Towards an optimal marked correlation function analysis for modified gravity

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Sesto

Kärcher+ A&A, 694, A253 (2025)

Arxiv: 2406.02504v1

16th of July 2025

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Why do we need non-standard statistics?

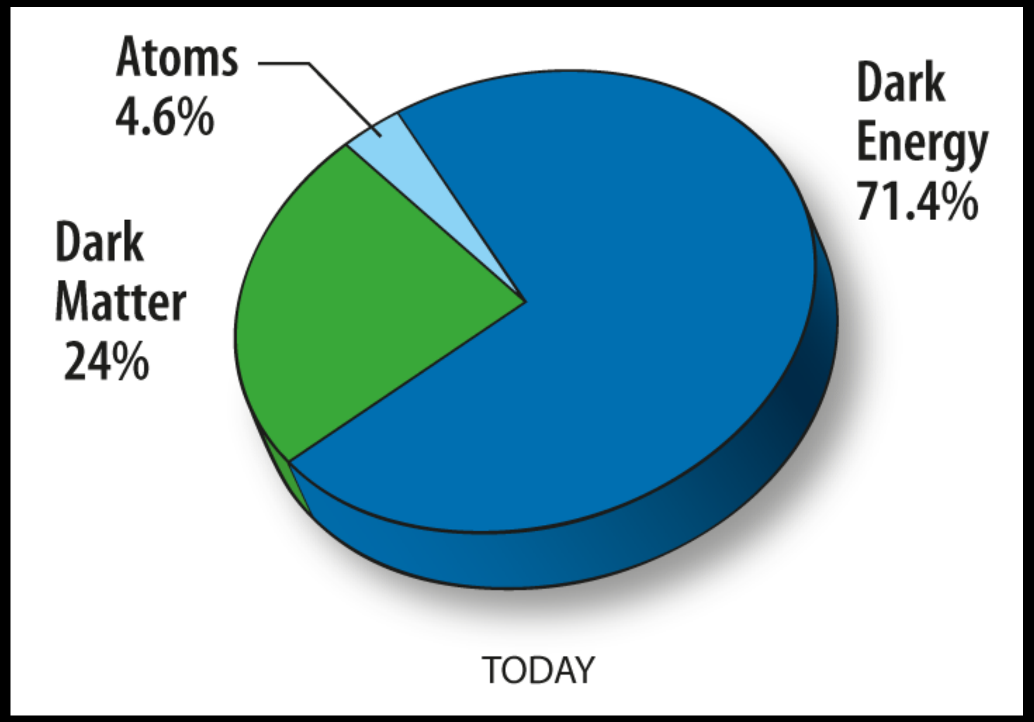
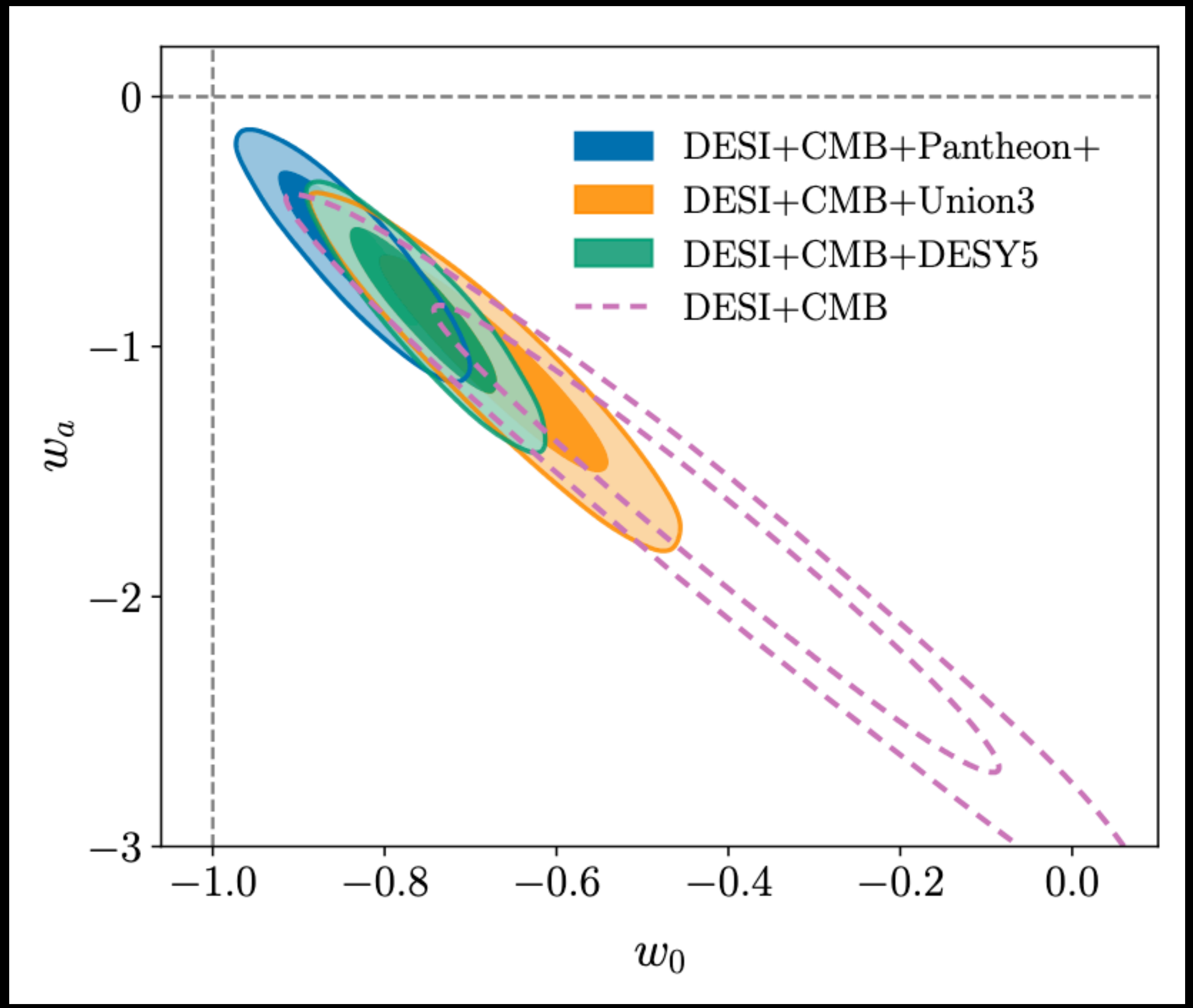


Image Credit: WMAP

What is the true nature of dark energy?



DESI Collaboration: Abdul Karim+ (2025)

Recent hints for deviations from Λ

Why do we need non-standard statistics?

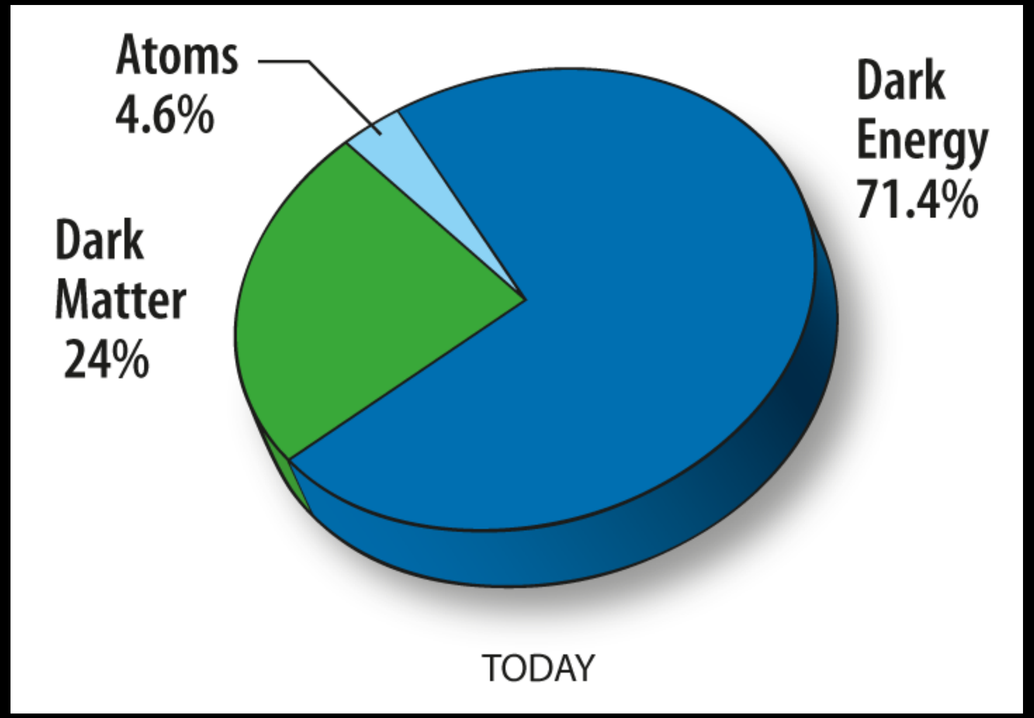
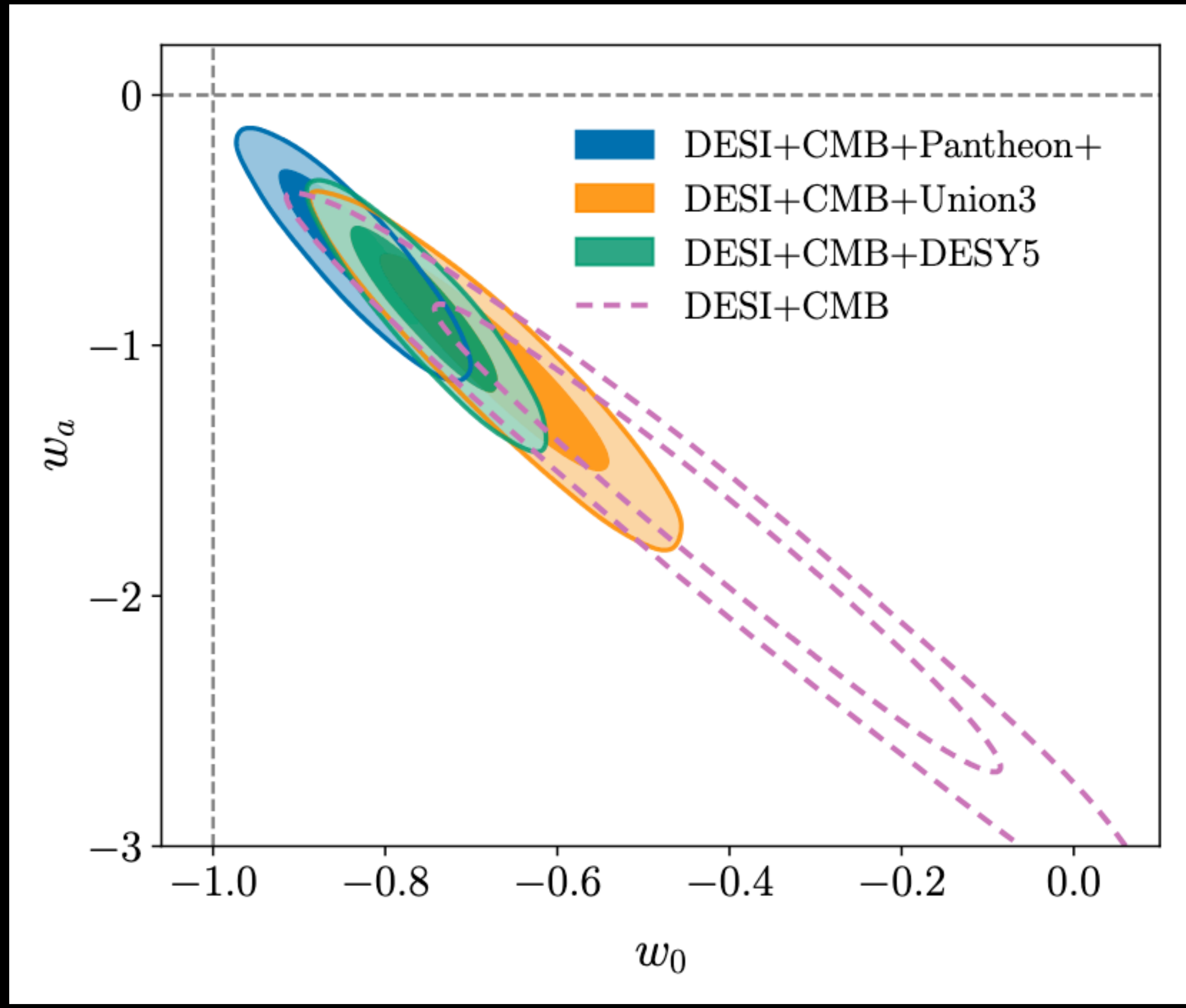


