Small scale baryon fluctuations from causal primordial magnetic fields

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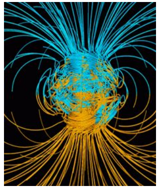
A preliminary snapshot of this work was presented at the international workshop **Cosmic Magnetism with Voids and Filaments**, DamsLab, Bologna, Italy, January 23-27, 2023, and at conference **Generation, evolution, and observations of cosmological magnetic fields**, Lausanne, May 15, 2024

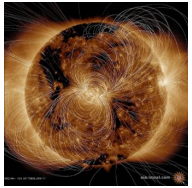
A. Ciabattoni, F. Finelli, D. Paoletti, K. Subramanian, 'Small scale baryon fluctuations from causal primordial magnetic fields', to appear (2025)

Introduction - Primordial Magnetic Fields (PMFs)

Magnetic fields are everywhere in the Universe, at all scales probed so far.

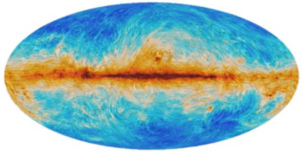


Earth's magnetic field
(Glatzmaier+ 1995)

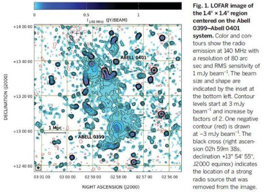


Sun magnetic field
(NASA/GSFC/Solar dynamics observatory)

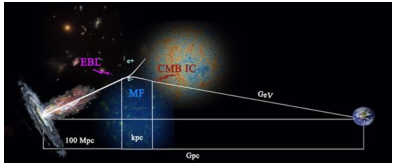
from small to large scales...



Map of Milky way MF
(Planck 2015 Results I)



MF in filaments
(Govoni+ 2019)



MF in voids

Origin? Possible generation by amplification of weak primordial magnetic fields.

Importance? PMFs strongly affect cosmological observables, in particular the *Cosmic Microwave Background* (CMB) (gravitational effects, ionization history, non-Gaussianities, Faraday rotation, parity-violating correlators etc.)

→ **fantastic probe for the physics of the early Universe!**

Introduction - Primordial Magnetic Fields (PMFs)

Standard approach for cosmological perturbations: PMF modelled as stochastic background, assumption of linearity in cosmological perturbations, quadratic terms in B and ideal magneto-hydrodynamics (MHD), with a spectrum described by a power-law up to a dissipation scale. These PMFs affect cosmological perturbations for different initial conditions (compensated, passive...) at CMB scales (k approximately smaller than 0.1/Mpc).

Small scales : PMFs components of the EMT computed as convolutions peak at the dissipation scale, much beyond the CMB scales

➡ **non-linearities in the baryon inhomogeneities induced by PMFs relevant for inhomogeneous recombination? Impact on the Hubble constant?** (Jedamzik Abel 2013, Jedamzik Saveliev 2018, Jedamzik Pogosian 2020)

The impact of PMFs into CMB anisotropy

Standard assumptions:

- ❑ **Stochastic background** of PMF

$$\langle B_i(\mathbf{k})B_j^*(\mathbf{k}')\rangle = \frac{(2\pi)^3}{2}\delta^{(3)}(\mathbf{k}-\mathbf{k}')(\delta_{ij}-\hat{k}_i\hat{k}_j)P_B(k)$$

$$P_B(k) = A\,k^{n_B}$$ *Magnetic power spectrum* *(viscous damping at kD)*

$E = 0$

$B \propto a^{-2}$

The (gravitational) impact of PMFs into CMB anisotropy

SB of PMFs influence the evolution of cosmological perturbations:

- they carry **energy-momentum tensor** (treated as a first-order quantity);
- they carry **anisotropic stress** ;
- they induce a **Lorentz force** on baryons;
- the PMF behaves as a stiff matter component which redshifts as radiation;
- the general relativistic Einstein-Boltzmann system of equations becomes inhomogeneous being **driven** by the PMF EMT system.

$$\tau^0_0 = -\rho_B = -\frac{\mathbf{B}^2(\mathbf{x})}{8\pi\, a^4} \,,$$

$$\tau^0_i = 0 \,,$$

$$\tau^i_j = \frac{1}{4\pi\, a^4}\left(\frac{\mathbf{B}^2(\mathbf{x})}{2}\,\delta^i_j - \mathbf{B}_j(\mathbf{x})\,\mathbf{B}^i(\mathbf{x})\right)$$

EMT quadratic in the field
→ *convolutions!*

$$|\rho_B(k)|^2 = \frac{1}{1024\,\pi^5}\int_\Omega \mathrm{d}^3p\, P_B(p)\, P_B(|\mathbf{k}-\mathbf{p}|)\,(1+\mu^2)$$

$$|L^{(S)}(k)|^2 = \frac{1}{128\,\pi^2\, a^8}\int_\Omega \mathrm{d}^3p\, P_B(p)\, P_B(|\mathbf{k}-\mathbf{p}|)\,[1+\mu^2+4\gamma\beta\,(\gamma\beta-\mu)]$$

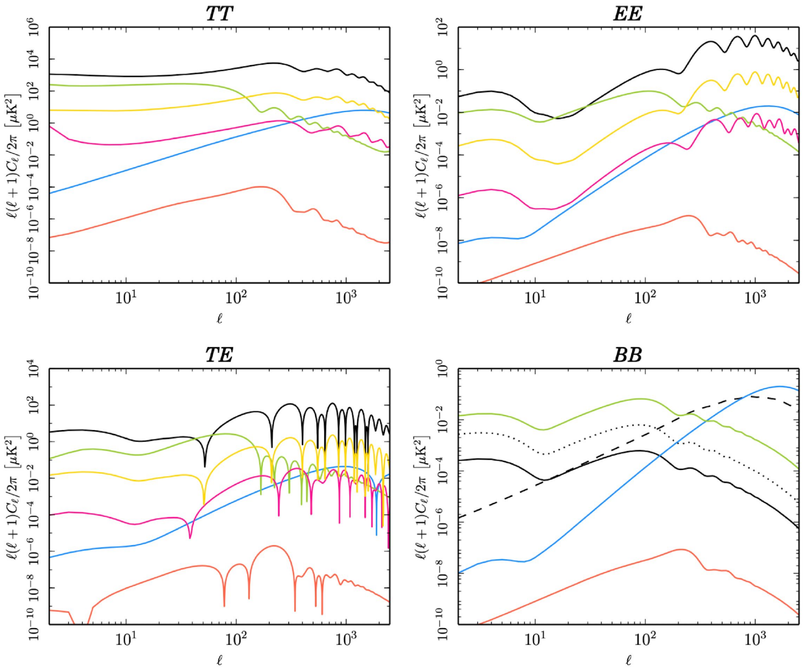
$$p_B = \frac{\rho_B}{3}$$ *Longitudinal pressure*

$$\nabla_\mu \tau^\mu_\nu = -F^\mu_\nu\, J_\nu$$ ➡ $$\sigma_B = \frac{\rho_B}{3} + L$$

Cosmological perturbations driven by a SB of PMFs

- **Tensor**
  - Durrer R., Ferreira P. G., Kahniashvili, 2000
  - Caprini C., Durrer R., Kahniashvili T., 2003
  - ....
- **Vector**
  - Mack A., Kahnashvili T., Kosowsky A., 2002
  - Subramanian K., Seshadri T. R., Barrow J. D., 2003
  - Lewis A., 2004
  - ...
- **Scalar**
  - Kahnashvili T., Ratra B., 2007
  - Giovannini M., Kunze K. E., 2008
  - Finelli F., Paci F., Paoletti D., 2008
  - ...
- **All**
  - Paoletti D., Finelli F., Paci F., 2009
  - Shaw, Lewis A., 2009, 2010
  - Zucca, Li, Pogosian, 2016
  - ...
- **All (including helicity)**
  - Ballardini M., Finelli F., Paoletti D., 2015
  - ...

