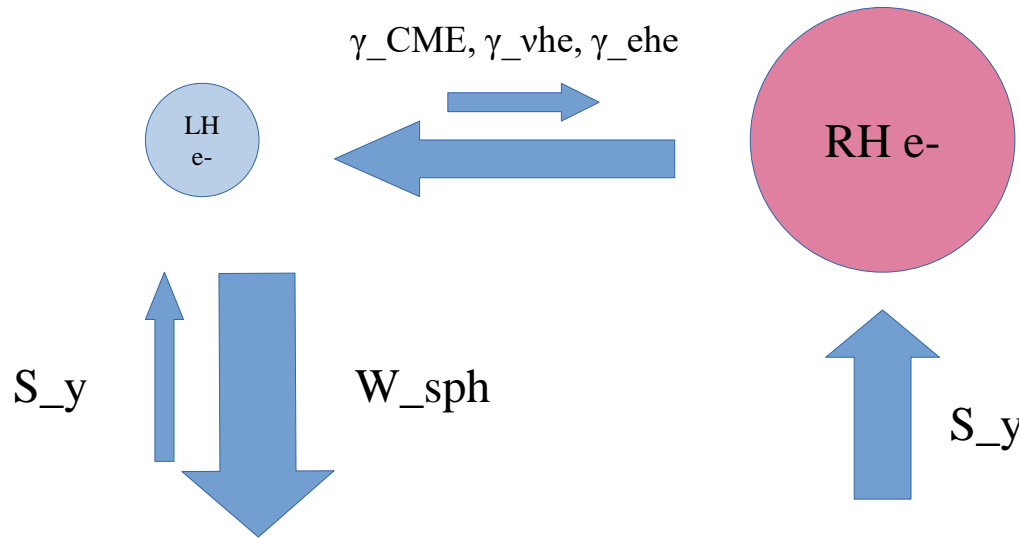
\mathbf{M} = transport coefficients for interacting particles
 \mathcal{S} = anomaly source terms

The diagram illustrates the kinetic equation $\frac{d\vec{\eta}}{dx} = \mathbf{M}\vec{\eta} + \vec{\mathcal{S}}$. A blue arrow points from the definition $x = M_0/T$ to the denominator dx . A red arrow points from the definition \vec{n}_B/s to the vector $\vec{\eta}$. A green arrow points from the definition \mathbf{M} = transport coefficients for interacting particles to the matrix \mathbf{M} . Another green arrow points from the definition \mathcal{S} = anomaly source terms to the vector $\vec{\mathcal{S}}$.

PMF and Baryogenesis

Key mechanism and equilibrium solution for Lepto(baryo)genesis **BEFORE EWPT** :

First generation electron has the smallest Yukawa coupling => last to be at equilibrium



Weak sphaleron is the dominant term but only affects LH particles

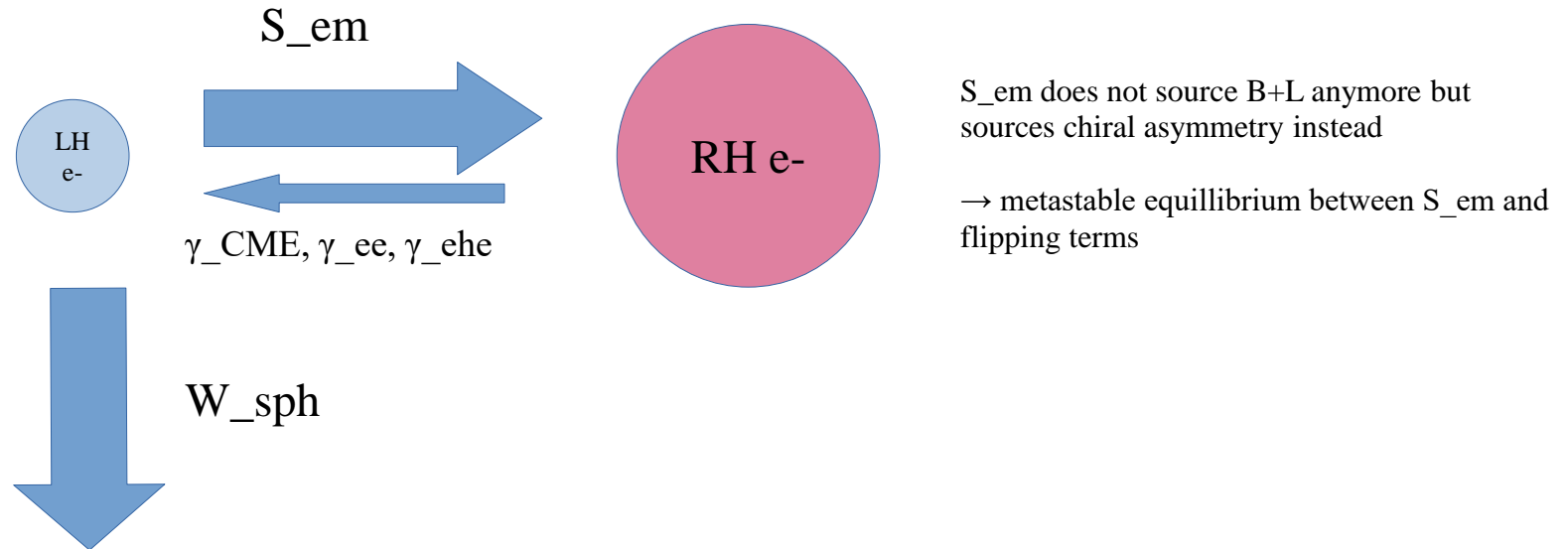
→ competition and equilibrium between the source term from decaying hyperhelical magnetic field S_y and flipping terms

