

New frontiers for the 3PCF

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in collaboration with

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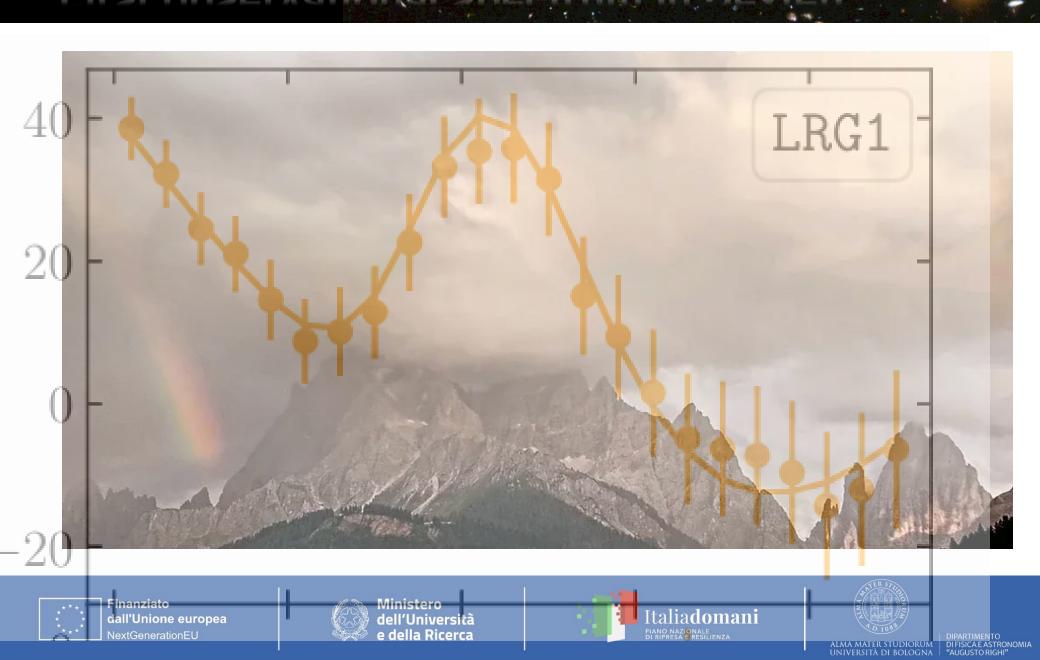




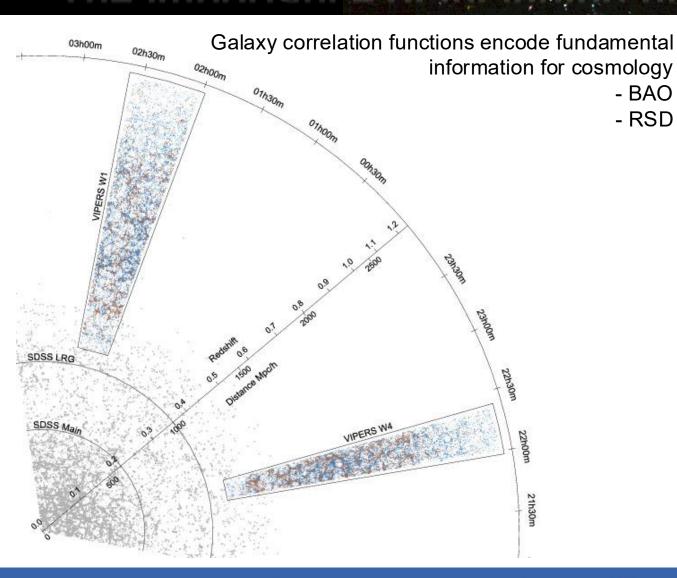


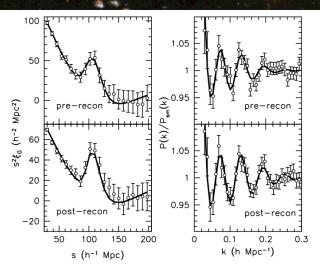


First observational spectrum in Sexten



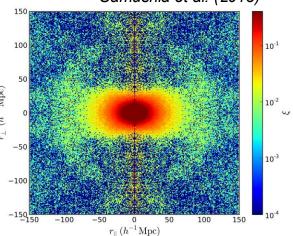
The importance of including higher-orders





Anderson et al. (2014)

Samushia et al. (2013)







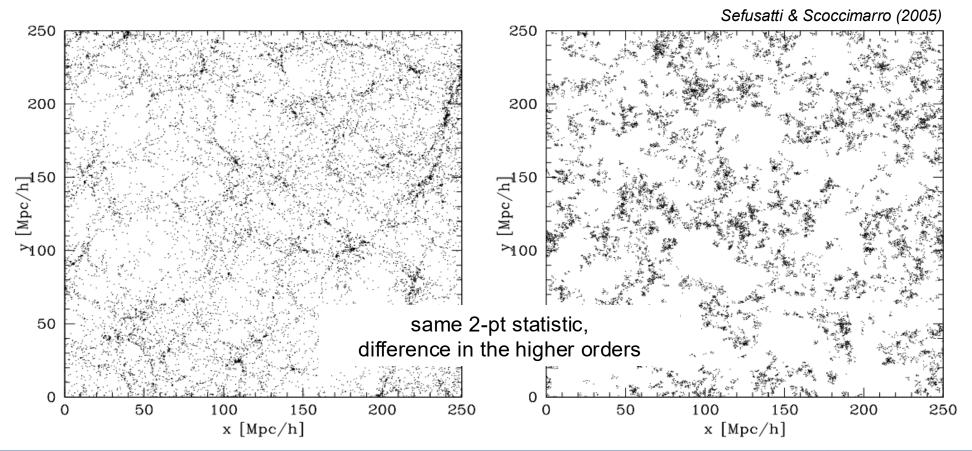




The importance of including higher-orders

For a Gaussian Random Field, 2PCF (and/or Pk) would be enough (mean and variance)

... but the Universe is just not like that!