Image Credit: WMAP

What is the true nature of dark energy?

Modified gravity (MG)?

Screening effect necessary to comply with GR in certain cases



DESI Collaboration: Abdul Karim+ (2025)

Recent hints for deviations from Λ

Goal: Use this environmental dependency to better detect MG

Marked correlation functions

Marked correlation functions

Weighted correlation function

Mark field

$$\mathscr{M}(r) \equiv \frac{1 + W(r)}{1 + \xi(r)} \quad \Rightarrow \quad 1 + W(\mathbf{r}) = \frac{1}{\bar{\rho}_M^{\,2}} \langle m(\mathbf{x})\rho(\mathbf{x})\, m(\mathbf{x}+\mathbf{r})\rho(\mathbf{x}+\mathbf{r})\rangle$$

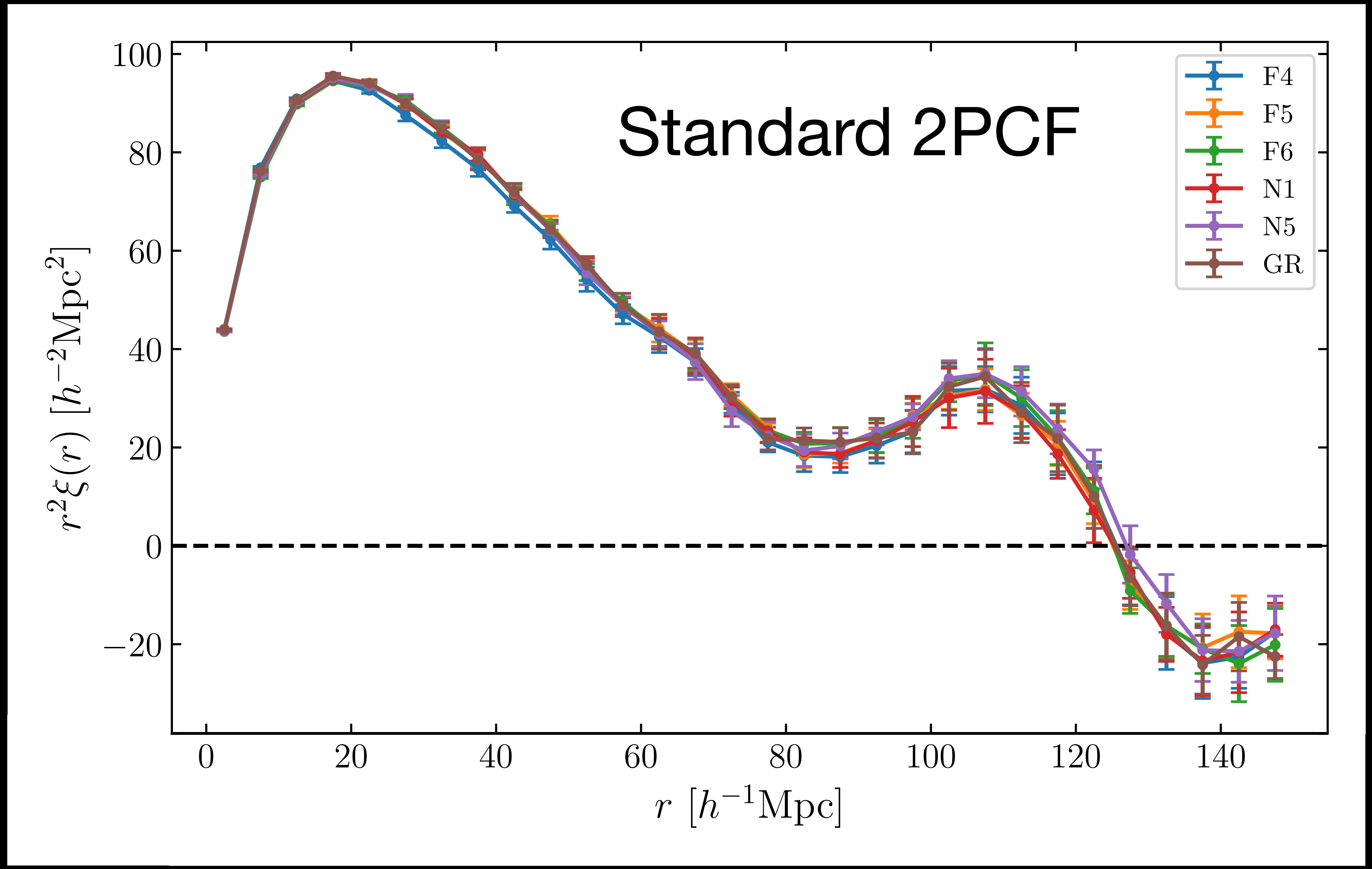
- mCF originally developed to investigate correlation of galaxy properties
- Mark function introduces environmental information as well as higher-order statistics
- General idea: up-weigh galaxies for which MG effects are more pronounced

$$m(\mathbf{x}) = \left(\frac{1 + \rho_*}{\rho_* + \rho(\mathbf{x})}\right)^p$$

White (2016)

ELEPHANT modified gravity simulations

- 5 realisations of GR, f(R)(3x) and nDGP(2x) gravity
- Side length $L = 1024\,\mathrm{h}^{-1}\mathrm{Mpc}$
- Mass resolution $7.8 \times 10^{10}\,h^{-1}M_{\odot}$
- HOD galaxies ⇒ Matched real-space two-point statistics
- Number density $\sim 3.2 \times 10^{-4} h^{3}\,\mathrm{Mpc}^{-3}$