CMB from SB of PMFs (gravitational effects)



Adiabatic (no PMF)

Scalar compensated

Vector

Tensor compensated

Scalar Passive

Tensor Passive

Dashed - B-mode lensing

Paoletti, Finelli, JCAP 2019

CMB from SB of PMFs

Other effects beyond the gravitational one:

- **heating (MHD decaying turbulence and ambipolar diffusion)**
- **Faraday rotation**
- **non Gaussianities**
- **parity violating correlators**

We restrict our study to **scalar gravitational effects**, assuming adiabatic initial conditions and causal primordial magnetic fields (**$n_B = 2$**)

Effects of PMFs on baryon fluctuations

Besides the impact onto CMB anisotropies, whose observational window lies on large and intermediate scales (k $<0.1/\mathrm{Mpc}$), the Lorentz force associated to PMFs enhance baryon fluctuations at smaller scales, comparable to the scale of dissipation (Wasserman 1978; Kim, Rosner, Olinto 1996; Subramanian, Barrow 1998; Finelli, Paoletti, Paci 2008; Jedamzik, Abel 2013, ...)

Convolution integrals

D. Paoletti, Master thesis, 2007; F. Finelli, Paci F., Paoletti D., 2008

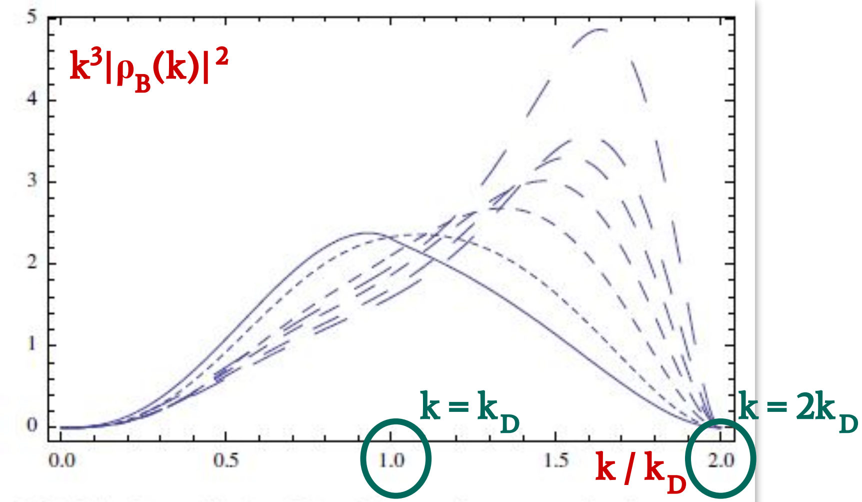


FIG. 2 (color online). Plot of magnetic energy density power spectrum $k^3|\rho_B(k)|^2$ in units of $\langle B^2\rangle^2/(1024\pi^3)$ versus k/k_D for different n_B for fixed $\langle B^2\rangle$. The different lines are for $n_B = -3/2, -1, 0, 1, 2, 3, 4$ ranging from the solid to the longest dashed.

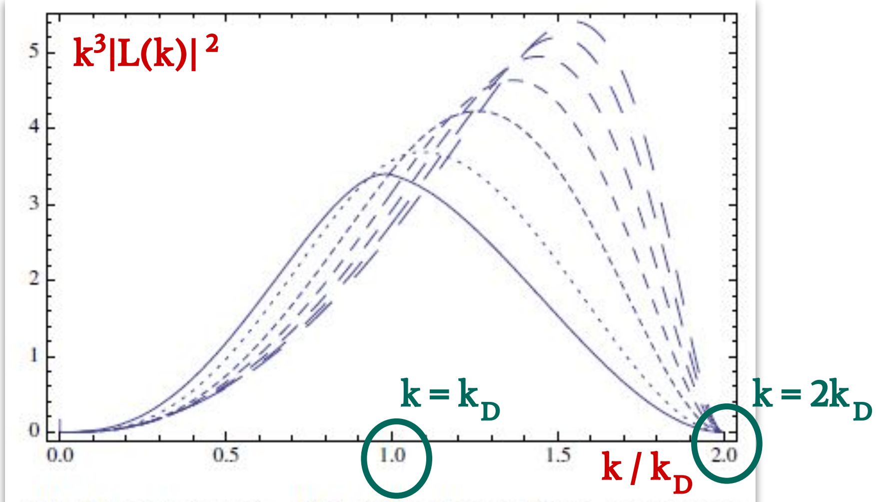


FIG. 4 (color online). Plot of the Lorentz force power spectrum $k^3|L(k)|^2$ in units of $\langle B^2\rangle^2/(1024\pi^3)$ versus k/k_D for different n_B for fixed $\langle B^2\rangle$. The different lines are for $n_B = -3/2, -1, 0, 1, 2, 3, 4$ ranging from the solid to the longest dashed.

$$|\rho_B(k)|^2_{n_B=2} = \frac{A^2 k_D^7}{512\pi^4 k_*^4}\left[\frac{4}{7} - \tilde{k} + \frac{8\tilde{k}^2}{15} - \frac{\tilde{k}^5}{24} + \frac{11\tilde{k}^7}{2240}\right],$$

$$|L(k)|^2_{n_B=2} = \frac{A^2 k_D^7}{512\pi^4 k_*^4}\left[\frac{44}{105} - \frac{2\tilde{k}}{3} + \frac{8\tilde{k}^2}{15} - \frac{\tilde{k}^3}{6} - \frac{\tilde{k}^5}{240} + \frac{13\tilde{k}^7}{6720}\right]$$

Cold dark matter and baryon perturbations

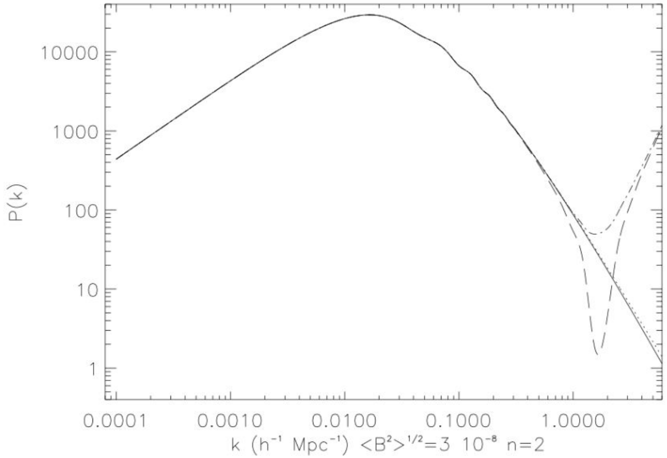


FIG. 13: Linear cold dark matter power spectrum obtained with fully correlated PMF with (dashed line) and without (dotted line) Lorentz term, with uncorrelated PMF and the Lorentz force (dot-dashed line) in comparison with the vanishing PMF (solid line). In the figure $\sqrt{\langle B^2 \rangle} = 3 \times 10^{-8}$ Gauss, $k_D = 2\pi$ and $n_B = 2$ are considered. The other cosmological parameters are the same as Fig. (6).

