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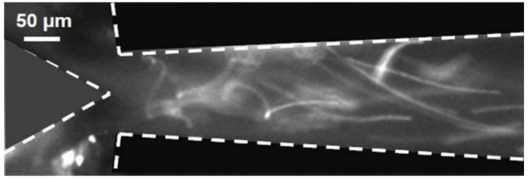
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
ICSC Centro Nazionale di Ricerca in HPC, Big Data and Quantum Computing

An application of TURBO: Scaling and Predictability in SQG Turbulence

Victor de J. Valadão
University of Turin

11th March 2025.

Brief Review on 3D turbulence



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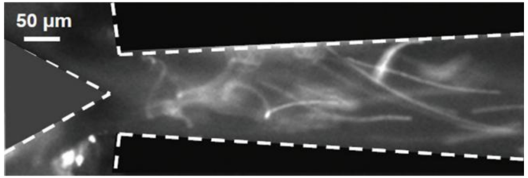

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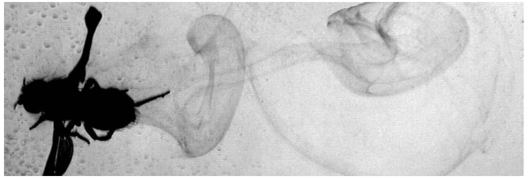

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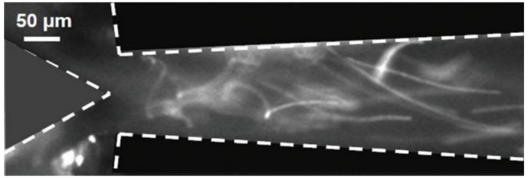

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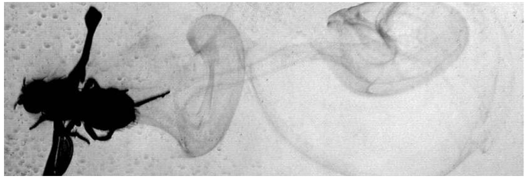

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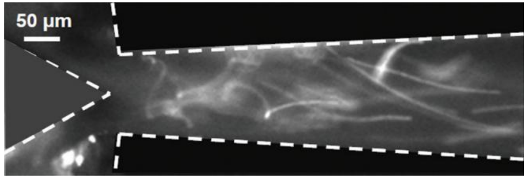

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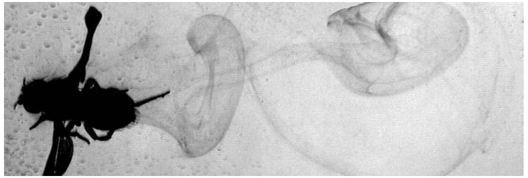

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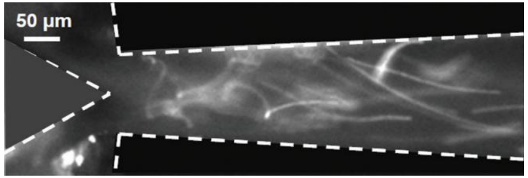

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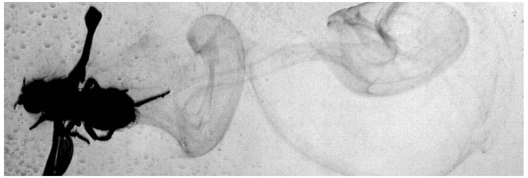

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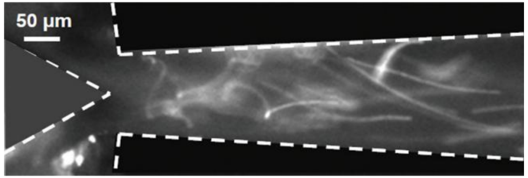

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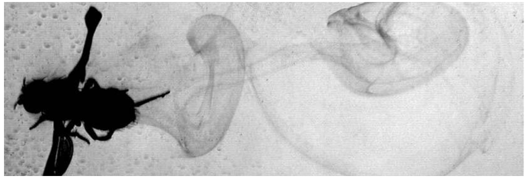

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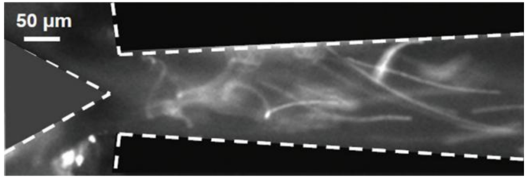

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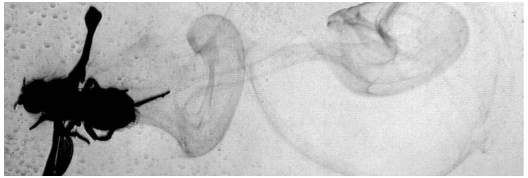

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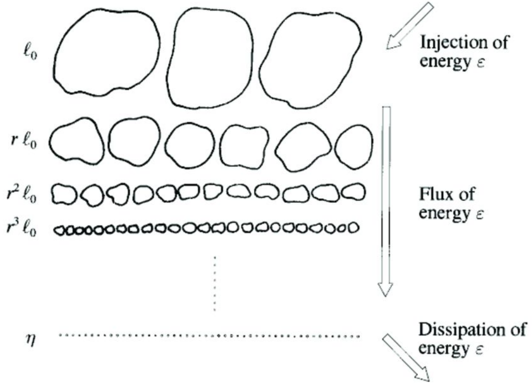

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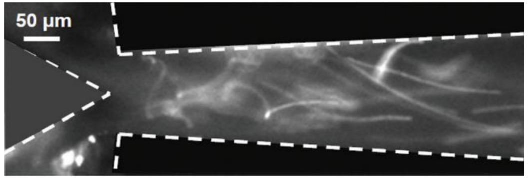

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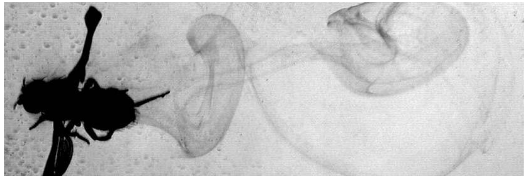

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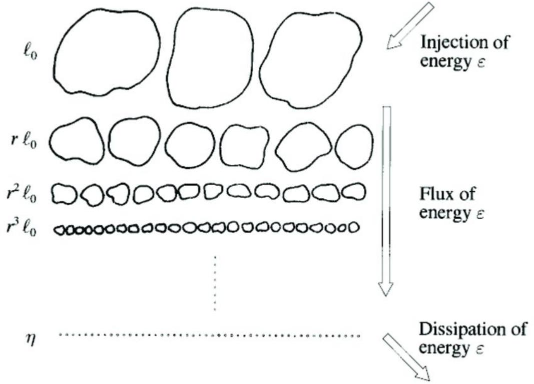

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Andrey N. Kolmogorov

$$E(k) = \frac{\langle |\vec{v}_k|^2 \rangle}{2} \propto \varepsilon^{2/3} k^{-5/3}$$

$$\ell_f^{-1} < k < \eta_\kappa^{-1}$$

$$\eta_\kappa = \left(\frac{\nu^3}{\varepsilon} \right)^{1/4}$$

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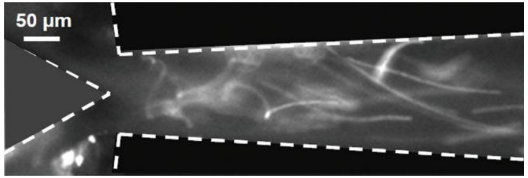

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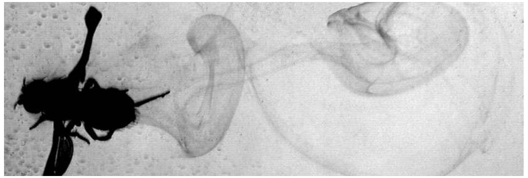