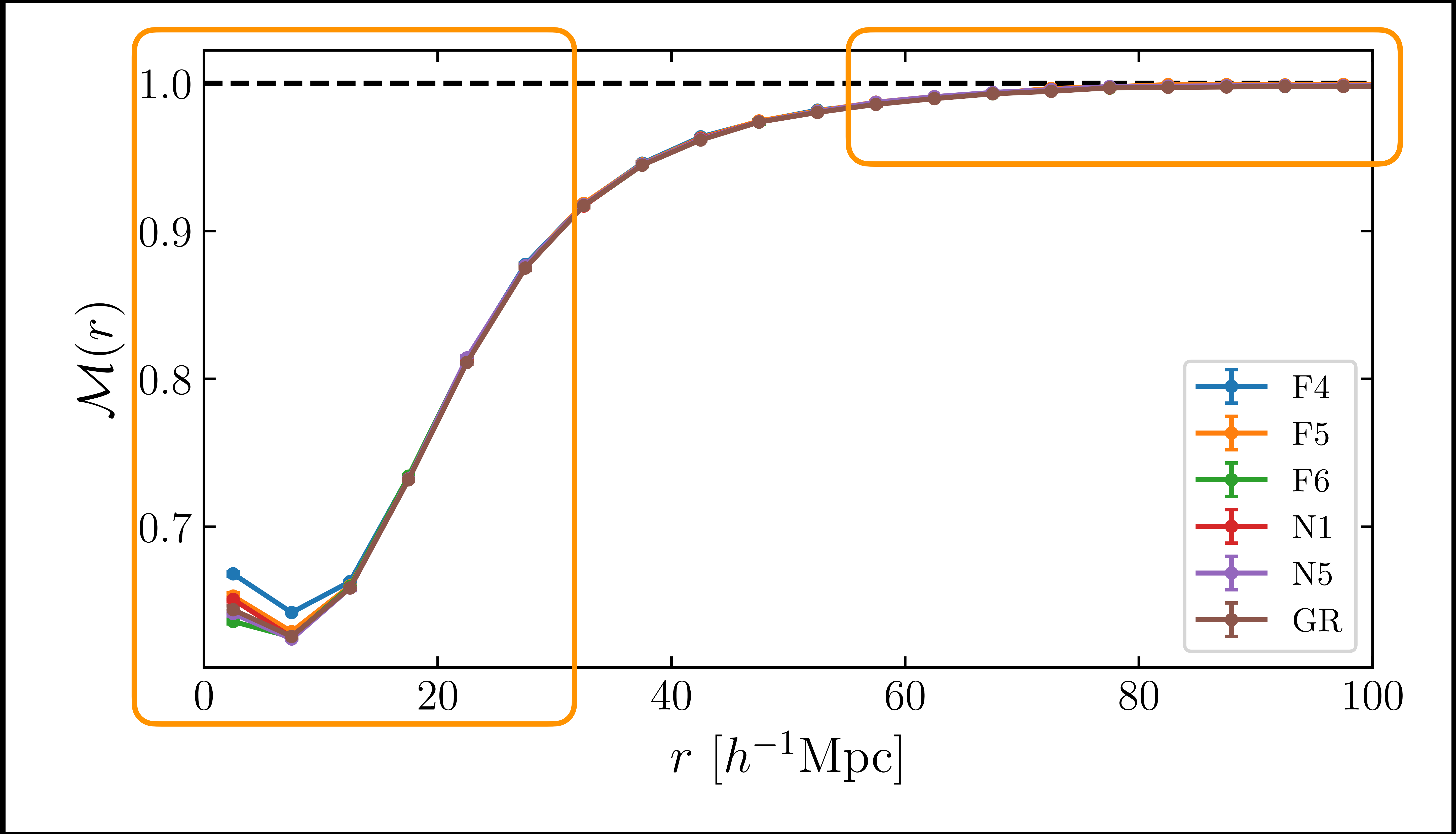


Alam+ (2021)

Marked correlation functions

Signal on small scales

Correlation of marks



Convergence to 1 on large scales

Marked field gets uncorrelated

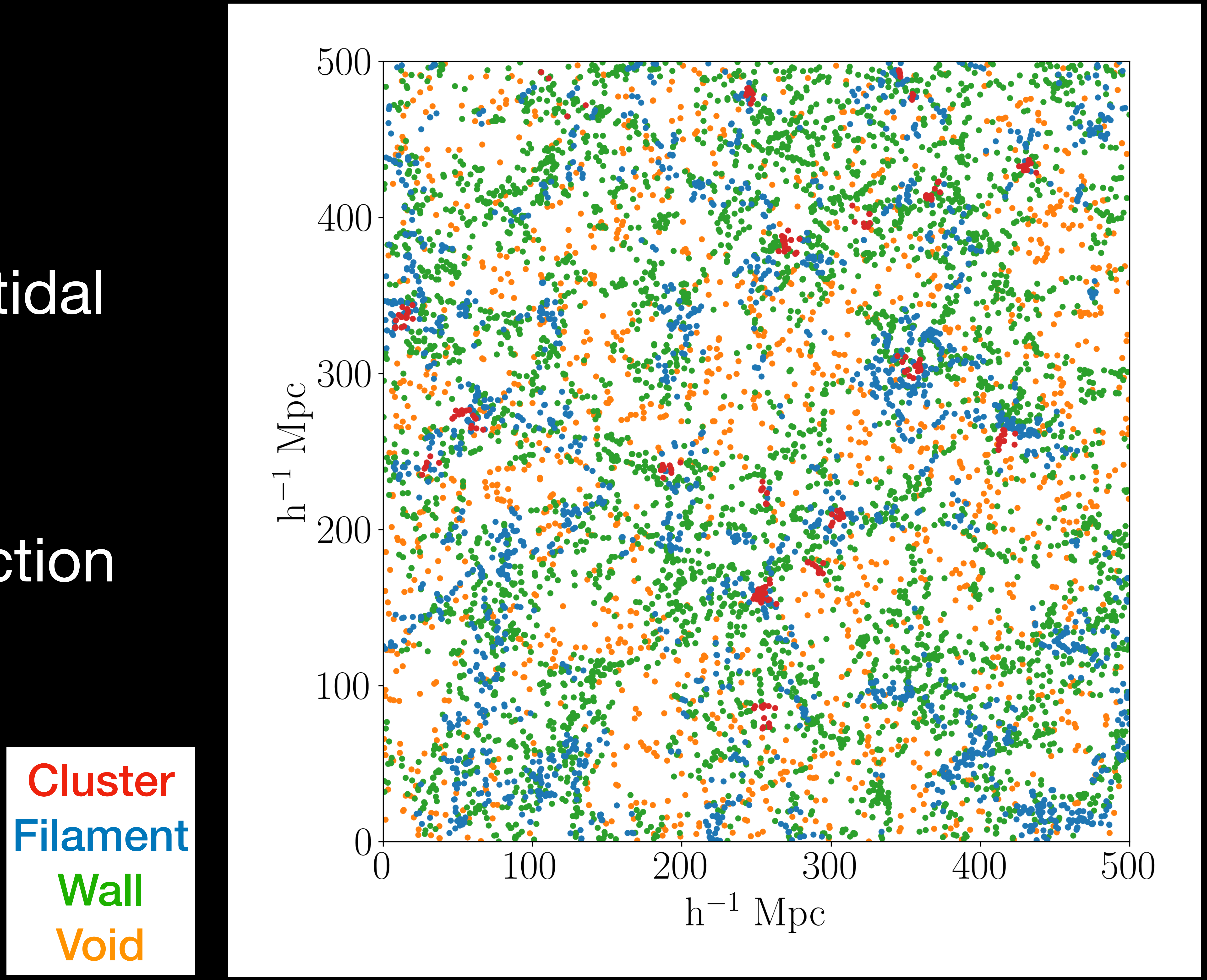
$$m(\mathbf{x}) = \left(\frac{1+\rho_*}{\rho_* + \rho(\mathbf{x})}\right)^p$$ with $\rho_* = 10^{-6}$ and $p = 1$

Large-scale environment

- Large-scale environment contains information beyond local density
- Use eigenvalues of reconstructed tidal tensor to classify environment

$\Rightarrow$ Large-scale environmental information can be used in mark function

Environmental classification in ELEPHANT



Performance of large-scale environment marks

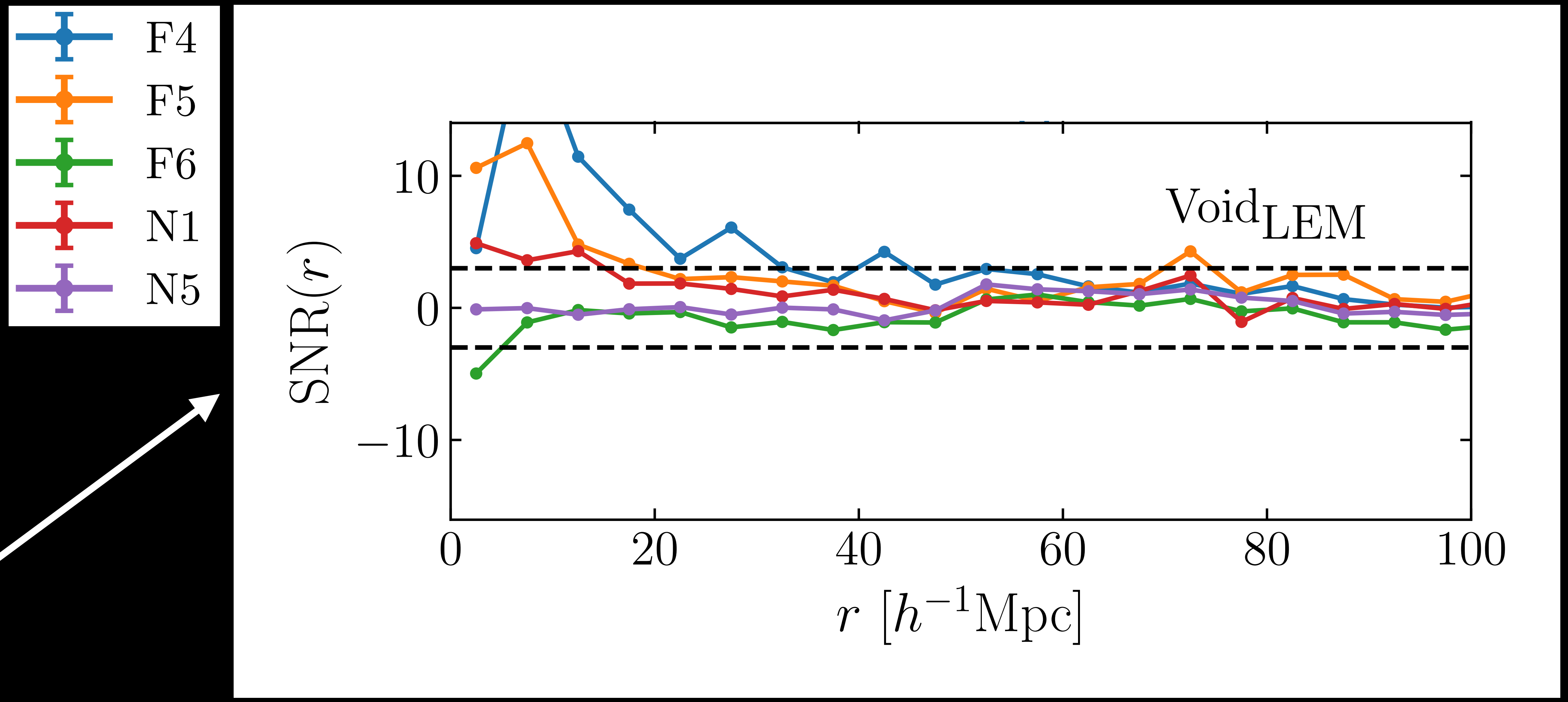
$$m(\mathbf{x}) = \begin{cases} 4 & \text{if void} \\ 3 & \text{if wall} \\ 2 & \text{if filament} \\ 1 & \text{if cluster} \end{cases}$$

