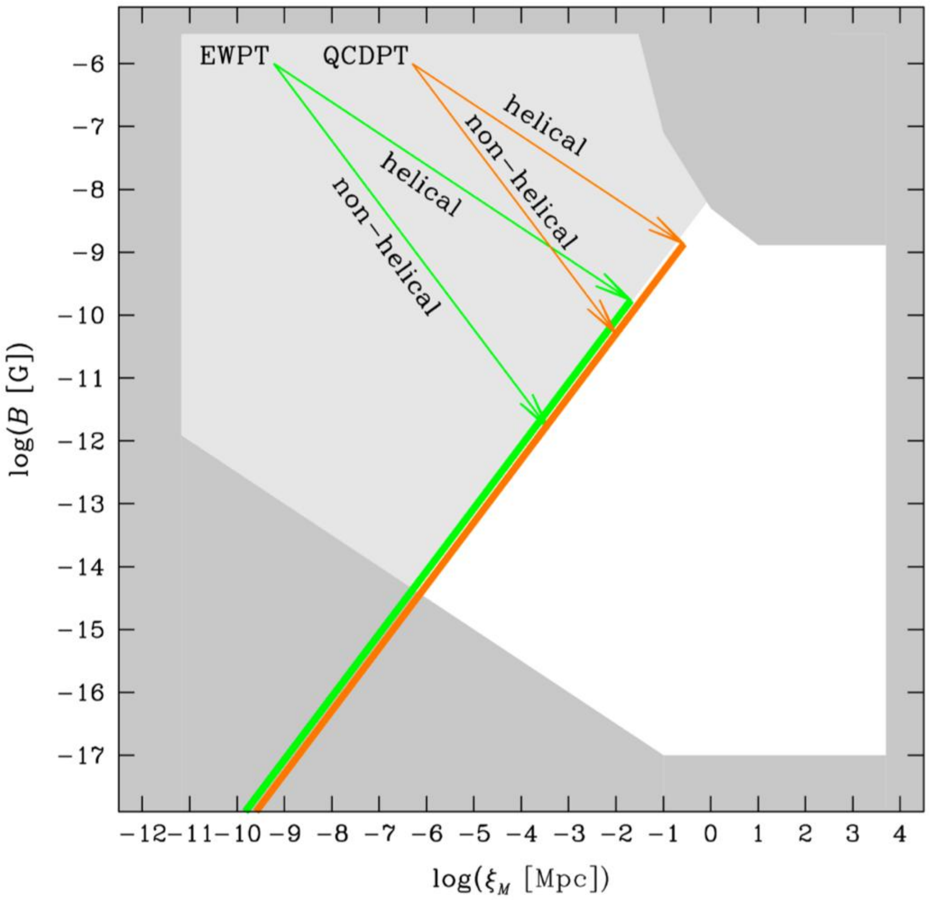
How can we pinpoint the start- and endpoints of primordial magnetic field evolution



Axel Brandenburg (Nordita/Stockholm)

- PhD (Helsinki) in solar & galactic dynamos
- Self-sustained magneto-rotational instability
- MHD turbulence, chaos & fractals, helicity
- Inverse cascade of primordial B-field in 1996
- Pencil Code since 2001
- Relic gravitational waves from primordial MHD turbulence and inflationary fields

Overview

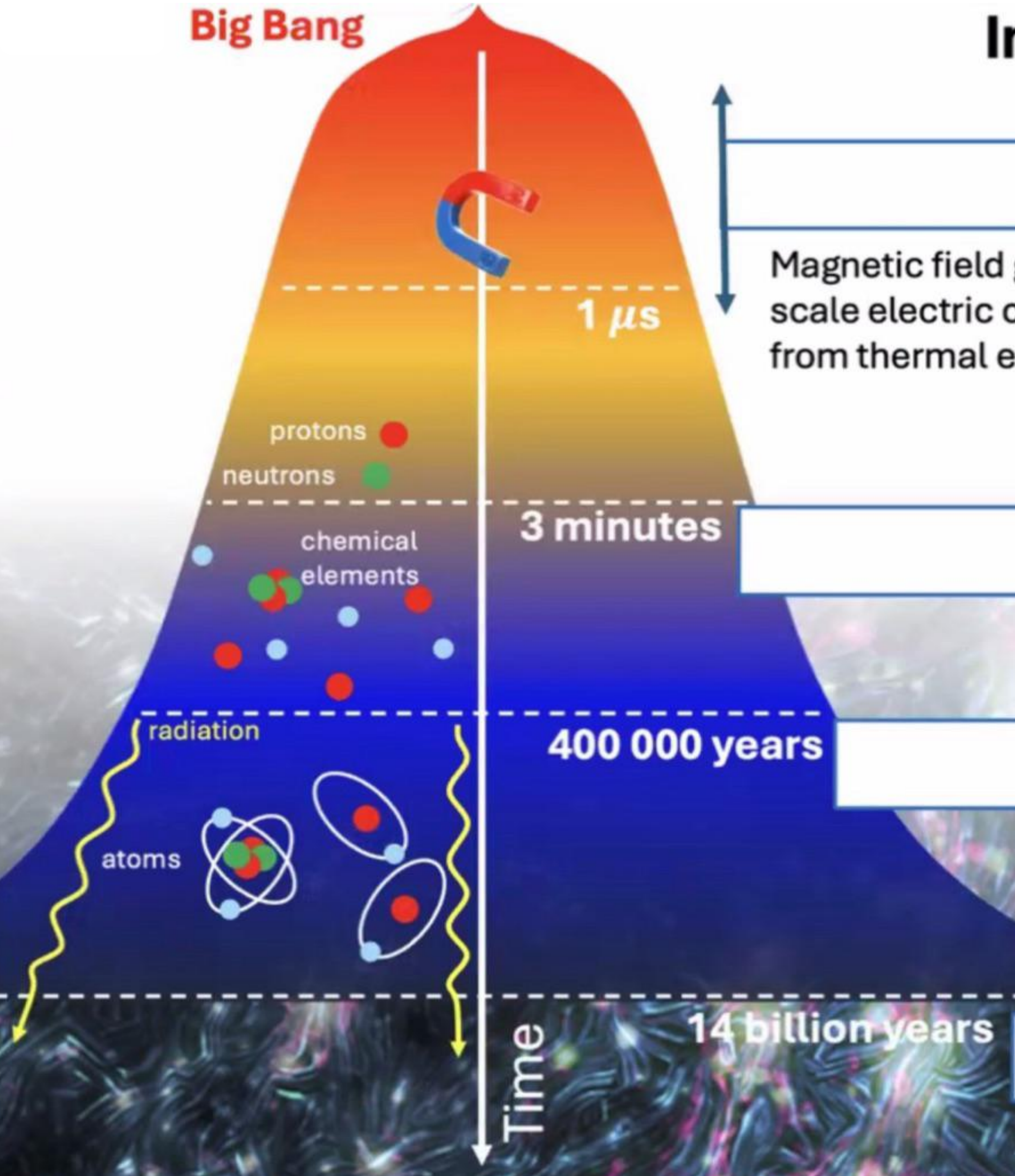
- Contemporary magnetic fields: dynamo action (kinetic → magnetic energy)
  - Works generically in turbulent flows (allows irreversible foldings of field lines)
  - In stars and galaxies: also large-scale fields (solar 11-yr cycle)
  - Typically in flows with helicity per hemisphere (EMF in direction of B-field: α effect)
  - Alternatively: just small-scale dynamos: probably in galaxy clusters
- Primordial magnetic fields: best contrained in voids (GeV gamma rays)
  - But: also contamination from outflows
- MHD: when electrically conducting (displacement current unimportant)
  - Different during inflation: electromagnetic waves (destabilized at large scales?)
  - Charge-separation almost always unimportant!
- Relic gravitational waves (GWs): they don’t decay
  - Direct probe of turbulence and magnetic fields at time of generation
  - GW spectrum related to turbulence spectrum
  - Circular polarization: related to kinetic and magnetic helicity

Magnetic field evolution

- During radiation-dominated era
  - Possibilities of kinetic energy from phase transitions → dynamo action (but need vorticity)
  - Conversion of chiral chemical potential to magnetic energy (chiral magnetic effect)
  - Higgs field
- Turbulent decay (unless always perfectly uniform)
  - Characterized by a spectral peak ($k_{\rm peak}$) → generic turbulence spectrum for higher k
  - Turnover time $(u_{\rm rms}\, k_{\rm peak})^{-1}$ and/or Alfven time $(v_{\rm A}\, k_{\rm peak})^{-1}$ govern speed of decay
  - But possibility of inverse cascade (increase of spectral energy at low k)
  - Most efficient for helical fields (also slower decay)
  - Even nonhelical decay faster than hydrodynamic decay
- Magnetic fields as a probe of the first microsecond of the universe
  - End points on a universal line $\boldsymbol{B}$ vs length scale

Introduction

(Andrii Neronov's slide)



Cosmological magnetic fields?

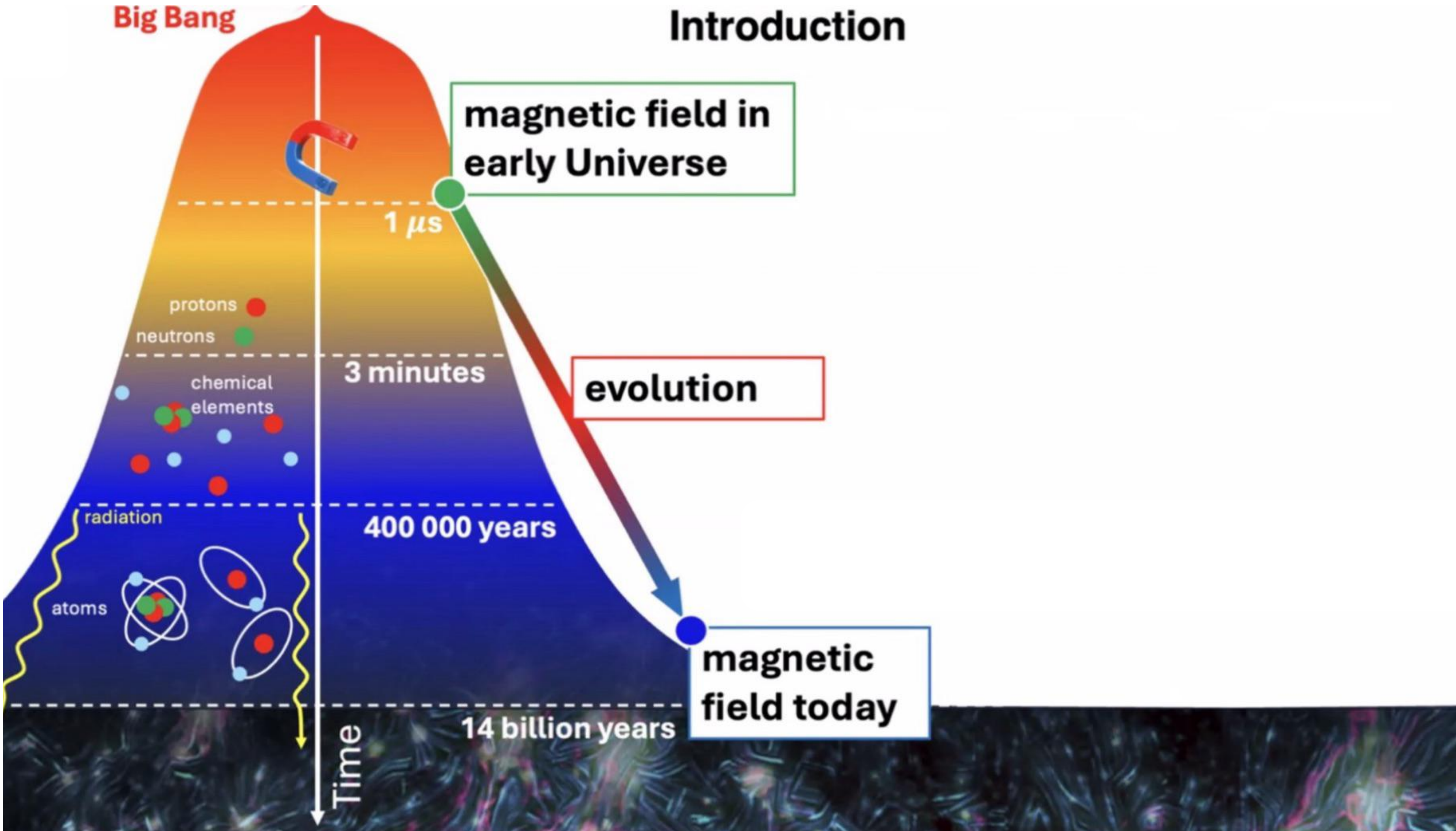
Magnetic field generation in the early universe requires large scale electric currents, possible during episodes of departure from thermal equilibrium, all within microseconds from Big Bang

Big Bang Nucleosynthesis

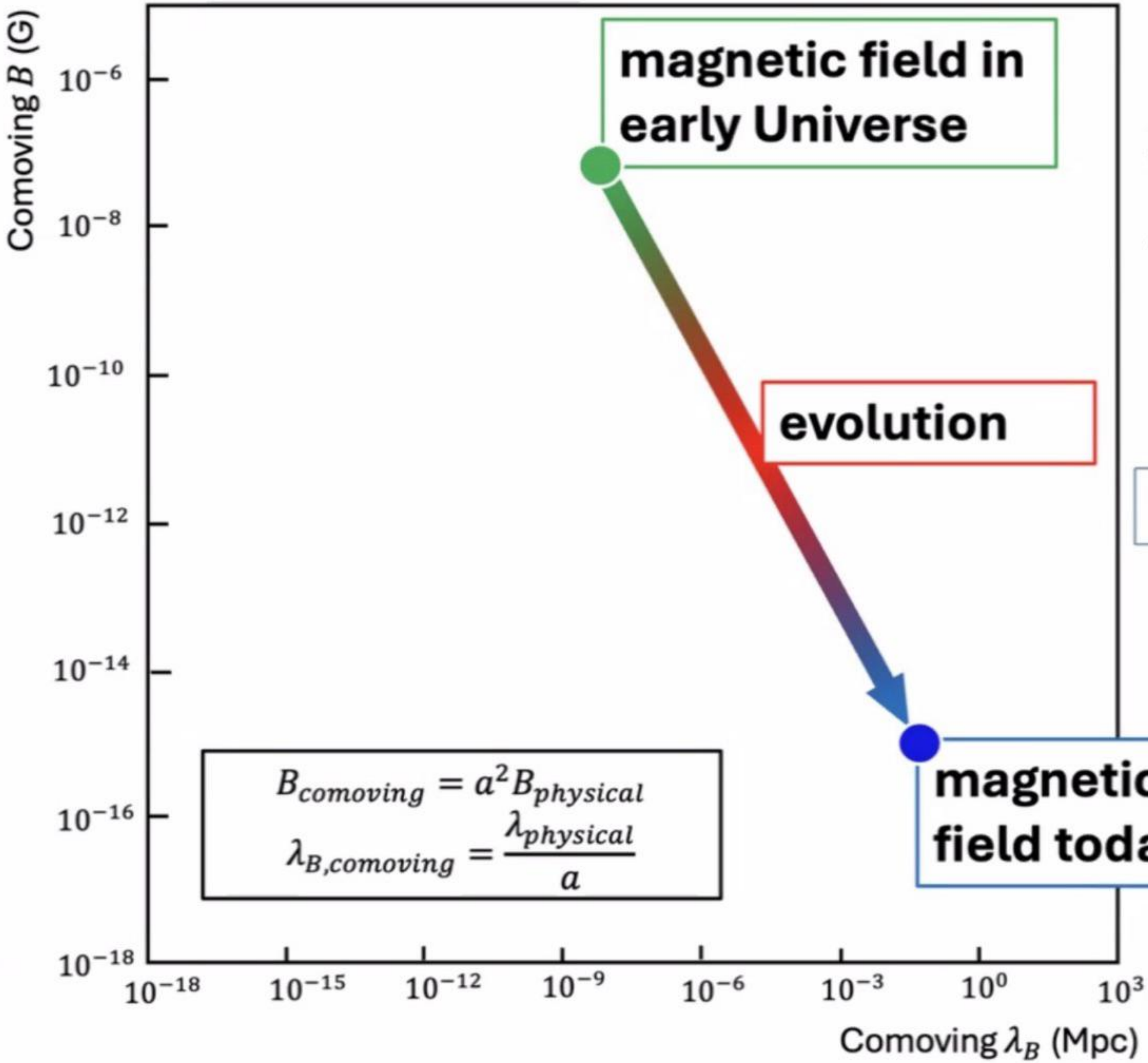
Cosmic Microwave Background

Large-Scale Structure

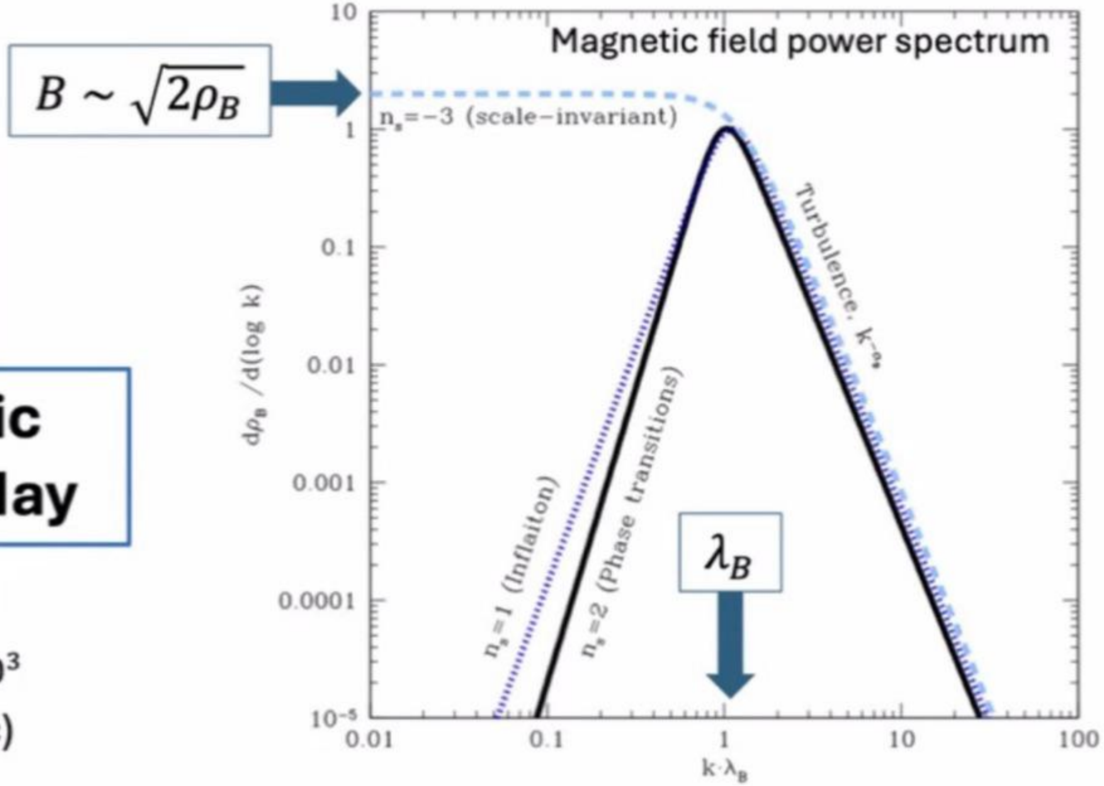
Introduction



Introduction



Present-day relic magnetic field parameters are related to the initial field characteristics. They potentially provide information on the mechanism of production of the field in the early Universe.