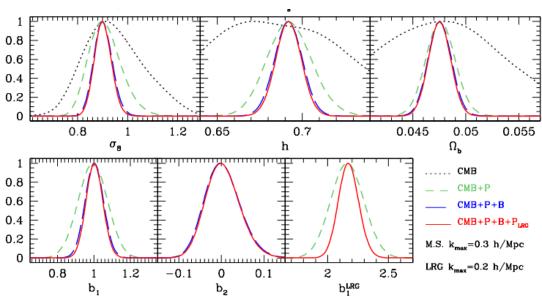
The gain and price to move to higher orders

PROs

- first significant order to detect non-Gaussian signals
- can probe both scale (as the 2PCF) and shape (unlike the 2PCF)
- in combination with the 2PCF can break the degeneracy between bias and s₈
- improve constraints on parameters in combination with other probes
- exploit additional information:
 fundamental for future surveys
 (Euclid, ...)

CONs

- scales as O(N³) (at least nominally)
- difficult to model (both theoretically and computationally)
- quite unexplored field (at least in configuration space)



Sefusatti et al. (2006)



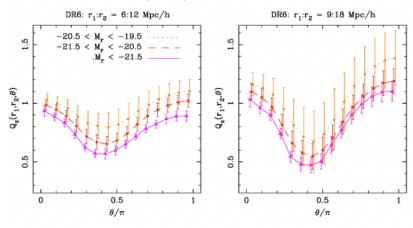




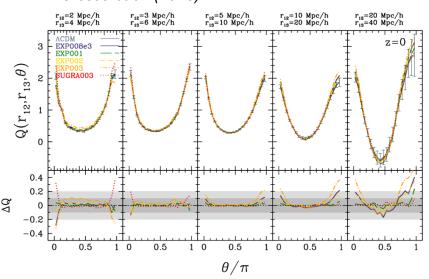


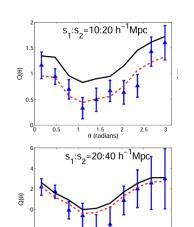
Quite novel field (in configuration space)

Mc Bride et al. (2011)



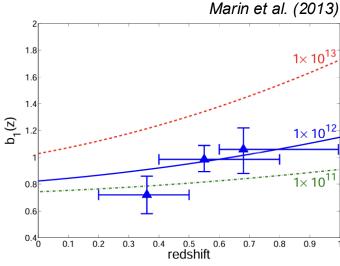
Moresco et al. (2013)

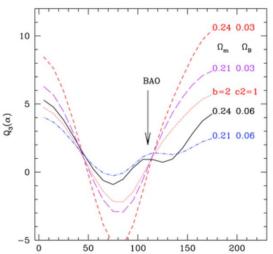




1.5

0.5





Gaztanaga et al. (2009)

Bottlenecks

- Computation time
- Modelling
- Covariance

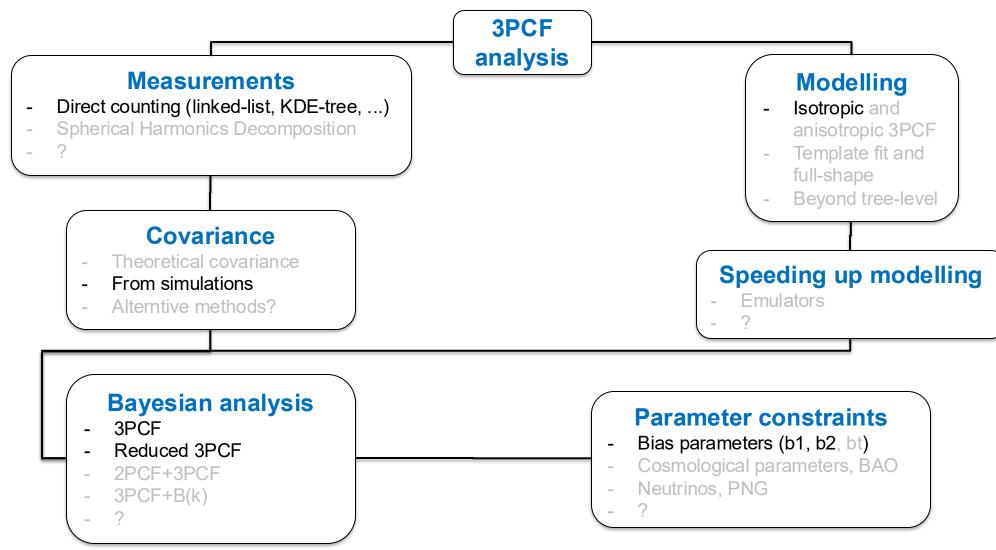








State of the art (pre-2015)