$\mathrm{Void}_{\mathrm{LEM}}$

More weight to successively less screened regions

Performance metric:

$$\mathrm{SNR}(r) = \frac{\overline{\Delta \mathscr{M}(r)}}{\sigma_{\mathrm{avg}}(r)}$$



⇒ Only small separations can distinguish between MG and GR

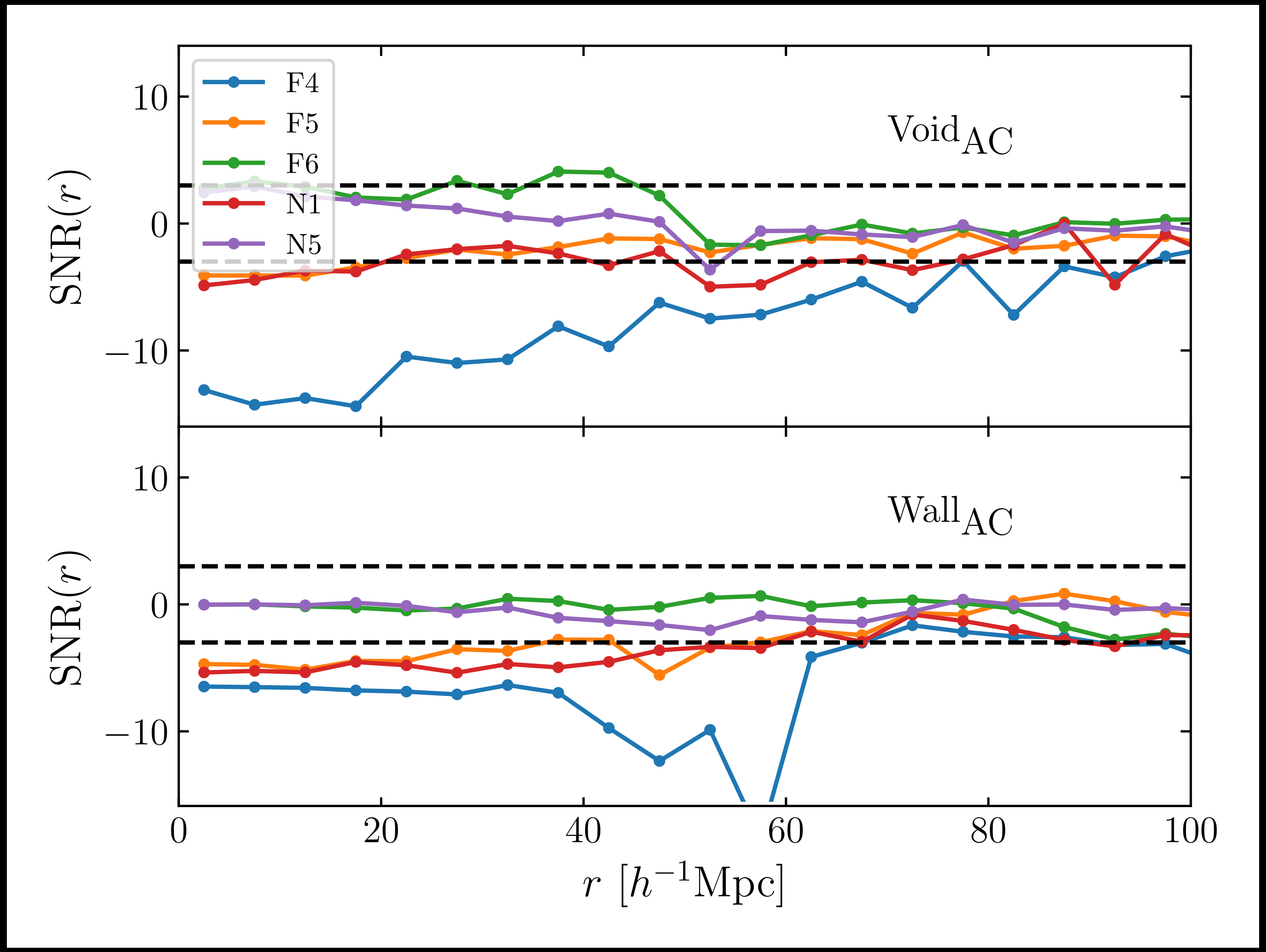
Performance of large-scale environment marks

$$m(\mathbf{x}) = \begin{cases} -1 & \text{if void} \\ 1 & \text{else} \end{cases} \quad \mathrm{Void_{AC}}$$

Enhance anti-correlation between low- and high-density regions

Significant difference ($> 3\sigma$) of F4 up to large separations

F5 and N1 exhibit significant differences up to around $50\,h^{-1}\mathrm{Mpc}$

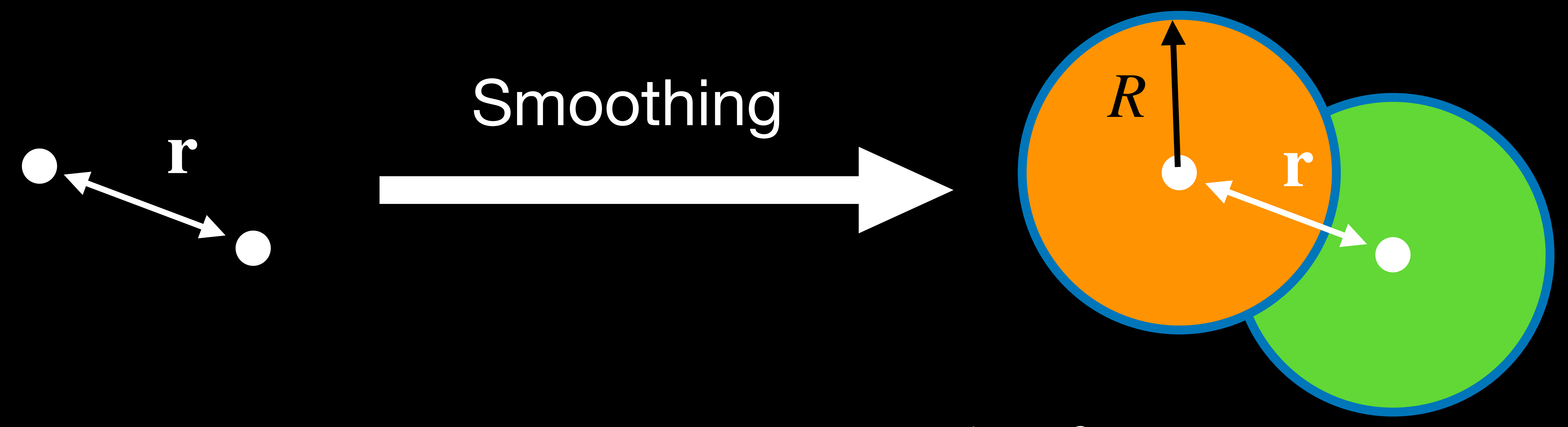


⇒Large-scale-environment marks are very powerful yet complex

Shot noise in clustering statistics

Standard Pk: $P_f(\mathbf{k}) = P_{\rm true}(\mathbf{k}) + \frac{1}{\bar{n}}$ $\Rightarrow$ Shot noise is constant

Standard 2PCF: $\xi_f(\mathbf{r}) = \xi_{\rm true}(\mathbf{r}) + \frac{\delta_D(\mathbf{r})}{\bar{n}}$ $\Rightarrow$ Shot noise at zero-lag only