→ Equilibrium reached between RH-LH conversion and sourcing by Hyperhelical fields

PMF and Baryogenesis

Key mechanism and equilibrium solution **AFTER EWPT** :

First generation electron has the smallest Yukawa coupling => last to be at equilibrium



→ quasi-equilibrium reached between the sourcing of R-H electrons and the weak sphaleron washout

PMF and Baryogenesis

We obtain an equilibrium solution in each phase

Symmetric Phase

$$\eta_{B,\text{eq}} = \eta_{L,\text{eq}} \approx \frac{\mathcal{S}_y}{\gamma_{h\leftrightarrow ee} + \gamma_y^{\text{CME}}}$$

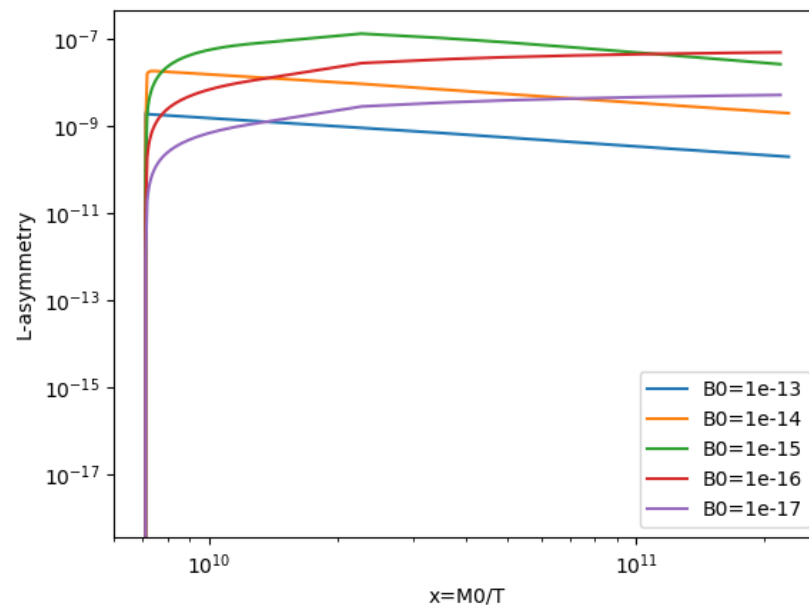
Broken phase

$$\eta_{B,\text{eq}} = \eta_{L,\text{eq}} \approx \frac{\mathcal{S}_{\text{em}}}{\gamma_{h\leftrightarrow ee} + \gamma_{\text{flip}} + \gamma_{\text{em}}^{\text{CME}}}$$