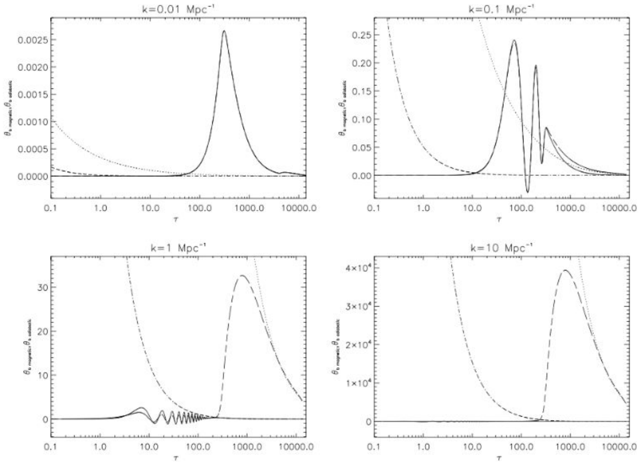


FIG. 6: Evolution of baryons velocity for 4 different wavenumbers with (dashed) and without (solid) PMF. $k^2 L/\rho_b$ (dot-dashed line) and the solution $\theta_b^{\rm late}$ (dotted line) are also plotted: note how the numerics agree with $\theta_b^{\rm late}$ at late times. The cosmological parameters of the flat ΛCDM model are $\Omega_b h^2 = 0.022$, $\Omega_c h^2 = 0.123$, $\tau = 0.04$, $n_s = 1$, $H_0 = 72\,{\rm km\,s^{-1}\,Mpc^{-1}}$.

Finelli, Paci, Paoletti 2008

Effects of PMFs on baryon fluctuations

We improved and extended the publicly available Einstein Boltzmann code **SONG**, based on CLASS, in order to include the PMF contributions to scalar perturbations. We aligned the code with the modified CAMB version of Paoletti used for CMB calculations with PMFs

The Lorentz force

Scalar Euler equation for baryons:

$$\dot{\theta}_b = -\mathcal{H}\,\theta_b + k^2\psi + k^2 c_s^2\delta_b + R\frac{d\kappa}{d\tau}(\theta_\gamma - \theta_b) - \frac{k^2}{4\pi\rho_b}L$$

Lorentz force term

Scalar Euler equation for photons:

$$\dot{\theta}_\gamma = k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) + k^2\psi + \frac{d\kappa}{d\tau}(\theta_b - \theta_\gamma)$$

The PMF energy-momentum tensor

Perturbed Einstein equations: $$\delta G_{\mu\nu} = 8\pi G\,(\delta T_{\mu\nu} + \tau_{\mu\nu})$$ → **PMF EMT**

$$6\mathcal{H}^2\Psi + 6\mathcal{H}\,\Phi' + 2k^2\Phi = -\,8\pi G a^2\left(\sum_n \bar{\rho}_n\,\delta_n + \rho_B\right),$$

$$6\ddot{\Phi} + \Psi(6\mathcal{H}^2 + 12\dot{\mathcal{H}}) + 6\mathcal{H}(\dot{\Psi} + 2\dot{\Phi}) + 2k^2(\Phi - \Psi) = 8\pi G a^2\left(3\sum_n c_{sn}^2\,\bar{\rho}_n\,\delta_n + \rho_B\right),$$

$$2k(\dot{\Phi} + \mathcal{H}\Psi) = 8\pi G a^2 \sum_n (\bar{\rho}_n + \bar{P}_n)\,\theta_n\,,$$

$$-\frac{2k^2}{3}(\Phi - \Psi) = 8\pi G a^2\left(\sum_n (\bar{\rho}_n + \bar{P}_n)\,\sigma_n + \sigma_B\right)$$

PMF energy density (← ρ_B)

PMF anisotropic stress (← σ_B)

PMFs are an extra source of anisotropic stress, together with that of neutrinos and photons.

Damping scale

$$k_D = \alpha \left(\frac{B_{rms}}{\text{nG}}\right)^{-1} h^{1/2} \left(\frac{\Omega_b h^2}{0.022}\right)^{1/2} \text{Mpc}^{-1}$$

The exact value of the damping scale kD is affected by theoretical uncertainty . This uncertainty is less relevant for large scales affecting CMB, however **it significantly affects the results on small scales** around the damping scale at the peak of the convolutions!

We parameterize this uncertainty with **α** and test three values:

α = 3, 30, 300

α = 3 is closer to values reported in Trivedi, Reppin, Banerjee, Chluba MNRAS 2018

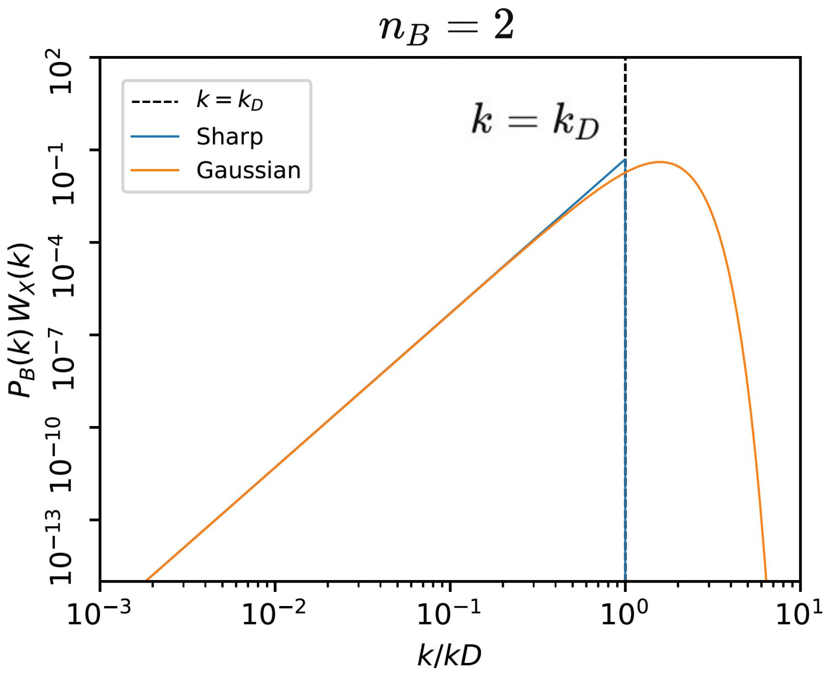
α = 300 is closer to estimate in Subramanian, Barrow 1998, Jedamzik Katalinic Olinto 1998