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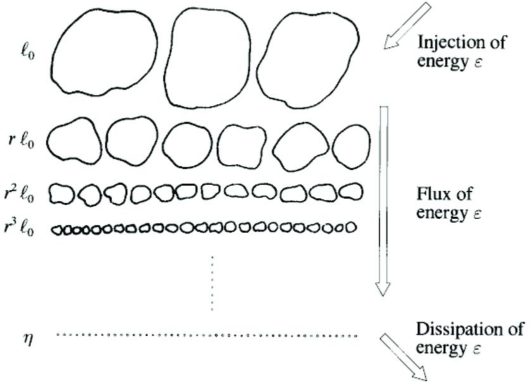

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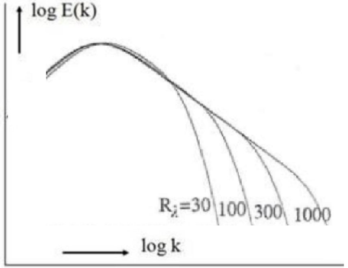

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Current Methodology and Paradigms

Pseudospectral approach

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- Solves NSE in the Fourier space;
- Linearity is easily solvable;
- Non-linear part is calculated in the physical space;
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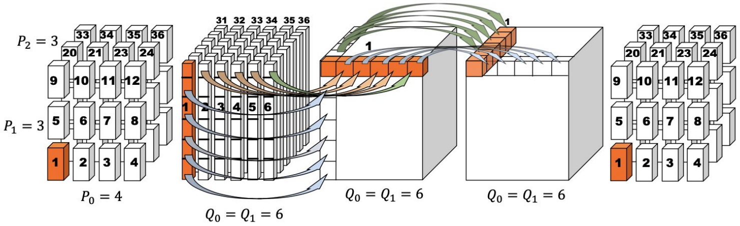

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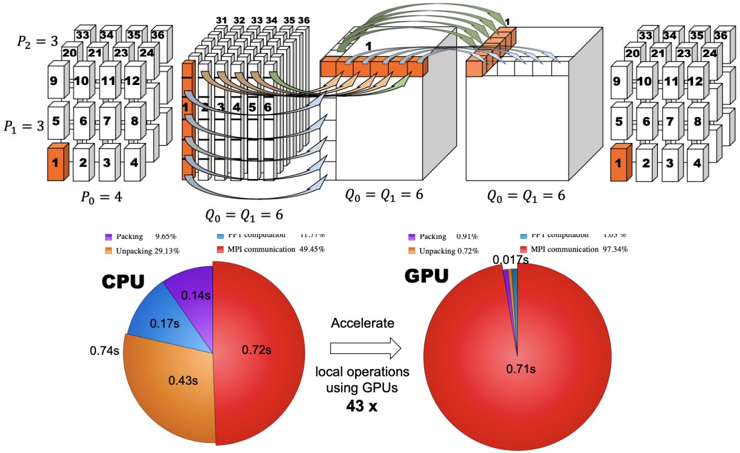

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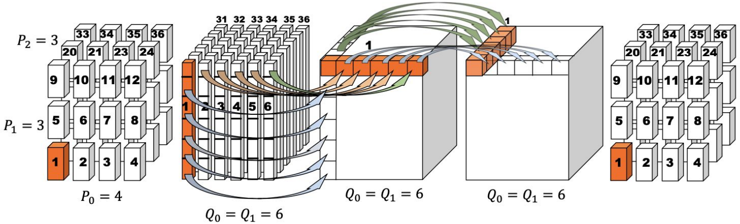

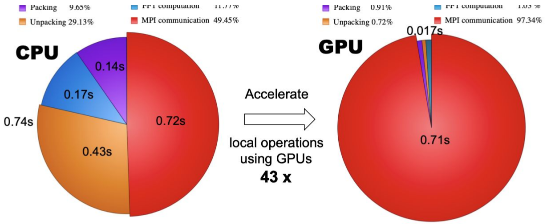

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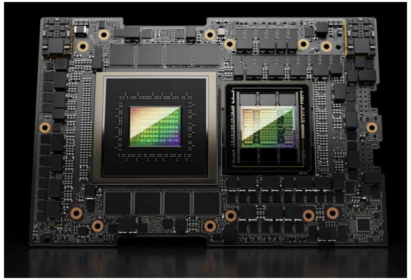

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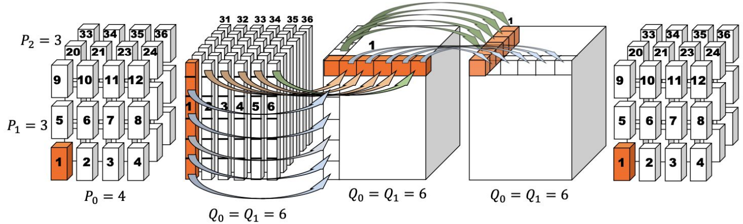

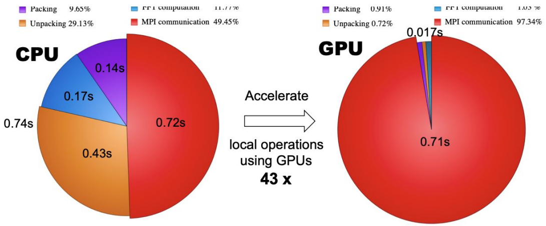

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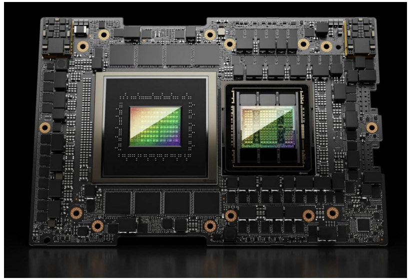

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"Small sized" problems: 2D turbulence models

$$\partial_t \omega + v_i \partial_i \omega + (-1)^n \nu \partial^{2n} \omega + (-1)^m \mu \partial^{2m} \omega = F$$

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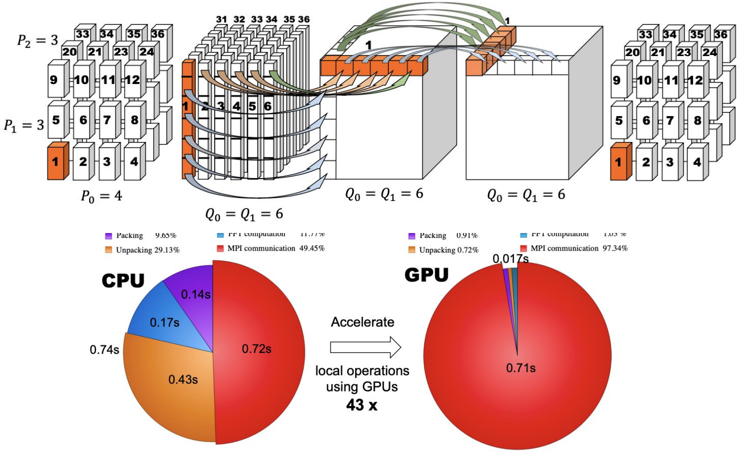

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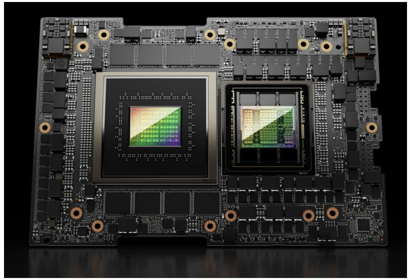