→ B asymmetry preserved at sphaleron freeze-out

T_{fo} ~135 GeV

Simulations are in agreement with the analytical solutions



Constraints on PMF/IGMF

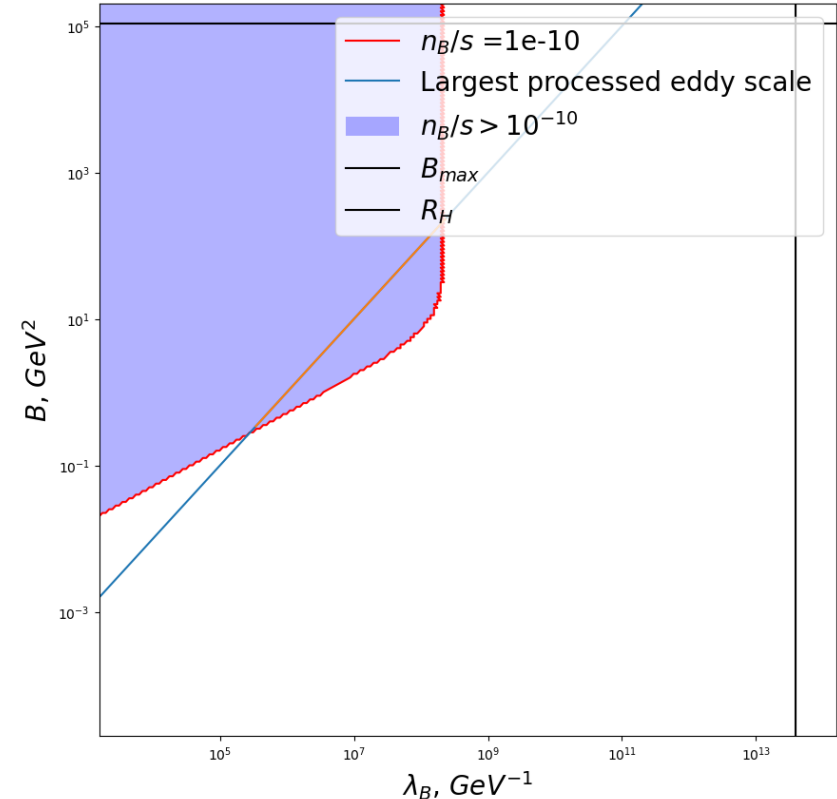
$$\eta_B = 7.3 \times 10^{-2} \frac{B_p^2}{\lambda_{B,p} T^5} \frac{1}{3.5 \times 10^{-8} \frac{m_b^2(T)}{T^2} + 2.1 \times 10^{-9} \frac{v(T)^2}{T^2} + \frac{B_p^2}{T^4}}$$

Constraints in the parameters space (B, λ_B) by overproduction of baryons at the time of freeze-out.

→ 2 distinct regimes : **strong field** and **weak field**

$$\lambda_B \gtrsim \begin{cases} 10^{10} T_*^{-1} & , B \gg 5 \times 10^{-4} T_*^2 \\ 10^{10} T_*^{-1} \left(\frac{B}{5 \times 10^{-4} T_*^2} \right)^2 & , B \ll 5 \times 10^{-4} T_*^2 \end{cases}$$

However, $B < B_{\max}$ (equipartition with photons)



Constraint on magnetic field parameters at the time of sphaleron freeze-out $T^* \sim 135$ GeV compared to the largest processed eddy relation

Evolution of PMF to nowadays

For a fully helical magnetic field, **helicity conservation** imposes :

$$\lambda_{B,c} B_c^2 = a^3 \lambda_{B,p} B_p^2 = \lambda_0 B_0$$

Once the **largest processed eddy scale** catches up the magnetic field correlation length, the system enters the inverse cascade process. From this point :

$$\lambda_{B,p} = v_A t = \frac{B_p}{\sqrt{\mu_0 \rho_{rad}}} t$$

We get the evolution law for the correlation length and the amplitude of a fully helical magnetic field :

$$B_p \simeq (1 \times 10^{20} \text{G}) \left(\frac{T}{100 \text{GeV}} \right)^{7/3} \left(\frac{B_0}{10^{-14} \text{G}} \right)$$
$$\lambda_B \simeq (2 \times 10^{-29} \text{Mpc}) \left(\frac{T}{100 \text{GeV}} \right)^{-5/3} \left(\frac{\lambda_0}{1 \text{pc}} \right)$$

PMF and Baryogenesis

Supposing an inverse cascade process :

→ We retrieve the results from [1606.08891](https://arxiv.org/abs/1606.08891)

→ Baryogenesis from decaying hypermagnetic fields not compatible with MHD constraints