Different damping profiles

$$\langle B_X^2 \rangle = \frac{1}{2\pi^2} \int_0^\infty dk\, k^2\, P_B(k)\, W_X(k)$$

$$W_S(k) = \Theta(k_D - k)$$ usually assumed for CMB

$$W_G(k) = e^{-k^2/k_D^2}$$



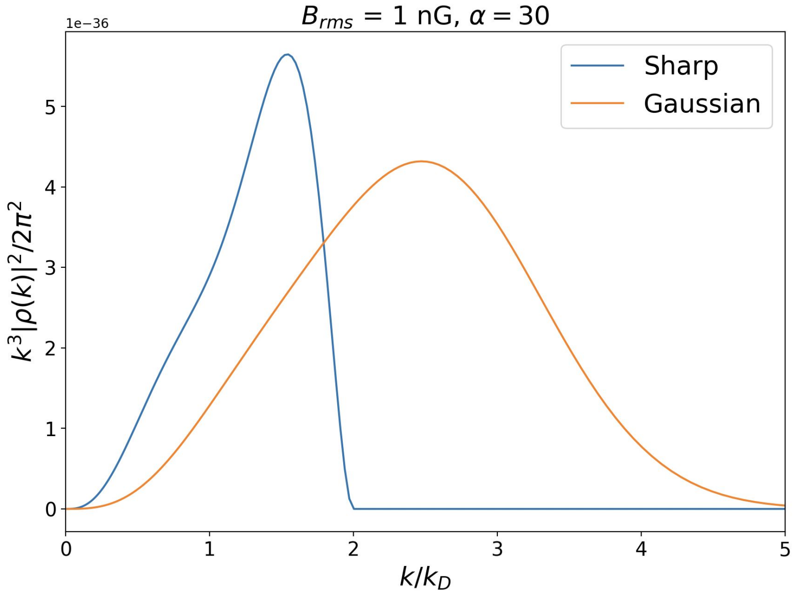
Magnetic power spectrum for the two damping profiles

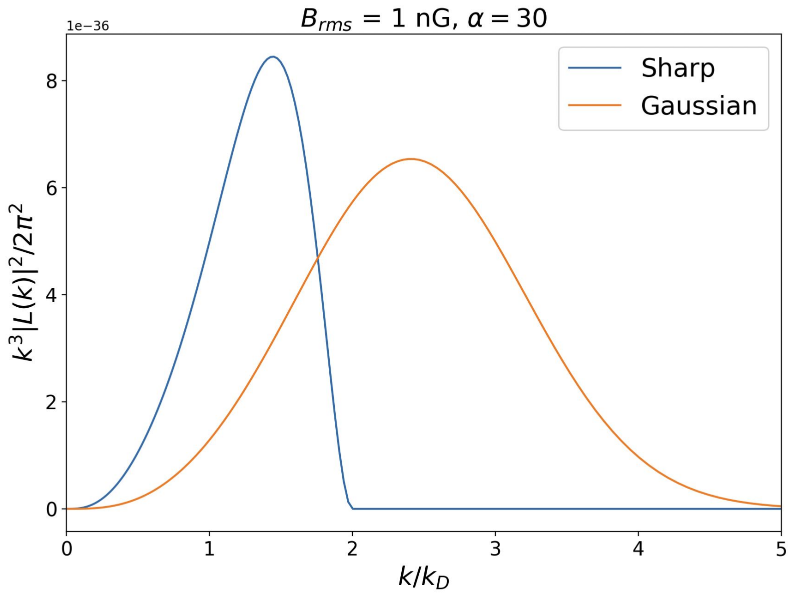
Different damping profiles

$$\langle B_X^2 \rangle = \frac{1}{2\pi^2} \int_0^\infty dk\, k^2\, P_B(k)\, W_X(k)$$

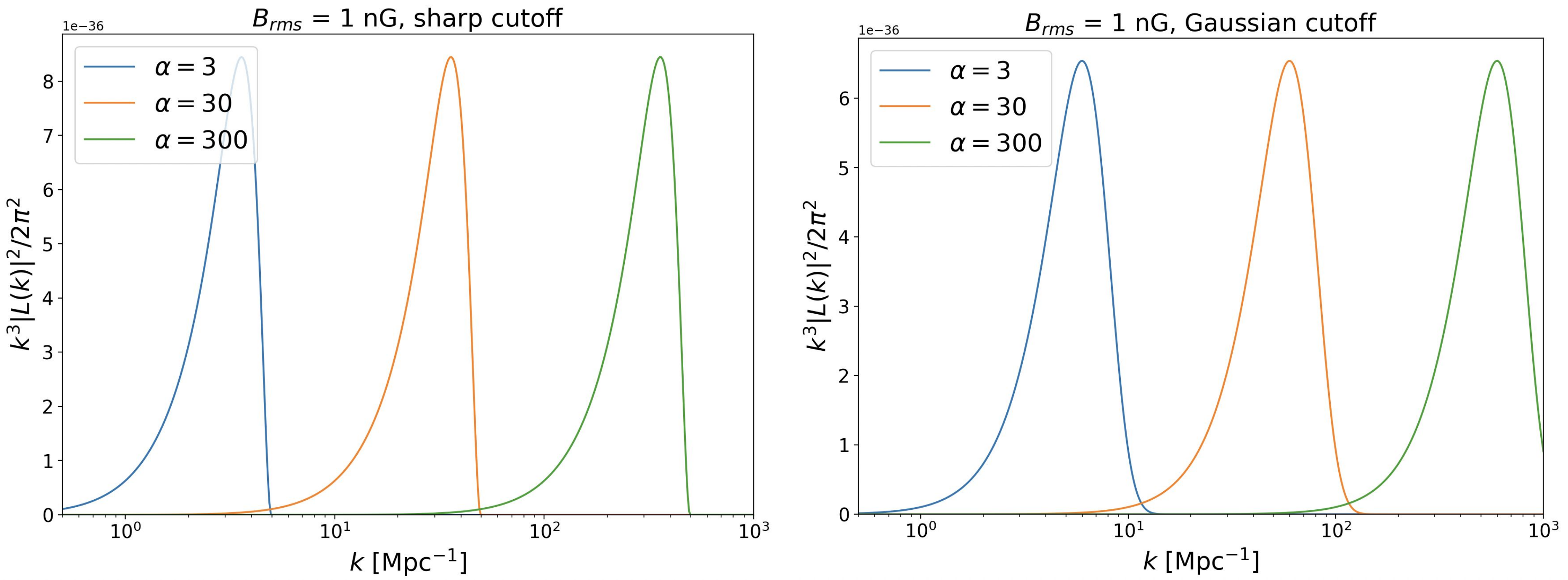
$$W_S(k) = \Theta(k_D - k)$$ usually assumed for CMB