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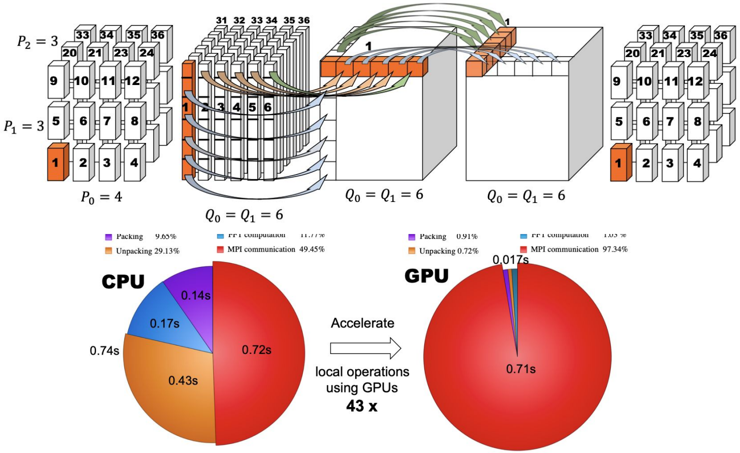

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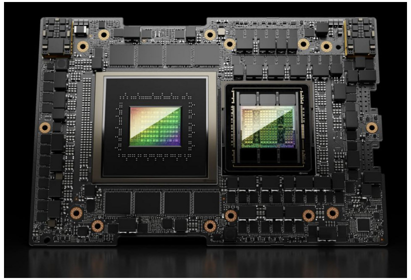

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$$v_i = \epsilon_{ij} \partial_j \psi \qquad \hat{\omega}(k,t) = k^\alpha \hat{\psi}(k,t)$$

- Same degree of physical/mathematical complexity;
- Reduces the spatial dimensions of all 2;
- Reduces dimensionality by solving scalar equations;
- Same algorithm with very little adaptations;

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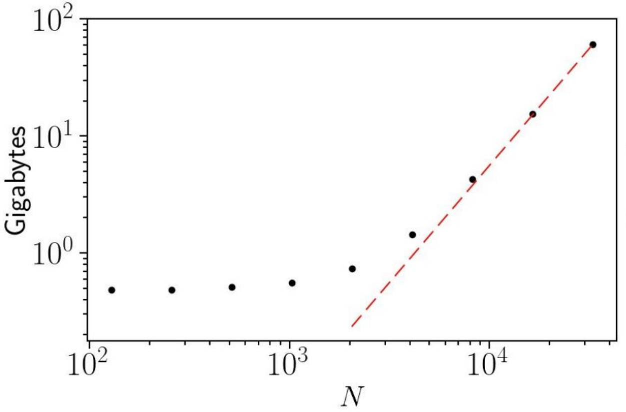
Performance tests @ Leonardo



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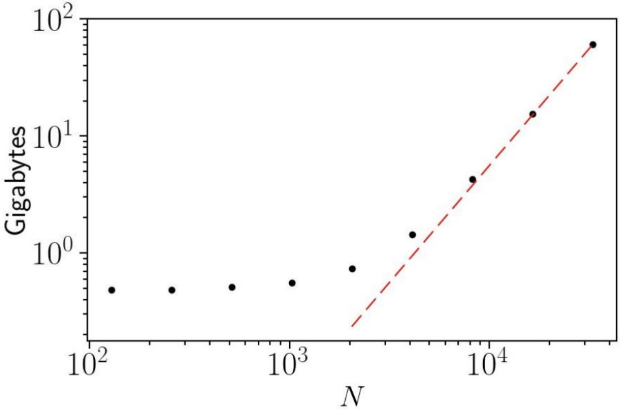

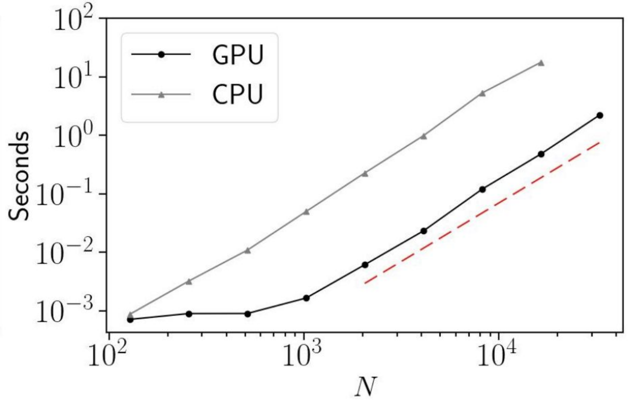

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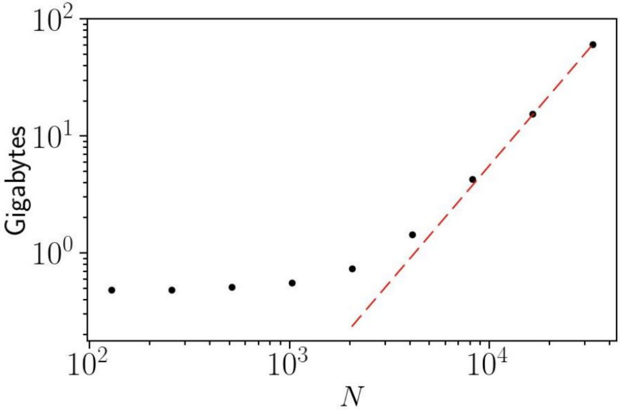

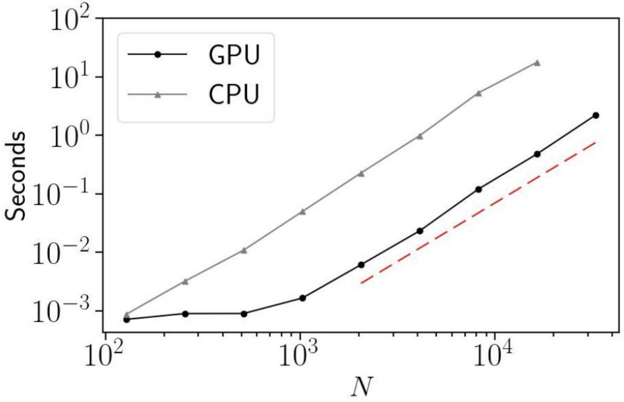

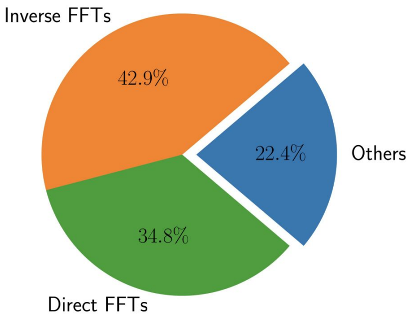

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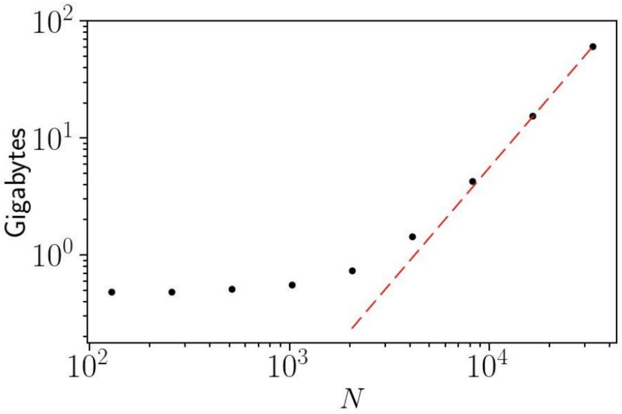
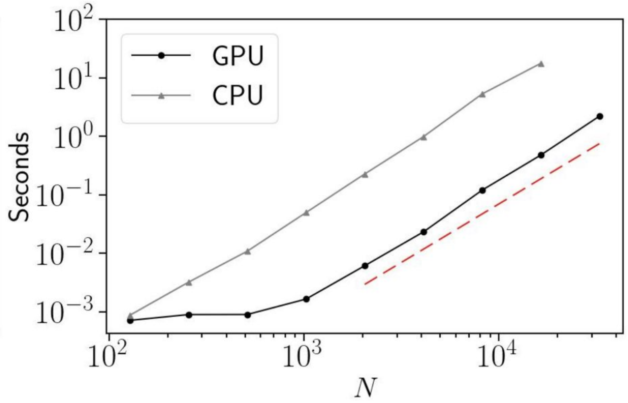
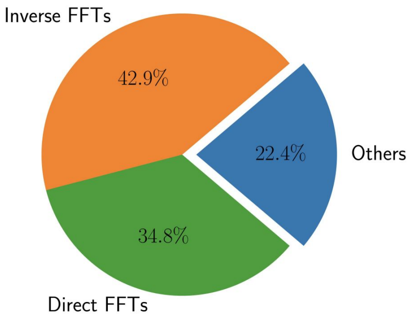
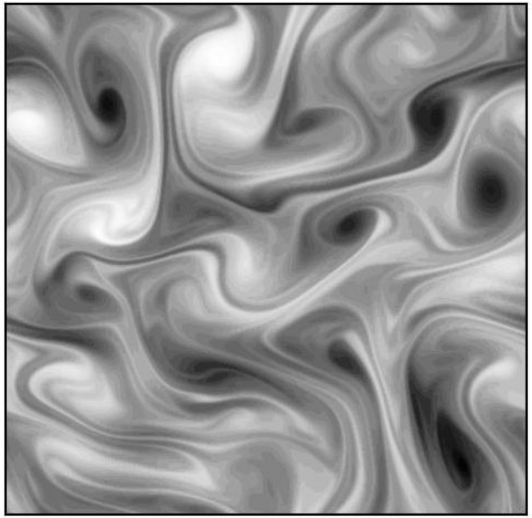