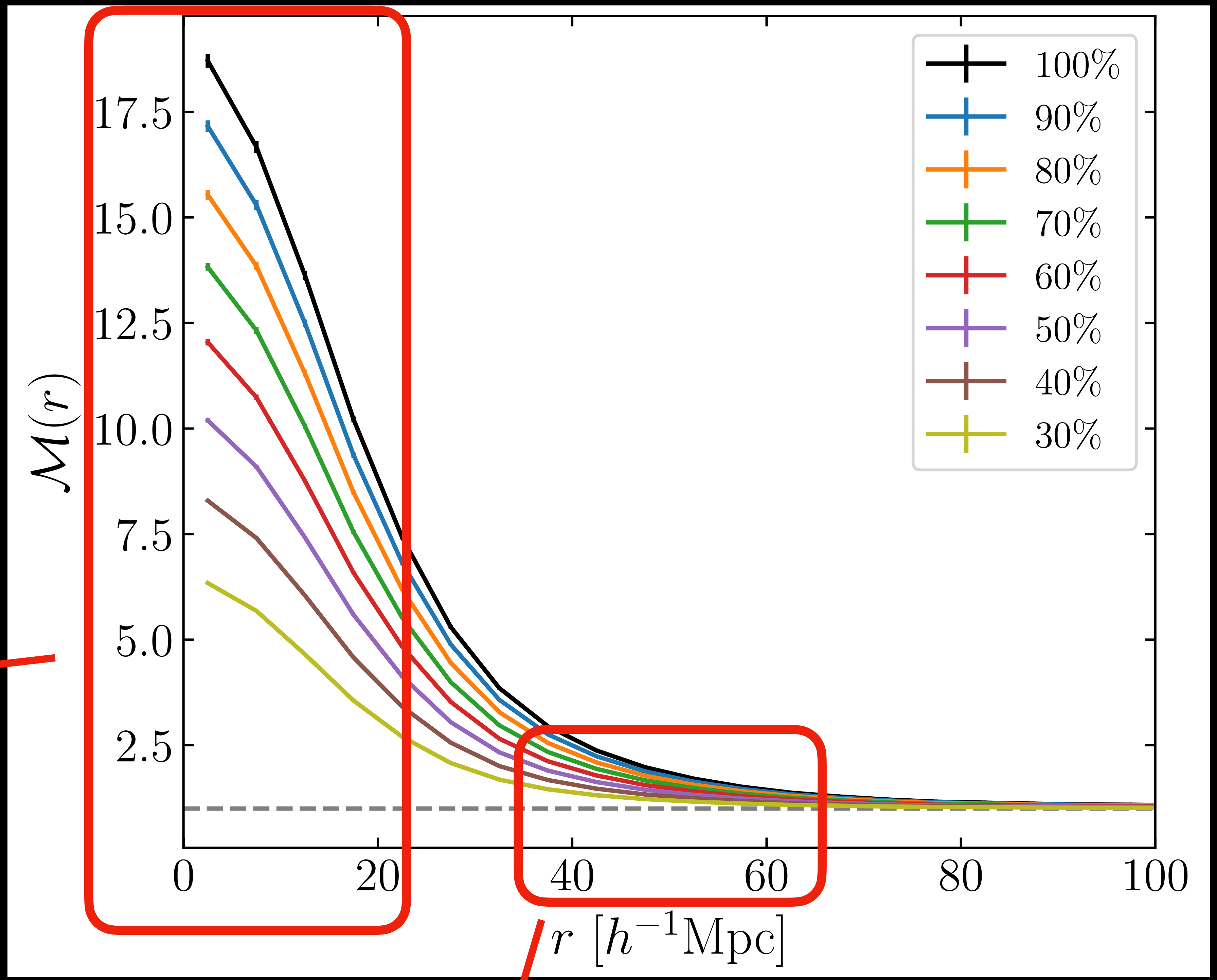


Smoothed 2PCF: $\xi_{RR,f}(\mathbf{r}) = \xi_{RR,{\rm true}}(\mathbf{r}) + \frac{1}{\bar{N}V_R}\int F(\mathbf{r}-\mathbf{y})F(\mathbf{y})\,{\rm d}^3y$

$\Rightarrow$ **Shot noise scale dependent**

Do we need to worry about this?

Simple test: deplete catalogue and recompute mCF



Significant effects on lower scales

Affects scales up to $\sim 60\,h^{-1}\mathrm{Mpc}$

Shot-noise is non-trivial (i.e. non-constant)

How to correct for it?

Shot noise - weighted correlation function

Weighted CF: $1 + W(\mathbf{r}) = \dfrac{w(\mathbf{r})}{\bar{m}^2}$

$$w(\mathbf{r}) = \sum_{i,j} \frac{c_i c_j}{i!j!} \langle \delta_R^i(\mathbf{x})(1+\delta(\mathbf{x}))\delta_R^j(\mathbf{x}+\mathbf{r})(1+\delta(\mathbf{x}+\mathbf{r}))\rangle \qquad \bar{m} = \sum_i \frac{c_i}{i!} \left\langle \delta_R^i(\mathbf{x}) \frac{\rho(\mathbf{x})}{\bar{\rho}} \right\rangle$$

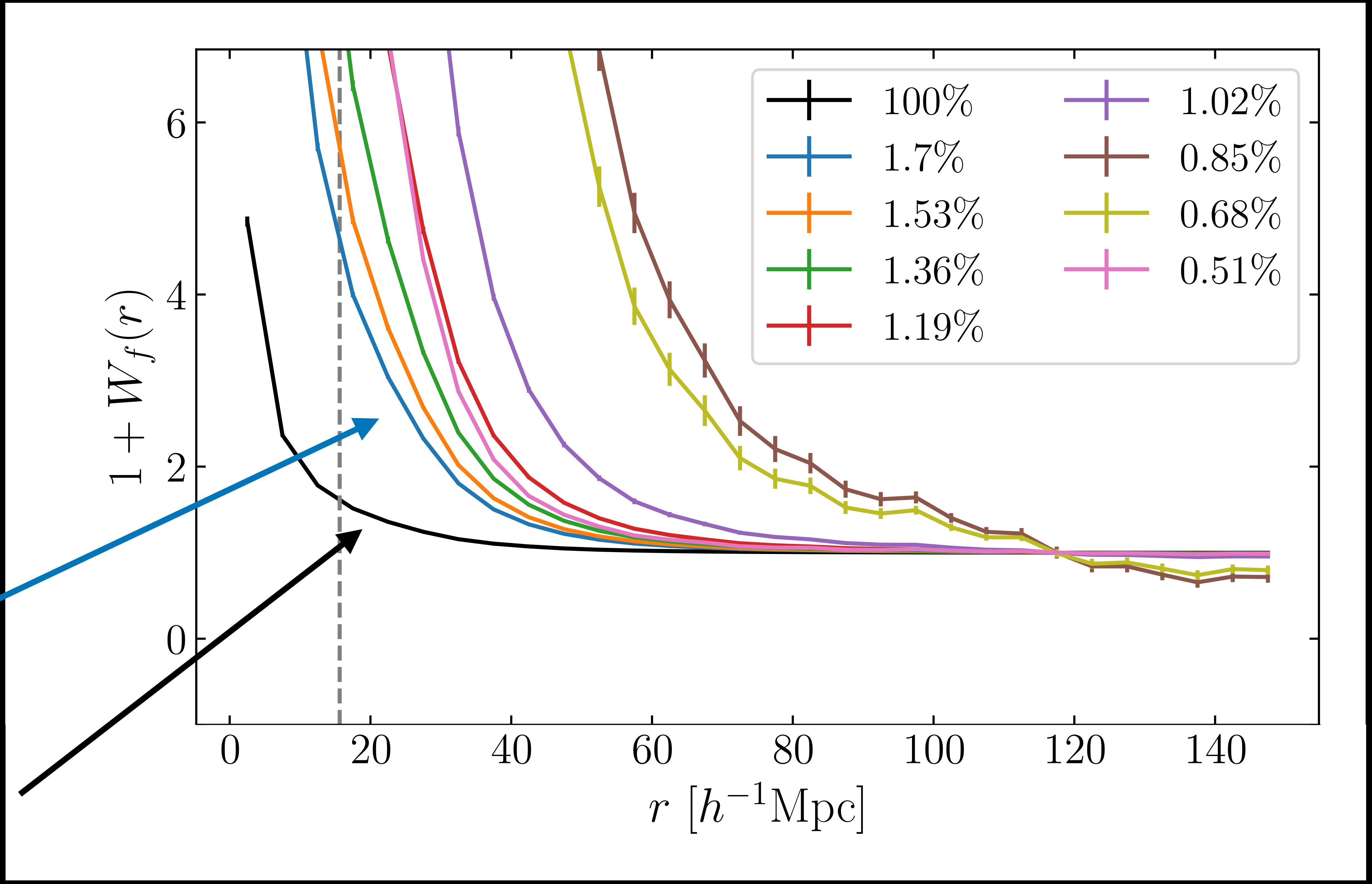
- Weighted correlation function is an infinite series of smoothed N-point correlators
- Resummation of shot-noise terms into power series in $\bar{N}^{-1}$

$$w_f(\mathbf{r}, \bar{N}) = w_{\text{true}}(\mathbf{r}) + A(\mathbf{r})\frac{1}{\bar{N}} + B(\mathbf{r})\frac{1}{\bar{N}^2} + C(\mathbf{r})\frac{1}{\bar{N}^3} + \mathcal{O}(\bar{N}^{-4})$$

$\Rightarrow$**Fit low-order polynomial in** $\dfrac{1}{\bar{N}}$

Testing the shot-noise correction

- Test methodology on high-density Covmos realisations
- Deplete to galaxy number density of ELEPHANT
- Deplete further to $\{90\,\%, 80\,\%, \ldots, 30\%\}$

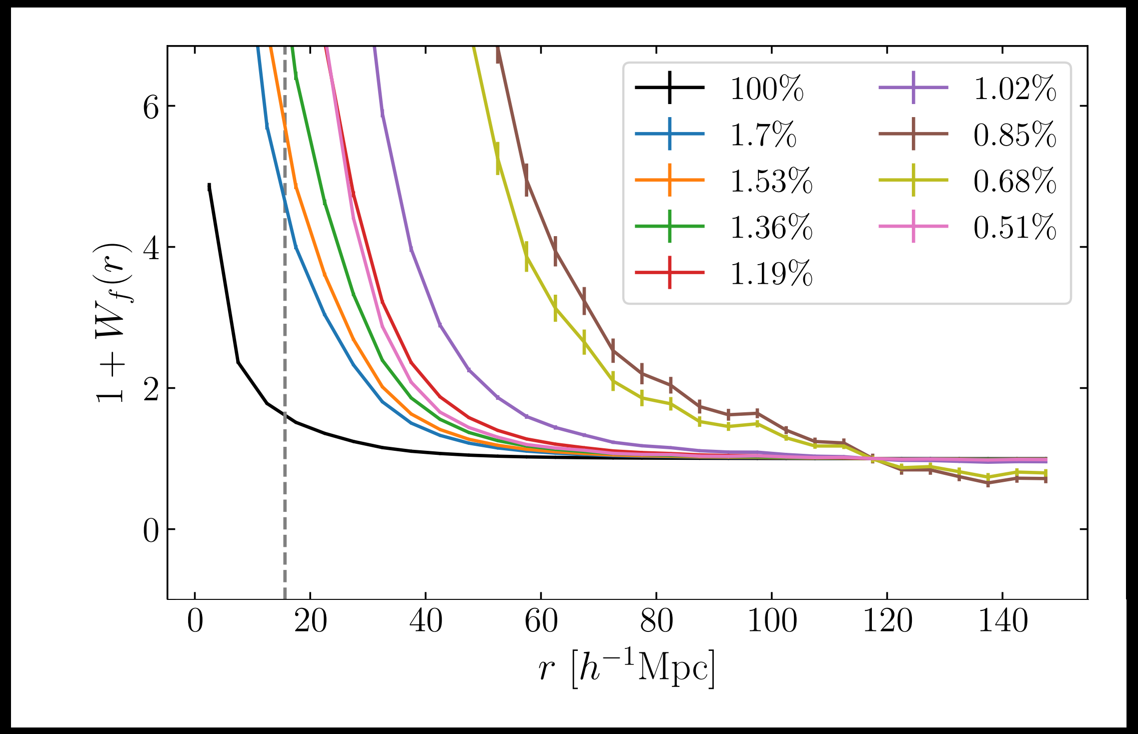


Galaxy number density of ELEPHANT

True signal (shot-noise free)