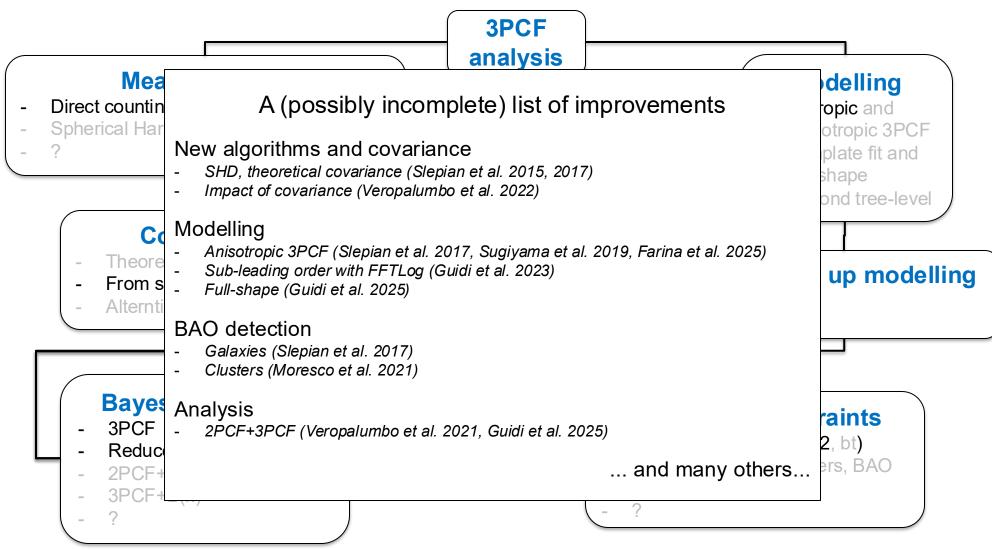








State of the art (pre-2015)



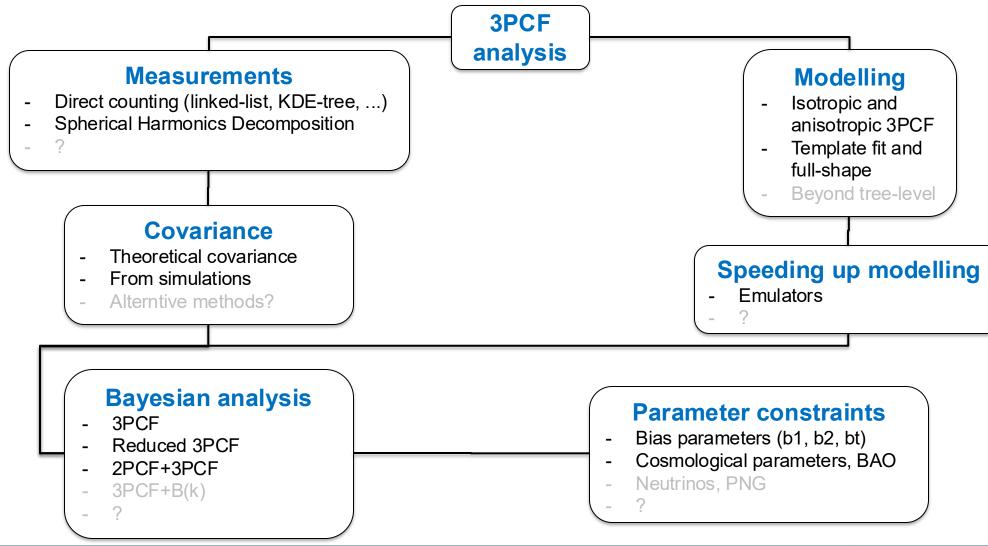








State of the art (now)











How to move forward?

- ☐ Computational improvements <u>vs</u> approximations: direct counting, spherical harmonics decomposition, what else?
- Improvements in codes? GPU?
- Modelling at the level of multiples vs resummed 3PCF vs reduced 3PCF?
- Push models to small scales?
- Models beyond tree-level: is it feasible? How much gain?
- Improving covariance?
- Configuration + Fourier space?
- Modelling 3PCF in the analysis of real data (Euclid, DESI, ...)
- ☐ 3PCF to provide constraints beyond standard (neutrinos, PNG, ...)
- □ Others?







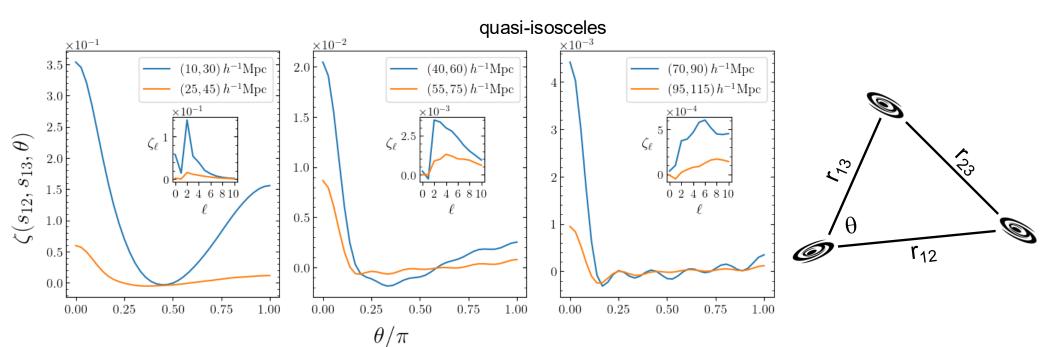


The three-point correlation function

3PCF:

$$\hat{\zeta}(r_{12}, r_{13}, \theta)$$

$$Q(r_{12}, r_{13}, r_{23}) \equiv \frac{\zeta(r_{12}, r_{13}, r_{23})}{\xi_0(r_{12})\xi_0(r_{13}) + \xi_0(r_{13})\xi_0(r_{23}) + \xi_0(r_{23})\xi_0(r_{12})}$$