$\eta_{B_max} \sim$

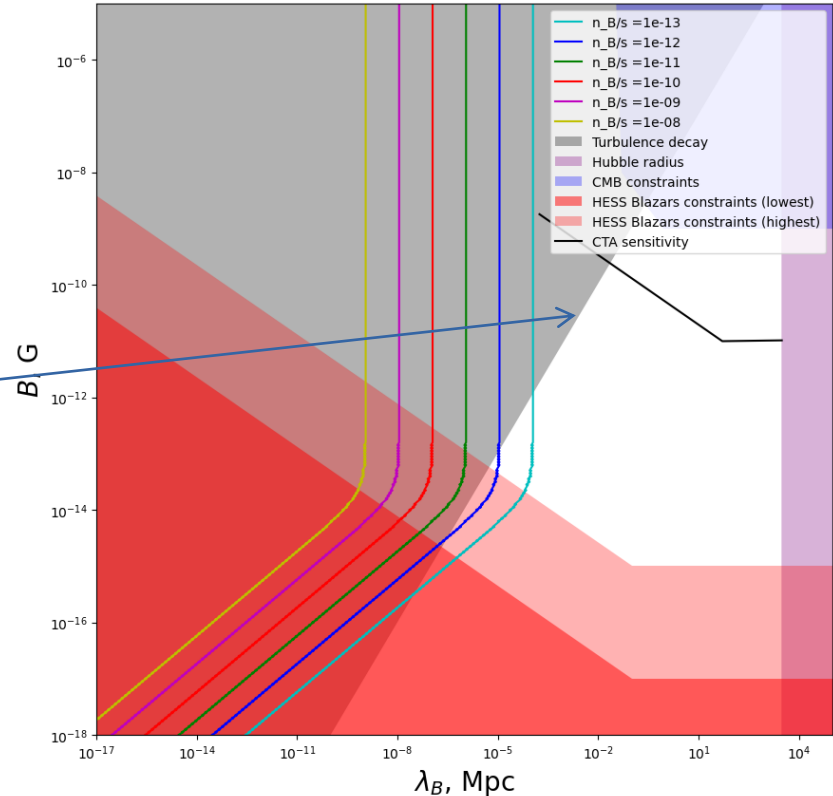
$$10^{-11}$$

$$\left(\frac{\lambda_0}{1 \text{ pc}} \right) \sim \left(\frac{B_0}{10^{-14} \text{ G}} \right)$$

However if the hypermagnetic field is sourced on a **fixed scale** (for example from a 1st order phase transition)

→ no constraints from MHD turbulence

→ Baryogenesis can be made and current magnetic fields can satisfy the MHD constraint at the same time



Values of B , λ_B nowadays compatible with different amounts of baryon asymmetry, supposing an inverse cascade process

PMF and Baryogenesis

Supposing an inverse cascade process :

→ We retrieve the results from [1606.08891](https://arxiv.org/abs/1606.08891)

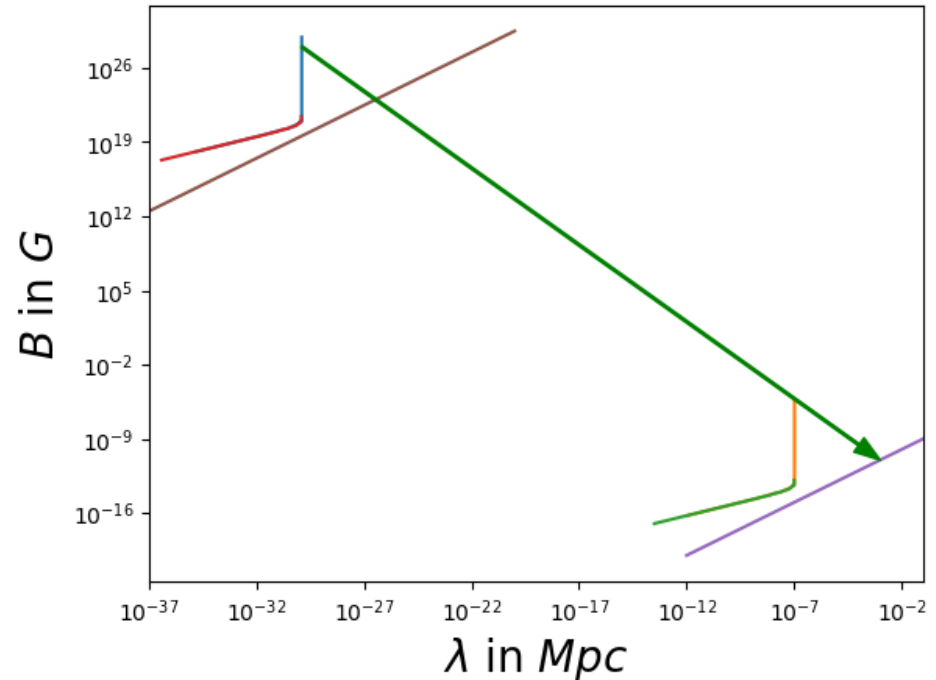
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$$\eta_{\mathbf{B}_{\max}} \sim 10^{-11}$$