Comoving horizon scale today

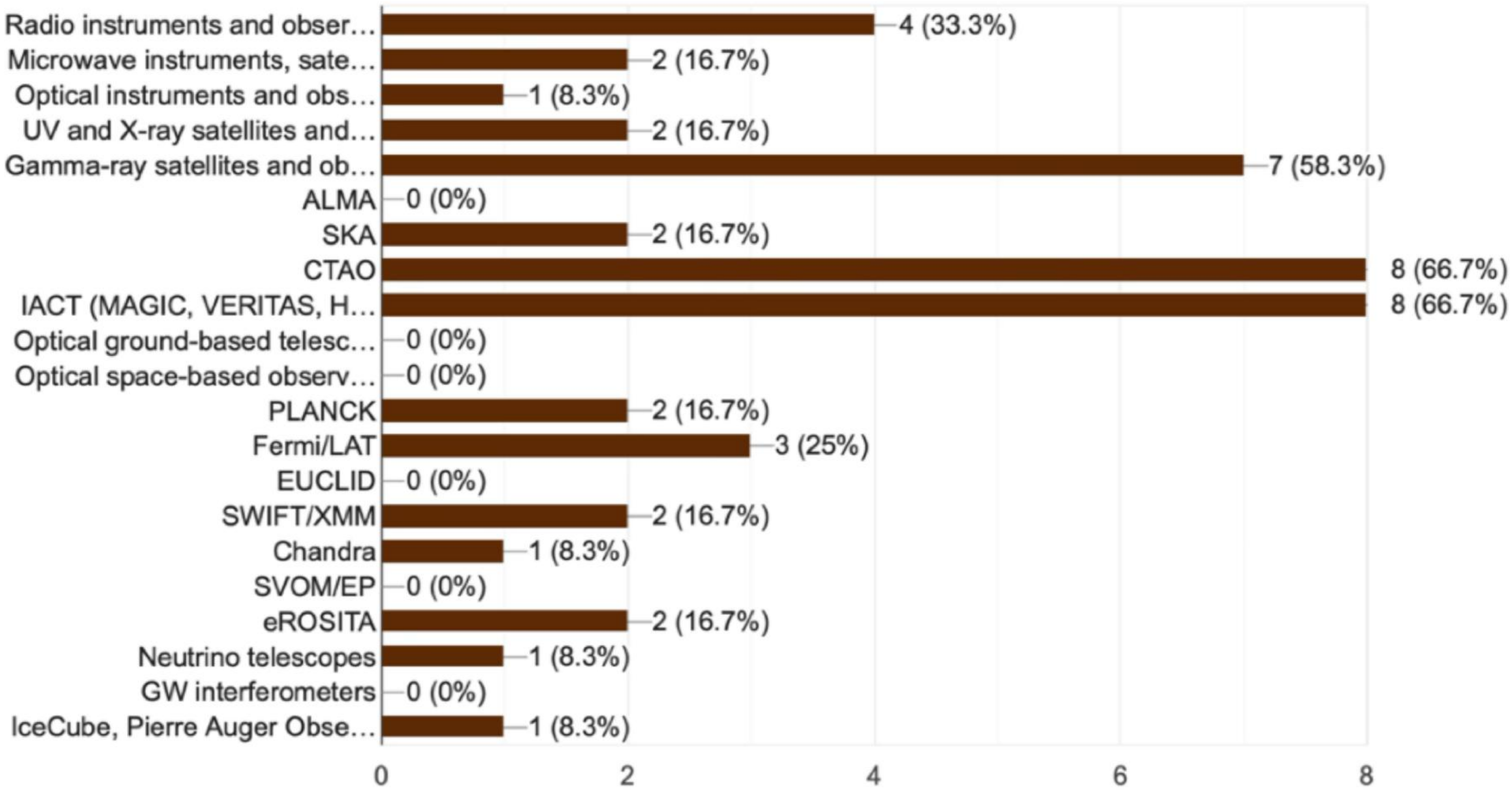
$$\lambda_{H_\star} = 5.8 \times 10^{-10}\ \mathrm{Mpc}\left(\frac{100\ \mathrm{GeV}}{T_\star}\right)\left(\frac{100}{g_\star}\right)^{1/6}$$

- Electroweak (EW) energy scale
  - 5.8x10^{-10} Mpc ~ 100 AU
  - Unless inflationary field, sausally generated fields always smaller
- QCD (quark confinement) energy scale (T_*=0.15 GeV, g_*=15)
  - 0.5 pc ~ 100 000 AU

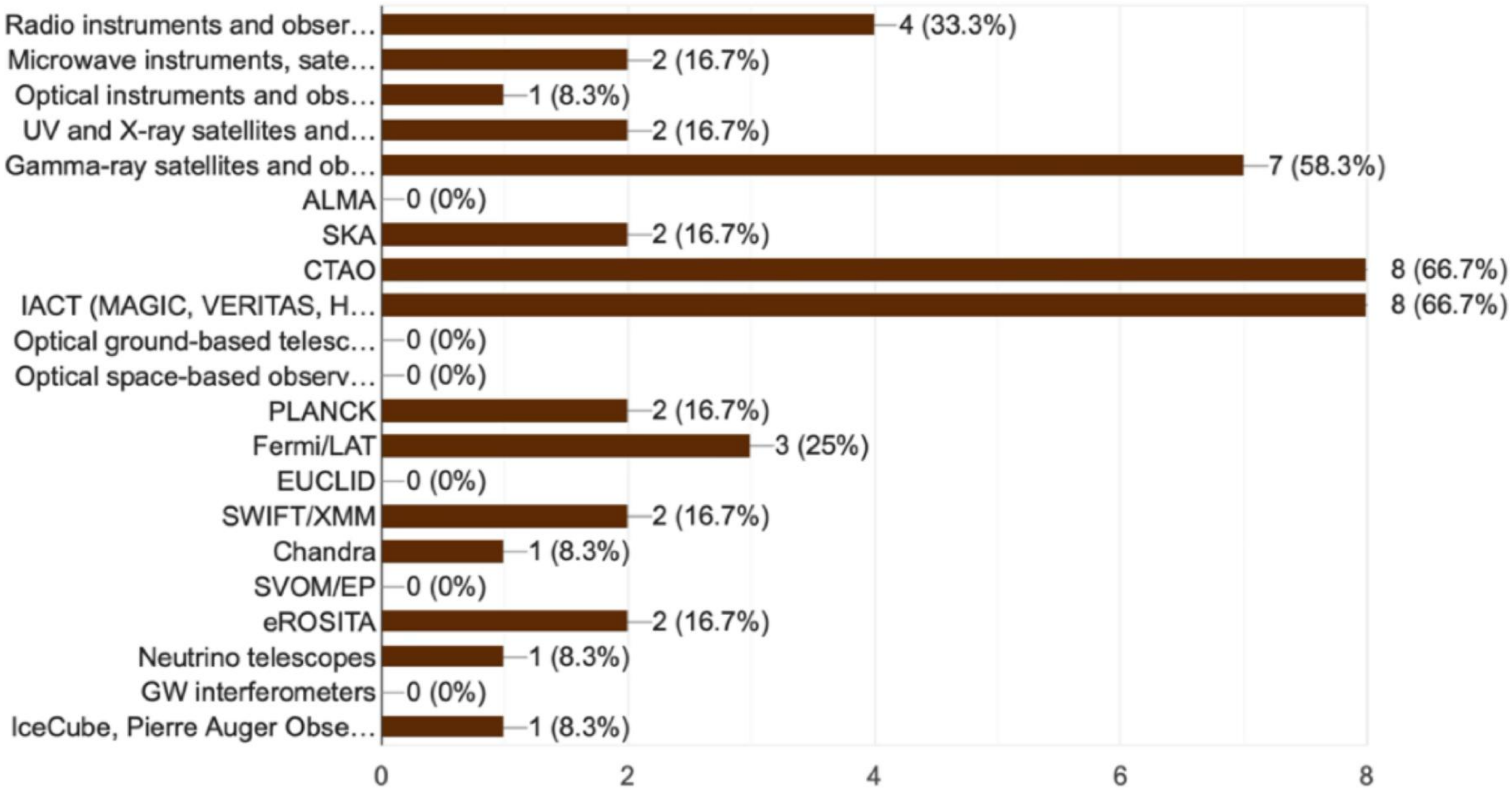
$$f_* = \frac{a_* H_*}{a_0} \simeq (1.8 \times 10^{-8}\ \mathrm{Hz})\left(\frac{g_*}{15}\right)^{1/6}\left(\frac{T_*}{150\ \mathrm{MeV}}\right).$$

- Use GWs to pinpoint starting point of magnetic field evolution
  - End points on a universal line ***B*** vs length scale
  - EW enery scale corresponds to 0.2 mHz

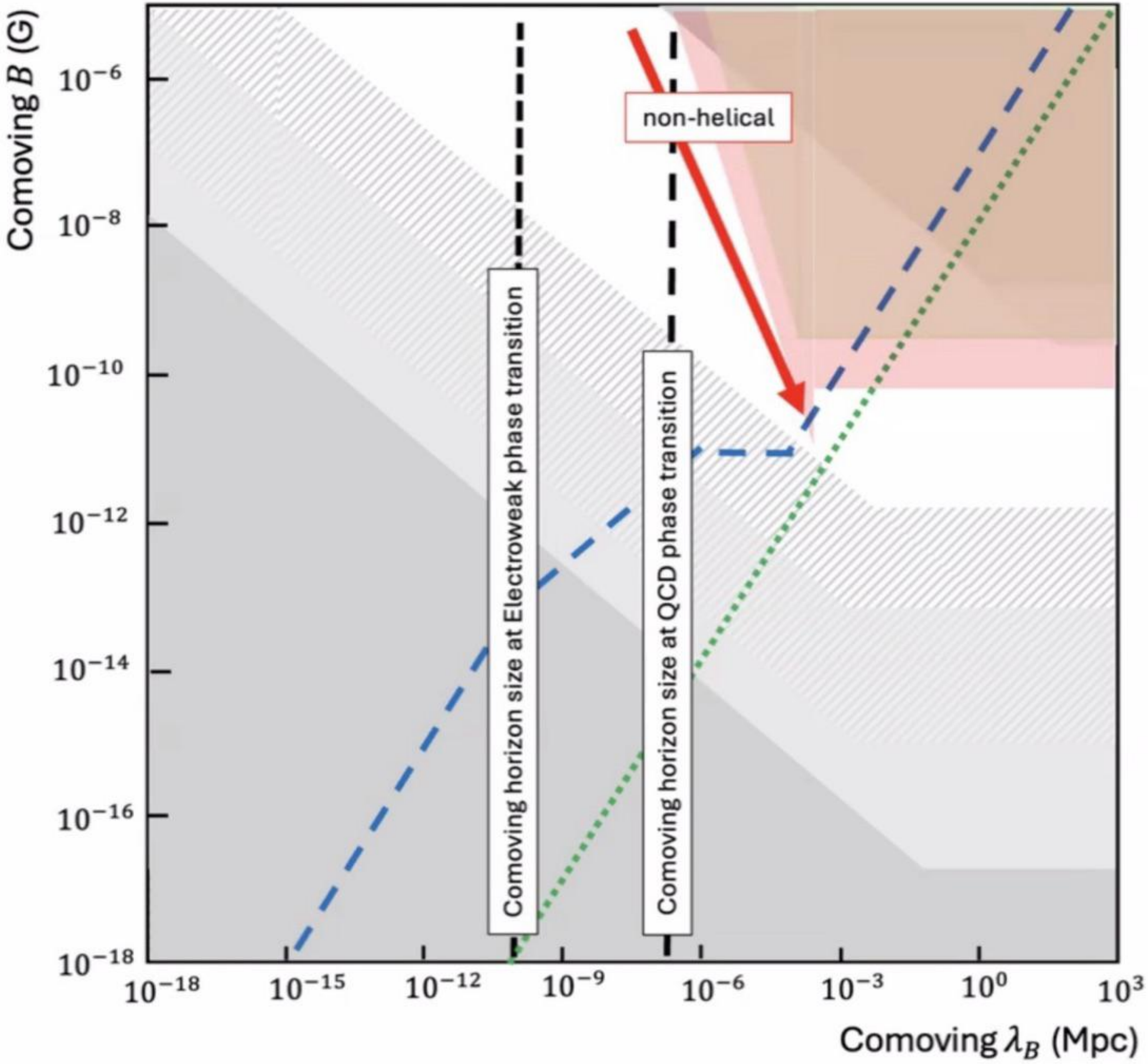
To pinpoint magnetic fields at QCD scale, need to use…

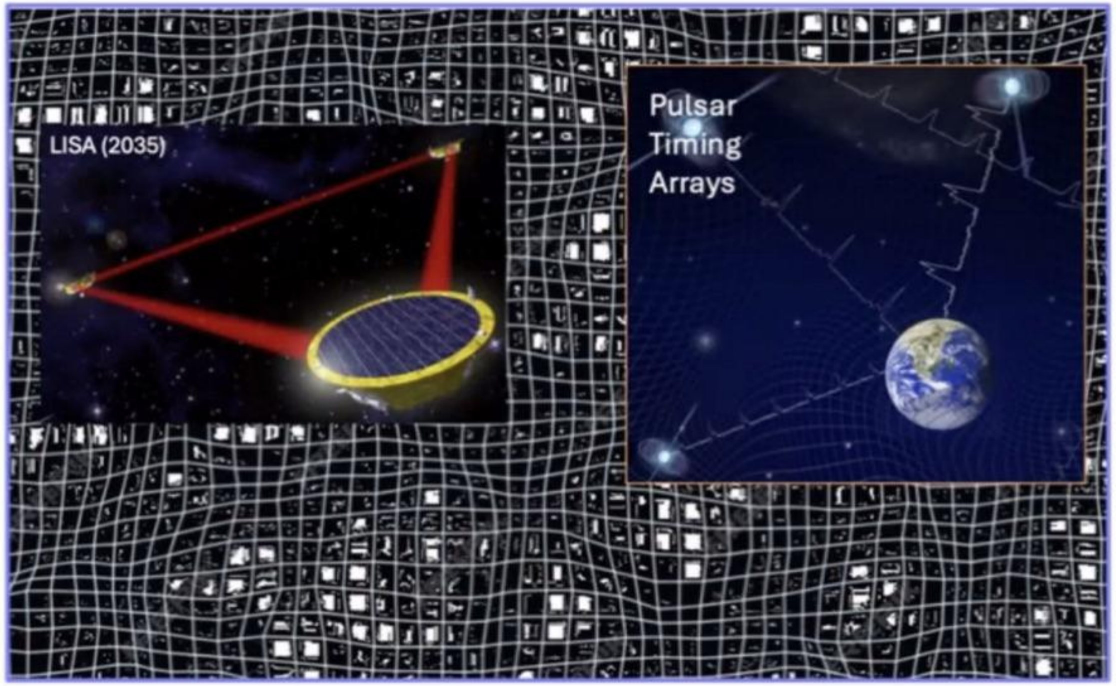


To pinpoint magnetic fields at QCD scale, need to use PTA



Magnetic field at the moment of generation: gravitational waves





Gravitational wave background

Magnetic field has stress-energy tensor that is a source in the gravitational wave equation.

Magnetic field generates motions of plasma that has stress-energy tensor that is also a source for the gravitational waves

Plasma motions that generate magnetic fields may also source gravitational waves.

Gravitational waves & polarization

$$\left(\partial_t^2 + 3H\partial_t - c^2\nabla^2\right) h_{ij}(\boldsymbol{x},t) = \frac{16\pi G}{c^2}\, T_{ij}^{\rm TT}(\boldsymbol{x},t)$$

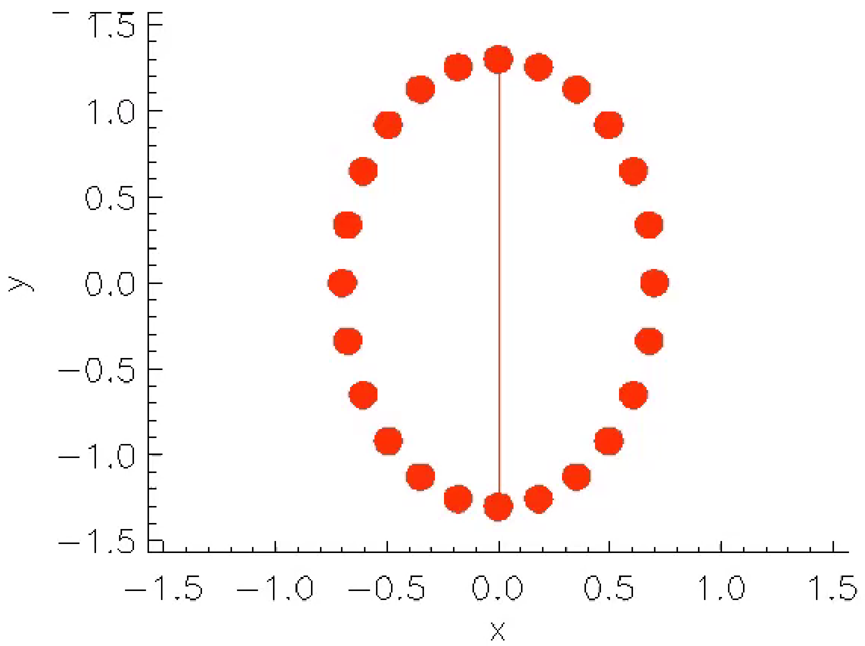
$$T_{ij}(\boldsymbol{x},t) = \left(p/c^2 + \rho\right)\gamma^2 u_i u_j - B_i B_j + (\boldsymbol{B}^2/2 + p)\delta_{ij}$$

Example

$$\boldsymbol{B} = \begin{pmatrix} 0 \\ \sigma \sin kx \\ \cos kx \end{pmatrix} \longrightarrow \boldsymbol{\nabla}\times\boldsymbol{B} = \begin{pmatrix} \partial_x \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ \sin kx \\ \cos kx \end{pmatrix} = k \begin{pmatrix} 0 \\ \sin kx \\ \cos kx \end{pmatrix} = k\boldsymbol{B}$$

Traceless-transverse

$$T_{ij}(x) = \mathcal{E}_{\rm M} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\cos 2kx & \sigma \sin 2kx \\ 0 & \sigma \sin 2kx & \cos 2kx \end{pmatrix}$$



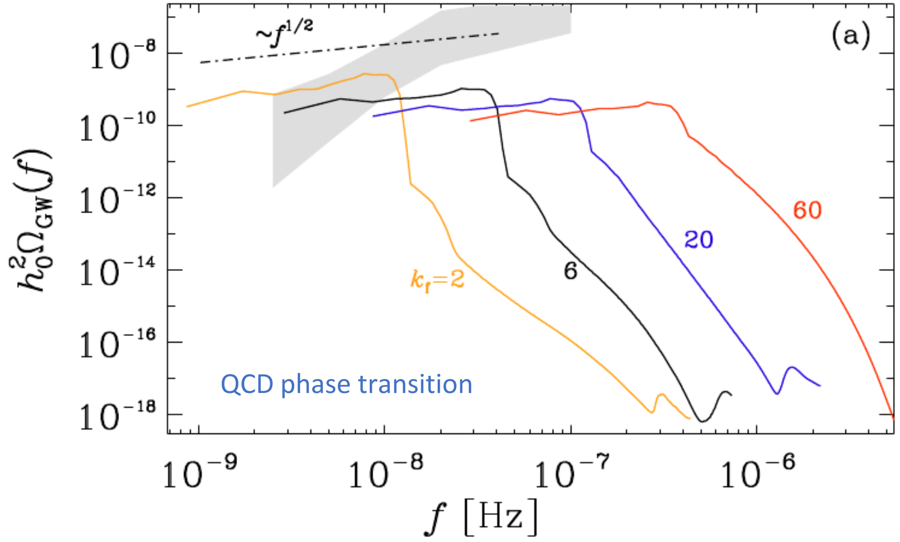
GW energy dependence on magnetic energy and wavenumber k0.

$$\bar{\Omega}_{\rm GW} = \frac{3H_*^2}{c^2 k_0^2}\,\Omega_{\rm M}^2$$

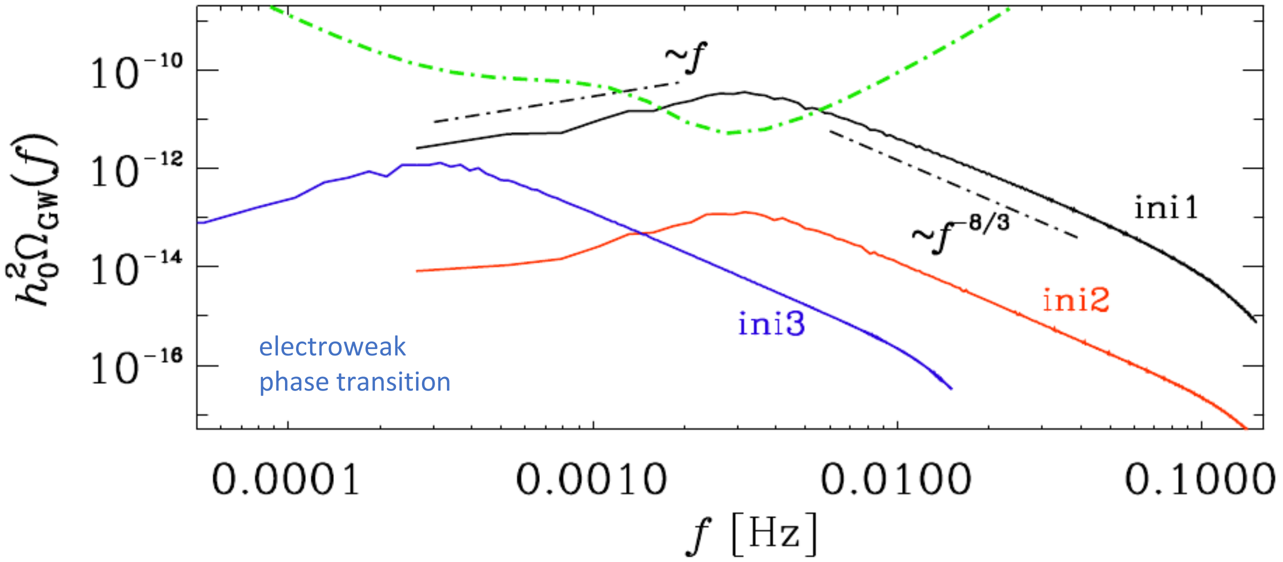
Roper Pol et al. (2020, GAFD 114, 130)