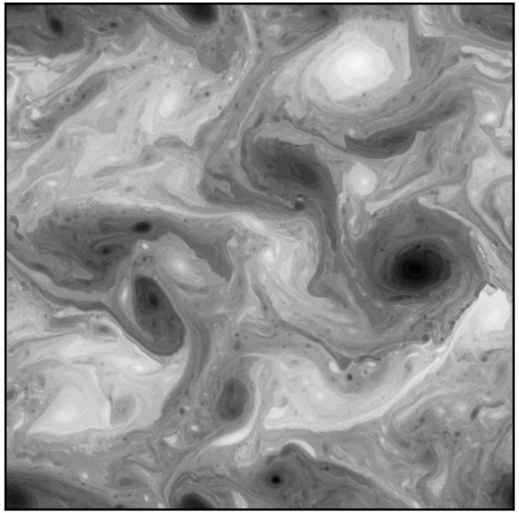


- About 40x speed up respect to 32 cores serial code;
- 24 hour use: 9.6 kWh (GPU) vs 6.0 kWh (CPU);
- It consumes 25 times more energy to run the same simulation time in the CPU;
- Budget consuming reduced by 4 times because 1 GPU = 8 CPU cores at Leonardo;

Some nice images

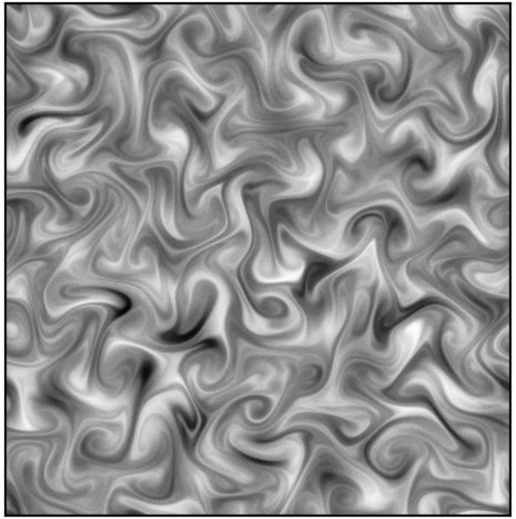


Navier-Stokes Turbulence



Surface Quasi-geostrophic Turbulence

Up to N^2=(32768)^2



Passive-scalar advection

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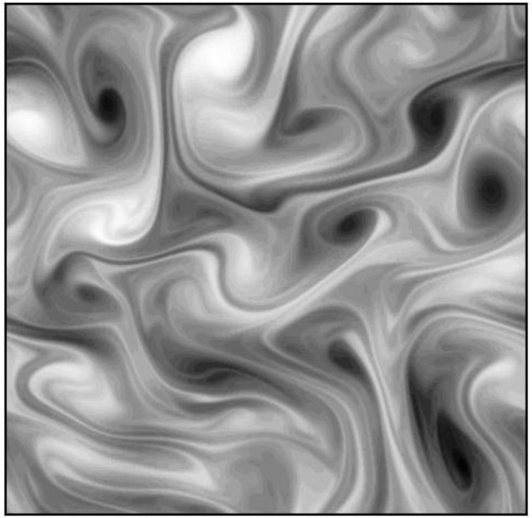

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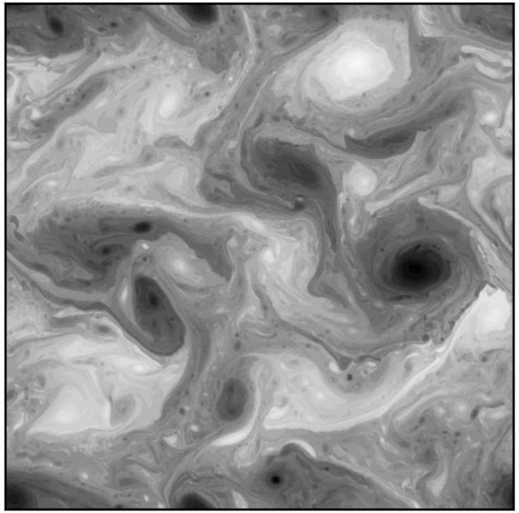

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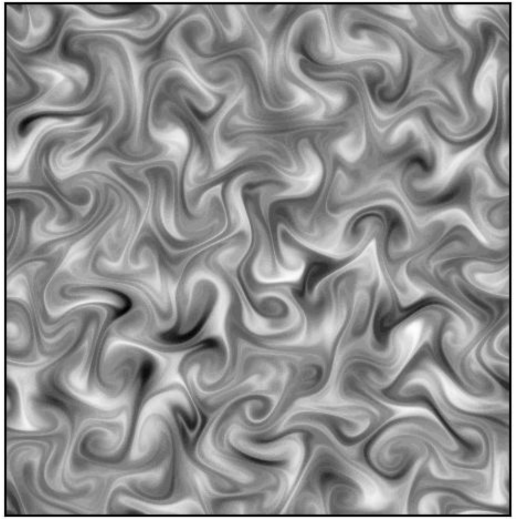

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Surface Quasi Geostrophic

QG, primitive equations and Geophysics

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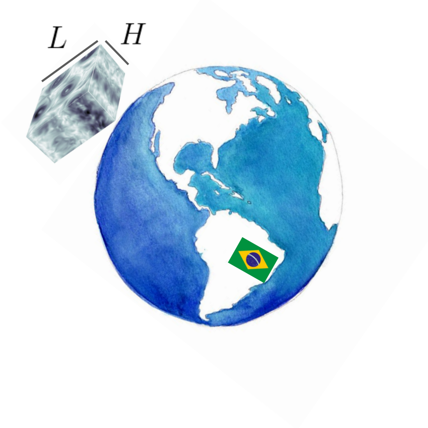

Navier-Stokes +
Rotation +
Gravity +
Shallow water +
Stable Stratification

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QG, primitive equations and Geophysics

Navier-Stokes +
Rotation +
Gravity +
Shallow water +
Stable Stratification

Suppression of vertical motion

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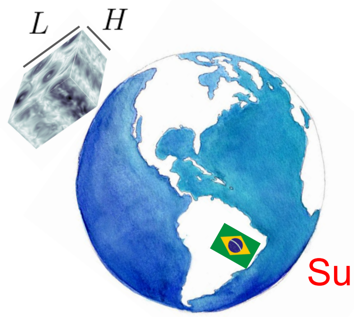
QG, primitive equations and Geophysics