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Trajectory of evolution of a magnetic field generated at EWPT- on a fixed scale

Evolution of PMF to nowadays

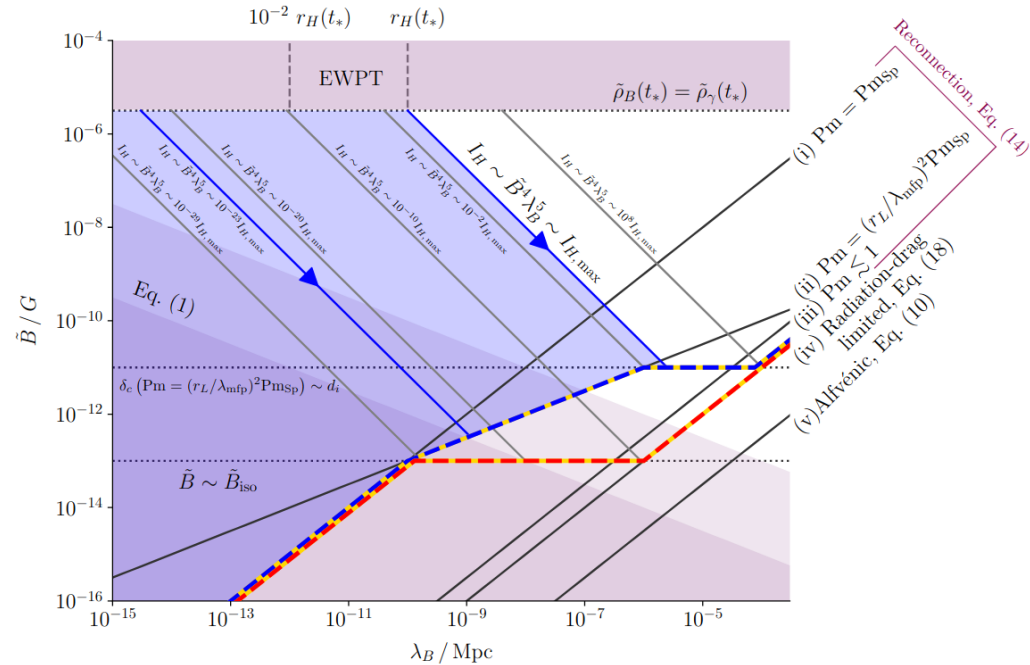
However, the alfvenic decay of magnetic field is questionable :

From [2203.03573](#) : reconnection seems to be the dominant process in the evolution behavior of magnetic fields.

$$\tau_{\text{rec}} = (1 + \text{Pm})^{1/2} \min \left\{ S^{1/2}, S_c^{1/2} \right\} \frac{\lambda_B}{\tilde{v}_A}$$

To simplify the situation and express our ignorance on the behavior of decaying magnetic fields we introduce the free parameter C_{rec} such that :

$$\lambda_{B,p} = C_{\text{rec}} v_A t = C_{\text{rec}} \frac{B}{\sqrt{\mu_0 \rho_{\text{rad}}}} t$$