Polarization in turbulent cases:
Kahniashvili et al. (2021, PRR 3, 013193)

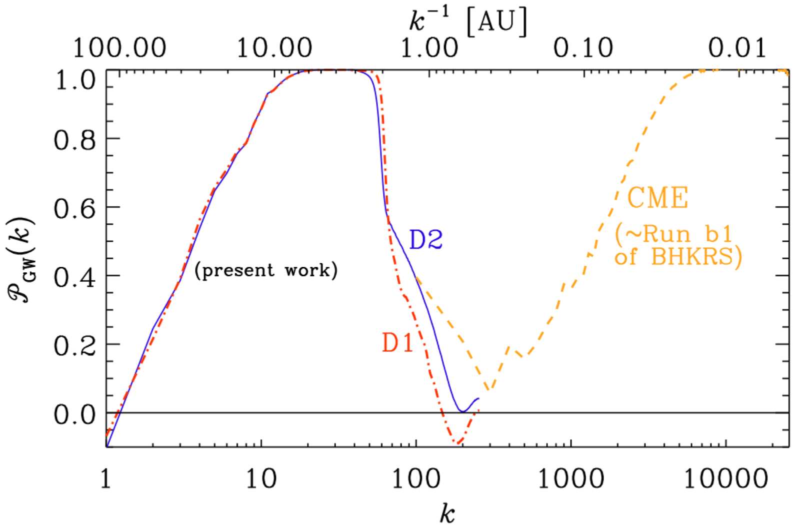
Observability of relic GWs



NANOGrav = North American nHz Obs for GWs



LISA = Laser Interferometer Space Antenna
Roper Pol et al. (2020, PRD 102, 083512



$$\mathcal{P}(k) = \int 2\,\mathrm{Im}\,\tilde{h}_+\tilde{h}_\times^*\,k^2\mathrm{d}\Omega_k \Big/ \int \left(|\tilde{h}_+|^2 + \tilde{h}_\times|^2\right) k^2\mathrm{d}\Omega_k$$

- GWs driven by magnetic stress, B ~ 1 μG
  - 1 μG would have decayed to 0.3 nG at 30 kpc
- Lower limits from Fermi LAT (Large Area Telesc)
  - 10^{-15} G at 1 Mpc (Neronov & Vovk 2010)
  - Already well above chiral B-field limit of 10^{-18} G
- B-fields driven at hoc (no magnetogenesis)

Nonhelical & helical magnetic fields at the QCD energy scale

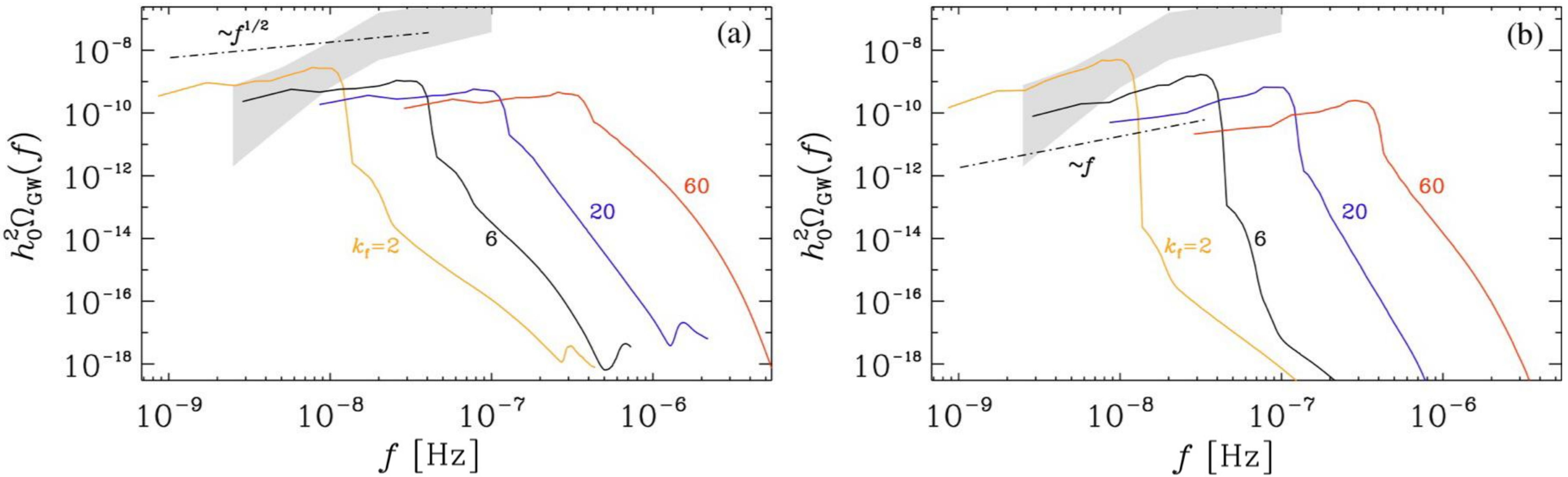


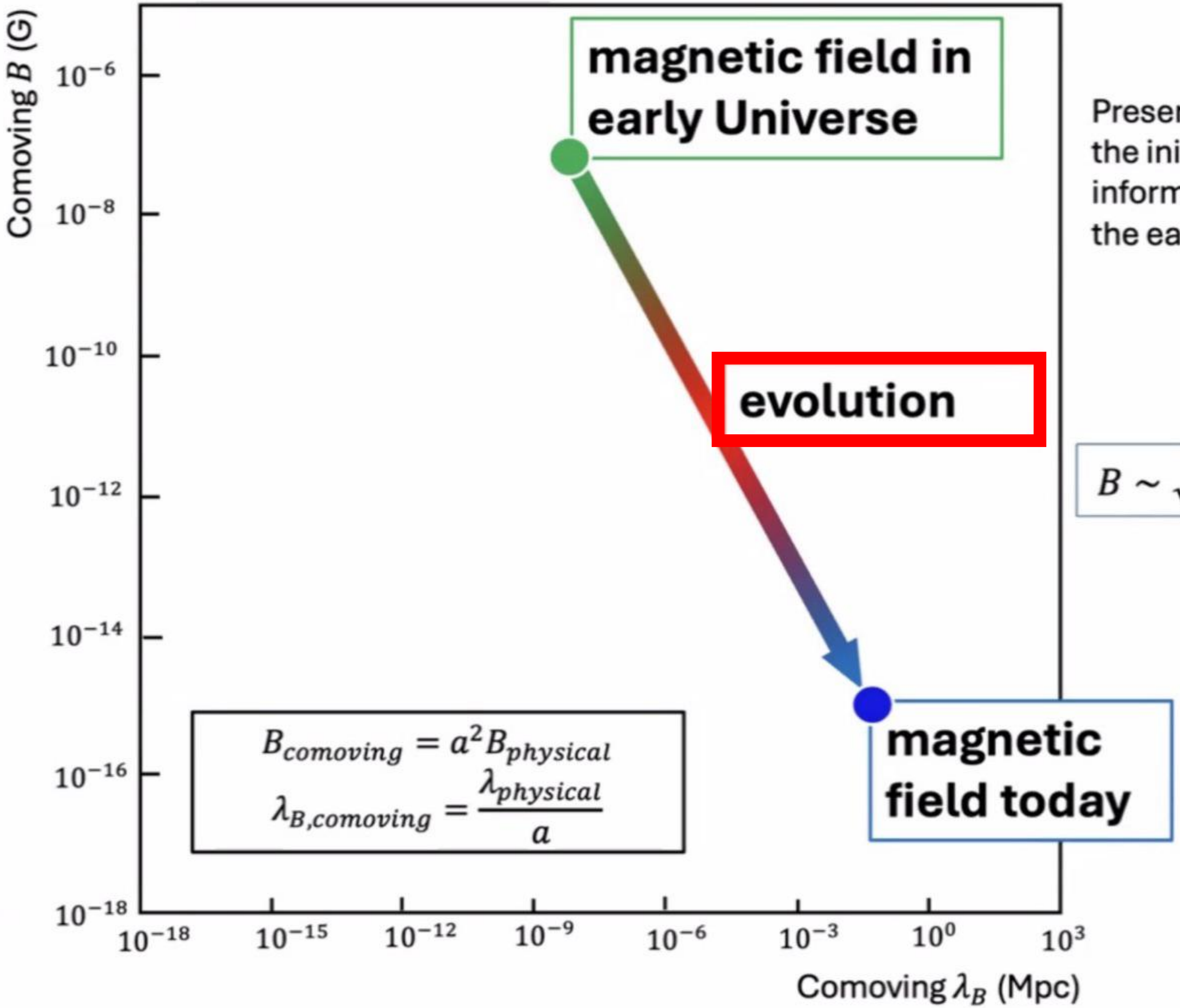
TABLE I. Summary of runs with nonhelical turbulence.

Run	k_f	k_1	f_0	p	τ	$\mathcal{E}_M^{max}$	$\mathcal{E}_{GW}^{sat}$	h_{rms}^{sat}	B [μG]	$h_0^2\Omega_{GW}(f)$	h_c
noh1	2	0.3	1.9×10^{-1}	1.0	16	3.83×10^{-2}	3.53×10^{-4}	4.83×10^{-2}	0.78	1.09×10^{-8}	4.83×10^{-14}
noh2	6	1	6.0×10^{-2}	1.0	4.5	3.75×10^{-2}	5.61×10^{-5}	7.06×10^{-3}	0.78	1.73×10^{-9}	7.07×10^{-15}
noh3	20	3	2.3×10^{-2}	1.3	2.0	3.81×10^{-2}	1.11×10^{-5}	1.15×10^{-3}	0.78	3.44×10^{-10}	1.15×10^{-15}
noh4	60	10	1.0×10^{-2}	1.4	0.43	3.93×10^{-2}	2.62×10^{-6}	1.65×10^{-4}	0.79	8.10×10^{-11}	1.65×10^{-16}
noh5	2	0.3	1.0×10^{-1}	…	…	1.06×10^{-2}	2.70×10^{-5}	1.40×10^{-2}	0.41	8.37×10^{-10}	1.40×10^{-14}
noh6	2	0.3	3.0×10^{-1}	…	…	9.48×10^{-2}	2.08×10^{-3}	1.02×10^{-1}	1.2	6.42×10^{-8}	1.02×10^{-13}
noh7	6	1	2.0×10^{-2}	…	…	4.63×10^{-3}	6.56×10^{-7}	8.10×10^{-4}	0.27	2.03×10^{-11}	8.11×10^{-16}
noh8	6	1	1.0×10^{-1}	…	…	8.90×10^{-2}	3.89×10^{-4}	1.67×10^{-2}	1.2	1.20×10^{-8}	1.67×10^{-14}

Helical fields produce steeper spectra and a sharper cutoff

cutoff due to short to turnover time

Introduction



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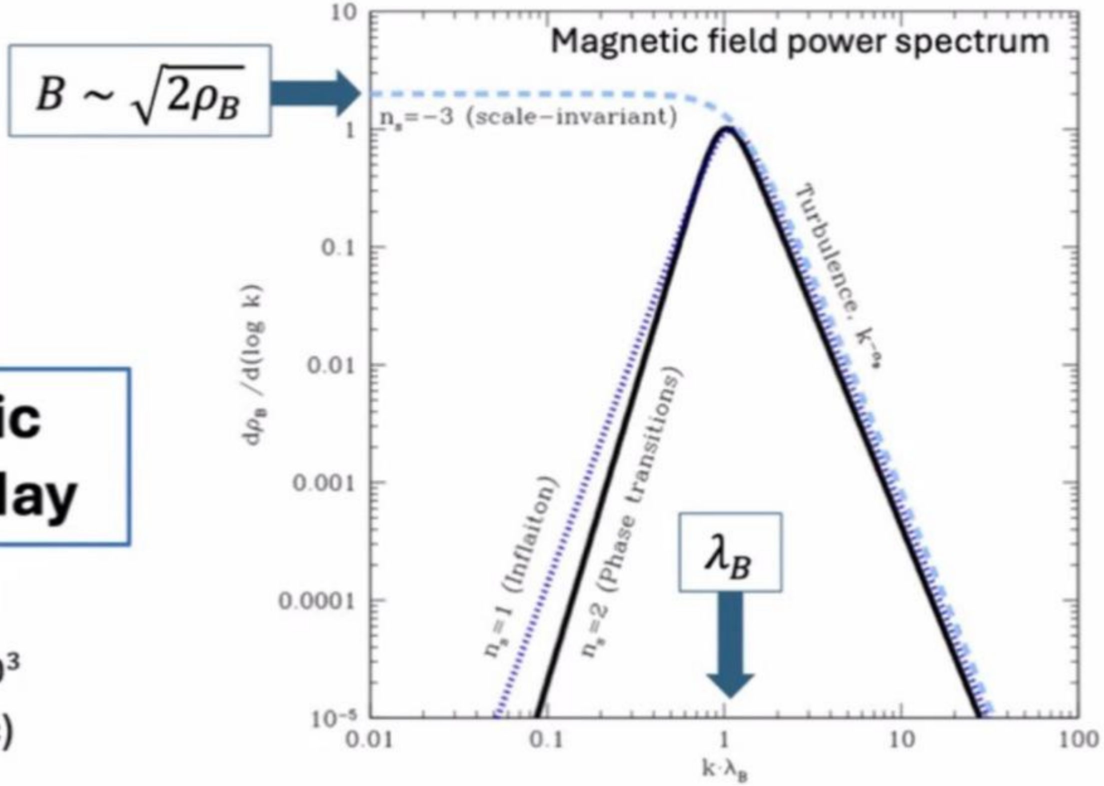
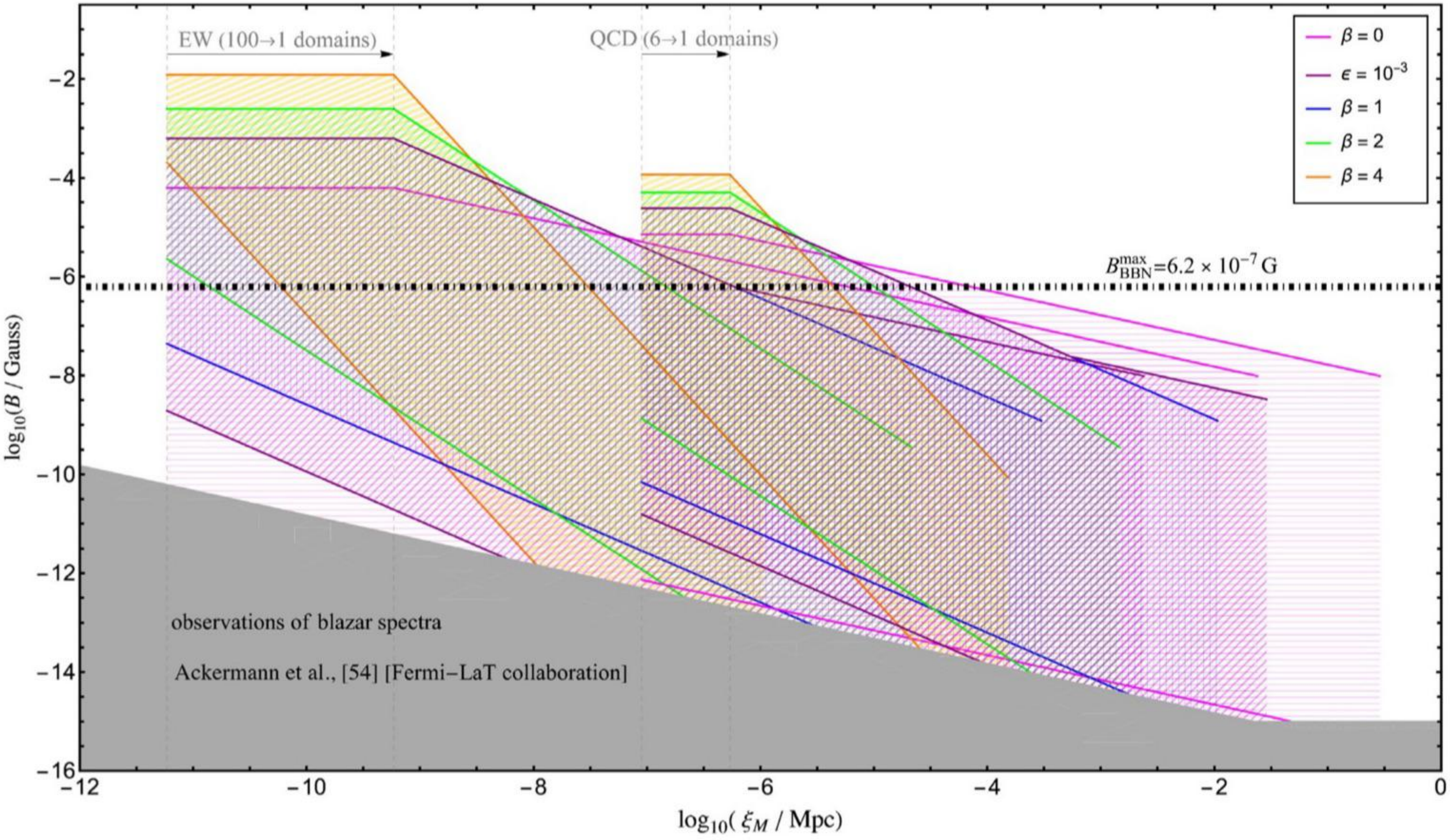


Big Band Nucleosynthesis (BBN) bound relaxed



Kahniashvili+22 (PRL 128, 221301)

Two examples of magnetogenesis in cosmology

(i) Chiral magnetic effect

(electroweak epoch)

$$\frac{\partial \boldsymbol{A}}{\partial t} = \frac{c^2}{\sigma}(\mu_5 \boldsymbol{B} - \boldsymbol{\nabla} \times \boldsymbol{B}) + \boldsymbol{u} \times \boldsymbol{B}$$

$$\mu_5 = 24\,\alpha_{\rm em}\,(n_{\rm L} - n_{\rm R})\,(\hbar c/k_{\rm B}T)^2$$

“Battery” still needed

$$\frac{\partial \mathbf{A}}{\partial t} = \frac{c}{q n_{\rm e}} \boldsymbol{\nabla} p_{\rm e},$$

(ii) Conformal invariance breaking

(during inflation)

$$f^2 F_{\mu\nu} F^{\mu\nu}$$

$$\tilde{\mathbf{A}}'' + \left(\mathbf{k}^2 - \frac{f''}{f}\right)\tilde{\mathbf{A}} = 0$$

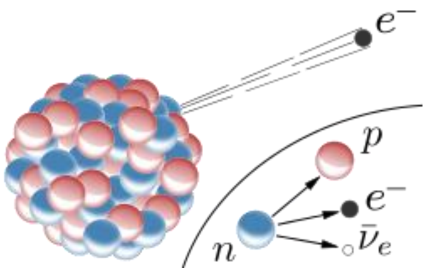
$$f(a) = a^{-\beta}, \quad \text{where } a = (\eta + 1)^2/4$$