Navier-Stokes +
Rotation +
Gravity +
Shallow water +
Stable Stratification

Suppression of vertical motion

$$\mathcal{O}(Ro^0, \alpha^0) \qquad Ro \equiv \frac{U}{fL} \qquad \alpha \equiv \frac{H}{L}$$

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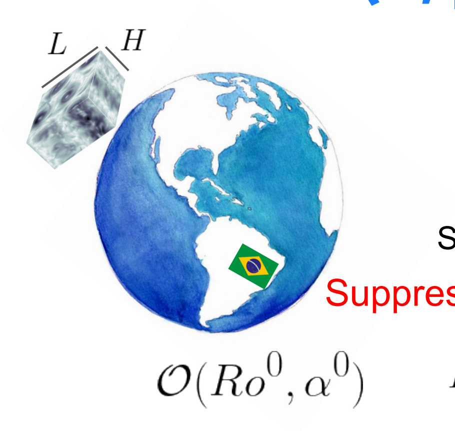

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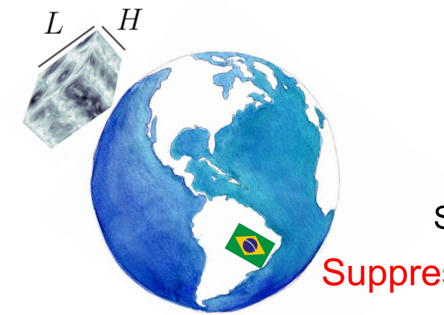

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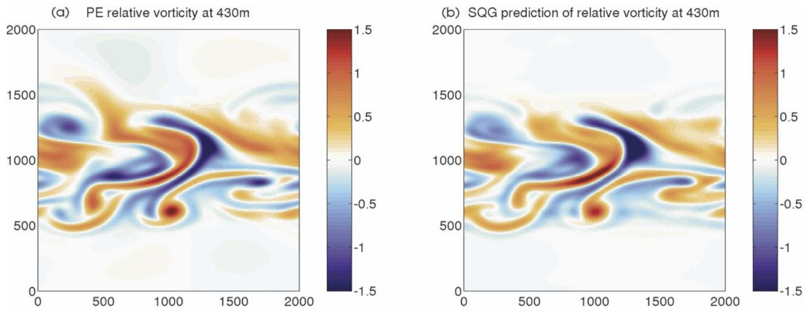

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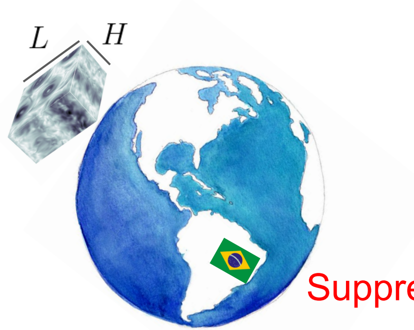

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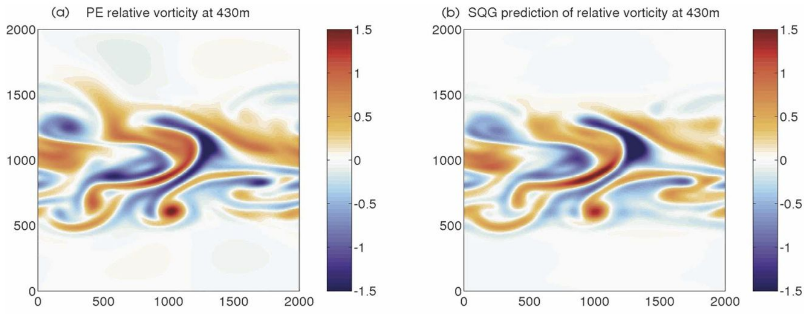

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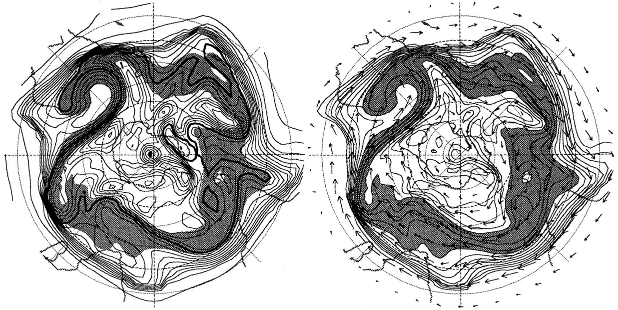

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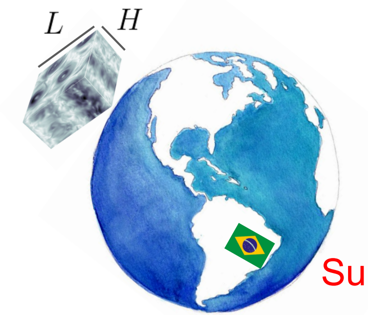

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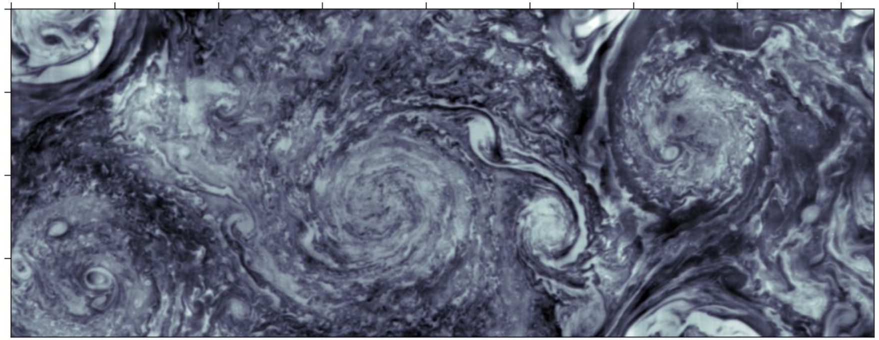

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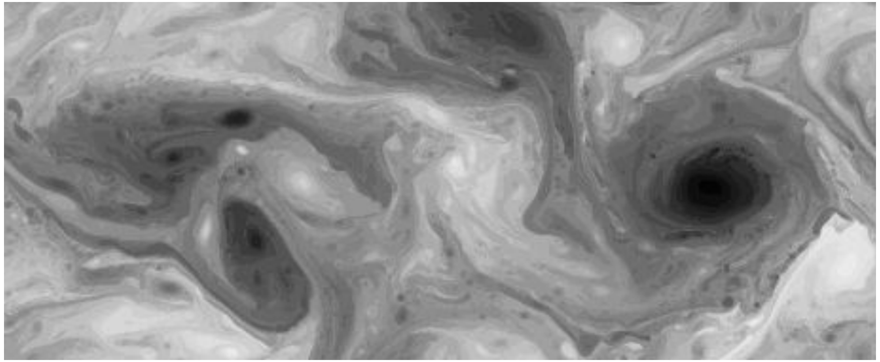

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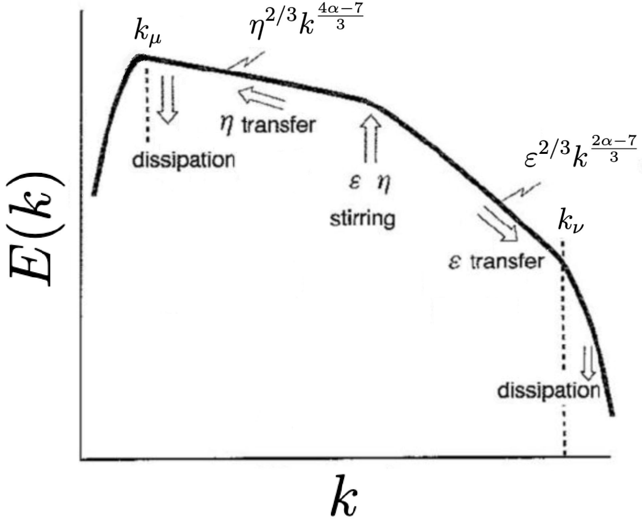