⇒ Clear evidence for shot-noise at non-zero separation r

Testing the shot-noise correction

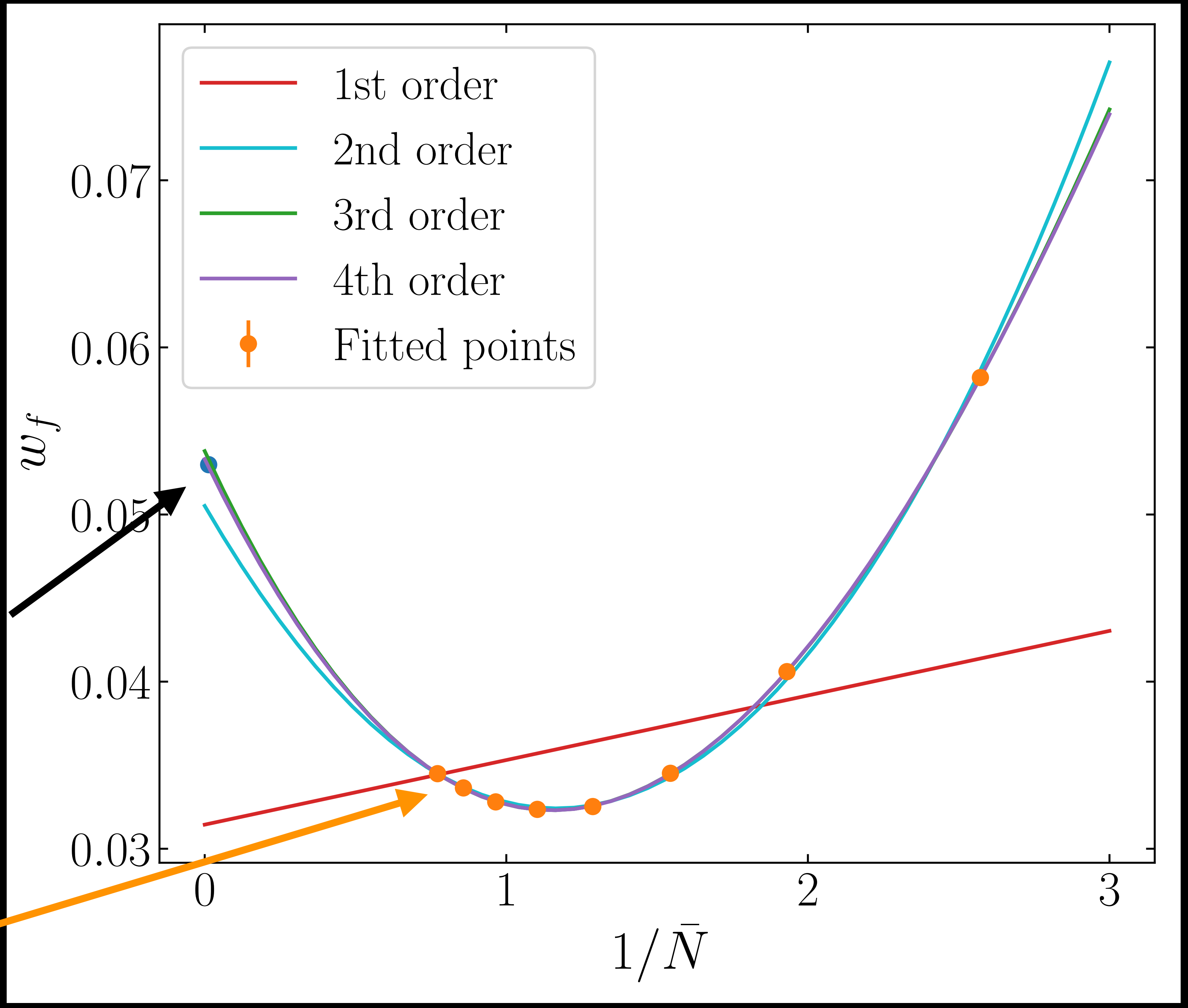


Let's apply the polynomial fit

$$1 + W(\mathbf{r}) = \frac{w(\mathbf{r})}{\bar{m}^2}$$

True signal (shot-noise free)