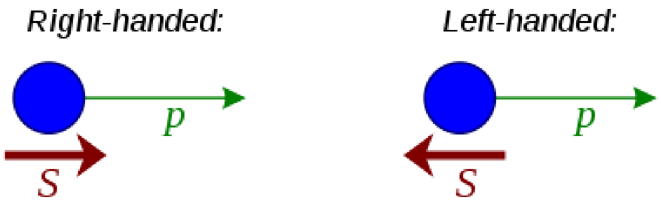
Quantum fluctuation

(i) Chiral magnetic effect: introduces pseudoscalar

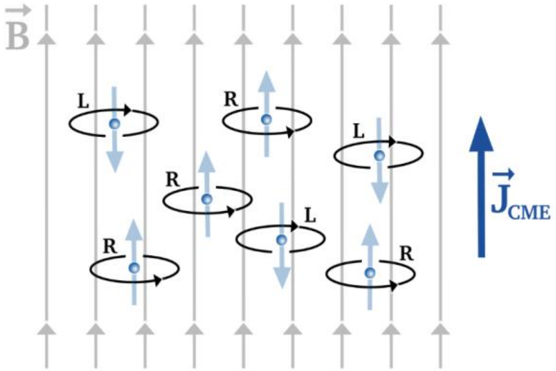
- Mathematically identical to α effect in mean-field dynamos
- Comes from chiral chemical potential μ (or μ_5)
- Number differences of left- & right-handed fermions

$$\mu_5 = 24\,\alpha_{\rm em}\,(n_{\rm L} - n_{\rm R})\,(\hbar c/k_{\rm B}T)^2,$$





- In the presence of a magnetic field, particles of opposite charge have momenta
- → electric current
- Self-excited dynamo
- But depletes μ



$$\frac{\partial \boldsymbol{A}}{\partial t} = \eta(\mu \boldsymbol{B} - \boldsymbol{\nabla} \times \boldsymbol{B}) + \boldsymbol{U} \times \boldsymbol{B}$$

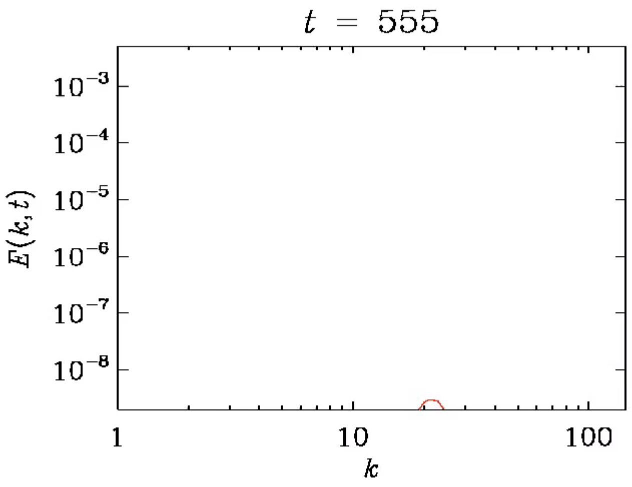
$$\sigma = |\mu k| - \eta k^2$$

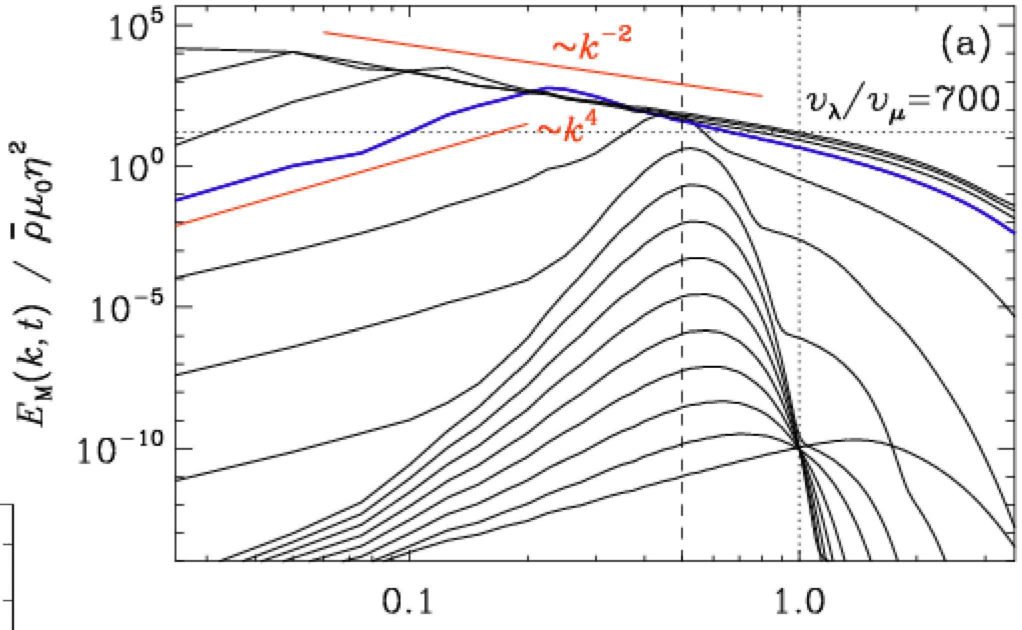
$\boldsymbol{B}$=curl$\boldsymbol{A}$

Discovered originally by Vilenkin (1980); application to magnetogenesis in early Universe by Joyce & Shaposhnikov (1997)

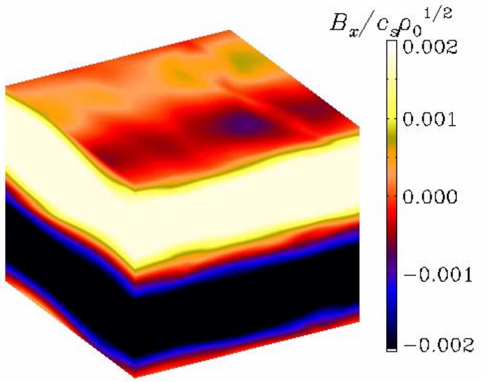
Time dependence from chiral magnetic effect (CME)

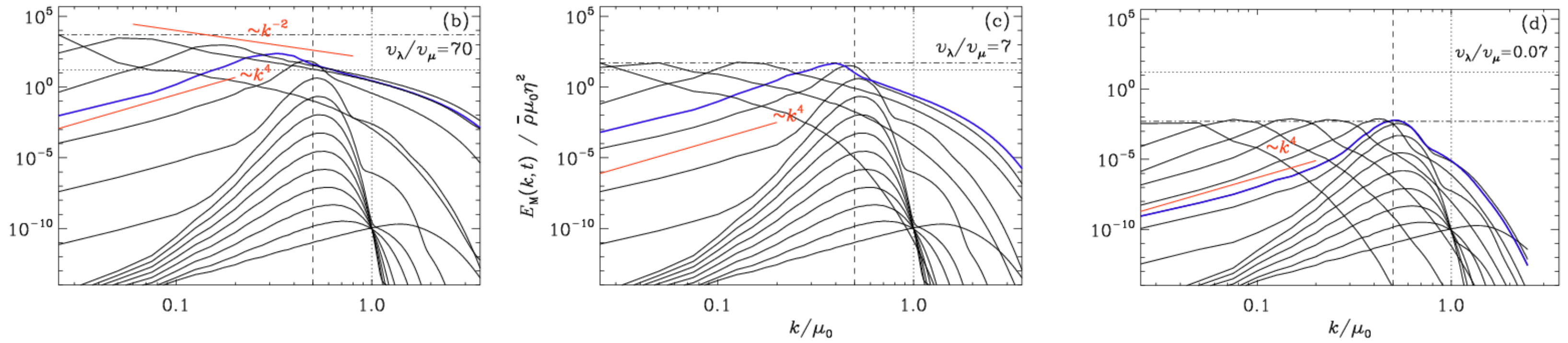
- Exponential growth at one k
- Subsequent inverse cascade
- Always fully helical





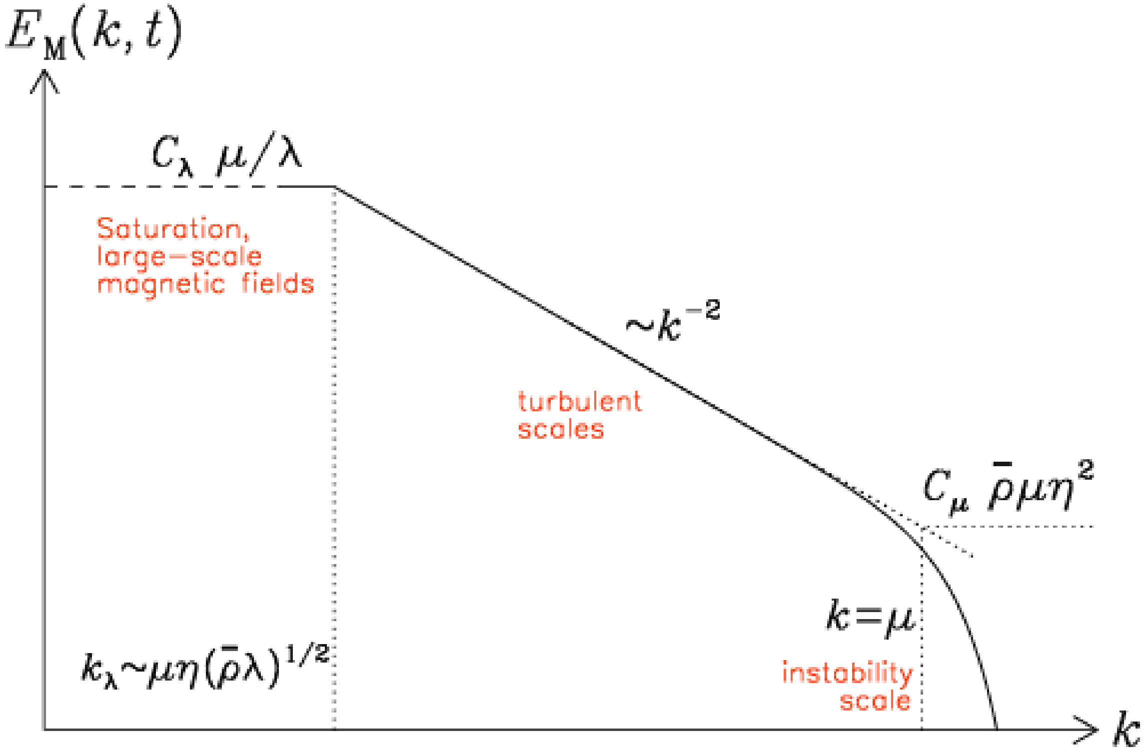
Growth at one wavenumber
Then: saturation caused by initial chemical potential





Brandenburg et al. (2017, ApJL 845, L21)

Many details are known by now



- Regime I is when turbulent subrange is long
- In regime II, just inverse cascading

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times [\boldsymbol{u} \times \boldsymbol{B} + \eta(\mu_5 \boldsymbol{B} - \boldsymbol{J})], \quad \boldsymbol{J} = \boldsymbol{\nabla} \times \boldsymbol{B},$$

$$\frac{\mathrm{D}\mu_5}{\mathrm{D}t} = -\lambda\,\eta\,(\mu_5 \boldsymbol{B} - \boldsymbol{J}) \cdot \boldsymbol{B} + D_5 \nabla^2 \mu_5 - \Gamma_{\mathrm{f}} \mu_5,$$

$$v_\lambda = \mu_{50}/\lambda^{1/2}, \qquad v_\mu = \mu_{50}\eta. \tag{6}$$

We recall that we have used here dimensionless quantities. We can identify two regimes of interest:

$$\eta k_1 < v_\mu < v_\lambda \quad \text{(regime I)}, \tag{7}$$

$$\eta k_1 < v_\lambda < v_\mu \quad \text{(regime II)}, \tag{8}$$

- Instability just η dependant
- Saturation governed by λ

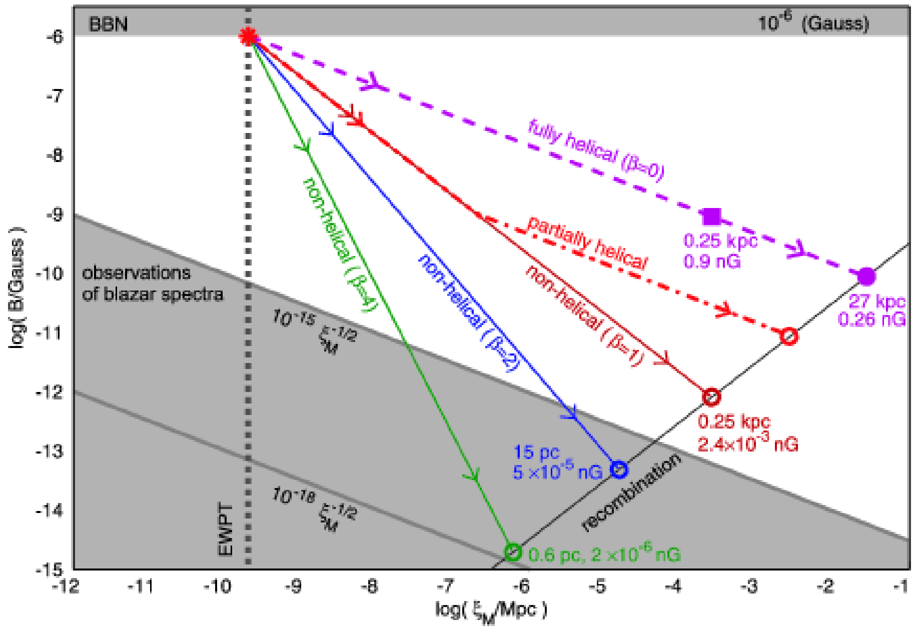
Strength of chiral magnetic effect

- Inverse turbulent cascade
  - $\langle \boldsymbol{B}^2 \rangle \sim t^{-2/3}$ length scale: $\xi_M \sim t^{+2/3}$
- Dimensional arguments give

$$\langle \boldsymbol{B}^2 \rangle \, \xi_M = \epsilon \, (k_B T_0)^3 (\hbar c)^{-2},$$

- Inserting T=3K gives 10^{-18} G on 1 Mpc
- Consequence of conservation law

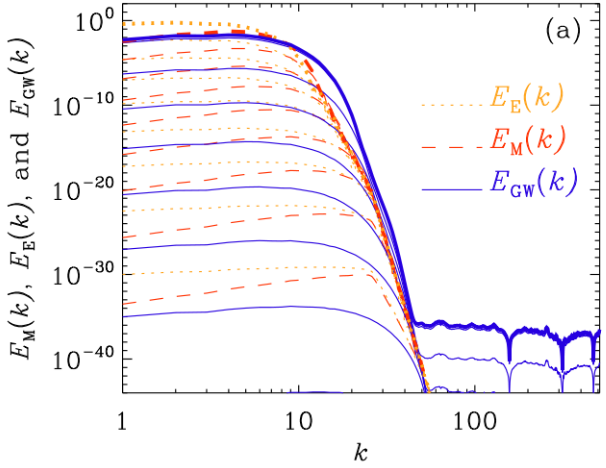
$$(n_L - n_R) + \frac{4\alpha_{em}}{\hbar c} \langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle = \text{const.}$$



- But starting length scale very small → 12 cm
- Compared with horizon scale at that time (electroweak) of ~1 AU
- Other dimensional argument:

$$\langle \boldsymbol{B}^2 \rangle \xi_M \lesssim \epsilon_3 (a_\star / a_0)^3 G^{-3/2} \hbar^{-1/2} c^{11/2},$$

(ii) Inflationary magnetogenesis



- Early Universe Turbulence
  - Source of gravitational waves
  - Information from young universe
- Magnetogenesis
  - Inflation/reheating
  - No particles yet, no conductivity
  - Coupling with electromagn field

$$f^2 F_{\mu\nu} F^{\mu\nu},$$

  - Breaking of conformal invariance
  - Quantum fluct → field stretched

$$\tilde{\mathbf{A}}'' + \left(\mathbf{k}^2 - \frac{f''}{f}\right)\tilde{\mathbf{A}} = 0,$$

$$\tilde{h}''_{+/\times} + \left(\mathbf{k}^2 - \frac{a''}{a}\right)\tilde{h}_{+/\times} = \frac{6}{a}\tilde{T}_{+/\times},$$

$$\iota f^2 F_{\mu\nu}{}^{*}F^{\mu\nu}$$ Coupling to pseudo-scalar (axion)

$$f(a) = a^{-\beta}, \quad \text{where } a = (\eta+1)^2/4$$

$$\tilde{A}''_{\pm} + \left(k^2 \pm 2\iota k\frac{f'}{f} - \frac{f''}{f}\right)\tilde{A}_{\pm} = 0,$$

$$\frac{f'}{f} = -\frac{2\beta}{\eta+1}, \quad \frac{f''}{f} = \frac{2\beta(2\beta+1)}{(\eta+1)^2}.$$

Brandenburg & Sharma 2106:03857

Large-scale magnetic fields from hydromagnetic turbulence in the very early universe

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(Received 1 February 1996)

We investigate hydromagnetic turbulence of primordial magnetic fields using magnetohydrodynamics (MHD) in an expanding universe. We present the basic, covariant MHD equations, find solutions for MHD waves in the early universe, and investigate the equations numerically for random magnetic fields in two spatial dimensions. We find the formation of magnetic structures at larger and larger scales as time goes on. In three dimensions we use a cascade (shell) model that has been rather successful in the study of certain aspects of hydrodynamic turbulence. Using such a model we find that after $\sim 10^9$ times the initial time the scale of the magnetic field fluctuation (in the comoving frame) has increased by 4–5 orders of magnitude as a consequence of an inverse cascade effect (i.e., transfer of energy from smaller to larger scales). Thus *at large scales* primordial magnetic fields are considerably stronger than expected from considerations which do not take into account the effects of MHD turbulence. [S0556-2821(96)02712-9]

Inverse cascade since the 1970s (*driven* turbulence)

J. Fluid Mech. (1975), *vol.* 68, *part* 4, *pp.* 769–778 769
Printed in Great Britain

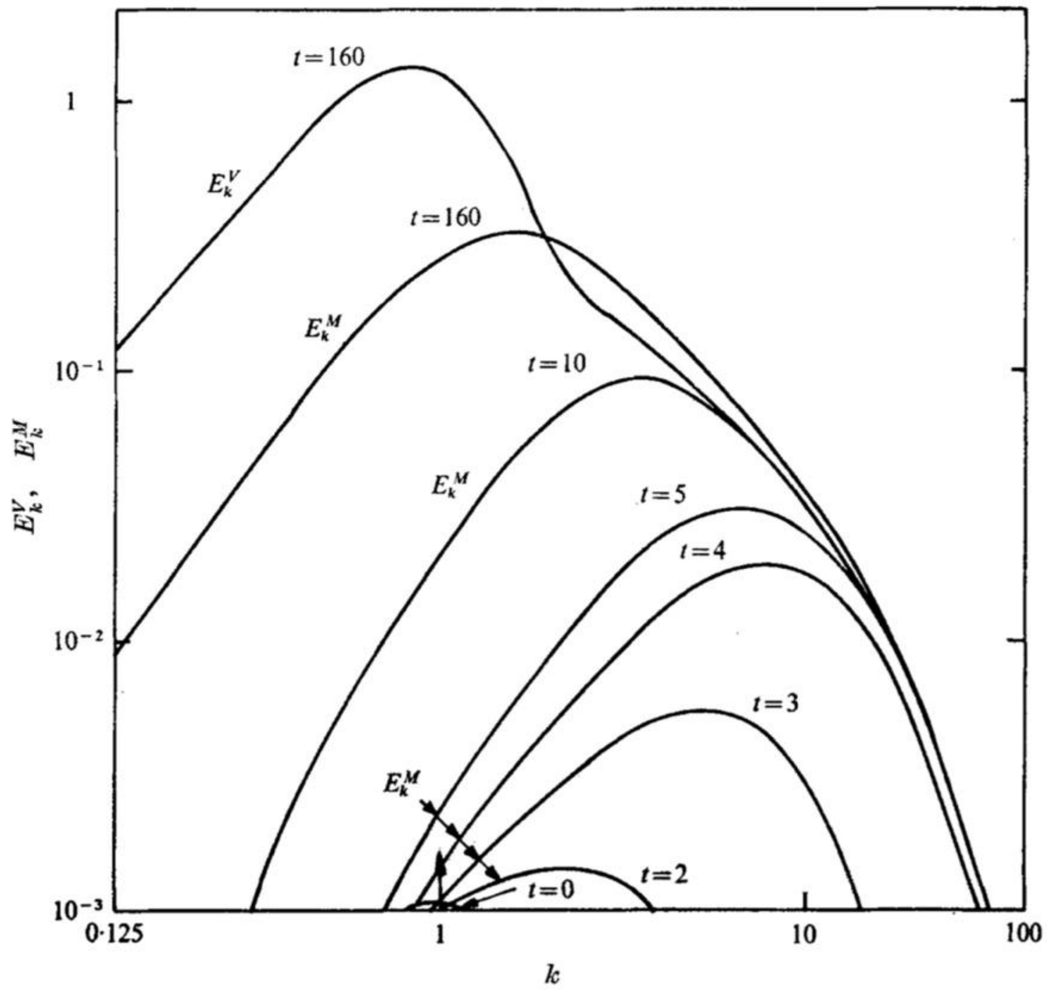
Possibility of an inverse cascade of magnetic helicity in magnetohydrodynamic turbulence