The impact of redshift interlopers on the 3PCF

and how to deal with it



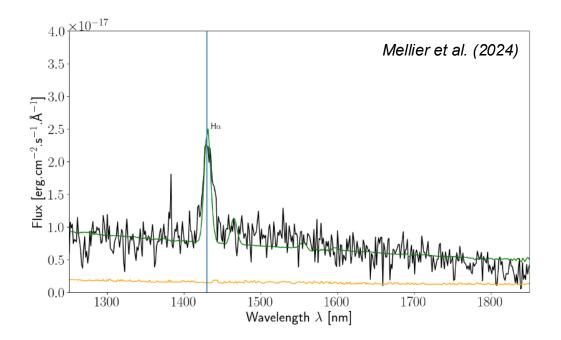




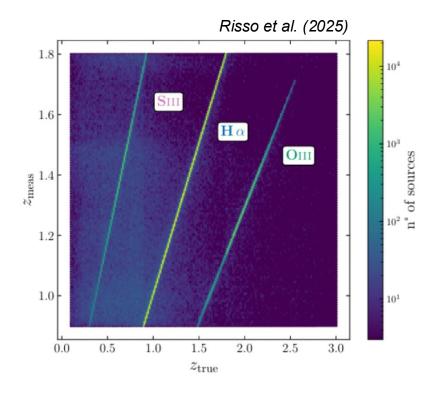


The issue of redshift interlopers

The spectroscopic part of the Euclid ESA mission will base the determination of redshifts on $H\alpha$ emitters. In many cases, spectra will show only one line. While the spectroscopic pipeline is developed to minimize as much as possibile the contamination, **line interlopers** and **noise interlopers** may be present in the sample.



Different redshift intervals, different contaminations











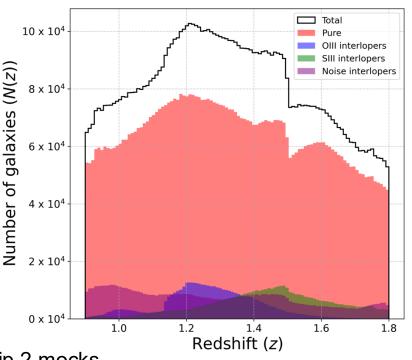
Euclid Large Mocks

To assess their impact, mocks have been created reproducing the expected behaviour of contaminants in Euclid samples (see Risso et al. 2025)

- 1000 Euclid mock samples
- Area: 2763 square degrees
 (similar to Euclid Data Release 1)
- Flux limit = 10⁻¹⁶ erg/s/cm²
 (1/2 of Euclid nominal limit)
- Redshift from the probabilistic model in Granett et al. (2025, in prep), calibrated on:
 - spectra simulated with fastSpec (Granett et al. 2025 in prep)
 - redshift derived with the official Euclid pipeline (Le Brun et al. 2025)

Risso et al. (2025)

| | $z \in [0.9, 1.1]$ | $z \in [1.1, 1.3]$ | $z \in [1.3, 1.5]$ | $z \in [1.5, 1.8]$ |
|-------|--------------------|--------------------|--------------------|--------------------|
| OIII | 0.03 | 0.12 | 0.09 | 0.01 |
| SIII | 0.01 | 0.03 | 0.08 | 0.07 |
| noise | 0.12 | 0.08 | 0.08 | 0.06 |