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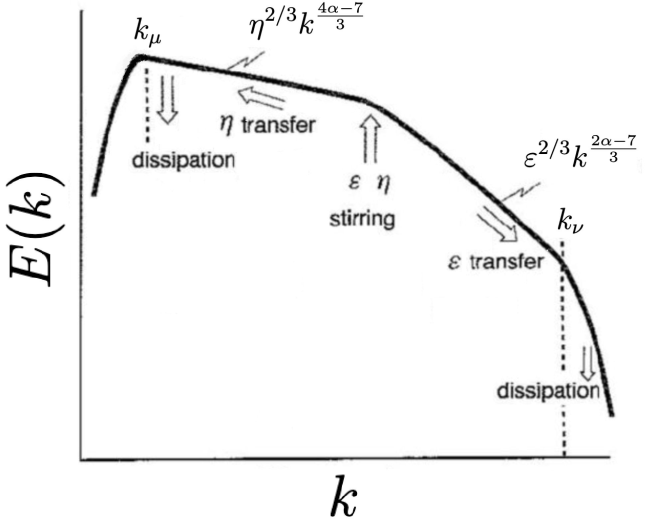

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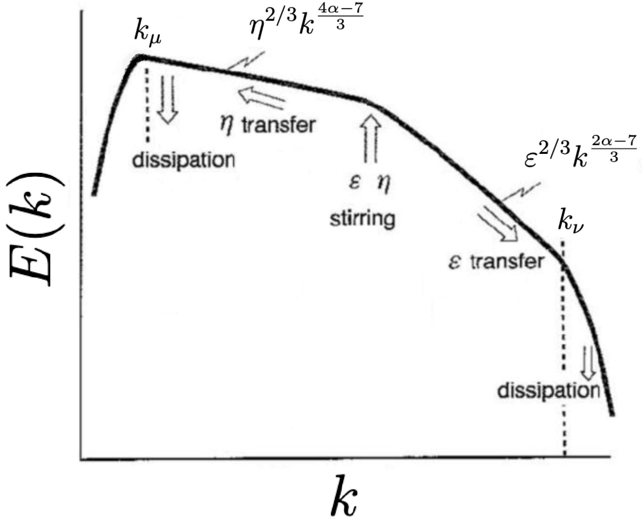

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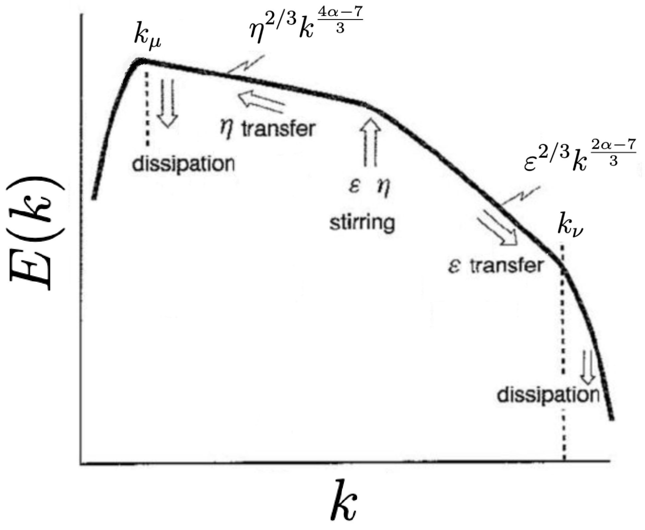

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Prototype of 3D turbulence in 2 dimensions.

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33 simulation
>300000 Cpu hours

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$$N = 1024 — 16384$$

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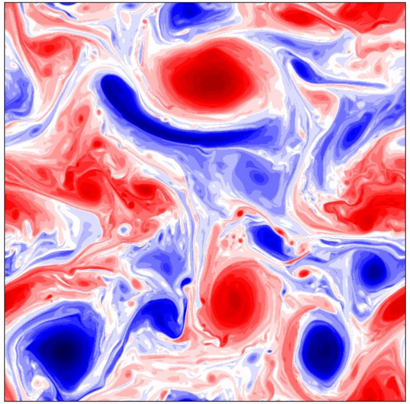

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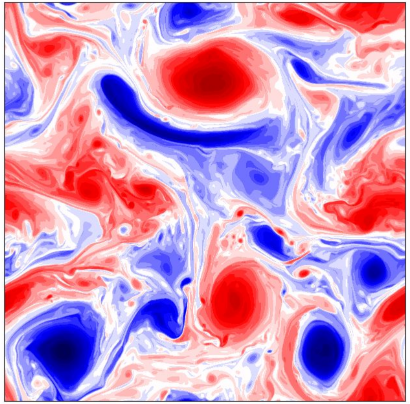

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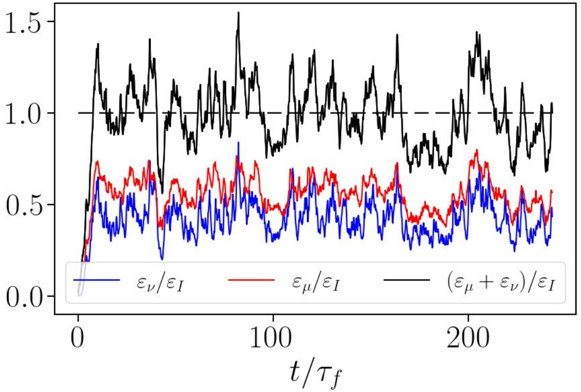

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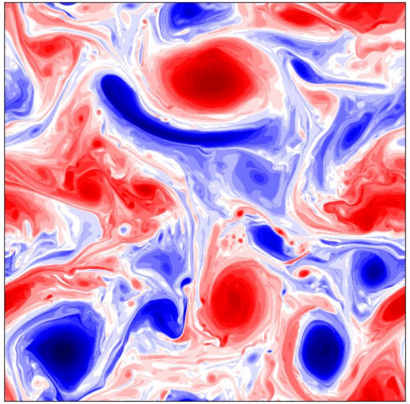

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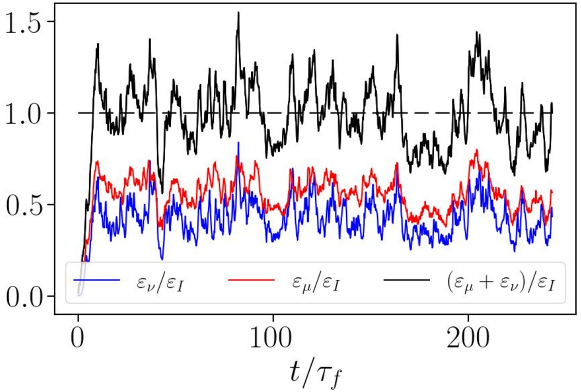

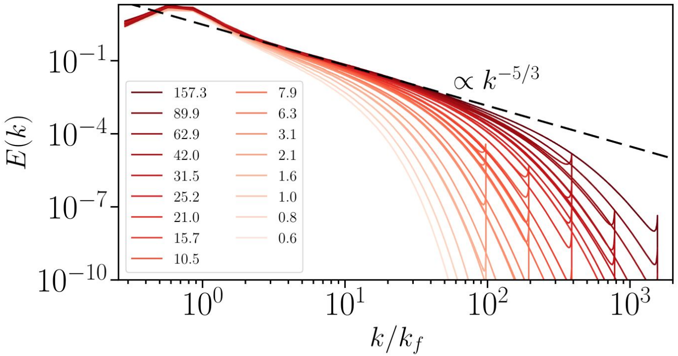

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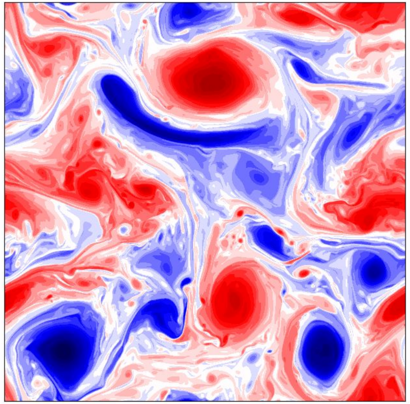