Plot from 2203.03573 David N. Hosking and Alexander A. Schekochihin

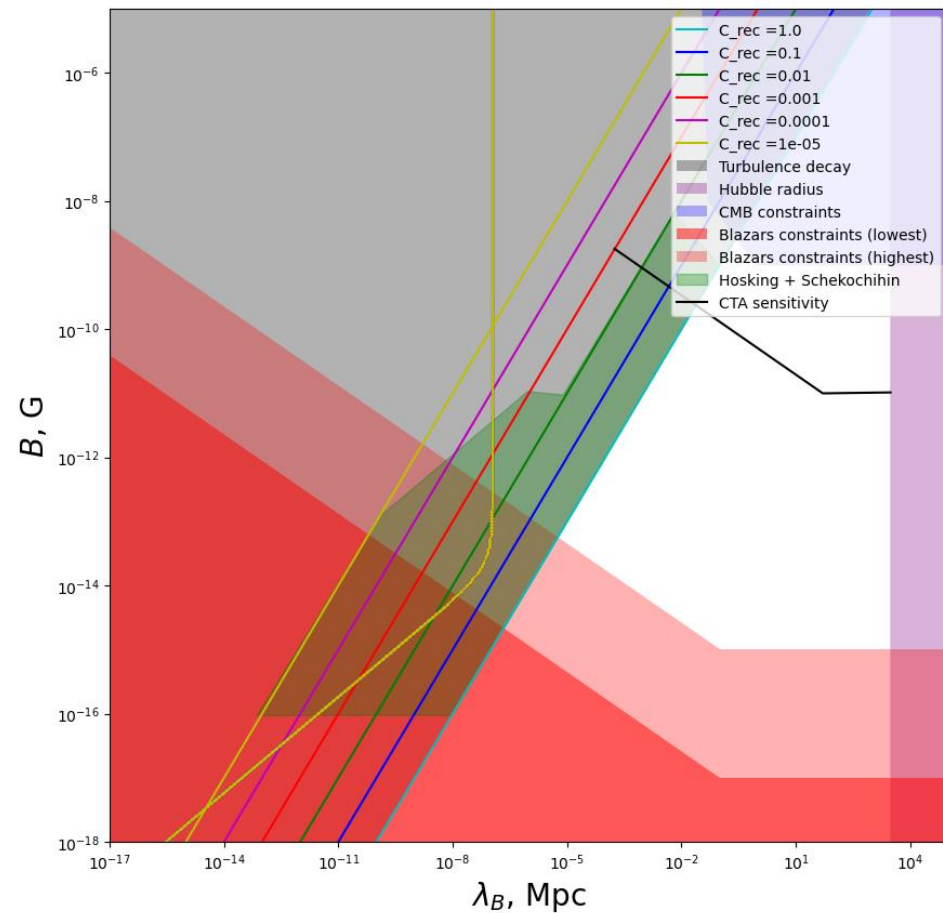
PMF and Baryogenesis

→ Range of values for C_{rec} compatible with the amount of baryon asymmetry observed

→ Intersection with plot from gives us the range of values for C_{rec} compatible with Hyperhelical baryogenesis and MHD evolution constraints.

$$\rightarrow 5 \times 10^{-4} < C_{rec} < 0.06$$

Different points of compatibility between MHD evolution and baryogenesis before EWPT



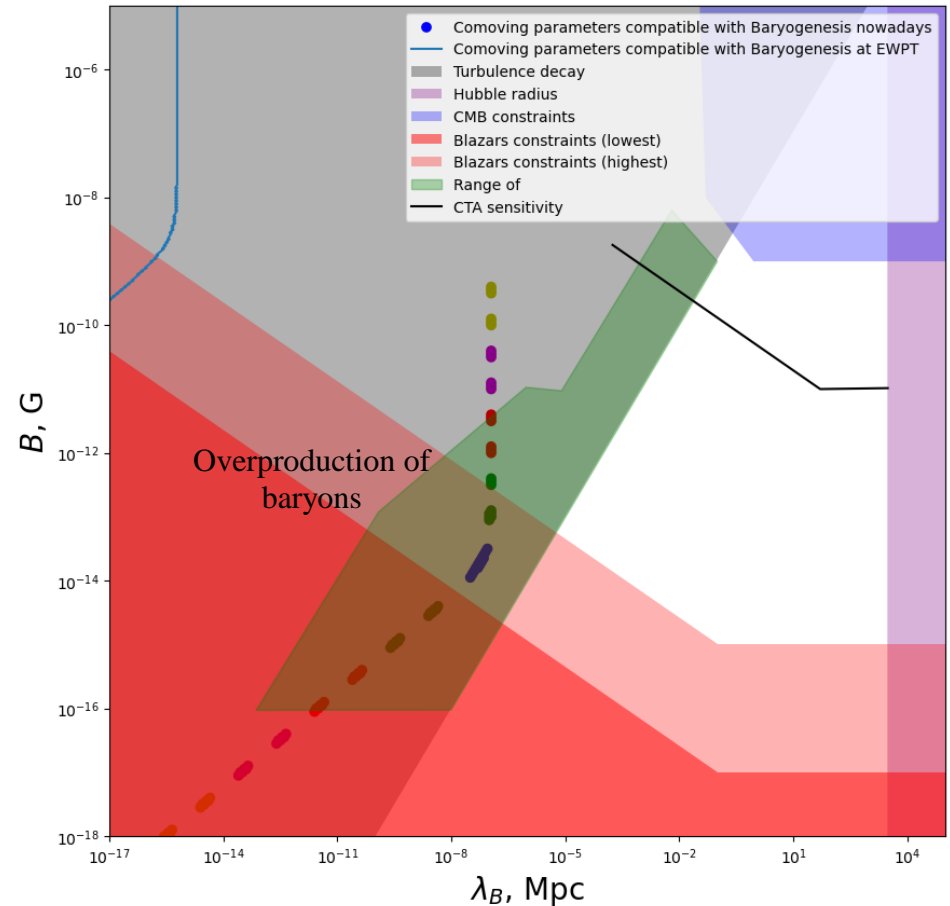
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Different points of compatibility between MHD evolution and baryogenesis before EWPT