By U. FRISCH, A. POUQUET,
Centre National de la Recherche Scientifique, Observatoire de Nice, France
J. LÉORAT AND A. MAZURE
Université Paris VII, Observatoire de Meudon, France

J. Fluid Mech. (1976), *vol.* 77, *part* 2, *pp.* 321–354 321
Printed in Great Britain

Strong MHD helical turbulence and the nonlinear dynamo effect

By A. POUQUET, U. FRISCH
Centre National de la Recherche Scientifique,
Observatoire de Nice, France
AND J. LÉORAT
Université Paris VII, Observatoire de Meudon, France

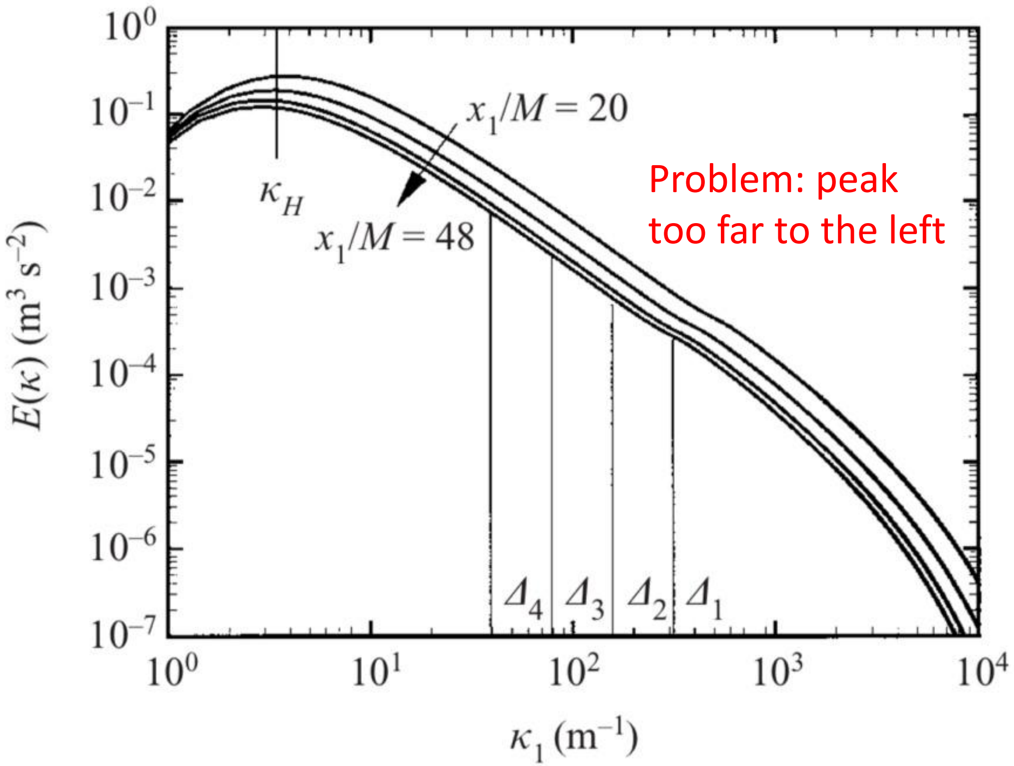


Turbulent decay: early results & expectations

J. Fluid Mech. (2003), vol. 480, pp. 129–160. © 2003 Cambridge University Press 129
DOI: 10.1017/S0022112002003579 Printed in the United Kingdom

Decaying turbulence in an active-grid-generated flow and comparisons with large-eddy simulation

By HYUNG SUK KANG[1], STUART CHESTER[1] AND CHARLES MENEVEAU[1,2]

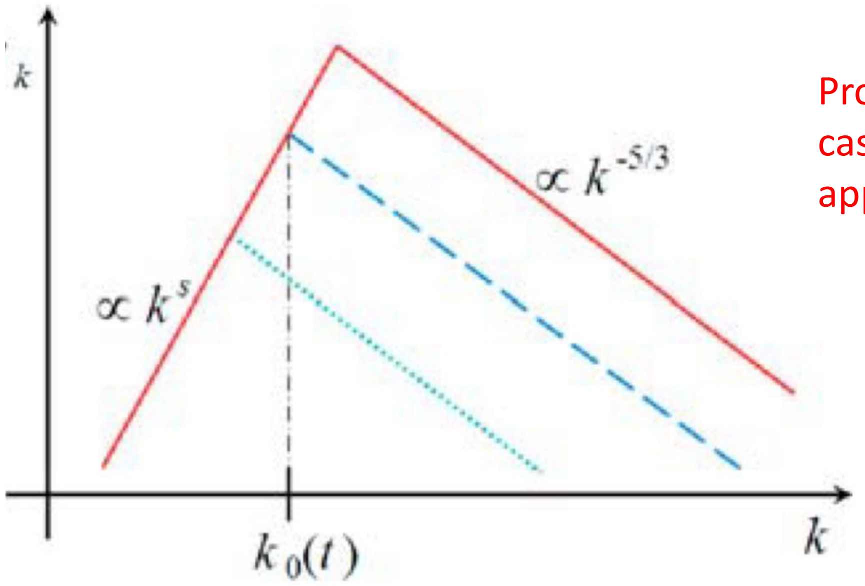


Problem: peak too far to the left

Mon. Not. R. Astron. Soc. 366, 1437–1454 (2006) doi:10.1111/j.1365-2966.2006.09918.x

Evolving turbulence and magnetic fields in galaxy clusters

Kandaswamy Subramanian,[1,4★] Anvar Shukurov[1,2,4★] and Nils Erland L. Haugen[3,5★]



Problem: inverse cascade not appreciated

Increase at small wavenumbers already in 2000

Need large scale separation: peak far to the right

THE DYNAMO EFFECT IN STARS

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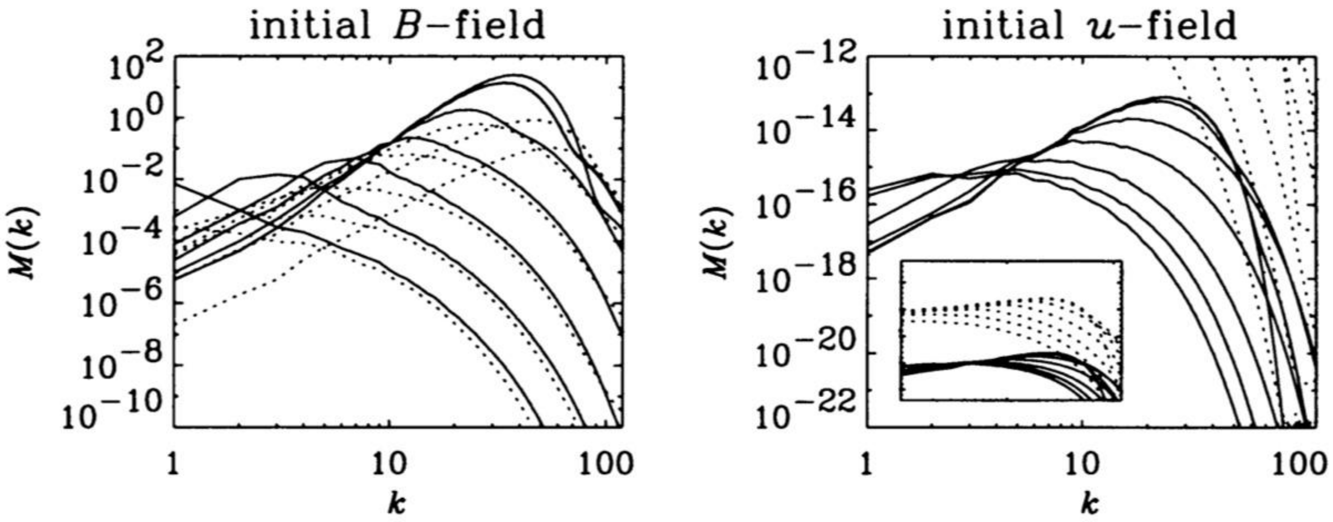


K.S. Cheng et al. (eds.), Stellar Astrophysics, 1–8.

- Magnetically dominated
  - Started from random vector potential
  - k^4 spectrum for magnetic energy
  - Kinetic energy (dotted) similar, but without the peak
- Kinetically dominated
  - Very similar inverse transfer
  - But kinetic energy much larger

3-D decay simulations with & without helicity

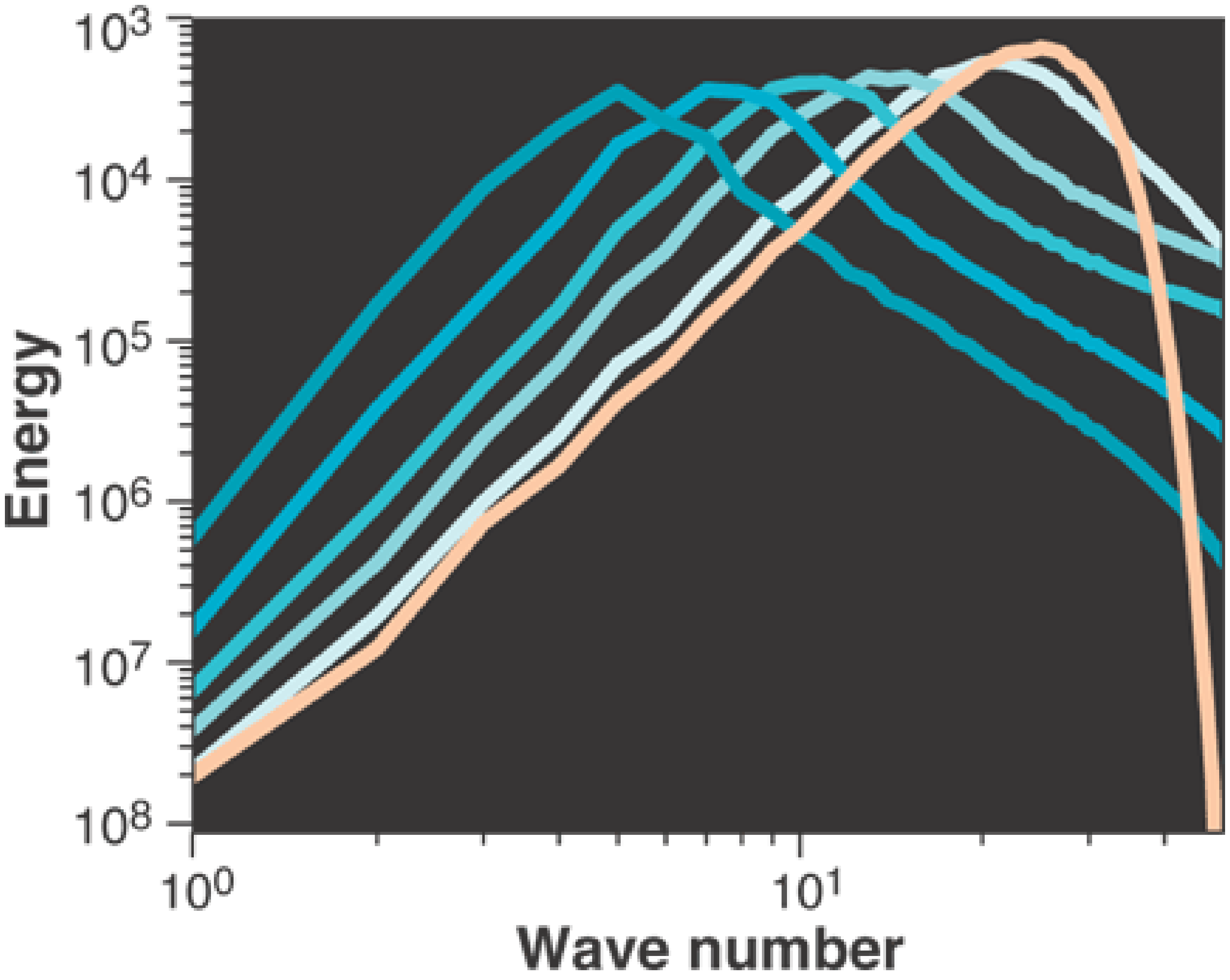
Initial slope
$E \sim k^4$

Causality (Durrer & Caprini 2003)
shell-integrated spectra
δ-correlated vector potential

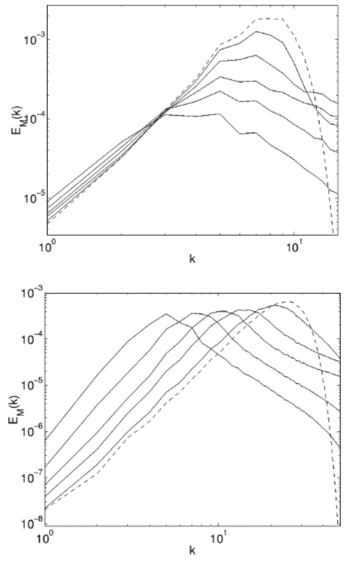
$$E_{\rm M}(k) = \frac{1}{2} \sum_{k_- < |\boldsymbol{k}| \leqslant k_+} |\tilde{\boldsymbol{B}}(\boldsymbol{k})|^2,$$

$$H_{\rm M}(k) = \frac{1}{2} \sum_{k_- < |\boldsymbol{k}| \leqslant k_+} (\tilde{\boldsymbol{A}} \cdot \tilde{\boldsymbol{B}}^* + \tilde{\boldsymbol{A}}^* \cdot \tilde{\boldsymbol{B}}),$$

$k_\pm = k \pm \delta k/2$ and $\delta k = 2\pi/L$ is the

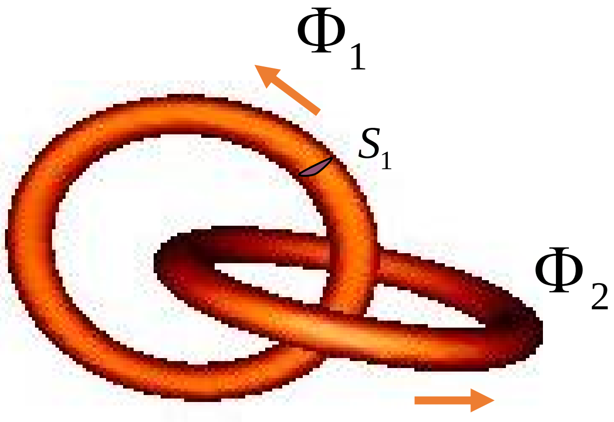


helical vs
nonhelical



Christensson et al.
(2001, PRE 64, 056405)

Magnetic helicity



$$H = \int_V \mathbf{A}\cdot\mathbf{B}\,\mathrm{d}V$$

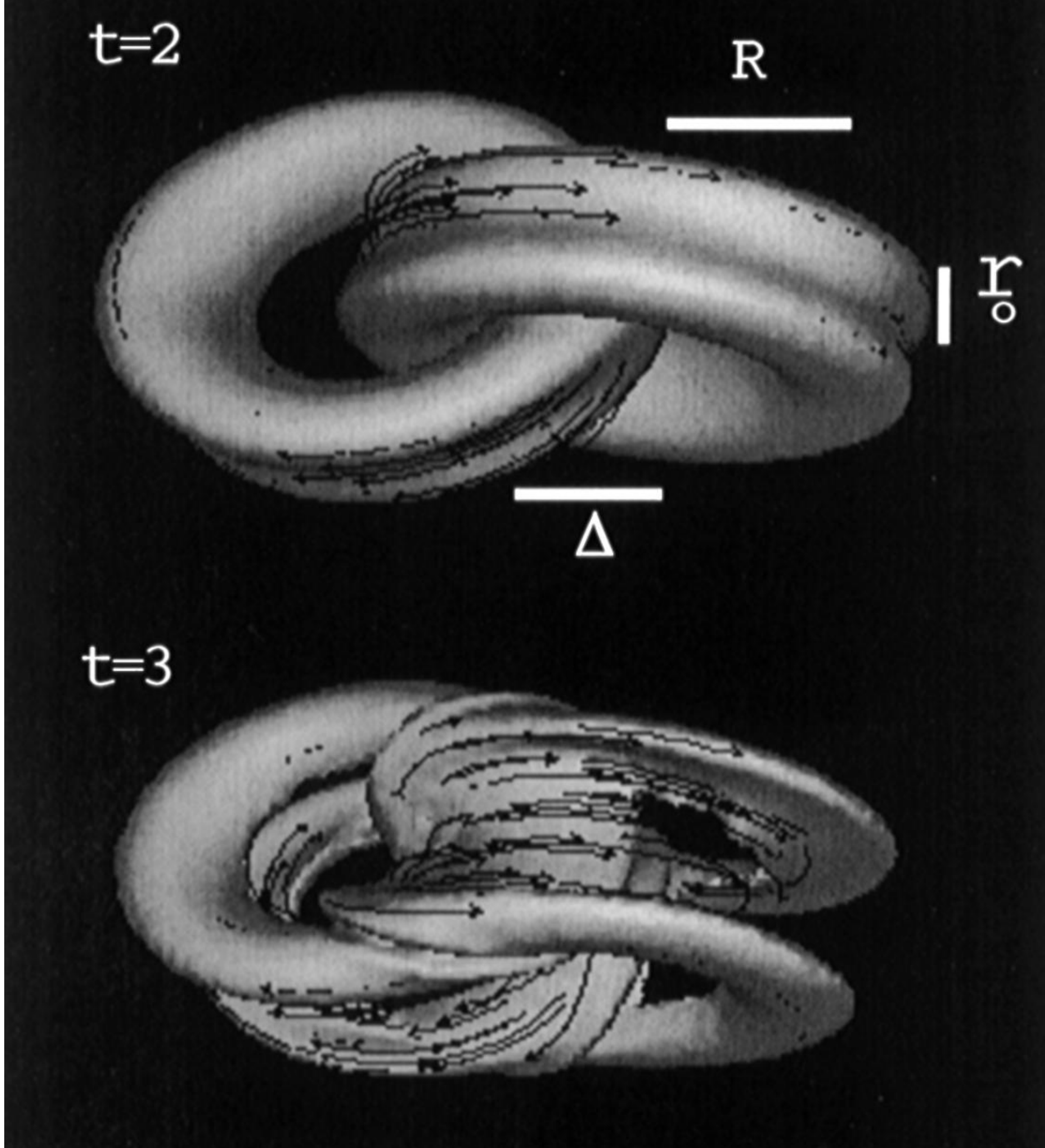
$$\mathbf{B} = \nabla\times\mathbf{A}$$

$$H_1 = \int_{L_1} \mathbf{A}\cdot\mathrm{d}\boldsymbol{\ell} \int_{S_1} \mathbf{B}\cdot\mathrm{d}\mathbf{S}$$

$$= \int_{S_2} \nabla\times\mathbf{A}\cdot\mathrm{d}\mathbf{S} = \Phi_2 \qquad = \Phi_1$$

$$H = \pm 2\Phi_1\Phi_2$$

Therefore the unit is Maxwell squared



Kerr & Brandenburg (1999)

Considerations

- Difficulties in seeing (nonhelical) inverse cascade
  - Must have: $k_{\text{peak}} >> k_{\text{min}}$ (enough k-range to the left of the peak)
  - Causal spectrum $E_M(k) \sim k^4$ (must be steep enough)
- Not seen for velocity spectrum
  - Even if incompressible
  - → long-range interactions immediately driven by ***B***-field
- Tools
  - *pq* diagram
  - conservation laws
  - study resistive effects

Different approaches to decays laws

- Initial slope matters
  - "selective decay"
- Olesen (1997)
  - Initial slope k^{α}
  - Invariance under rescaling: $x \rightarrow x\ell,\ t \rightarrow t\ell^{1/q}$
  - $\rightarrow q=2/(3+\alpha)$
- Inverse cascade criterion
  - $q>0$, so $\alpha > -3$