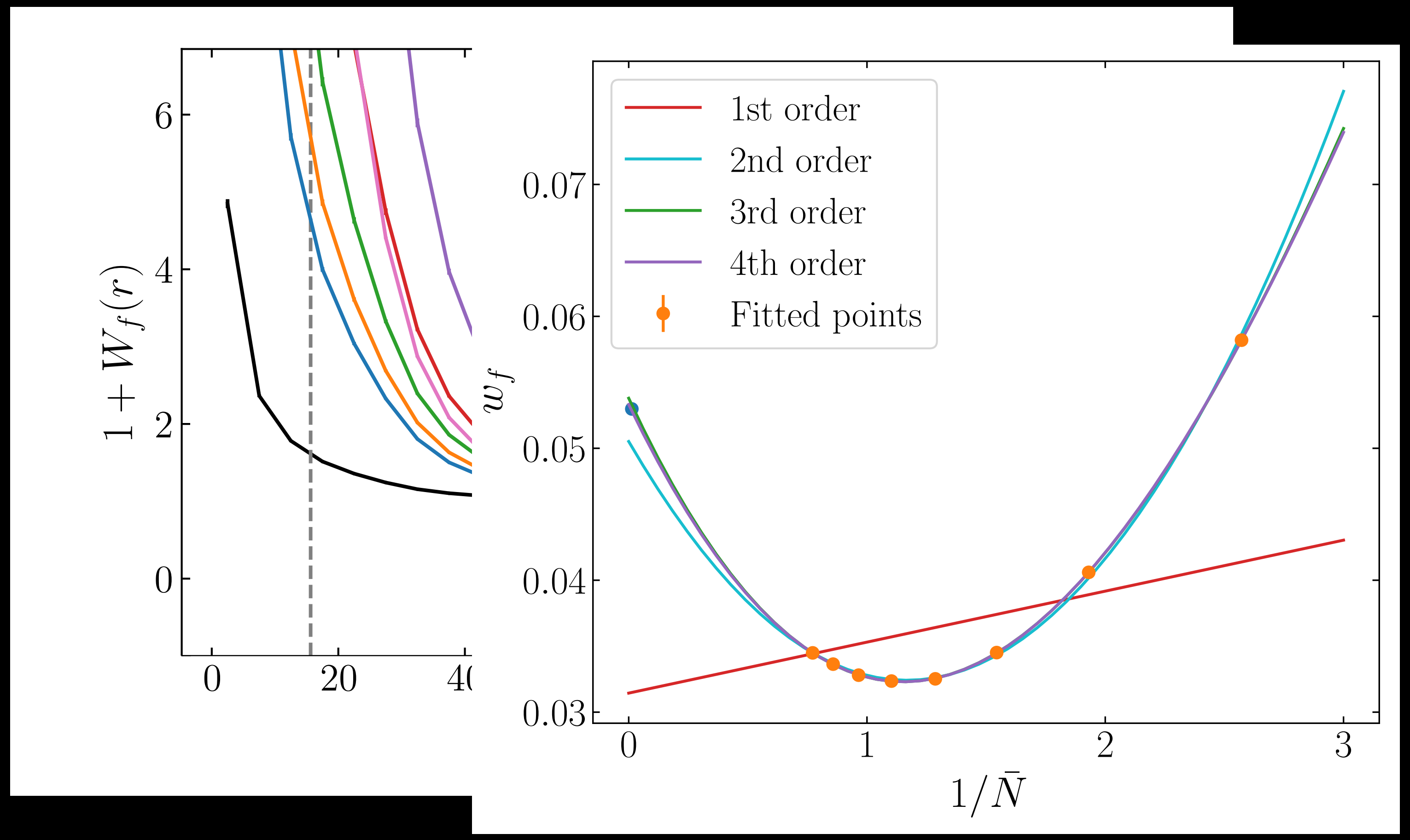
Galaxy number density of ELEPHANT



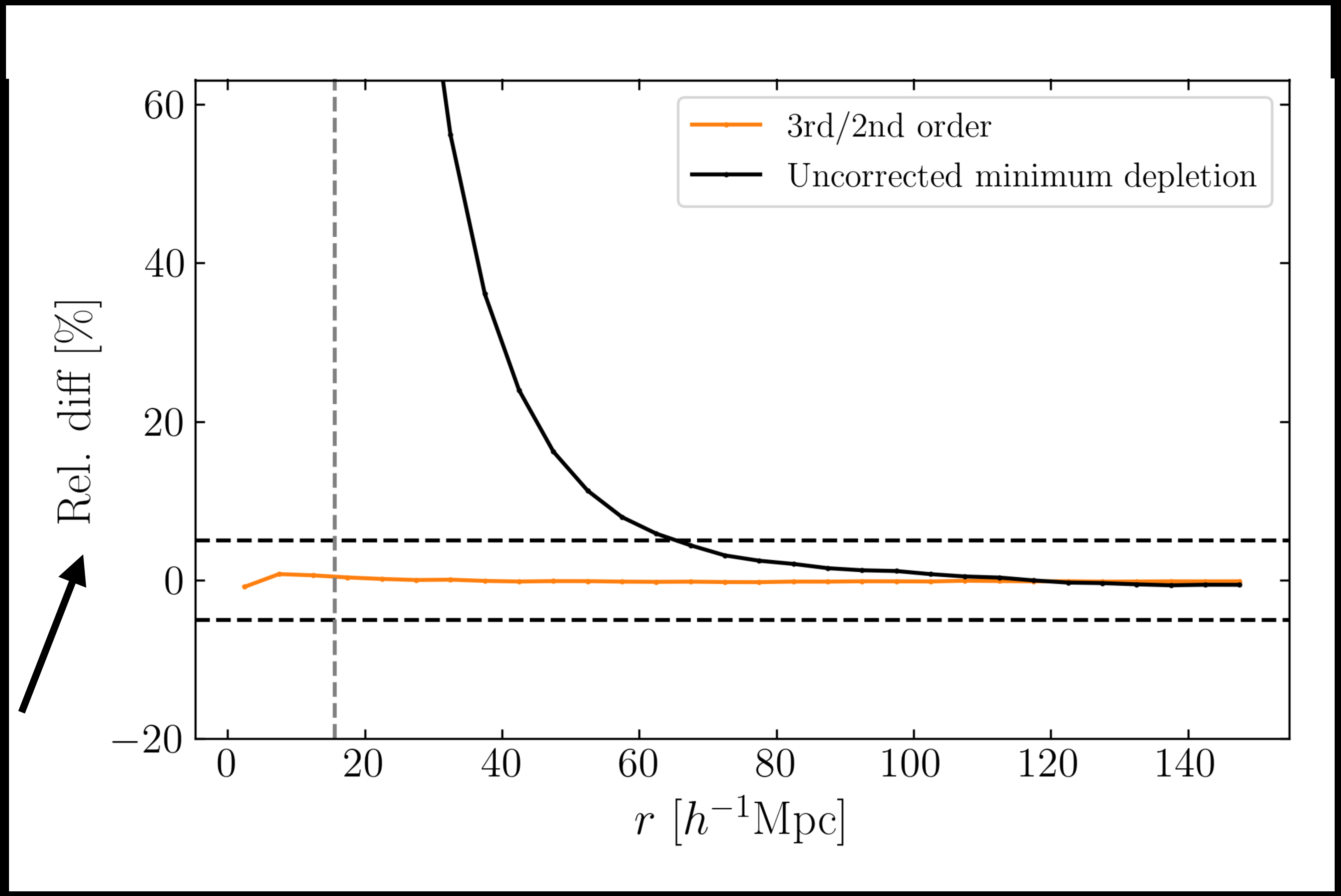
r fixed to around $22.5\ h^{-1}\ \mathrm{Mpc}$

⇒ Polynomial of order 3 necessary to recover true signal

Testing the shot-noise correction



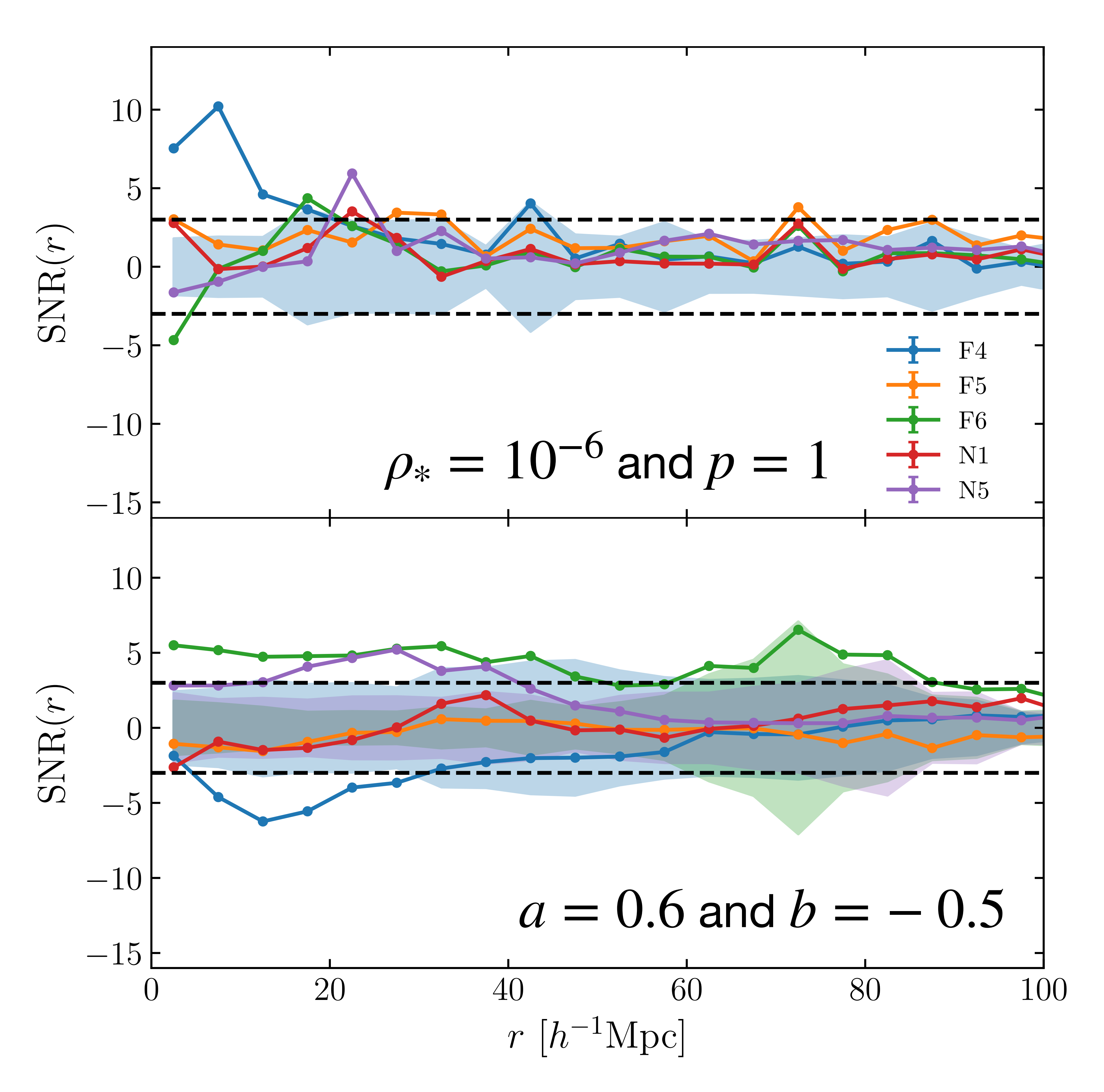
$$1 + W(\mathbf{r}) = \frac{w(\mathbf{r})}{\bar{m}^2}$$



Relative difference between truth and fitted in %

⇒ Recovery of true signal with $5\ \%$ accuracy for all considered scales

Peformance of local-density marks



$$m(\mathbf{x}) = \left(\frac{1+\rho_*}{\rho_* + \rho(\mathbf{x})} \right)^p$$

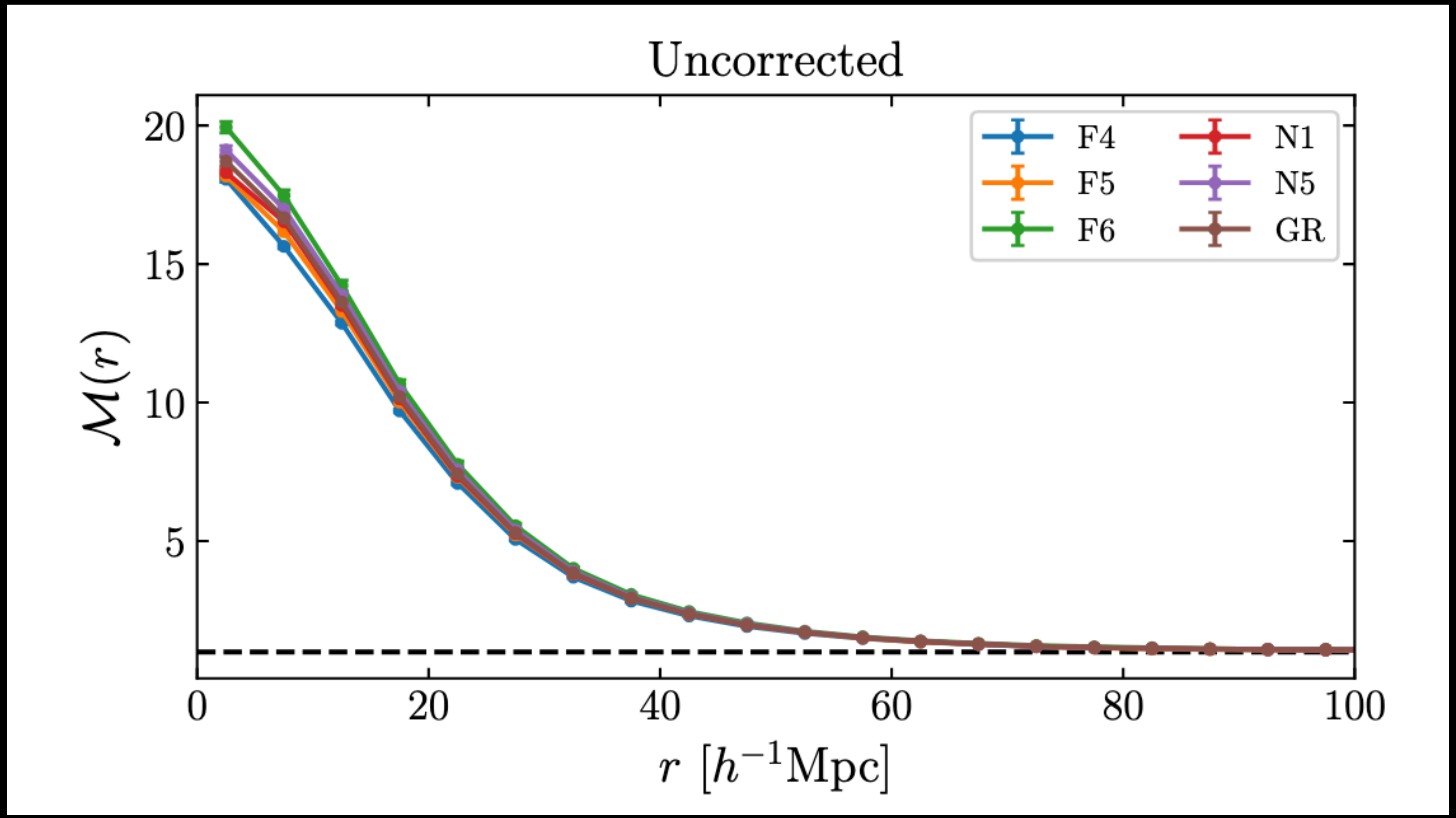
⇒ Significant differences at small separation below $20\,h^{-1}\mathrm{Mpc}$