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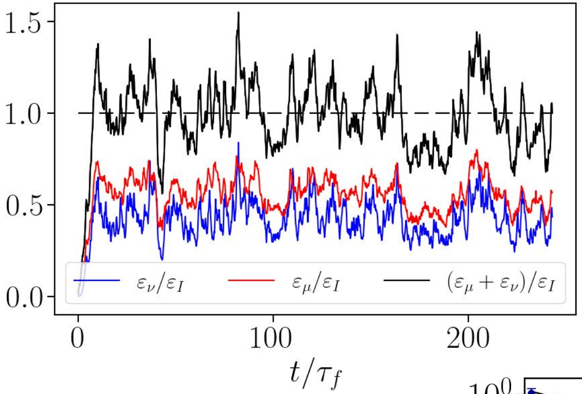

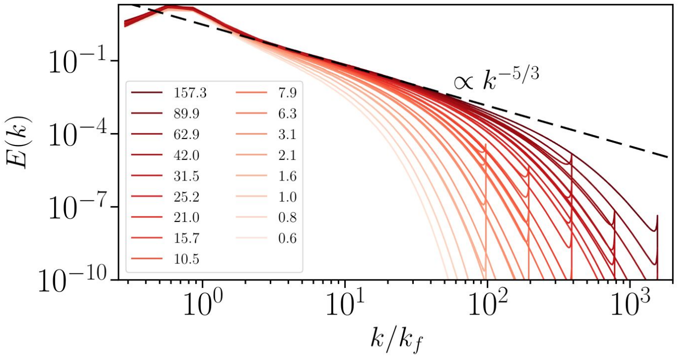

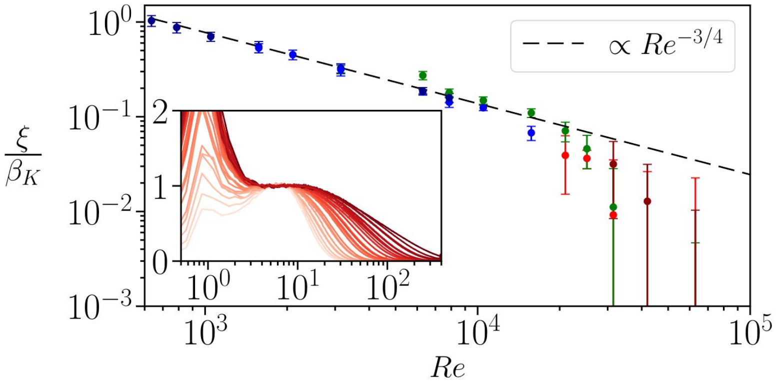

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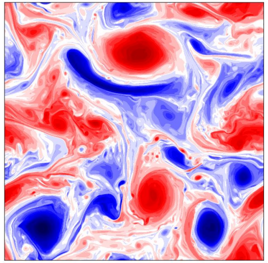

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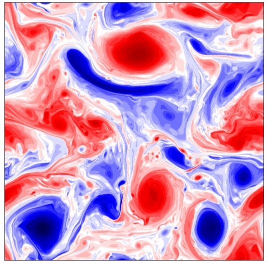

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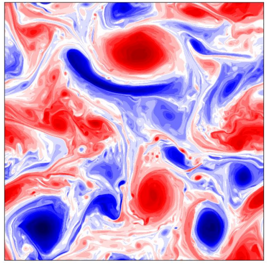

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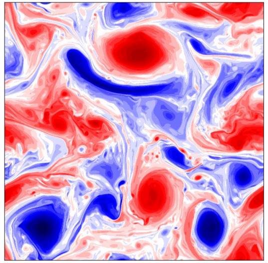

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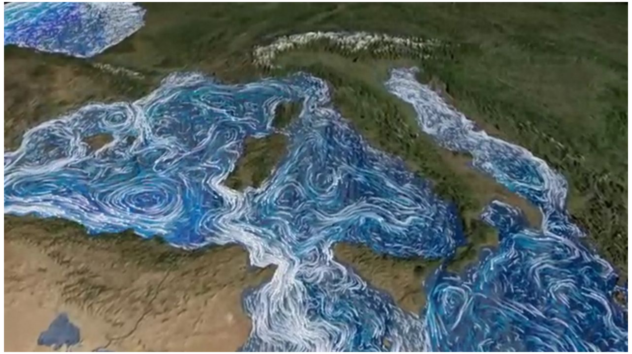

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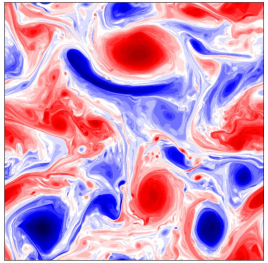

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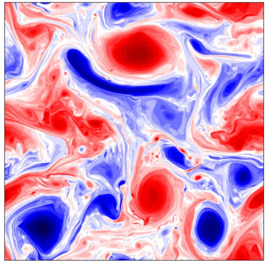

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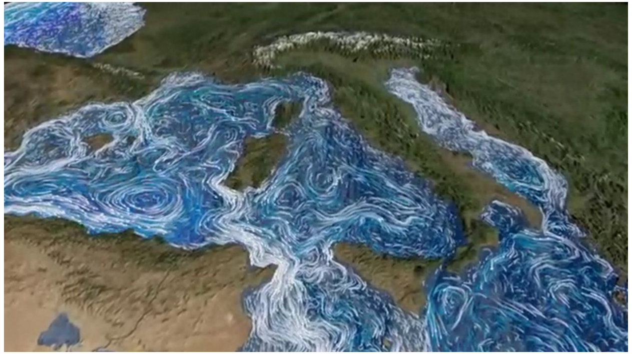

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$$\Delta\theta \sim e^{-\lambda t}$$

$$\Delta E \sim e^{-2\lambda t}$$

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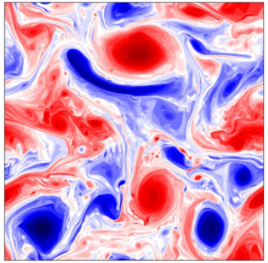

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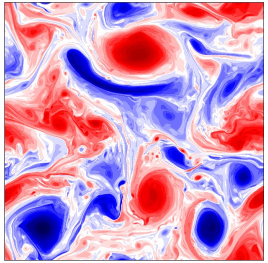

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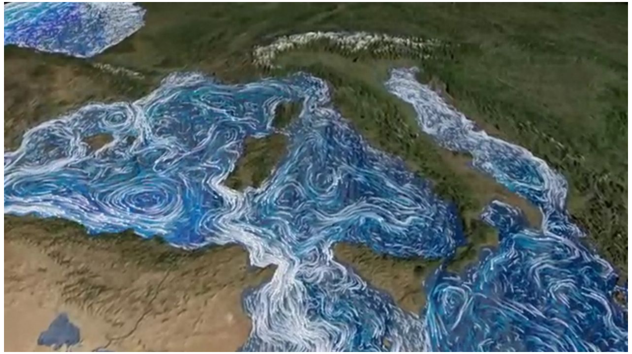