Analysis performed as cross-check also on Euclid Flagship 2 mocks



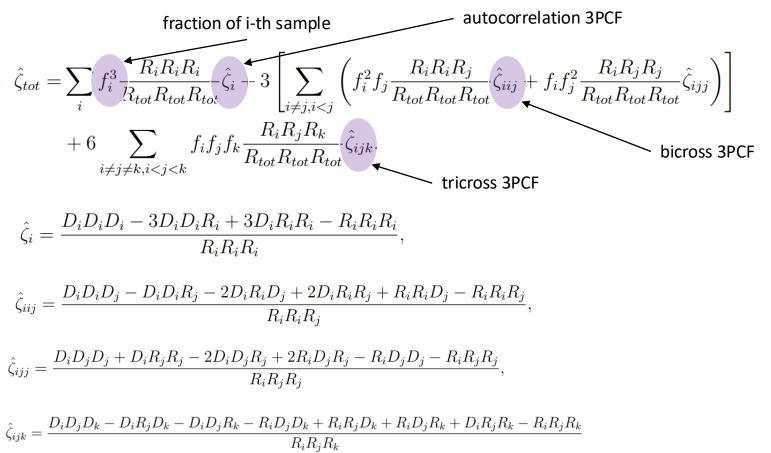






New cross 3PCF estimators

Work by Nicola Principi, with M. Moresco, F. Marulli, A. Veropalumbo



4 new classes and 19 new functions implemented in the CosmoBolognaLib suite









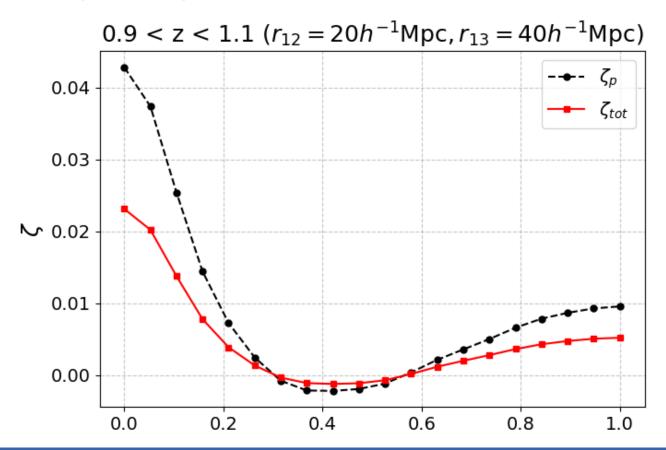


Autocorrelations and cross-correlations

Work by Nicola Principi, with M. Moresco, F. Marulli, A. Veropalumbo

Analysis on Flagship 2 catalog

Global effect: damping of the signal







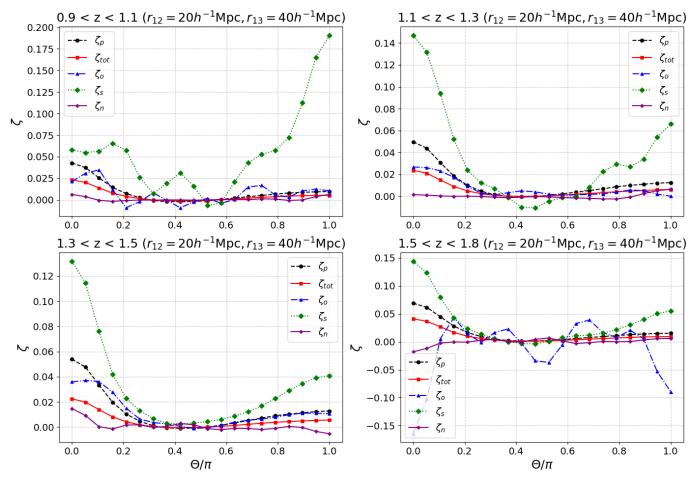




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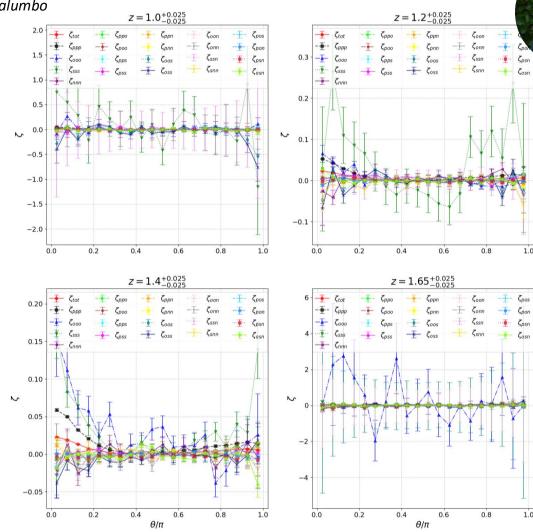




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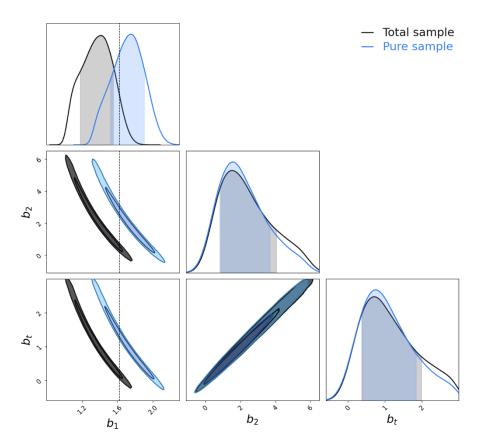


Impact of interlopers on the 3PCF

Work by Nicola Principi, with M. Moresco, F. Marulli, A. Veropalumbo

Analysis on Euclid Large Mocks

0.9 < z < 1.1



Direct analysis:

Offset in b₁, but not in b₂ and b_t

Could be due to a simple offset between contaminated and pure 3PCF









Can we retrieve the expected signal?

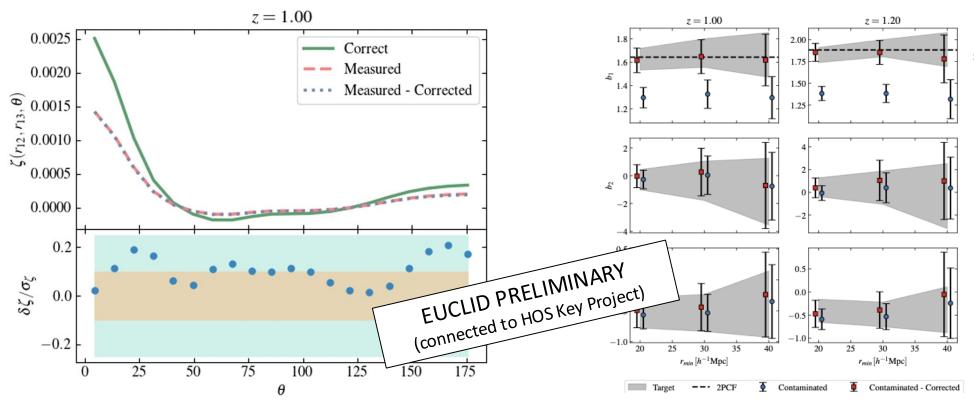
Work by A. Veropalumbo, Nicola Principi, M. Moresco, F. Marulli

Analysis on Euclid Large Mocks

$$\zeta_m = (1 - f_i) \frac{R_C R_C R_C}{R_m R_m R_m} \zeta_C + cross.corr.$$



Accounting for the leading term provide a very good approximation, with baseline contamination (f_i~20%)











3PCF as a tool to constrain neutrino masses



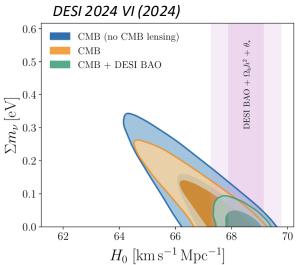


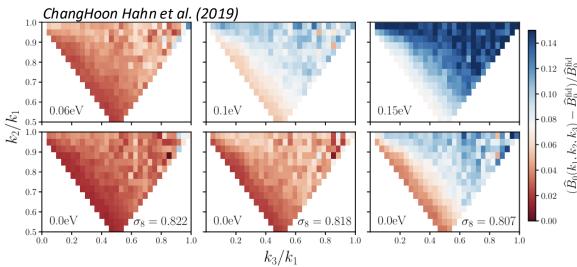


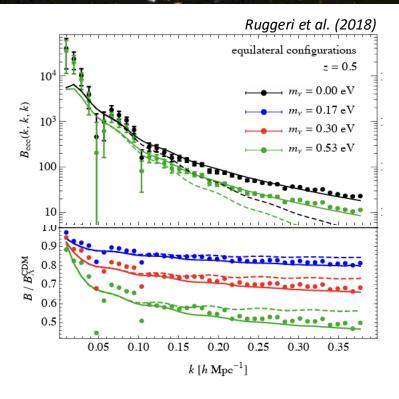


The effects of neutrinos on clustering

Not a novel topic, exploited extensively at 2-point level and in combination with other probes