Isocurvature Perturbations at BBN

Baryogenesis from hyperhelical magnetic fields can be inhomogeneous.

Locally we have :
$$\eta_B = \eta_B(\mathbf{r}) = \eta_L(\mathbf{r}) \approx \frac{11}{37} \frac{\mathcal{S}_{\text{em}}(\mathbf{r})}{\gamma_{\text{emE}}^{\text{CME}}(\mathbf{r})}$$

$$\mathcal{S}_{\text{em}}(\mathbf{r}) = \frac{\alpha_{\text{em}}}{\pi \sigma_{\text{em}} s T} \vec{B}_p \cdot \vec{\nabla} \times \vec{B}_p(\mathbf{r})$$

$$\gamma_{\text{em}}^{\text{CME}}(\mathbf{r}) = \frac{12}{\pi^2} \alpha_{\text{em}}^2 \frac{\vec{B}_p \cdot \vec{B}_p(\mathbf{r})}{\sigma_{\text{em}} T^3}$$

So finally :
$$\eta_B(r) = \mathcal{F} \frac{\vec{B}_p \cdot \vec{\nabla} \times \vec{B}_p}{B_p^2}$$

We are interested in spatial fluctuations of $\eta_B(\mathbf{r})$, following [9710234 \(M. Giovannini, M. E. Shaposhnikov\) 2012.14435 \(K. Kamada, F. Uchida, J. Yokoyama\)](#) :

$$\langle \eta_B(\vec{x}) \eta_B(\vec{x} + \vec{r}) \rangle = \mathcal{F}^2 \left\langle \frac{\vec{B}_p \cdot \nabla \times \vec{B}_p}{B_p^2}(\vec{x}) \frac{\vec{B}_p \cdot \nabla \times \vec{B}_p}{B_p^2}(\vec{x} + \vec{r}) \right\rangle$$

Isocurvature Perturbations at BBN

A recent constraint at 2σ on isocurvature perturbations at BBN was presented by K. Inomata, M. Kawasaki, A. Kusenko, L. Yang in [1806.00123](#) such that :

$$\overline{S_{B,\text{BBN}}^2}(\mathbf{x}) < 0.016 \quad (2\sigma) \quad \langle y_d \rangle = 18.754 - 1534.4 \bar{\omega}_B + 48656 \bar{\omega}_B^2 - 552670 \bar{\omega}_B^3 + (48656 \bar{\omega}_B^2 - 1658010 \bar{\omega}_B^3) \langle S_B^2 \rangle,$$

→ **constrained by Deuterium production** that depends on isocurvature perturbations to the 2nd order

We introduce the function

$$S_B(\vec{x}) = \frac{\delta\eta_B(\vec{x})}{\bar{\eta}_B} \quad \delta\eta_B = \eta_B(\vec{x}) - \bar{\eta}_B \quad \text{with}$$