$$W_G(k) = e^{-k^2/k_D^2}$$





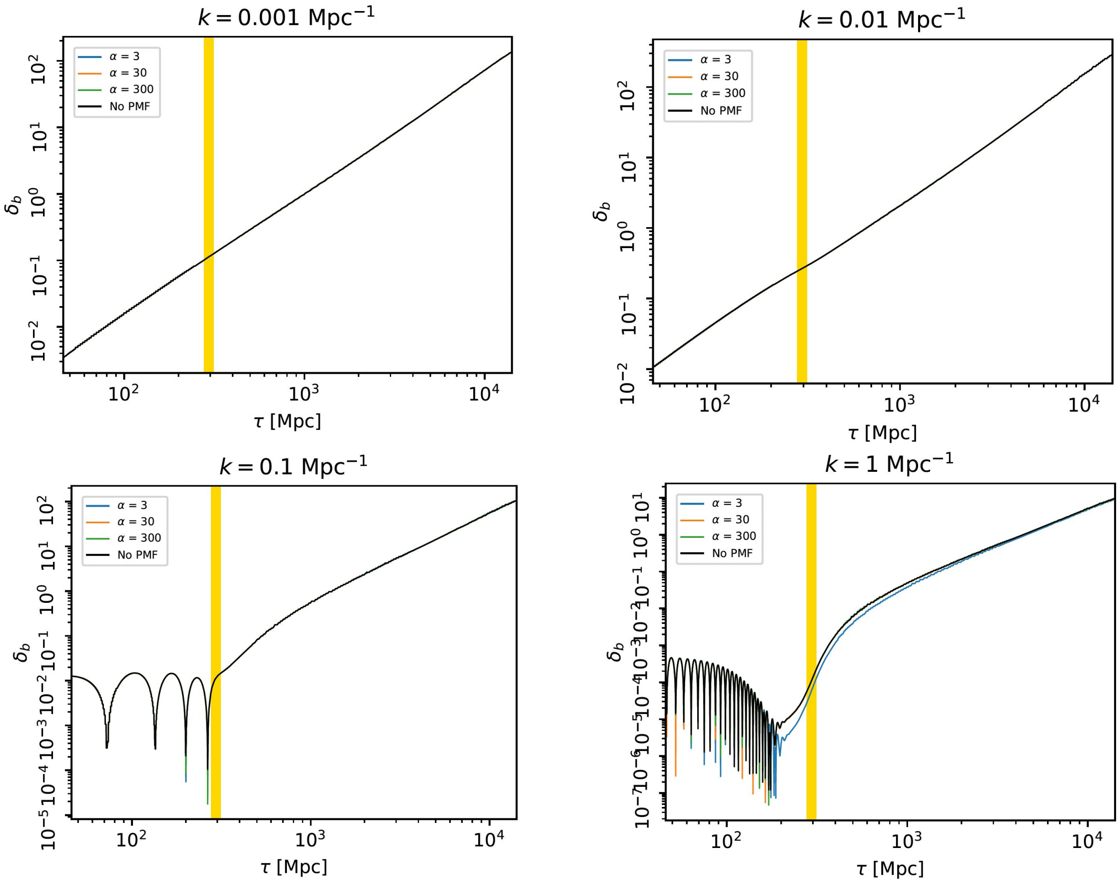
Different damping profiles



Evolution of baryon fluctuations across redshift

$$B_{\rm rms} = 1\,{\rm nG}$$

$$n_B = 2$$

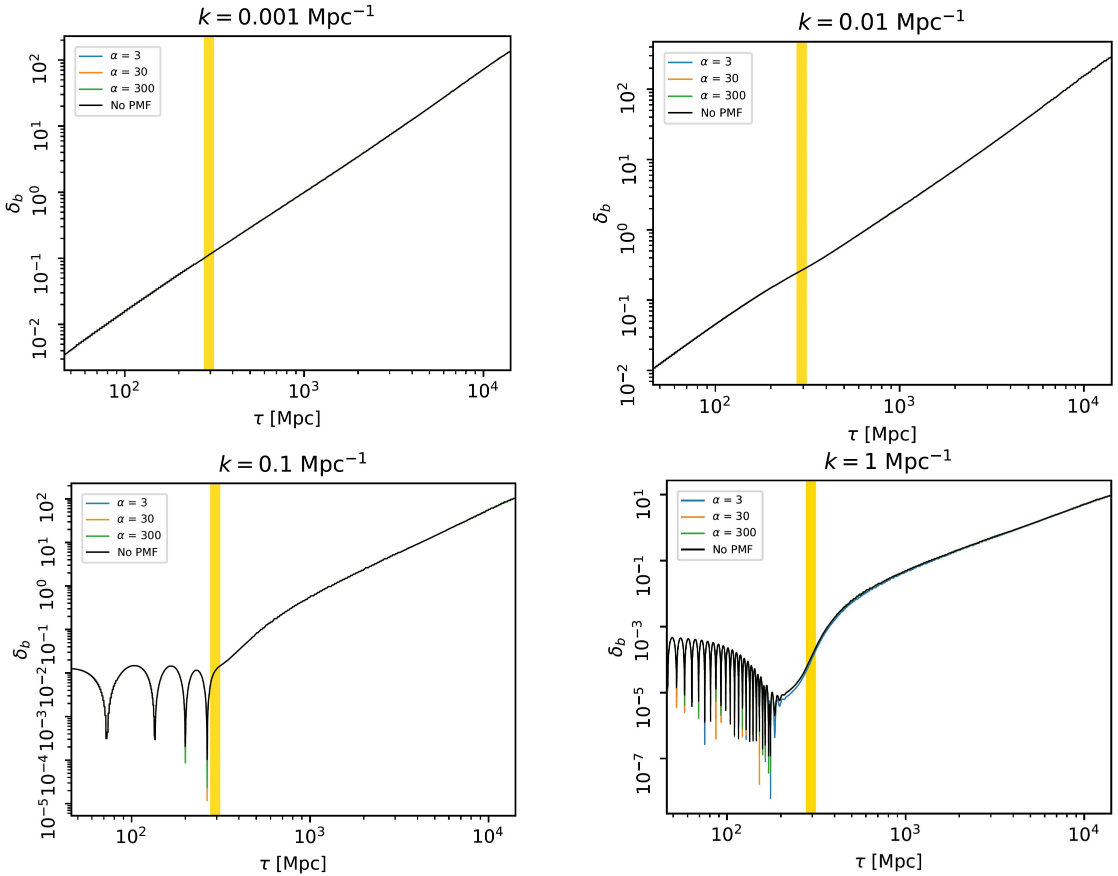


sharp cutoff

Evolution of baryon fluctuations across redshift

$$B_{\rm rms} = 1\,{\rm nG}$$

$$n_B = 2$$

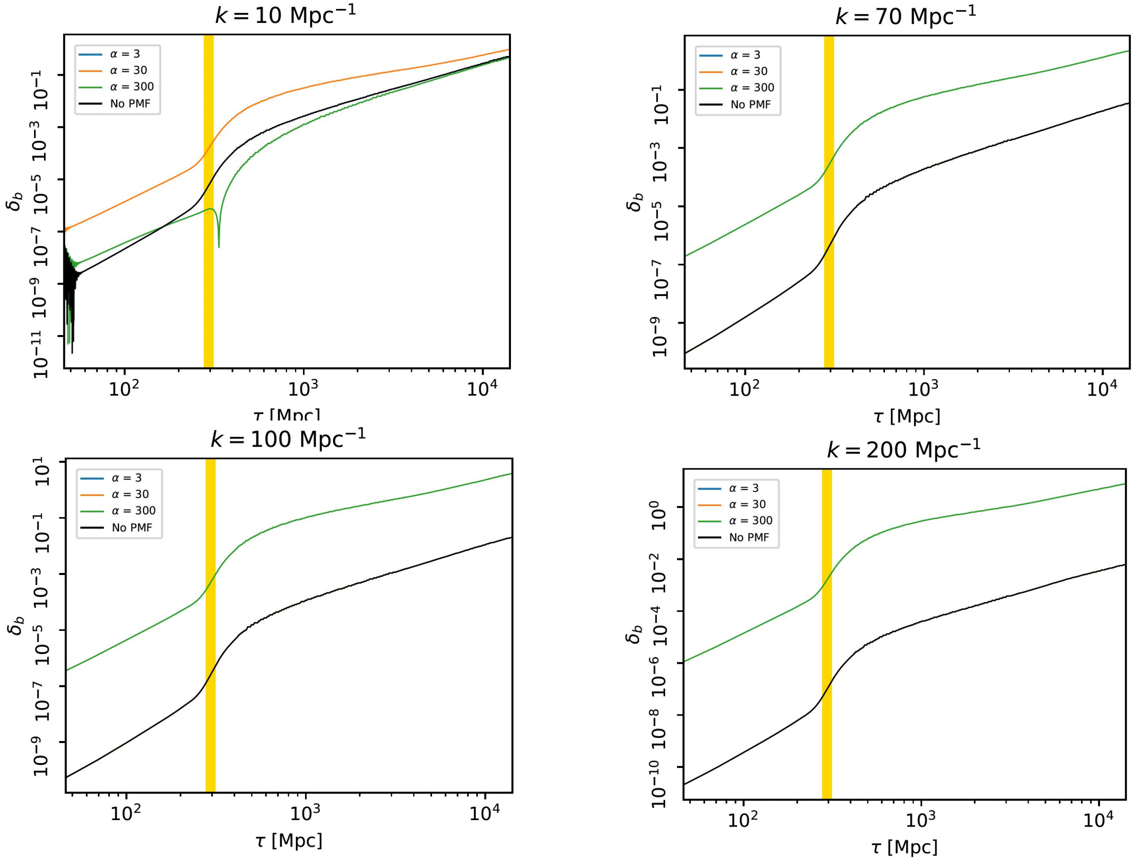


Gaussian cutoff

Evolution of baryon fluctuations across redshift

$B_{\rm rms} = 1\,{\rm nG}$

$n_B = 2$

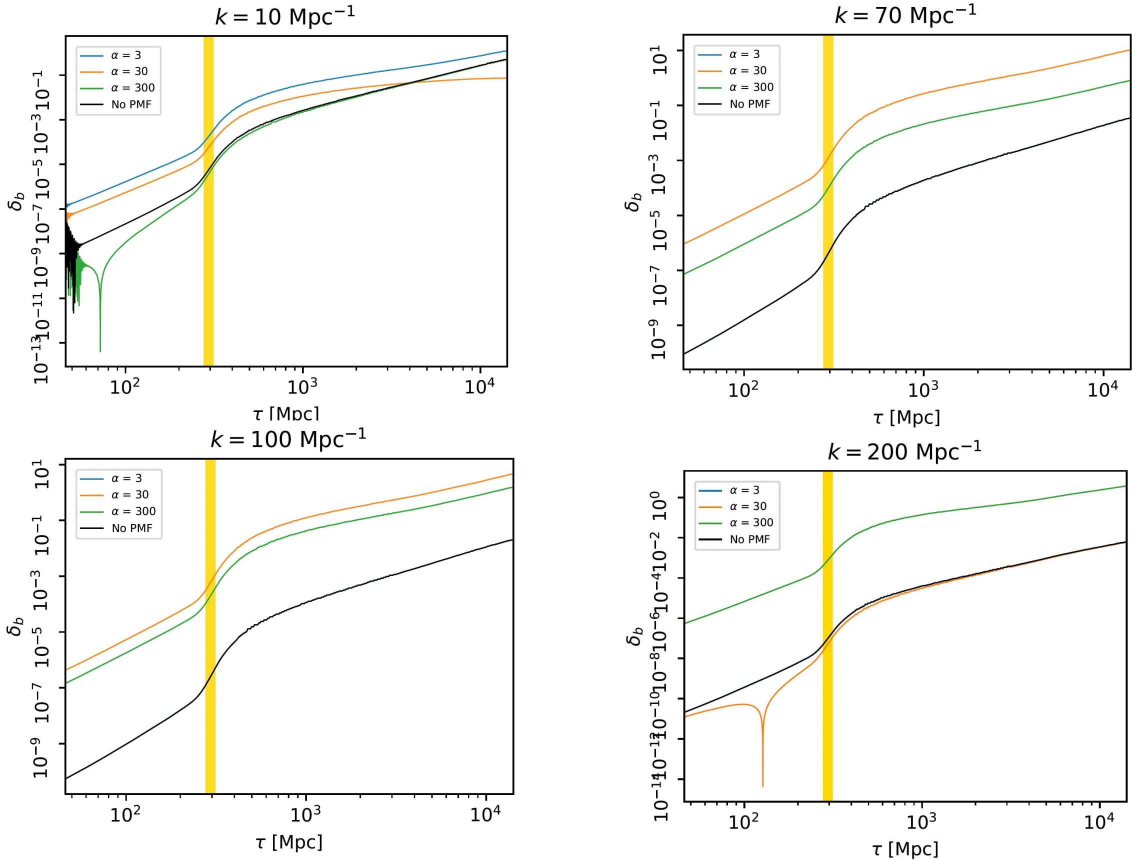


sharp cutoff

Evolution of baryon fluctuations across redshift

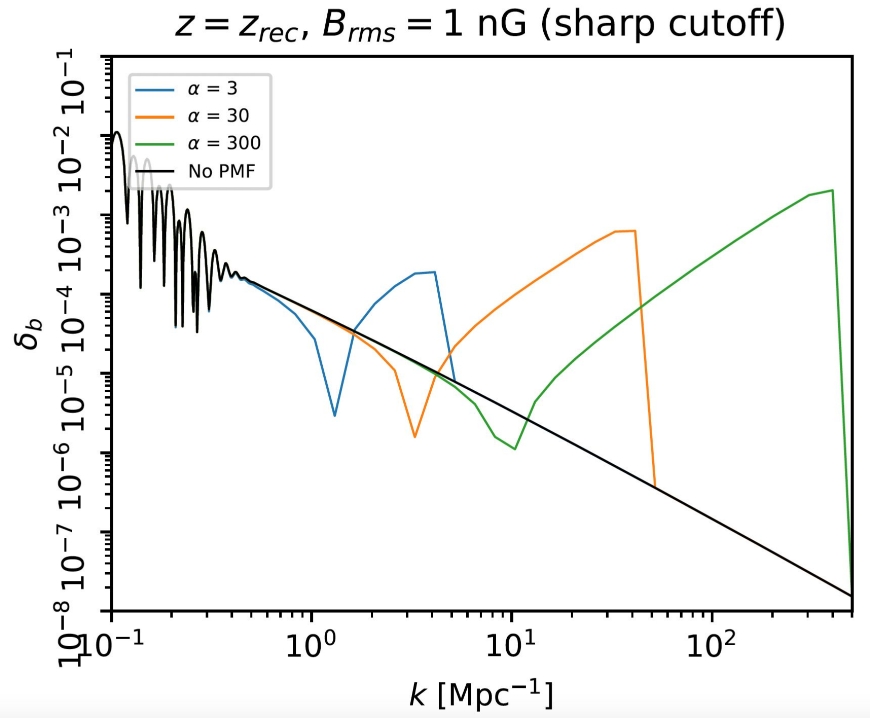
$B_{\rm rms} = 1\,{\rm nG}$

$n_B = 2$

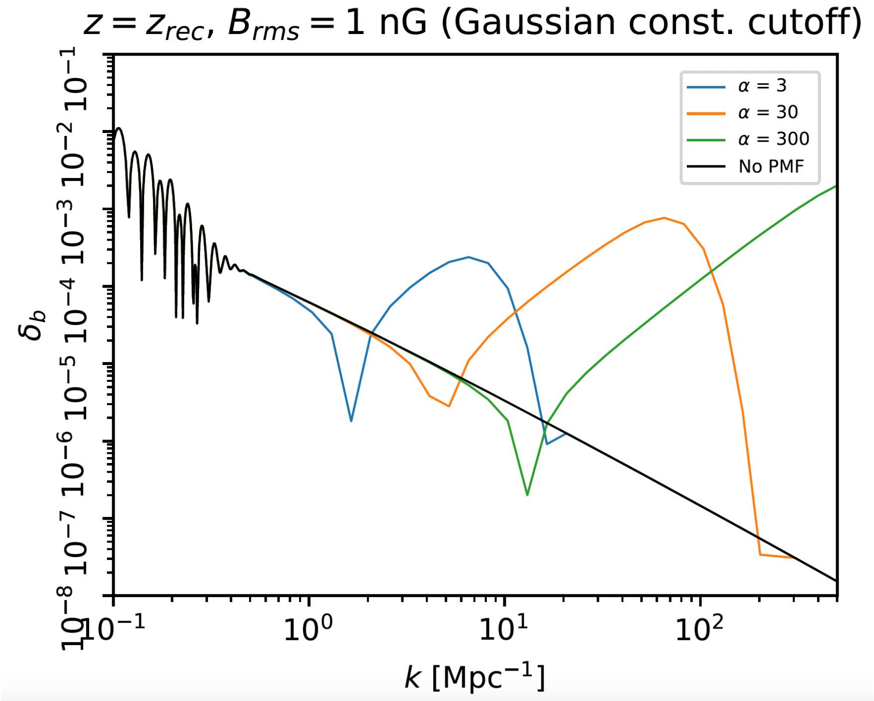


Gaussian cutoff

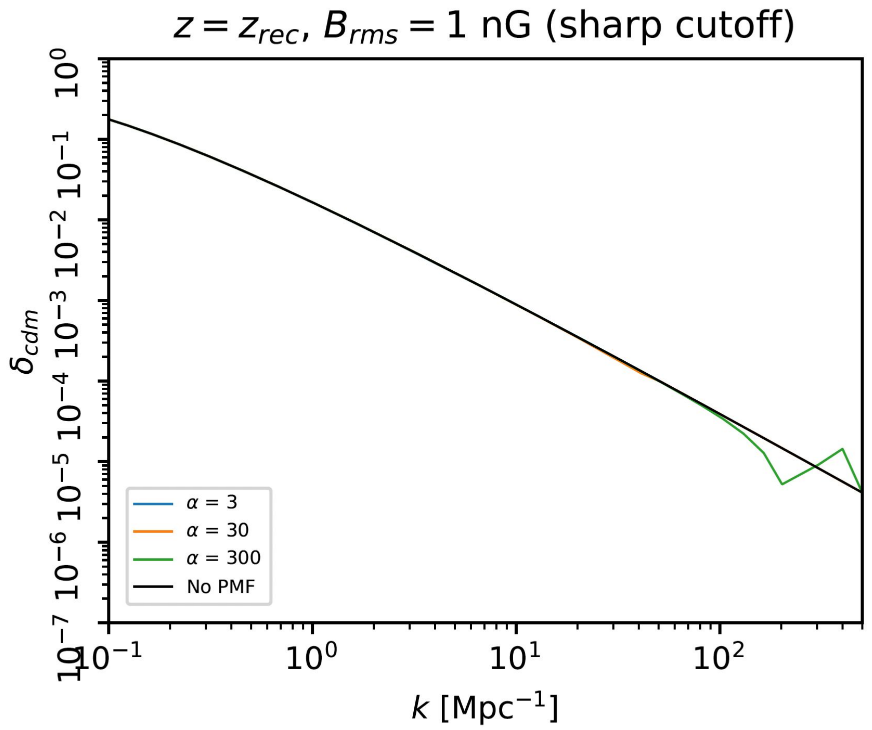
Baryon fluctuations at recombination

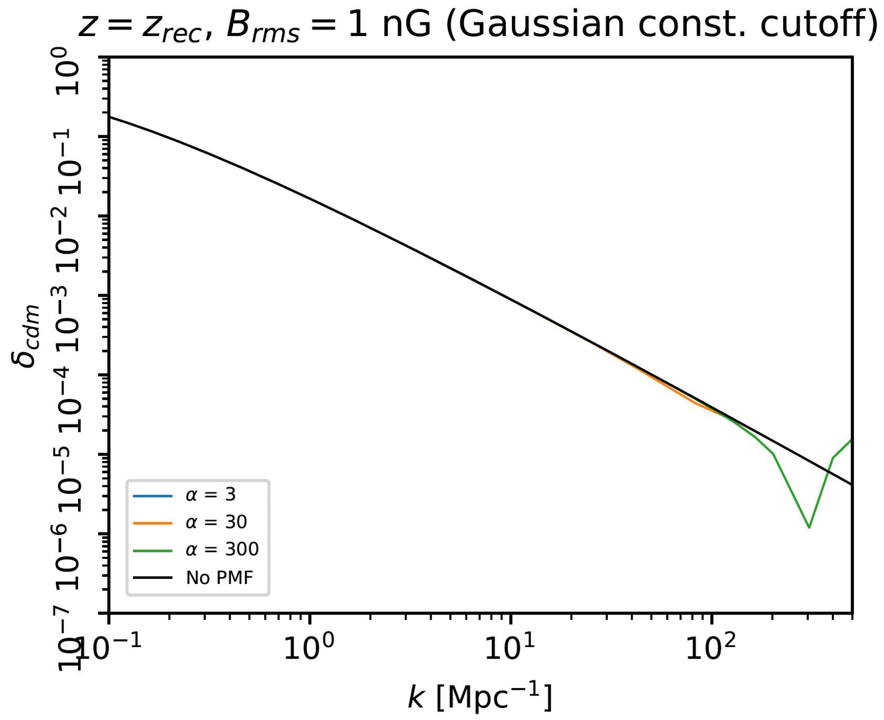