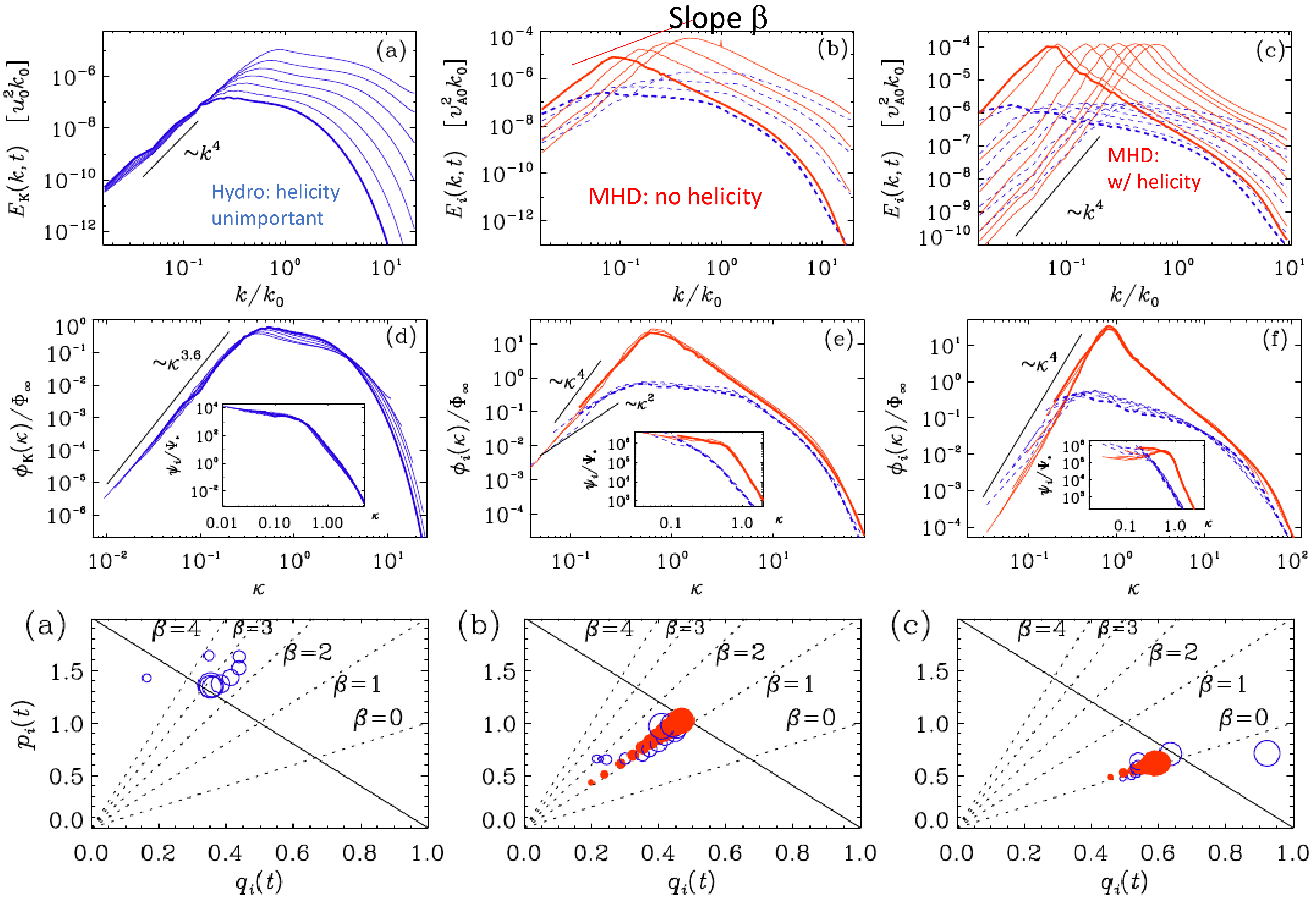
- Self-similarity matters
  - Measure empirically β
  - $\rightarrow q=2/(3+\beta)$
- Inverse cascade criterion
  - $\alpha-\beta > 0$, so
  - $\alpha > \beta$ cc
- Hosking integral:
  - $\beta = 3/2$

- Conservation law matters
  - Just dimensional arguments
  - Get nondim. prefactors from simulations

Collapsed spectra and *pq* diagrams

$-p_i(t) = d\ln\mathcal{E}_i/d\ln t, \quad q_i(t) = d\ln\xi_i/d\ln t,$

Brandenburg & Kahniashvili (2017)



Explanations for slope β

Exponents *p*,*q* (Hosking & Schekochihin 2021+2023)

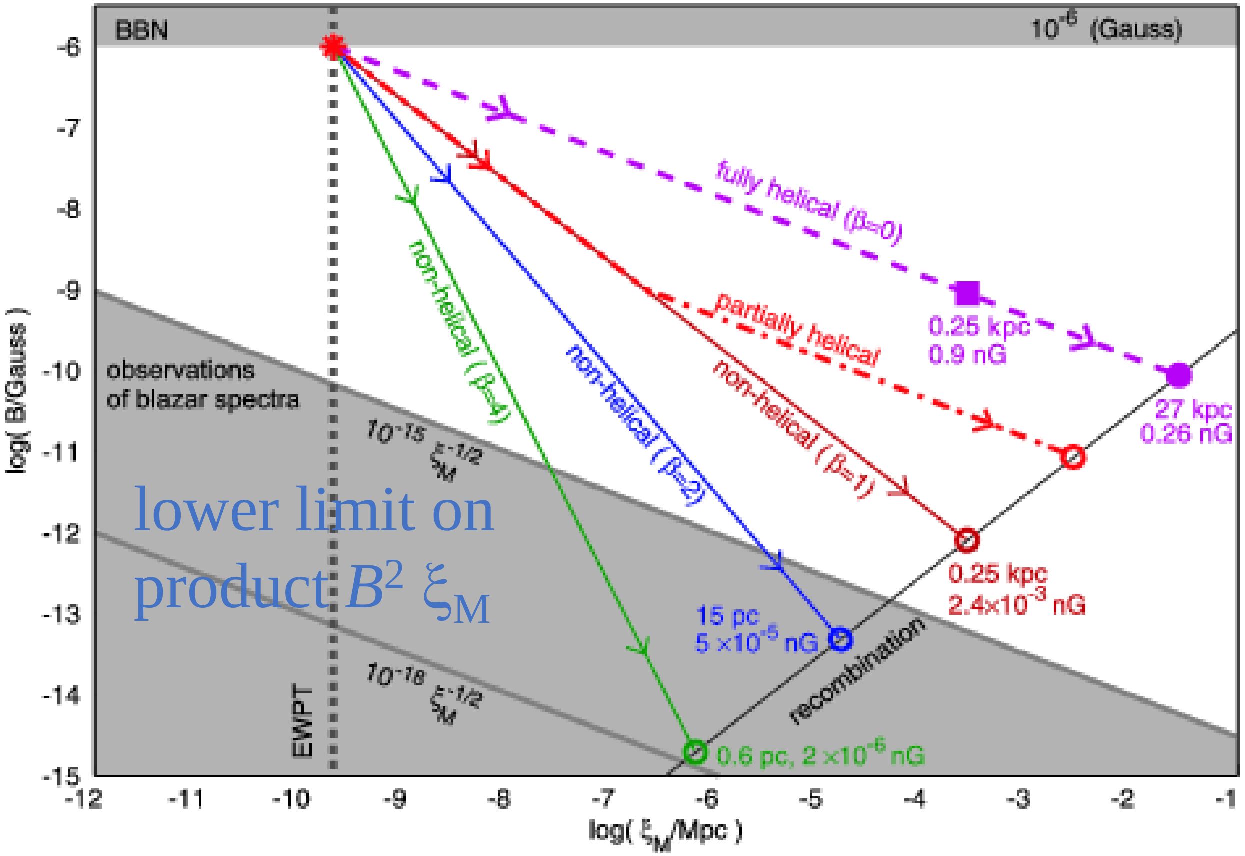
Conservation laws

$\xi_M \sim \langle A.B \rangle\, t^{2/3}$

cm ~ $(\mathrm{cm}^3/\mathrm{s}^2)\ \mathrm{s}^{2/3}$

Magnetic helicity	$\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle$	$\mathrm{cm}^3\,\mathrm{s}^{-2}$	$\xi_\mathrm{M}(t) \propto \langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle^{1/3} t^{2/3}$
Anastrophy (2-D)	$\langle A_z^2 \rangle$	$\mathrm{cm}^4\,\mathrm{s}^{-2}$	$\xi_\mathrm{M}(t) \propto \langle A_z^2 \rangle^{1/4} t^{1/2}$
Hosking integral	I_H	$\mathrm{cm}^9\,\mathrm{s}^{-4}$	$\xi_\mathrm{M}(t) \propto I_\mathrm{H}^{1/9} t^{4/9}$
Saffman integral	I_S	$\mathrm{cm}^5\,\mathrm{s}^{-2}$	$\xi_\mathrm{M}(t) \propto I_\mathrm{S}^{1/5} t^{2/5}$
Loitsyansky integral	I_L	$\mathrm{cm}^7\,\mathrm{s}^{-2}$	$\xi_\mathrm{M}(t) \propto I_\mathrm{S}^{1/7} t^{2/7}$

AB, Kahniashvili, ..., Vachaspati (2017)



Magnetic energy dependence
Parametric representation

magnetic energy	$\kappa = p/2q$
$\mathcal{E}_\mathrm{M}(t) \propto \langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle^{2/3} t^{-2/3}$	$\propto \xi_\mathrm{M}^{-1/2}$
$\mathcal{E}_\mathrm{M}(t) \propto \langle A_z^2 \rangle^{1/2} t^{-1}$	$\propto \xi_\mathrm{M}^{-1}$
$\mathcal{E}_\mathrm{M}(t) \propto I_\mathrm{H}^{2/9} t^{-10/9}$	$\propto \xi_\mathrm{M}^{-5/4}$
$\mathcal{E}_\mathrm{M}(t) \propto I_\mathrm{S}^{2/5} t^{-6/5}$	$\propto \xi_\mathrm{M}^{-3/2}$
$\mathcal{E}_\mathrm{M}(t) \propto I_\mathrm{S}^{2/7} t^{-10/7}$	$\propto \xi_\mathrm{M}^{-5/2}$

Nonhelical decay: mag helicity in patches conserved

h(x)=A.B

$$\mathcal{I}_H(R \ll \xi_M) \simeq \int_{V_R} \mathrm{d}^3 r \, \langle h(\boldsymbol{x}) h(\boldsymbol{x}) \rangle \propto R^3$$

Hosking integral

$$\mathcal{I}_H(R) = \int_0^\infty \mathrm{d}k \, w_{\mathrm{sph}}^{\mathrm{BC}}(k) \, \mathrm{Sp}\,(h)$$

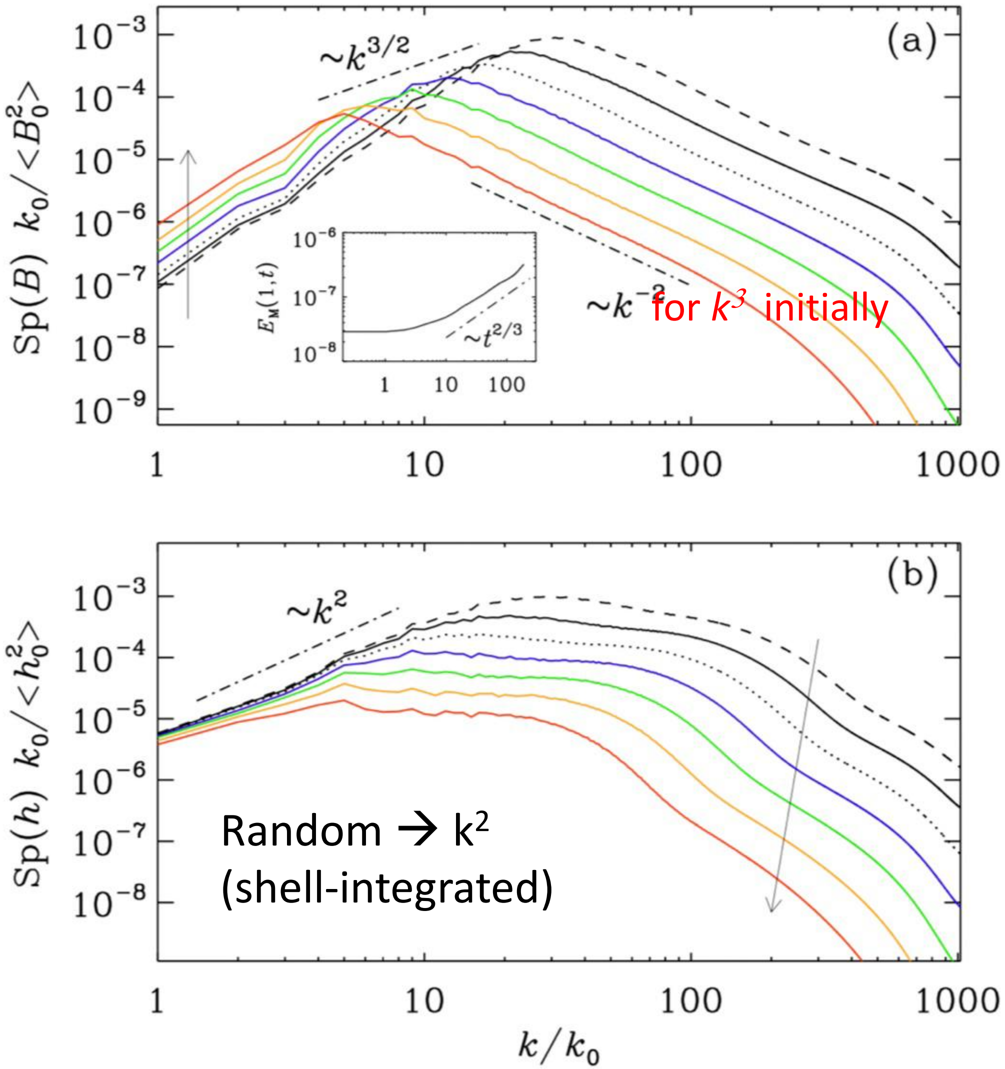
$$\mathrm{Sp}\,(h) = \frac{1}{V} \frac{k^2}{(2\pi)^3} \int_{|\boldsymbol{k}|=k} \mathrm{d}\Omega_k \, \tilde{h}^*(\boldsymbol{k}) \tilde{h}(\boldsymbol{k})$$

$$\mathrm{Sp}\,(h) = \frac{I_H}{2\pi^2} k^2 + \mathcal{O}(k^4)$$

$[I_H] = \mathrm{cm}^9 \ \mathrm{s}^{-4}$

$\xi_M = I_H{}^a \ t^b$

a=1/9, b=4/9



AB, Sharma, Vachaspati (2023)

$$\xi_{\mathrm{M}}(t) \approx 0.12 \, I_{\mathrm{H}}^{1/9} t^{4/9}, \quad \mathcal{E}_{\mathrm{M}}(t) \approx 3.7 \, I_{\mathrm{H}}^{2/9} t^{-10/9}, \quad E_{\mathrm{M}}(k,t) \lesssim 0.025 \, I_{\mathrm{H}}^{1/2} (k/k_0)^{3/2}$$

Universal coefficients?

$$\xi_{\rm M}(t) \approx 0.12\, I_{\rm H}^{1/9} t^{4/9}, \quad \mathcal{E}_{\rm M}(t) \approx 3.7\, I_{\rm H}^{2/9} t^{-10/9}, \quad E_{\rm M}(k,t) \lesssim 0.025\, I_{\rm H}^{1/2} (k/k_0)^{3/2}$$

If so, it would be in conflict with simple relationships of the form:

$$\frac{\xi_M}{\lambda_0} = \left(\frac{T}{T_\star}\right)^{-n_\xi},$$

$$\frac{B^{(\rm eff)}}{B_\star} = \left(\frac{T}{T_\star}\right)^{-n_E},$$

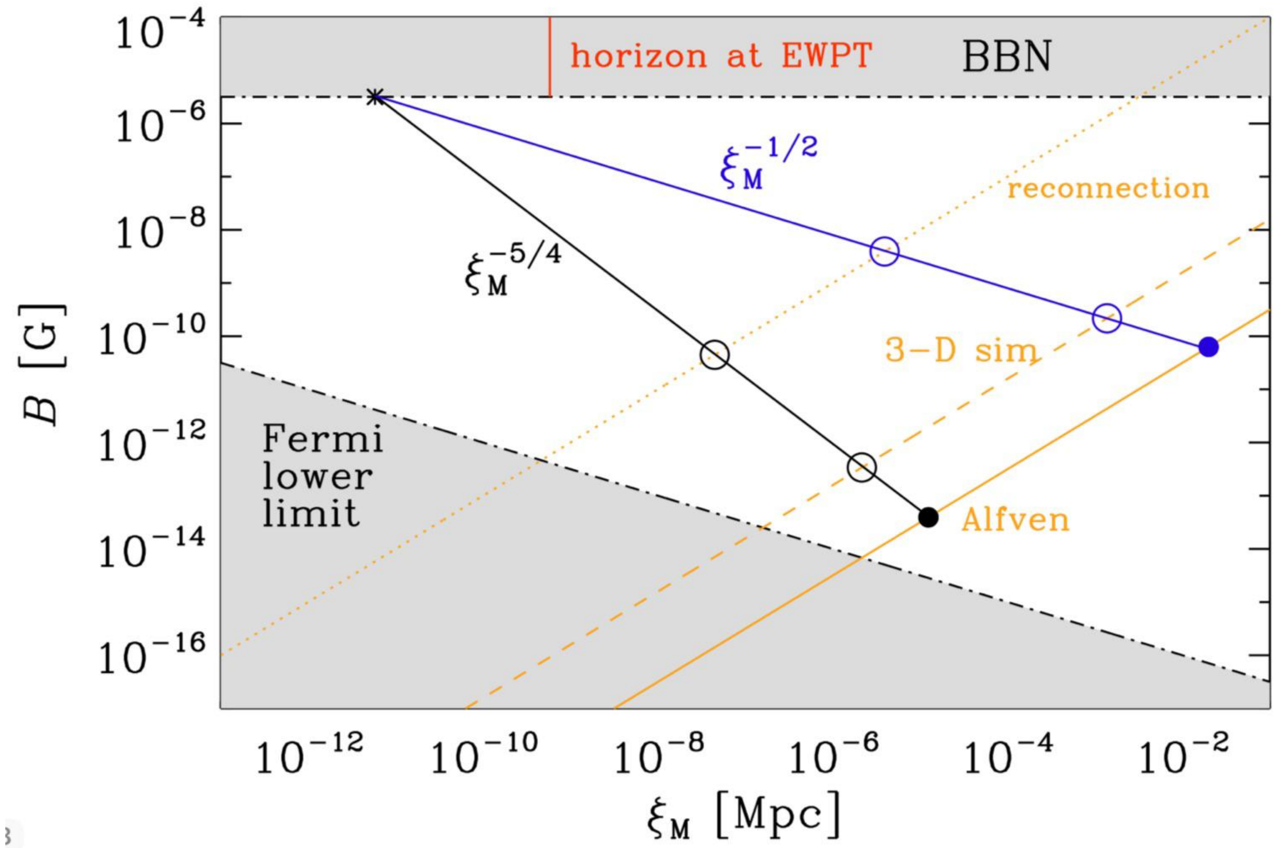
Which suggests that ξ_M and $B^{(\rm eff)}$ can be chosen freely and independently from each other!