More recently, expanded also at higher-order (but only for the bispetrum)

Never attempted fot the 3PCF









The Quijote simulations

Work by A. Labate, M. Moresco, M. Guidi

Data sample: Quijote simulations Villaescusa-Navarro et al., 2020

- 2000 fiducial simulations (for the covariance)
- 500 fiducial simulations (for control sample)
- 500 x 2 neutrino simulations
- 500 x 2 s8 simulations

| Name | $M_{ u} \ ({ m eV})$ | σ_8 | Realizations | Simulations | ICs | $N_c^{1/3}$ | $N_ u^{1/3}$ |
|-------------|----------------------|------------|--------------|---------------------------|------|-------------|--------------|
| fiducial | 0 | 0.834 | 2000 | $\operatorname{standard}$ | 2LPT | 512 | 0 |
| fiducial | 0 | 0.834 | 500 | paired fixed | 2LPT | 512 | 0 |
| fiducial_ZA | 0 | 0.834 | 500 | $\operatorname{standard}$ | ZA | 512 | 0 |
| Mnu_ppp | 0.4 | 0.834 | 500 | $\operatorname{standard}$ | ZA | 512 | 512 |
| Mnu_pp | 0.2 | 0.834 | 500 | standard | ZA | 512 | 512 |
| Mnu_p | 0.1 | 0.834 | 500 | standard | ZA | 512 | 512 |
| s8_p | 0 | 0.849 | 500 | paired fixed | 2LPT | 512 | 0 |
| s8_m | 0 | 0.819 | 500 | paired fixed | 2LPT | 512 | 0 |

Measurements (with MeasCorr):

- 2PCF from 1<r [Mpc/h]<150
- 3PCF from 2.5<r [Mpc/h]<147.5 up to I=10
 - Zeta multipoles
 - Zeta resummed (both single and all scales)
 - Reduced 3PCF (both single and all scales)

$$\hat{\zeta}(r_{12}, r_{13}, \theta) = \sum_{\ell=0}^{\ell_{\text{max}}} \bar{\hat{\zeta}}_{\ell}(r_{12}, r_{13}) P_{\ell}(\cos \theta)$$

$$Q(r_{12}, r_{13}, r_{23}) \equiv \frac{\zeta(r_{12}, r_{13}, r_{23})}{\xi_0(r_{12})\xi_0(r_{13}) + \xi_0(r_{13})\xi_0(r_{23}) + \xi_0(r_{23})\xi_0(r_{12})}$$

independent of s8









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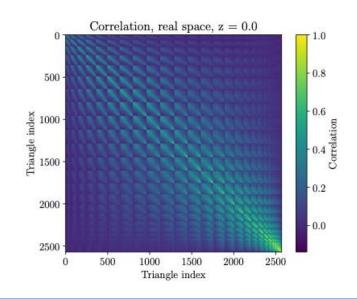
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Covariance estimated for all these datasets (rescaled for a volume of 10 h⁻³ Gpc³)











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Different estimators for the neutrino detectability

$$DET_i \equiv \frac{\hat{f}_i^{(\nu)} - \hat{f}_i^{(fiducial)}}{\sqrt{2} \, \sigma_{f,i}},$$

$$\chi^{2}(r_{12}, r_{13}) \equiv \sum_{i,j=1}^{N_{\theta}} \left(\hat{f}^{(\nu)} - \hat{f}^{(\text{fiducial})} \right) (r_{12}, r_{13}, \theta_{i}) \times \left[\hat{C}^{(\text{single-sc.})} \right]_{ij}^{-1} \times \left(\hat{f}^{(\nu)} - \hat{f}^{(\text{fiducial})} \right) (r_{12}, r_{13}, \theta_{j}),$$