Taking into account the **neutron diffusion** and recombination

$$\text{where } D=3/2 \quad \overline{S_{B,\text{BBN}}^2} = \int \frac{d^3k}{(2\pi)^3} e^{-\frac{k^2}{2D}} \mathcal{G}(\vec{k})$$

comoving neutron diffusion length

Where $\mathcal{G}(\vec{k})$ the Fourier transform of $\mathcal{G}(\vec{r}) = \langle S_B(\vec{x}) S_B(\vec{x} + \vec{r}) \rangle = \frac{\langle \eta_B(\vec{x}) \eta_B(\vec{x} + \vec{r}) \rangle}{\bar{\eta}_B^2} - 1$

Isocurvature Perturbations at BBN

We can express the magnetic field spectrum such as :

$$\langle B_i^*(\vec{k}) B_j(\vec{k}') \rangle = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') \tilde{\mathcal{F}}_{ij}^B(\vec{k})$$

With $\tilde{\mathcal{F}}_{ij}^B(\vec{k}) = P_{ij}(\vec{k}) \tilde{S}^B(k) + i\epsilon_{ijm} k_m \tilde{A}^B(k)$

$$A^B(k) = \epsilon S^B(k) \text{ fraction}$$

We assume that PMF are **not responsible** of the baryon asymmetry of the Universe. (for now)

Inserting this expression in the isocurvature fluctuation equation :

$$\begin{aligned} \overline{S_{B,\text{BBN}}^2} &= \frac{\mathcal{C}^2}{4\pi^4 \bar{\eta}_B^2} \int dk_1 dk_2 k_1^2 k_2^2 S^B(k_1) S^B(k_2) \\ &\left\{ \frac{(k_1 + k_2)^2}{2} (1 + \epsilon^2) \frac{D}{k_1 k_2} \left(1 - \frac{D}{k_1 k_2} \right) + \left[\frac{k_1^2 + k_2^2}{2} + \epsilon^2 k_1 k_2 \right] \left(\frac{D}{k_1 k_2} \right)^3 \right\} \exp \left[-\frac{(k_1 - k_2)^2}{2D} \right] \\ &- \left\{ \frac{(k_1 - k_2)^2}{2} (1 - \epsilon^2) \frac{D}{k_1 k_2} \left(1 + \frac{D}{k_1 k_2} \right) + \left[\frac{k_1^2 + k_2^2}{2} + \epsilon^2 k_1 k_2 \right] \left(\frac{D}{k_1 k_2} \right)^3 \right\} \exp \left[-\frac{(k_1 + k_2)^2}{2D} \right] \end{aligned}$$

Isocurvature Perturbations at BBN

To evaluate the isocurvature fluctuations we need a model for the magnetic field spectrum.

As in [2012.14435](#) we use a delta function model such that :

$$S^B(k) = \pi^2 \frac{B_0^2}{k_B^2} \delta(k - k_B)$$

where k_B is the wavenumber of the magnetic field.

By inserting this into the expression of the isocurvature fluctuations we get :

$$\langle S_{B,\text{BBN}}^2 \rangle = \frac{(1 + \epsilon^2)}{4\bar{n}_b^2 T_*^2} \left[D \left(1 - \frac{D}{k_B^2} \right) + k_B^2 \left(\frac{D}{k_B^2} \right)^3 \left(1 - \exp \left[-\frac{2k_B^2}{D} \right] \right) \right]$$

Isocurvature Perturbations at BBN

In the approximation where $k_B^2 \gg D$

$$\langle S_{B,\text{BBN}}^2 \rangle \simeq \frac{(1 + \epsilon^2)}{4\bar{\eta}_b^2 T_*^2 \lambda_n^2} \sim 10^{-11} (1 + \epsilon^2)$$

→ Well below the current constraint $\overline{S_{B,\text{BBN}}^2(\mathbf{x})} < 0.016 \quad (2\sigma)$

However interesting tool : also works for non helical magnetic fields → average helicity 0 but local helicity may vary

→ **constraints on the helicity fraction** of the field if this one is responsible for baryogenesis

Be carefull : magnetic helicity != current helicity ! Although maybe same sign ... (A. J. B. Russell and al. (2019))

+ Non-helical fields evolution (Hosking integral etc.)

Current investigations

[2012.14435 , 9707513 , T.Vashaspati \(1991\) ...](#)

→ Additional contribution during (2nd order / crossover) EWPT ? Term due to varying Weinberg angle ? Magnetic fields generated from Higgs gradients ?

→ Simulations of the EWPT through a toy model in the Cosmolattice code