Baryon fluctuations at recombination



Cold dark matter fluctuations at recombination





Semi-analytical approximation

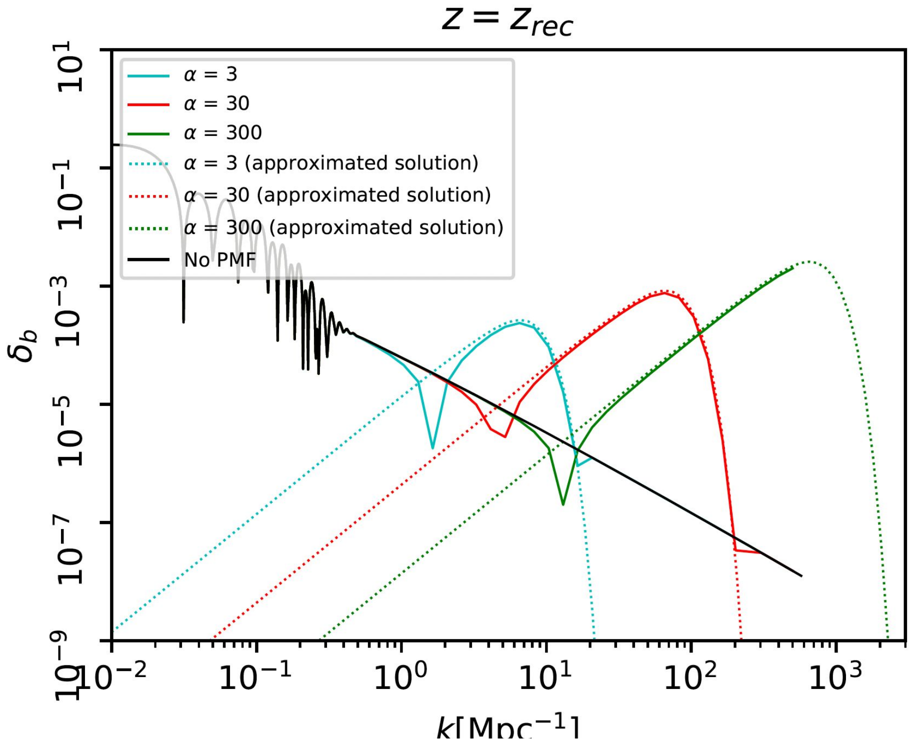
At scales much smaller than the photon mean free path (~ 1 Mpc at recombination), we can neglect gravitational driving and acceleration terms:

$$\dot{\delta}_b(Ran_e\sigma_T + \mathcal{H}) + k^2 c_{sb}^2 \delta_b = k^2 \frac{L}{\rho_b}$$

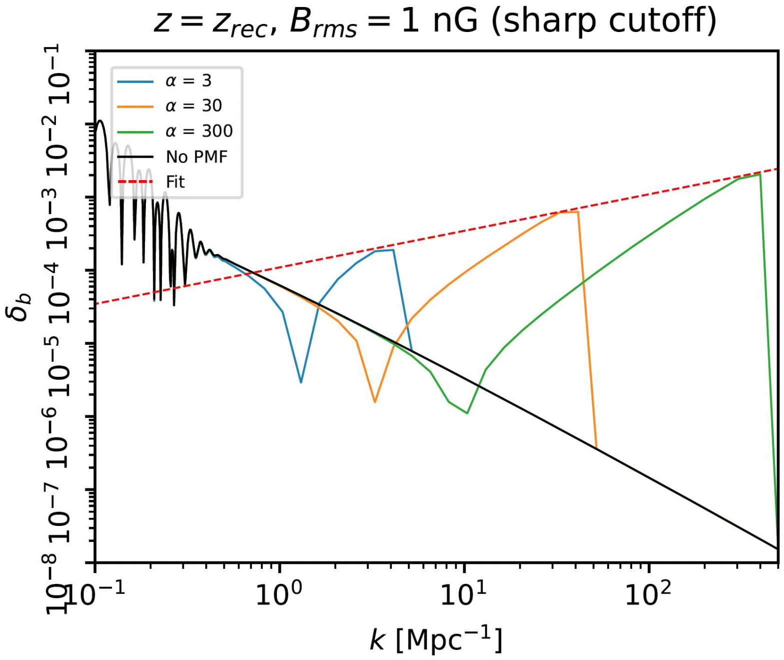
$$\delta_b(k,\tau) = e^{-k^2/k_b^2(\tau)} \int_{\tau_i}^{\tau} d\tau' \frac{3k^2 L}{4\rho_\gamma(an_e\sigma_T + \mathcal{H})}(\tau')\, e^{k^2/k_b^2(\tau')}$$

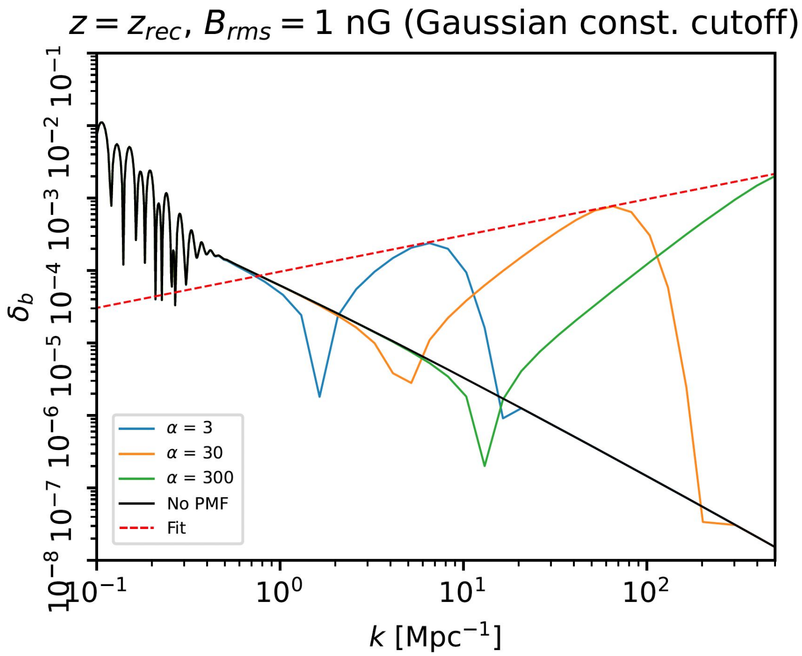
$$\frac{1}{k_b^2(\tau)} = \int^{\tau} d\tau' \frac{c_{sb}^2}{an_e\sigma_T + \mathcal{H}}$$

Semi-analytical approximation



Peak dependence on alpha





Constraints on PMF strength

Paoletti et al. 2018

n_B	2	-2.9	[-2.9,2]