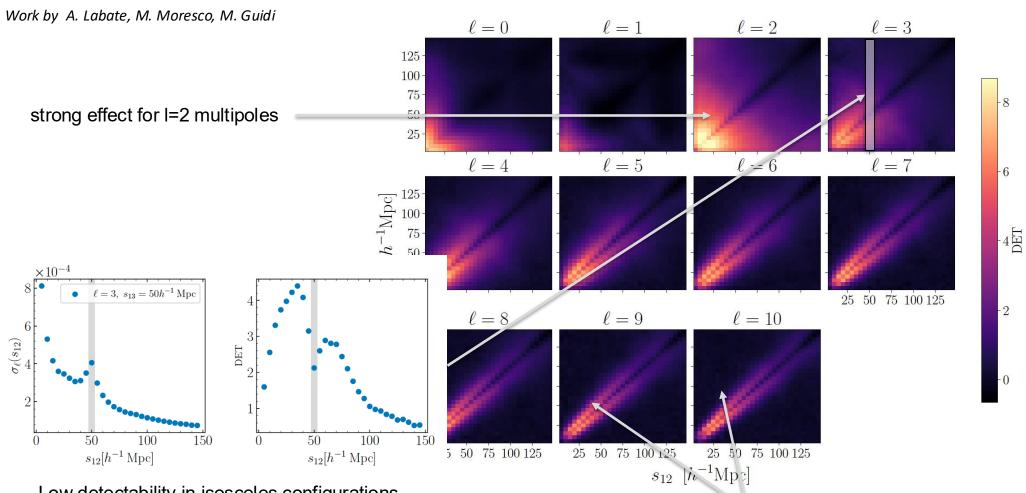








Effect of neutrinos on 3PCF multiples



Low detectability in isosceles configurations due to increased error

Many multipoles needed to reconstruct quasi-isosceles configurations



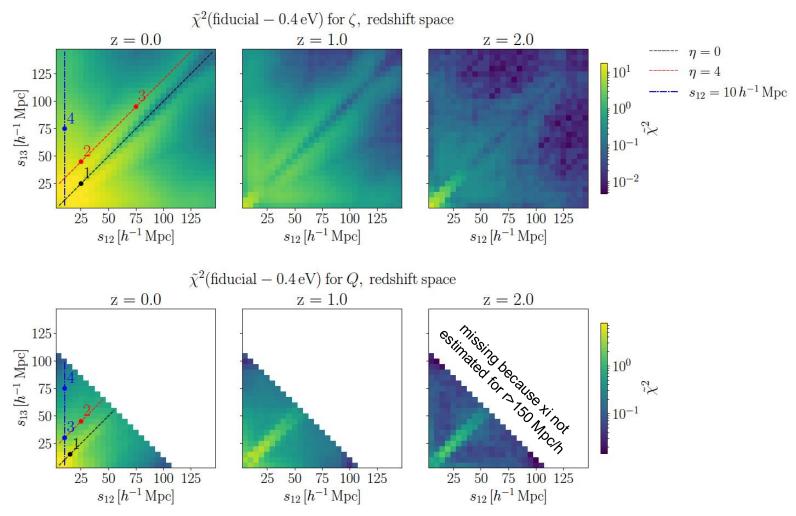






Effect of neutrinos on 3PCF

Work by A. Labate, M. Moresco, M. Guidi







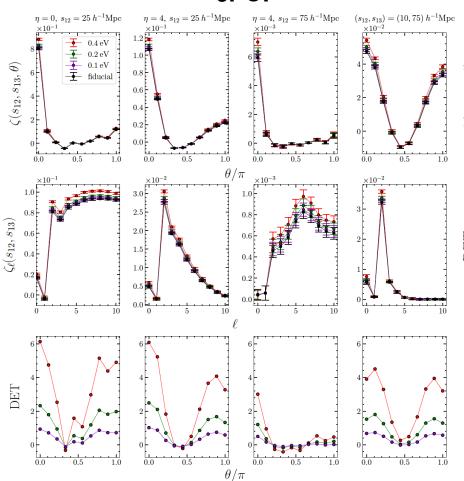




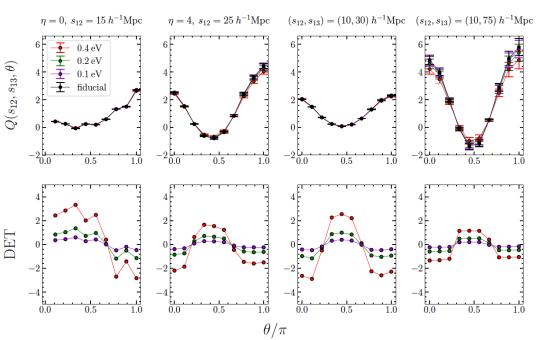
Effect of neutrinos on 3PCF

Work by A. Labate, M. Moresco, M. Guidi





reduced 3PCF



- larger ls are needed to reconstruct quasiisosceles configurations
- larger effect of neutrinos at smaller scales
- more significant impact for elongated configurations
- stronger effect on zeta