A&A, 687, A186 (2024)
https://doi.org/10.1051/0004-6361/202449267

Astronomy & Astrophysics

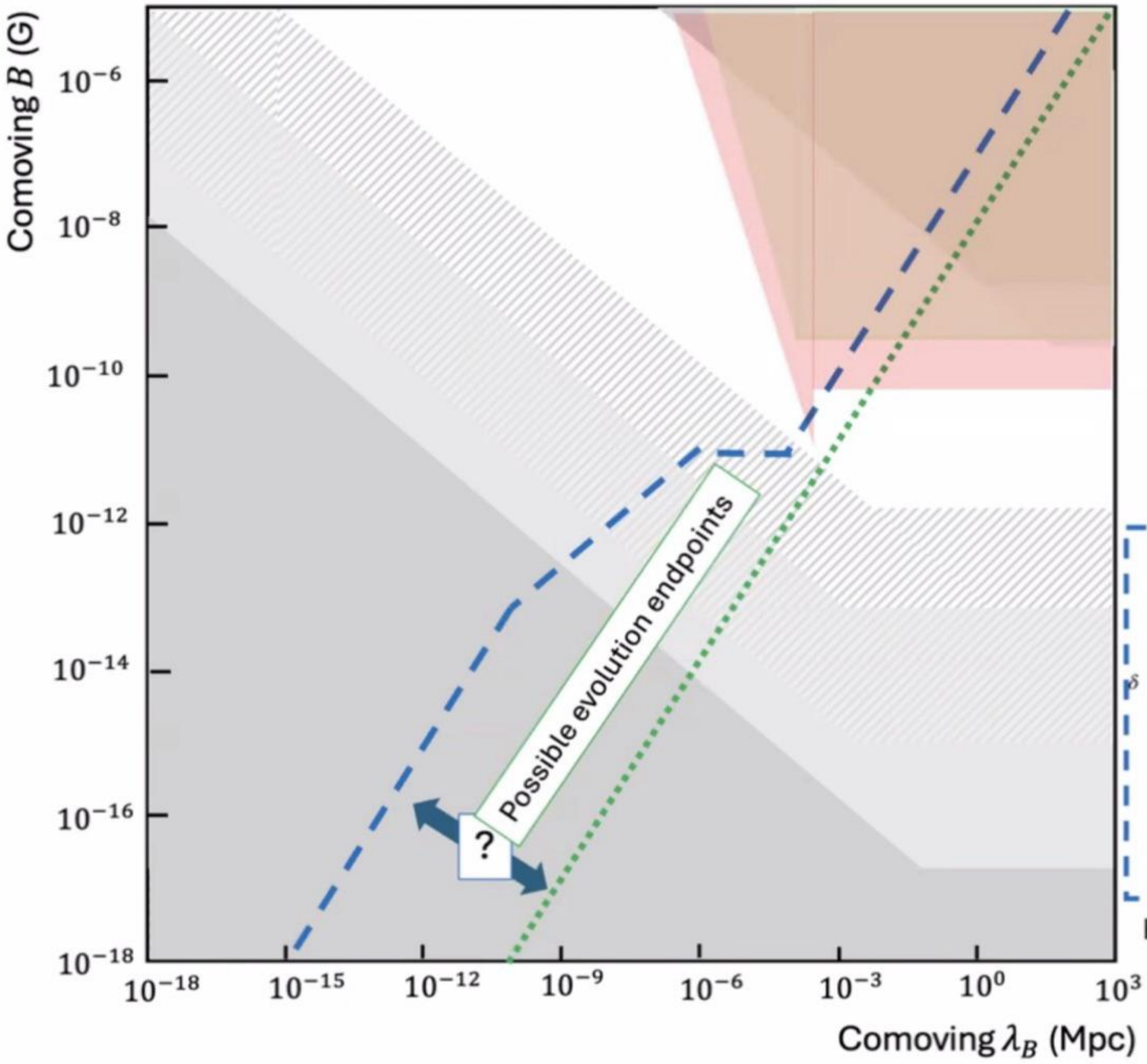
Resistively controlled primordial magnetic turbulence decay

A. Brandenburg[1,2,3,4,5], A. Neronov[6,7], and F. Vazza[8,9,10]



- Endpoints under assumption that decay time = Alfven time
- Use: decay time = recombination time
- Possibility: decay time >> Alfven time
- → Premature endpoint of evolution

Backtracing of magnetic field evolution



"Largest processed turbulent eddy" concept:

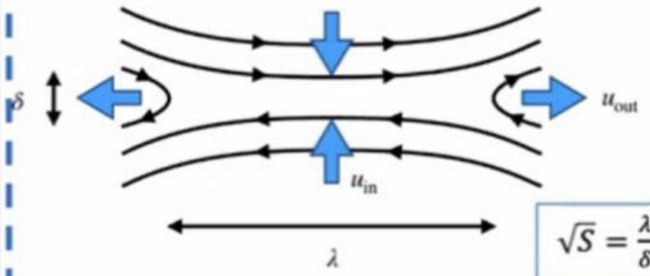
$$\lambda_B = Vt$$

Alfven velocity?

$$v_A = \sqrt{\frac{\rho_B}{\rho}}$$

Hubble time?

Banerjee, Jedamzik, astro-ph/0410032



$$; \quad \lambda = \frac{1}{\sqrt{S}\,(1 + P_m)}\, v_A t$$

$\sqrt{S} = \frac{\lambda}{\delta} \gg 1$; Prandtl number $P_m = \frac{\nu}{\eta} \gg 1$

Hosking, Schekochihin, 2203.03573

Resistively controlled primordial magnetic turbulence decay

A. Brandenburg[1,2,3,4,5], A. Neronov[6,7], and F. Vazza[8,9,10]

Relation between decay time

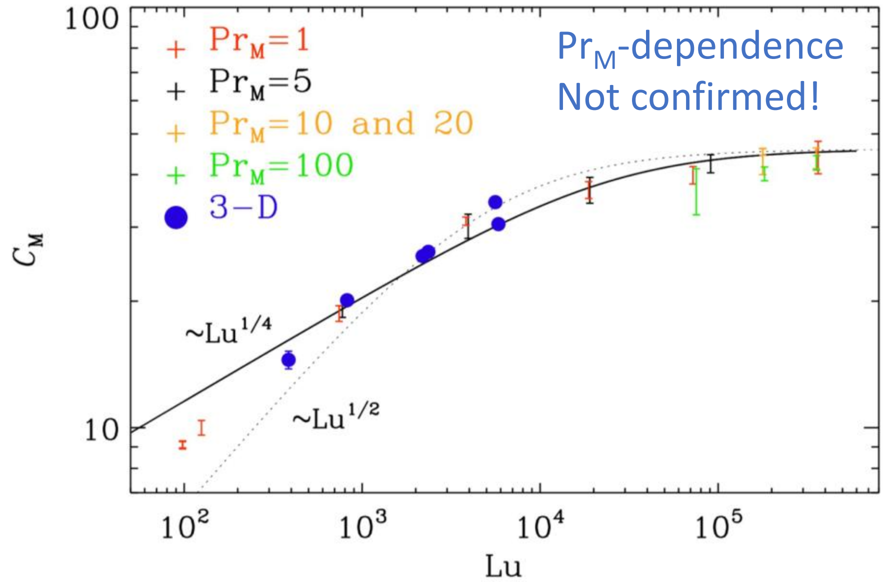
$$\tau^{-1} = -\mathrm{d}\ln\mathcal{E}_{\mathrm{M}}/\mathrm{d}t$$

and Alfven time

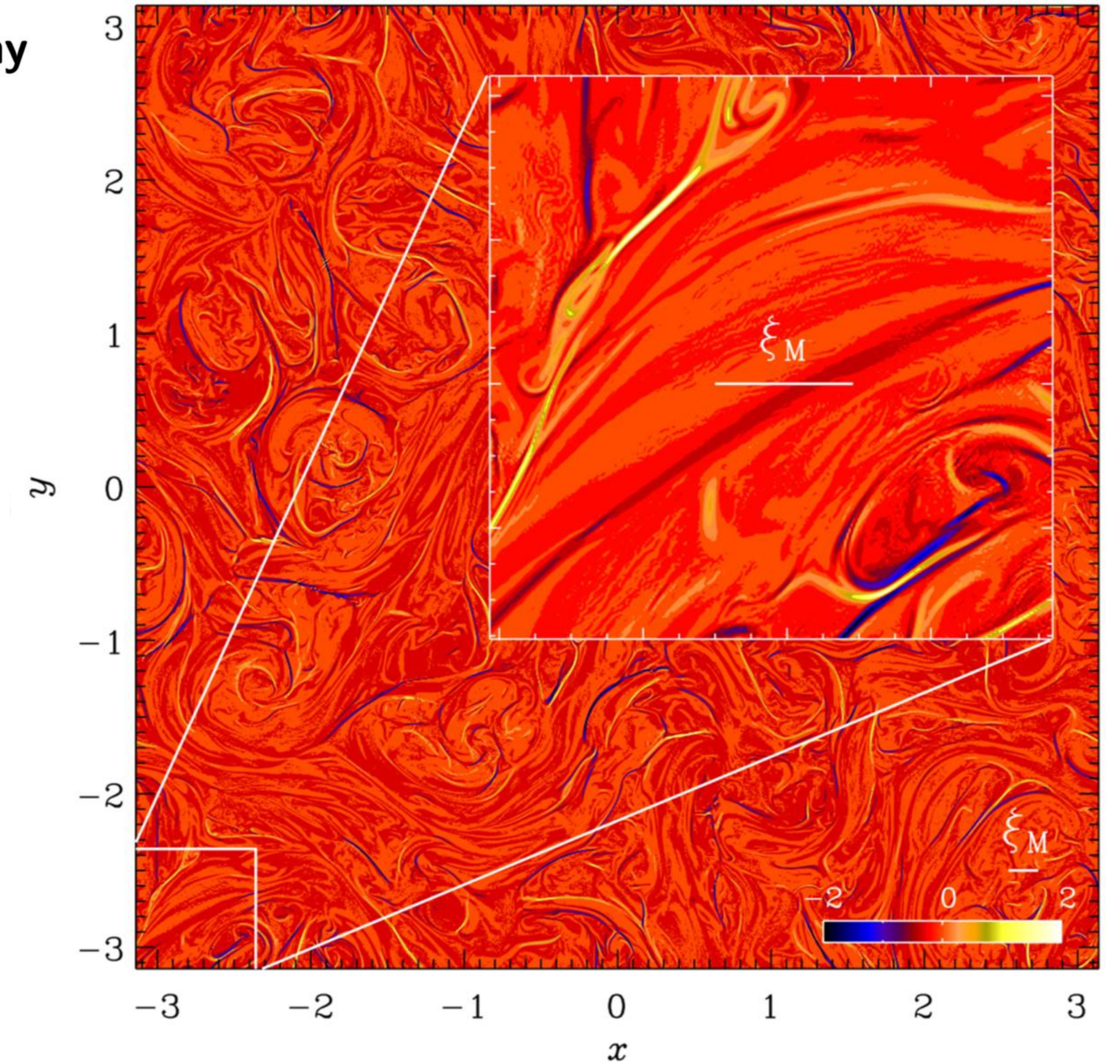
$$\tau_{\mathrm{A}} = \xi_{\mathrm{M}}/v_{\mathrm{A}} \qquad \mathcal{E}_{\mathrm{M}} = B_{\mathrm{rms}}^2/2\mu_0 = \rho v_{\mathrm{A}}^2/2$$

Determine C_{M} in relation:

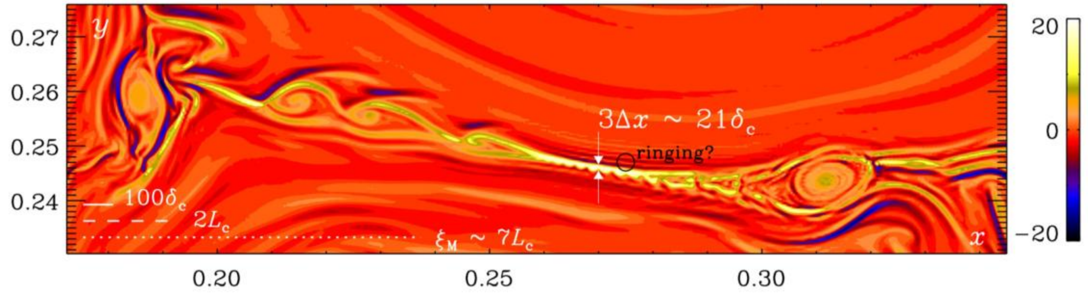
$$\tau = C_{\mathrm{M}}\xi_{\mathrm{M}}/v_{\mathrm{A}}$$



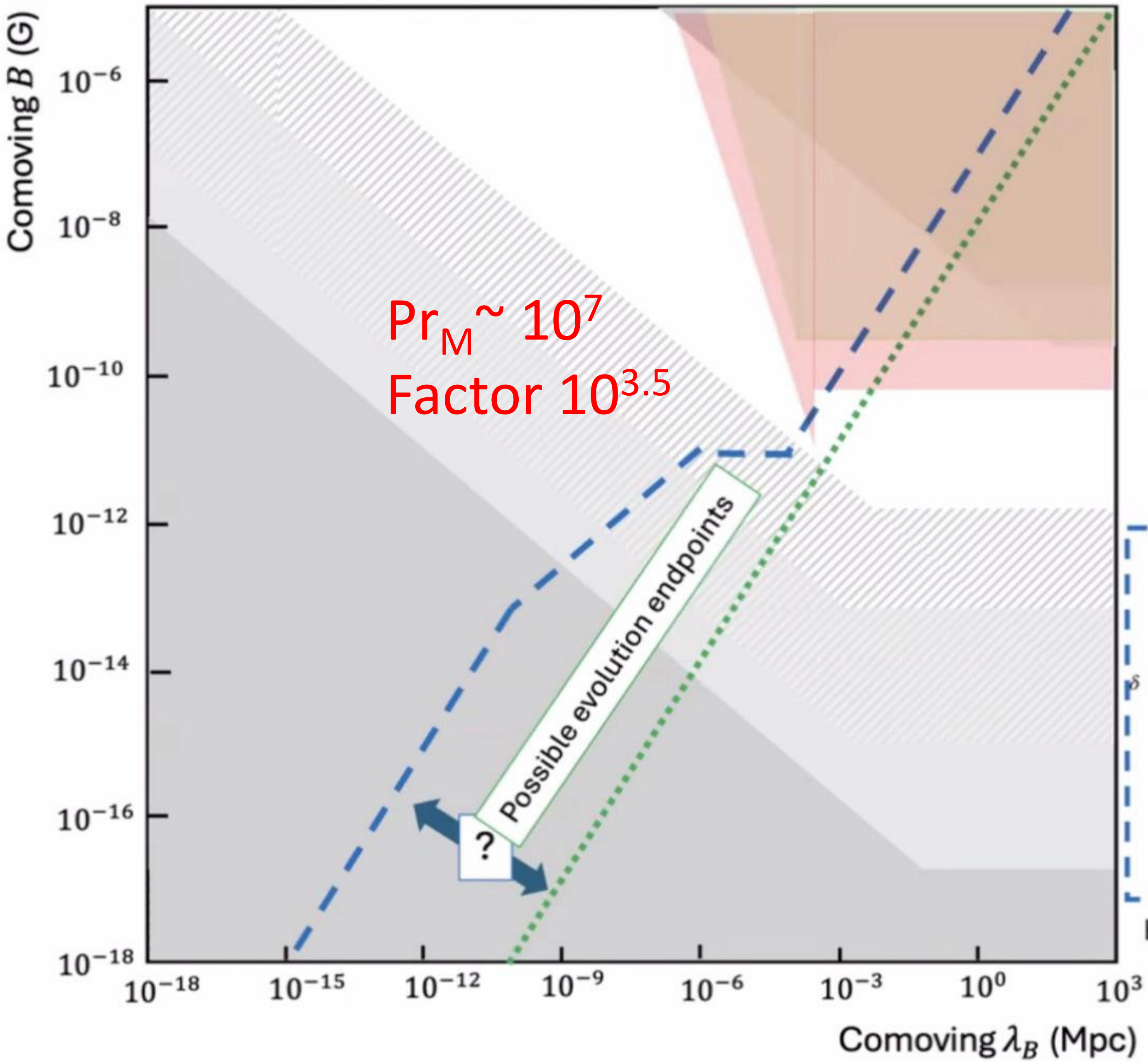
3-D



2-D



Backtracing of magnetic field evolution



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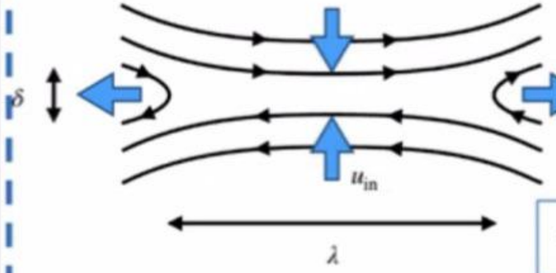

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$\sqrt{S} = \frac{\lambda}{\delta} \gg 1$; Prandtl number $P_m = \frac{\nu}{\eta} \gg 1$

Hosking, Schekochihin, 2203.03573

Structures highly dynamical:
outflow not opposed by viscosity

Summary:

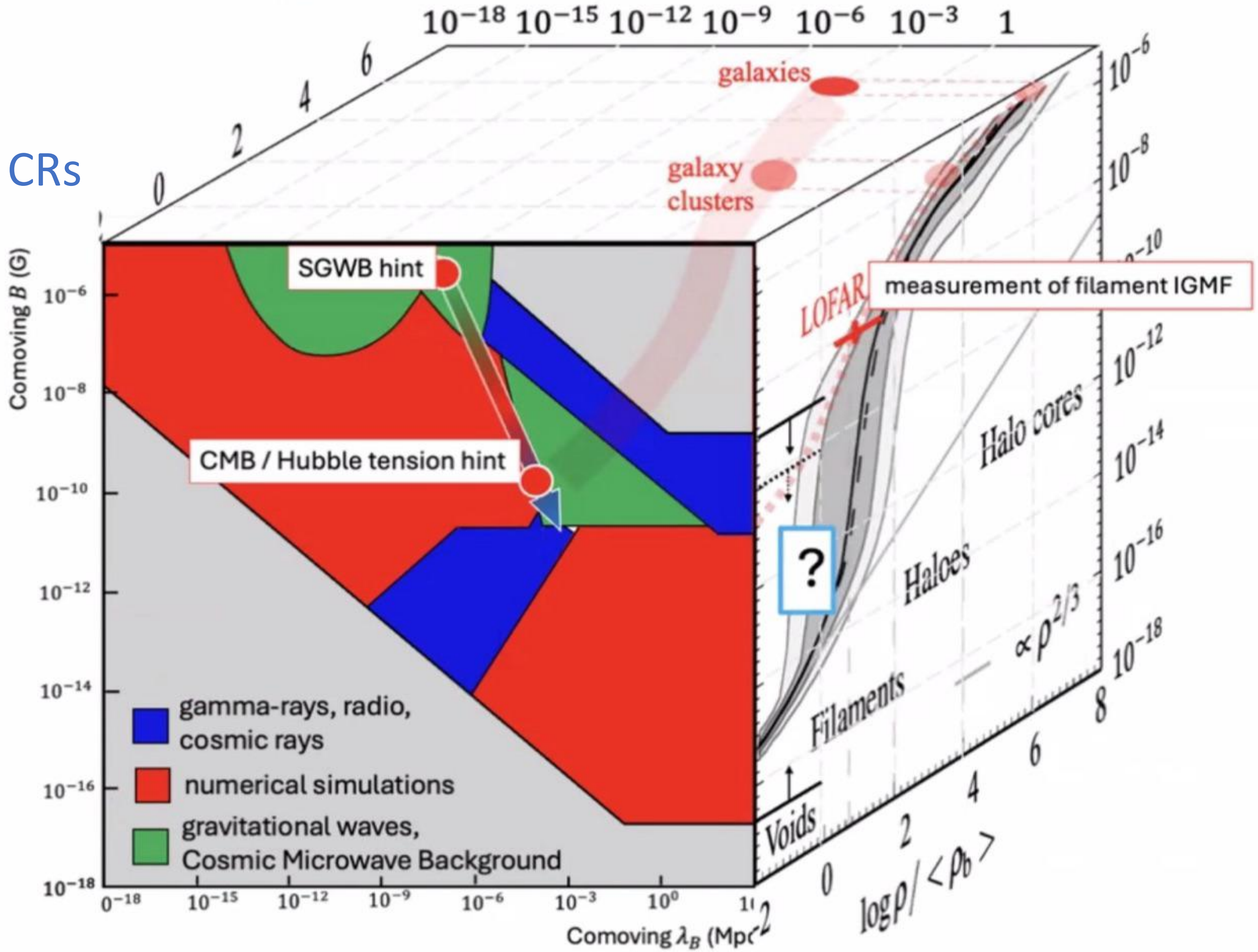
tools are available to explore "full" intergalactic / cosmological magnetic fields parameter space, from the moment of creation to recombination and throughout structure formation up to $z = 0$

Interplay:

Gamma-ray, radio, CRs

GWs, CMB

simulations



Conclusions (so far)

- Selfsimilar decay
  - Magnetic helicity plays a role even when it vanishes on average!
  - Hosking integral conserved relevant for early universe
  - Perhaps also for galaxy clusters (after mergers)
- Universe as a whole → primordial (non-astrophysical) fields
  - Decay till recombination: < 0.1 nG fields, 1 kpc scales at best (phase transitions)
  - Larger scales from reheating scenarios
  - If nonhelical: Hosking integral conserved
  - Also applies to fully helical, if balanced by fermion chirality
- Inflationary: large scales, often helical
  - Electric energy → kinetic energy
  - Circularly polarized waves
- What next?
  - Reconnection
  - Rm dependence
  - magnetic helicity fluxes

Note on the Pencil Code

JOSS
The Journal of Open Source Software

- 2001 started at Summer School
- 2004 First User Meeting
  - Annually since then
- 2016 Steering Committee
- 2020 Special Issue in GAFD
- 2020 Newletter
  - Good references to code updates
- 2020 Office hours
  - Second Thursday of the month
- JOSS=Journal for Open Source Software: code rather than paper

Open code: will one be scooped?
Negative press? Mistakes traced back..