B_0 (nG) [$\bar{k}_D$]	< 0.95	< 1.10	< 0.91

Table 2. Constraints from the combined effects for the alternative model of the damping profile, B_0.

PMF energy dissipation

for α ≃ 258

Using the relations for δ_b^{peak} and k_{peak} we have:

$$\delta_b^{\rm peak} \simeq 1.6\times 10^{-3} \quad \text{at} \quad k_{\rm peak} \simeq 377\,{\rm Mpc}^{-1} \quad \text{for sharp cutoff}$$

$$\delta_b^{\rm peak} \simeq 1.9\times 10^{-3} \quad \text{at} \quad k_{\rm peak} \simeq 587\,{\rm Mpc}^{-1} \quad \text{for Gaussian cutoff}$$

Conclusions

We have analysed the **baryon density fluctuations on small scales** in presence of a SB of causal PMF generated prior to nucleosynthesis to study large baryon fluctuations in the pre-recombination era with implications for inhomogeneous recombination.

We have used Einstein-Boltzmann codes employed in obtaining limits on PMFs by CMB anisotropy data and assumed a simple **power-law for the SB up to the dissipation scale**.

As happened for the effect of heating on CMB, the qualitative assessment of **the enhancement of small scale baryon fluctuations critically depend on the damping scale, whereas the CMB scales are less sensitive.**

We have considered two different approximations for the damping profile, i.e. **sharp** and **Gaussian**. Due to the **theoretical uncertainty in kD**, we parameterized it, covering a range of two orders of magnitude.

Conclusions

By a novel **semi-analytic approximation** which provide an excellent fit to numerical results, we have been able to extend the range of our theoretical predictions to the scales of interest.

Baryons overdensity reaches at most ~ 2×10^{-3} at k ~ 400-600 Mpc^{-1} for **observational allowed PMFs**.

Other spectral indices are under study.