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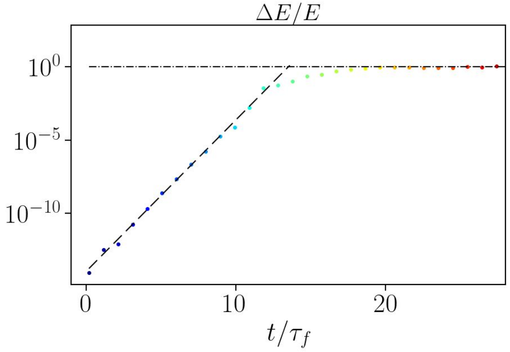
$$\Delta\theta \sim e^{-\lambda t}$$

$$\Delta E \sim e^{-2\lambda t}$$



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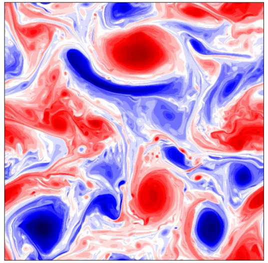
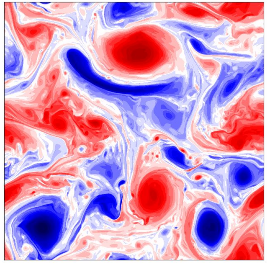
Predictability



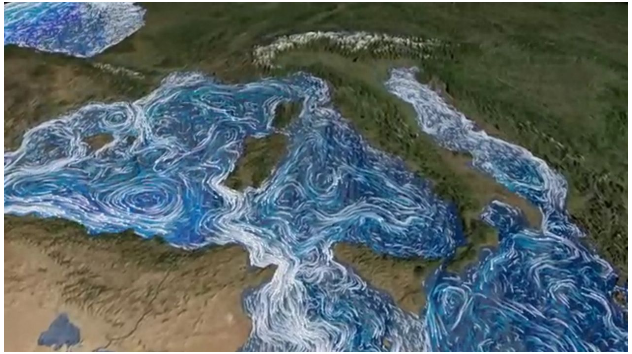
θ'

-



θ

$\sim \quad \mathcal{O}(\delta \ll \sqrt{E})$



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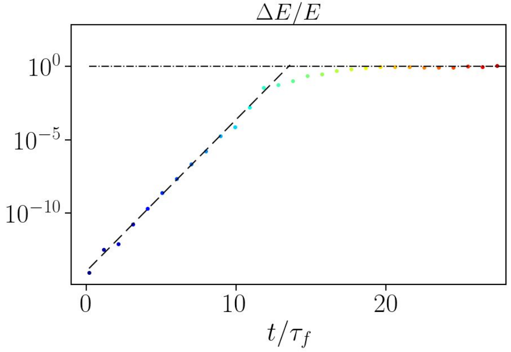

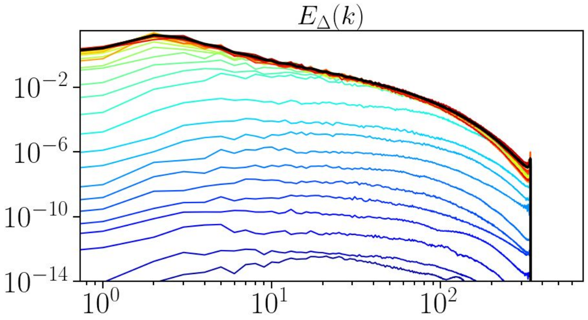

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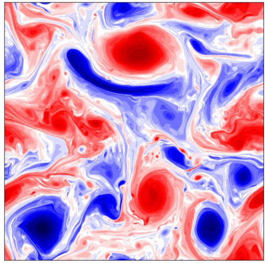
Predictability



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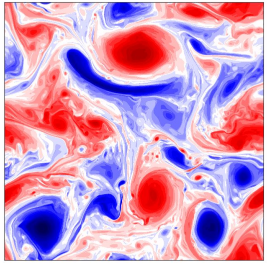
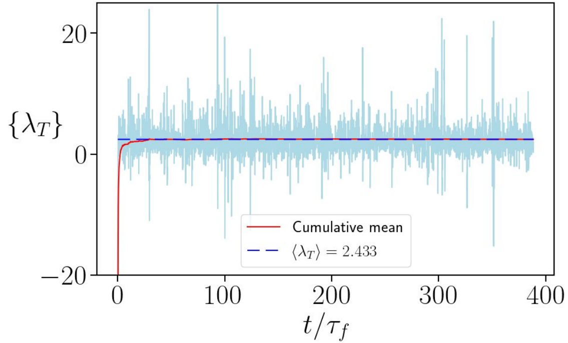
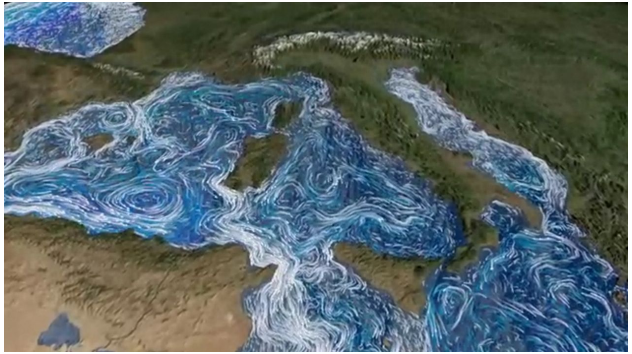


$\sim \quad \mathcal{O}(\delta \ll \sqrt{E})$

θ' $\qquad\qquad$ θ





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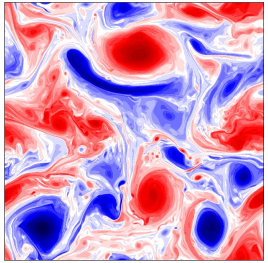
Predictability



θ'

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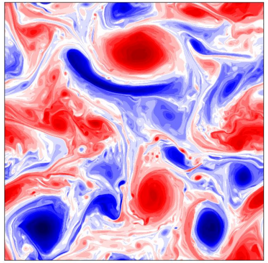
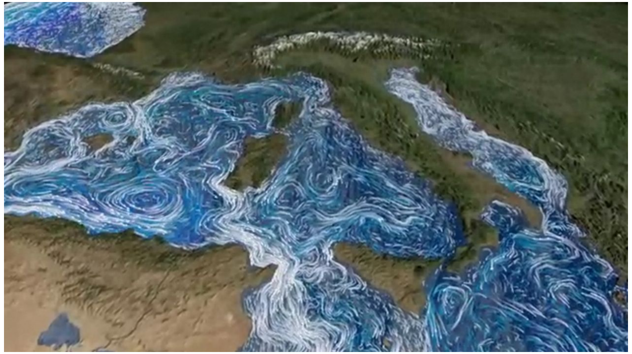


θ

$$\sim \quad \mathcal{O}(\delta \ll \sqrt{E})$$



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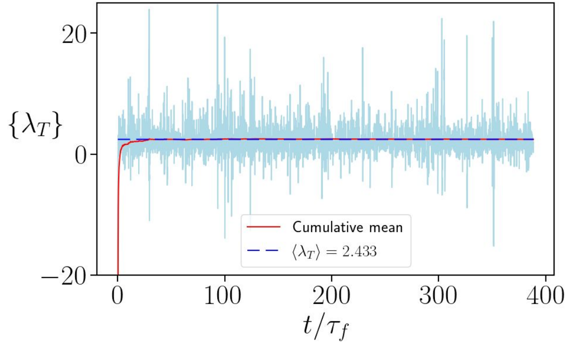
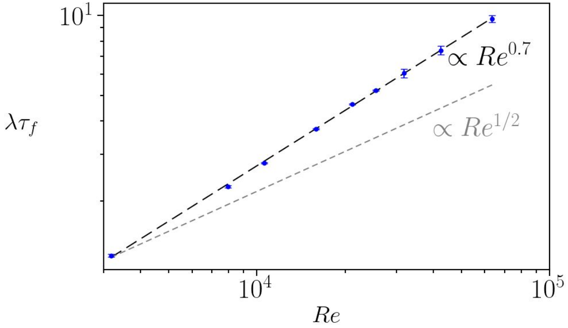


By the definition of Kolmogorov time

$$\frac{\tau_f}{\tau_\kappa} \propto Re^{1/2}$$

$$\lambda\tau_f \propto Re^{1/2}$$



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Thank you for your attention!

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