DOI: 10.21105/joss.02807

Software

- Review
- Repository
- Archive

Editor: Arfon Smith
Reviewers:

- @zingale
- @rtfisher

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Published: 21 February 2021

The Pencil Code, a modular MPI code for partial differential equations and particles: multipurpose and multiuser-maintained

The Pencil Code Collaboration[1], Axel Brandenburg[1, 2, 3], Anders Johansen[4], Philippe A. Bourdin[5, 6], Wolfgang Dobler[7], Wladimir Lyra[8], Matthias Rheinhardt[9], Sven Bingert[10], Nils Erland L. Haugen[11, 12, 1], Antony Mee[13], Frederick Gent[9, 14], Natalia Babkovskaia[15], Chao-Chin Yang[16], Tobias Heinemann[17], Boris Dintrans[18], Dhrubaditya Mitra[1], Simon Candelaresi[19], Jörn Warnecke[20], Petri J. Käpylä[21], Andreas Schreiber[15], Piyali Chatterjee[22], Maarit J. Käpylä[9, 20], Xiang-Yu Li[1], Jonas Krüger[11, 12], Jørgen R. Aarnes[12], Graeme R. Sarson[14], Jeffrey S. Oishi[23], Jennifer Schober[24], Raphaël Plasson[25], Christer Sandin[1], Ewa Karchniwy[12, 26], Luiz Felipe S. Rodrigues[14, 27], Alexander Hubbard[28], Gustavo Guerrero[29], Andrew Snodin[14], Illa R. Losada[1], Johannes Pekkilä[9], and Chengeng Qian[30]

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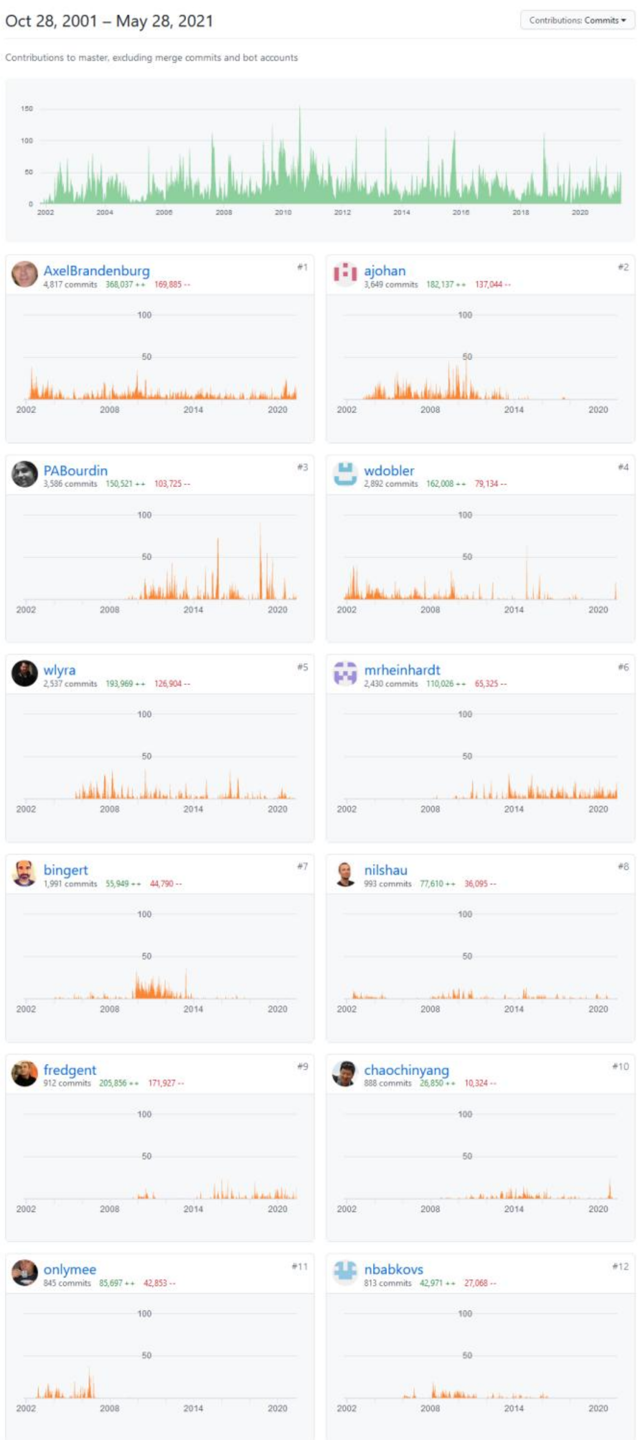
Summary

The Pencil Code is a highly modular physics-oriented simulation code that can be adapted to a wide range of applications. It is primarily designed to solve partial differential equations (PDEs) of compressible hydrodynamics and has lots of add-ons ranging from astrophysical magnetohydrodynamics (MHD) (A. Brandenburg & Dobler, 2010) to meteorological cloud microphysics (Li et al., 2017) and engineering applications in combustion (Babkovskaia et al., 2011). Nevertheless, the framework is general and can also be applied to situations not related to hydrodynamics or even PDEs, for example when just the message passing interface or input/output strategies of the code are to be used. The code can also evolve Lagrangian (inertial and noninertial) particles, their coagulation and condensation, as well as their interaction with the fluid. A related module has also been adapted to perform ray tracing

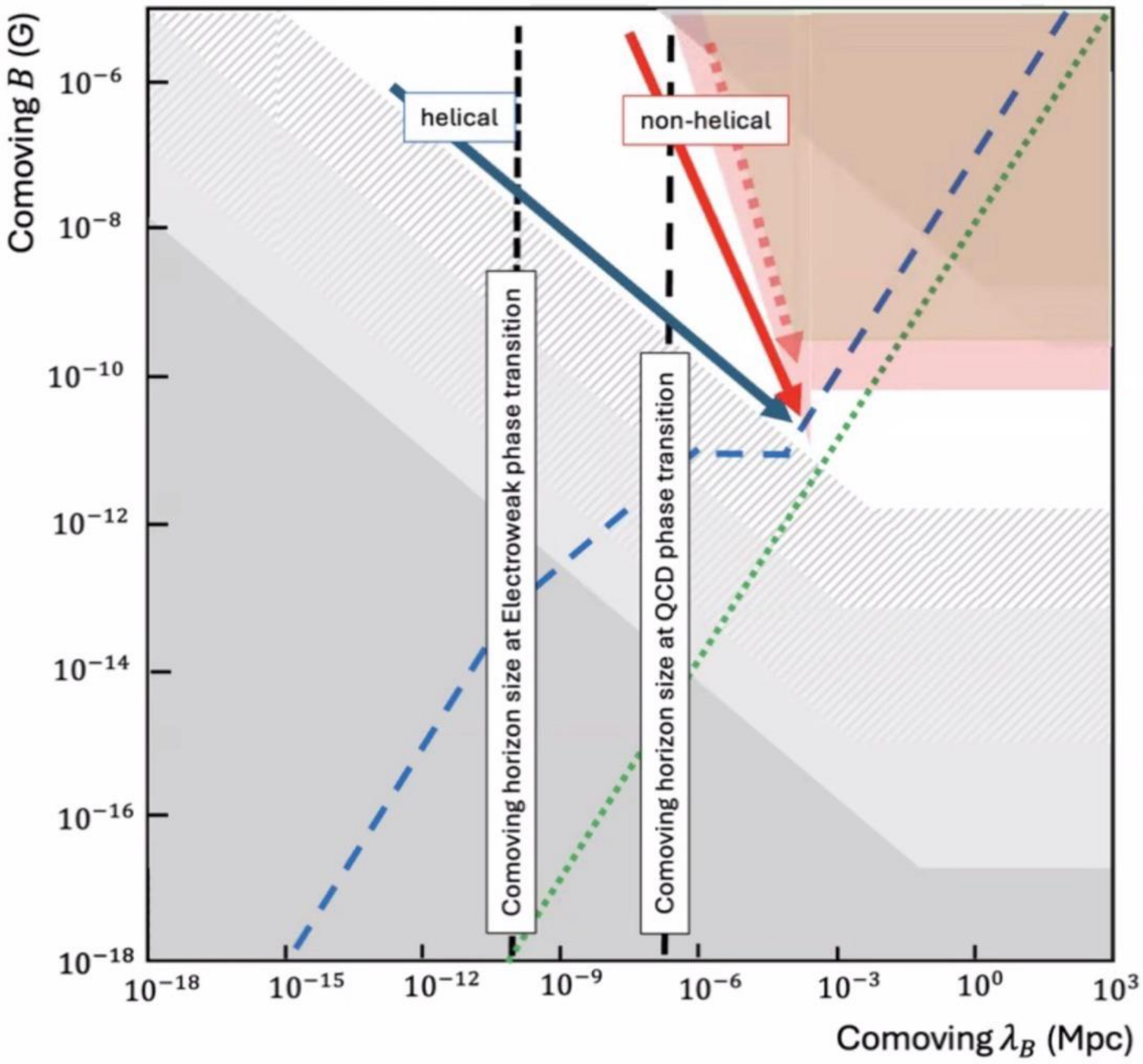
H=37 people have done > 37 commits



Further todos

- Ionization evolution during recombination
  - How important is departure from equilibrium?
  - Can we use Saha equation?
- How are the endpoints affected by this
  - Positive or negative shift?
- Clumping factor
  - Affects sound horizon
  - Hubble tension
- Including dark Matter evolution
  - Selfgravity and particles already in the Pencil Code
  - But nobody used it yet for dark matter modeling

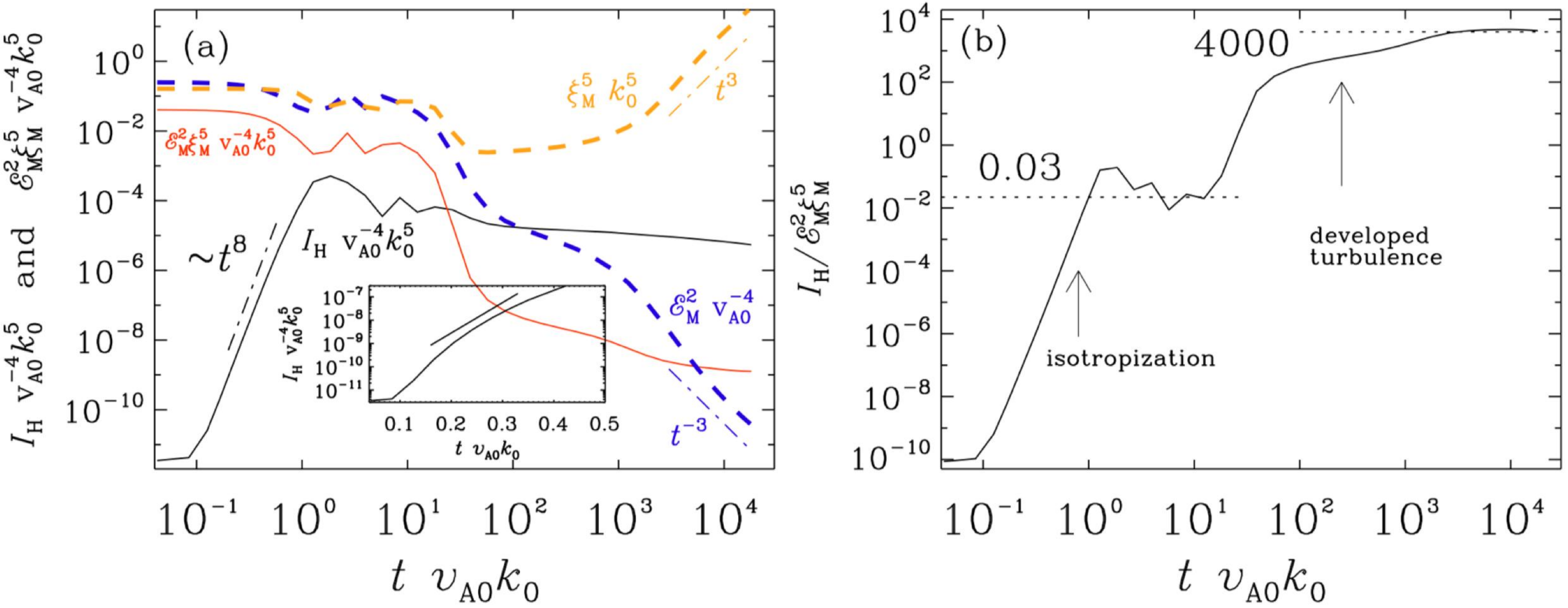
Magnetic field at the moment of generation



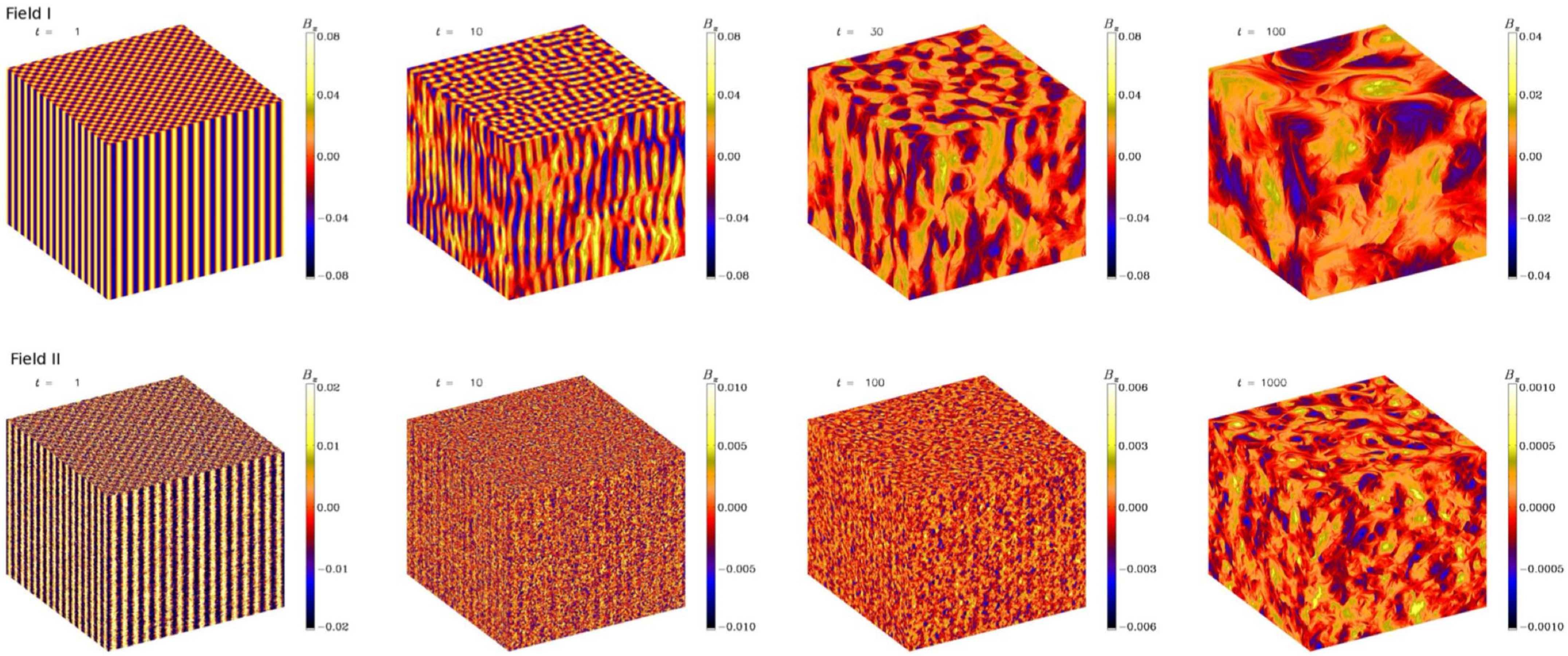
Backtracing the magnetic field trajectory we can guess from which epoch does magnetic field originate.

Example: non-helical magnetic field consistent with the CMB / Hubble tension hint has to originate from the QCD epoch.

Piecewise nonhelical initial field

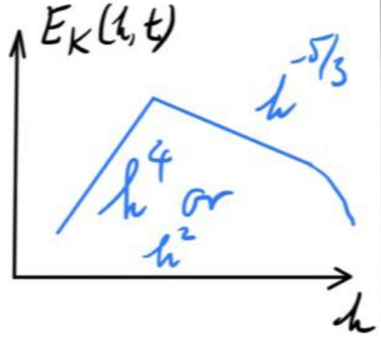


Columnar initial fields

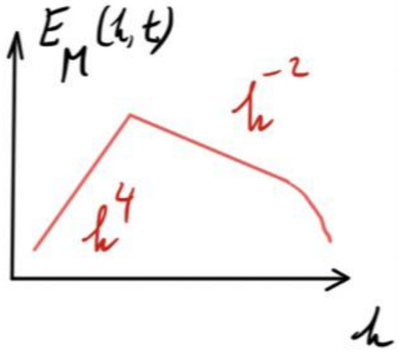


Cascades (periodic box)

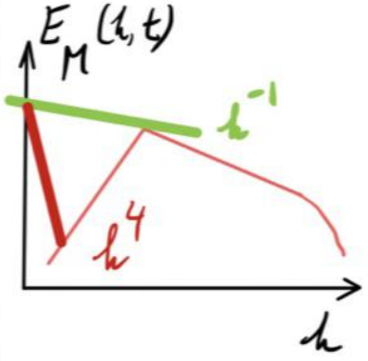
forced turbulence

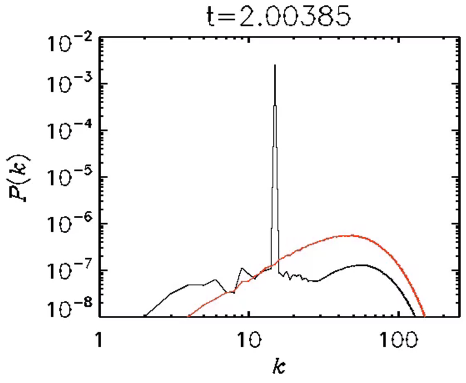


MHD nonhel

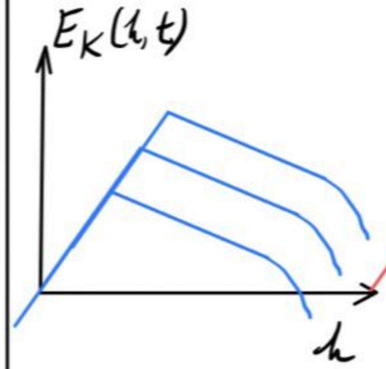


MHD hel



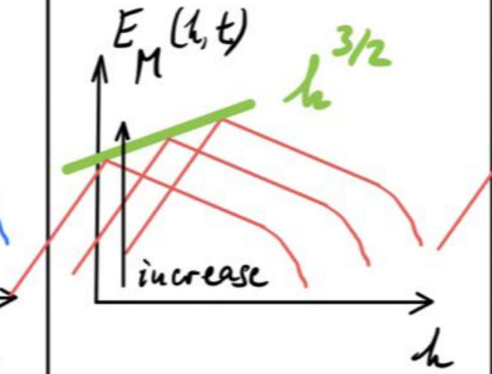


decaying turbulence



just decay
no increase at small k

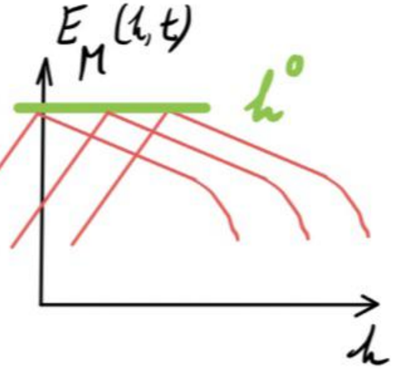
MHD nonhel



decay & growth
shift and decay

$E_M(k_1,t) \sim t^{(\alpha-\beta)q}$

MHD hel



selfsimilar shift to the left