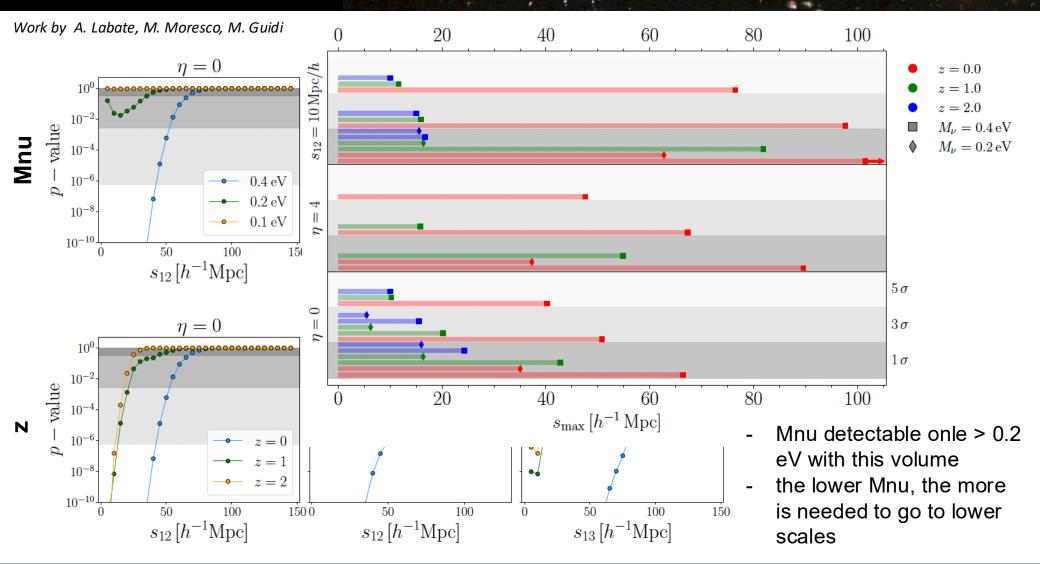








Quantifying the detectability





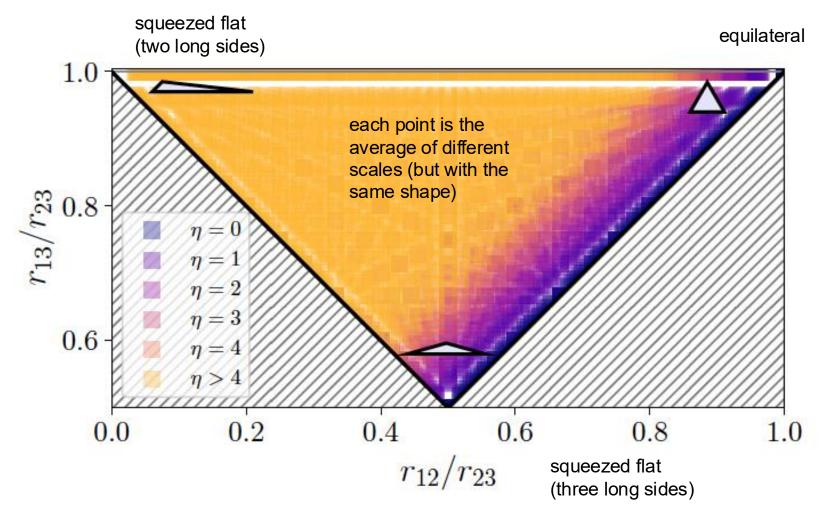






Which kind of configurations are better?

Work by A. Labate, M. Moresco, M. Guidi





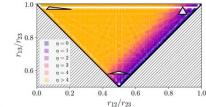


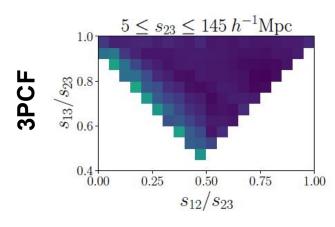


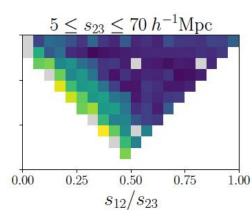


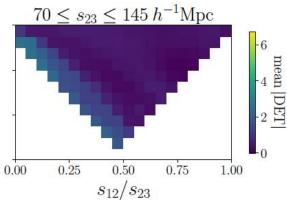
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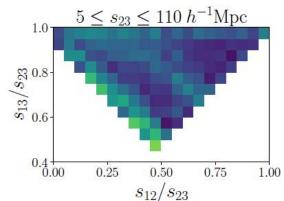


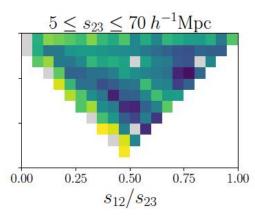


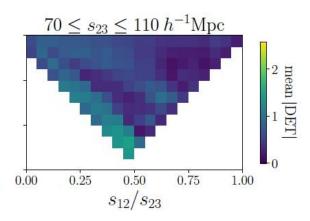
















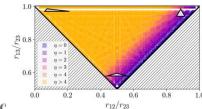




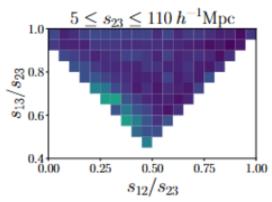
Are there degeneracies?

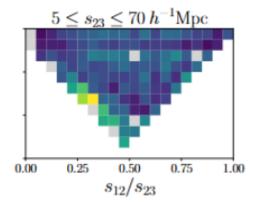
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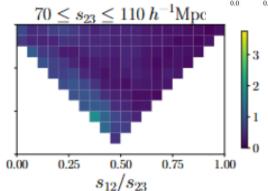
 $\langle |{\rm DET}| \rangle$ for $\sigma_8=0.819,\ \eta\geq 0,\ z=0.0,$ redshift space





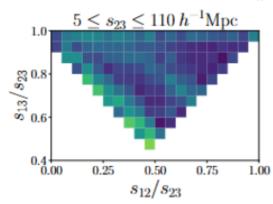


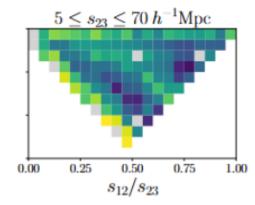


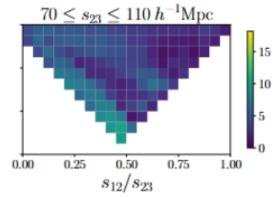


 $\langle |\text{DET}| \rangle$ for $M_{\nu} = 0.4 \,\text{eV}, \ \eta \geq 0, \ z = 0.0, \ \text{redshift space}$











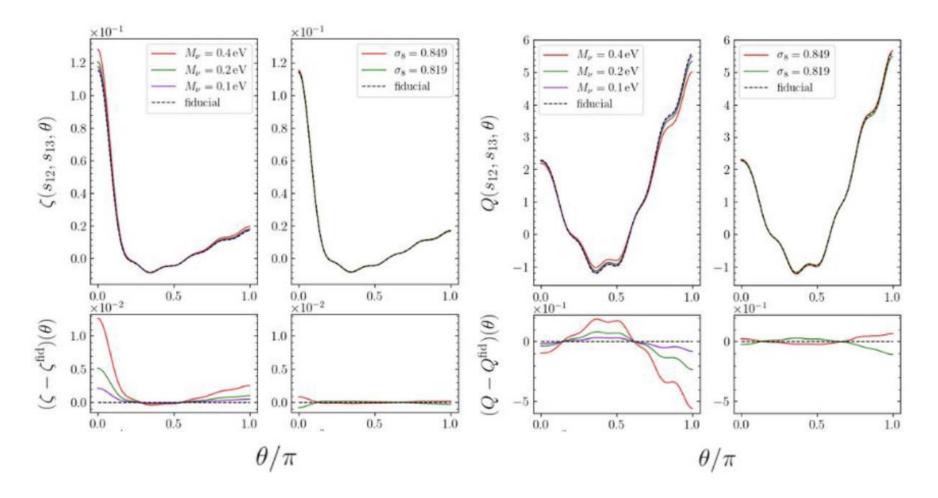






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