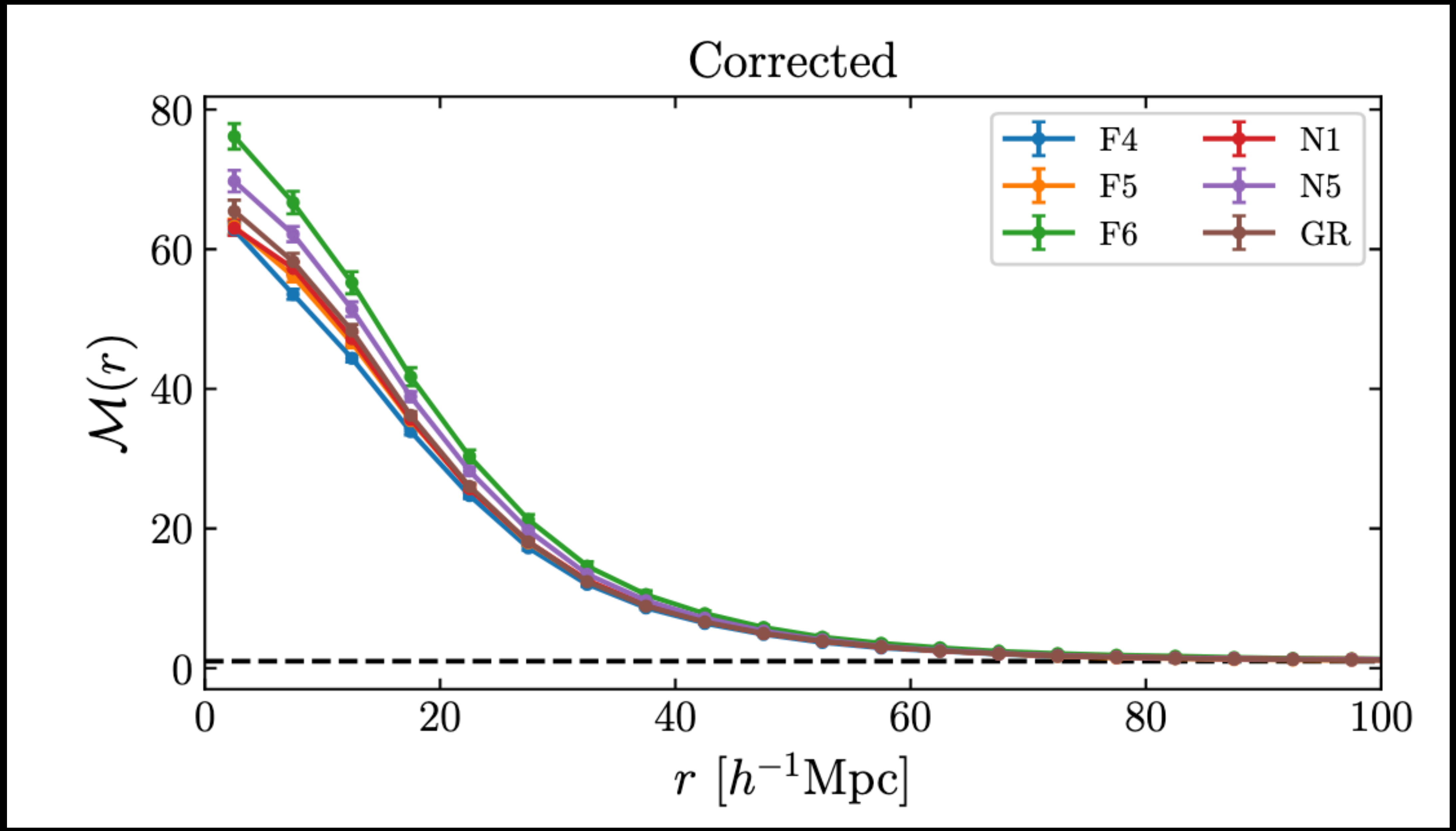
Propose new mark that incorporates anti-correlation:

$$m(\mathbf{x}) = \tanh(a\,(\delta_R(\mathbf{x}) + b))$$

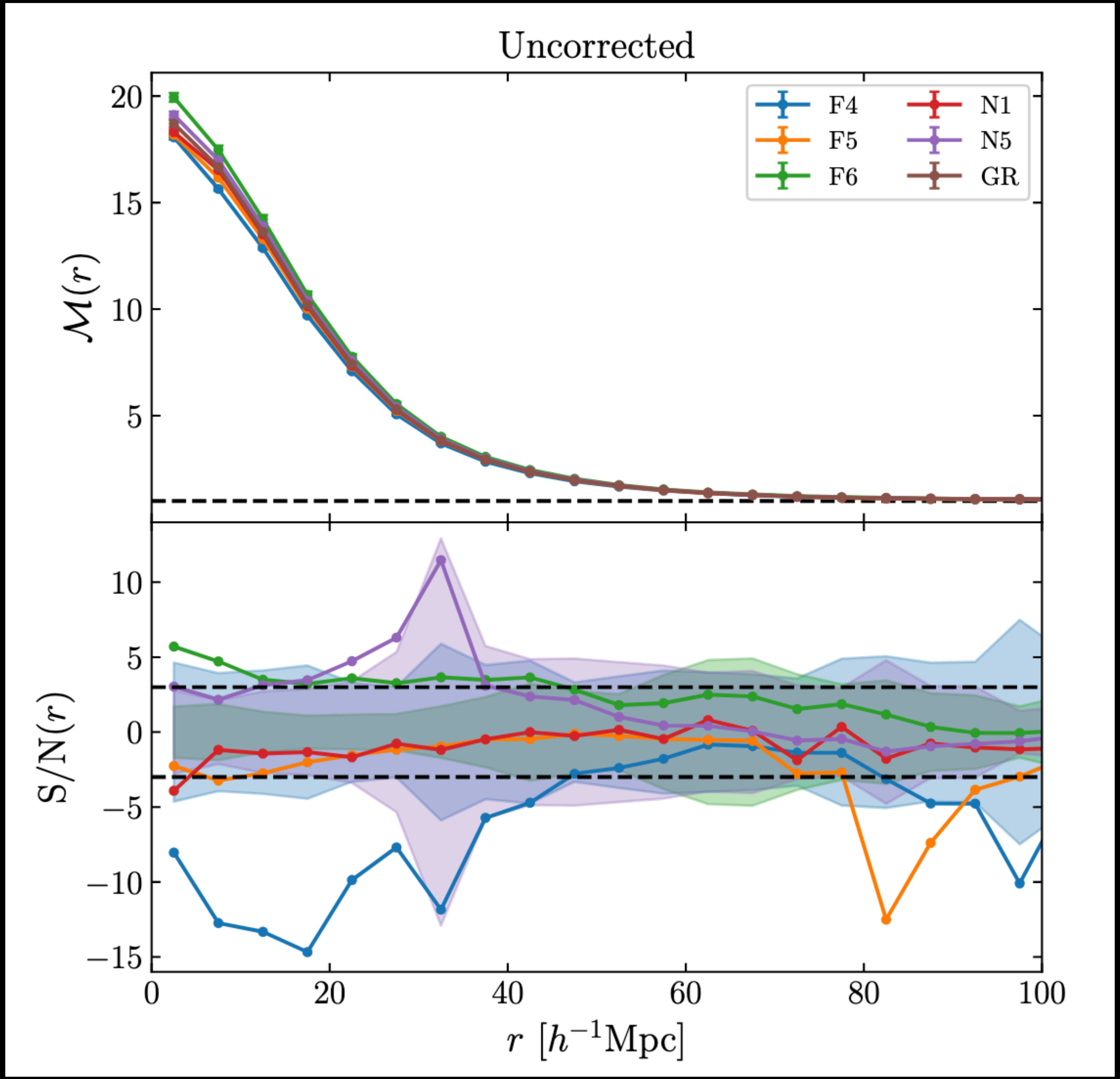
⇒ **Stable SNR up to scales of** $60 - 80\,h^{-1}\mathrm{Mpc}$ **for F6**

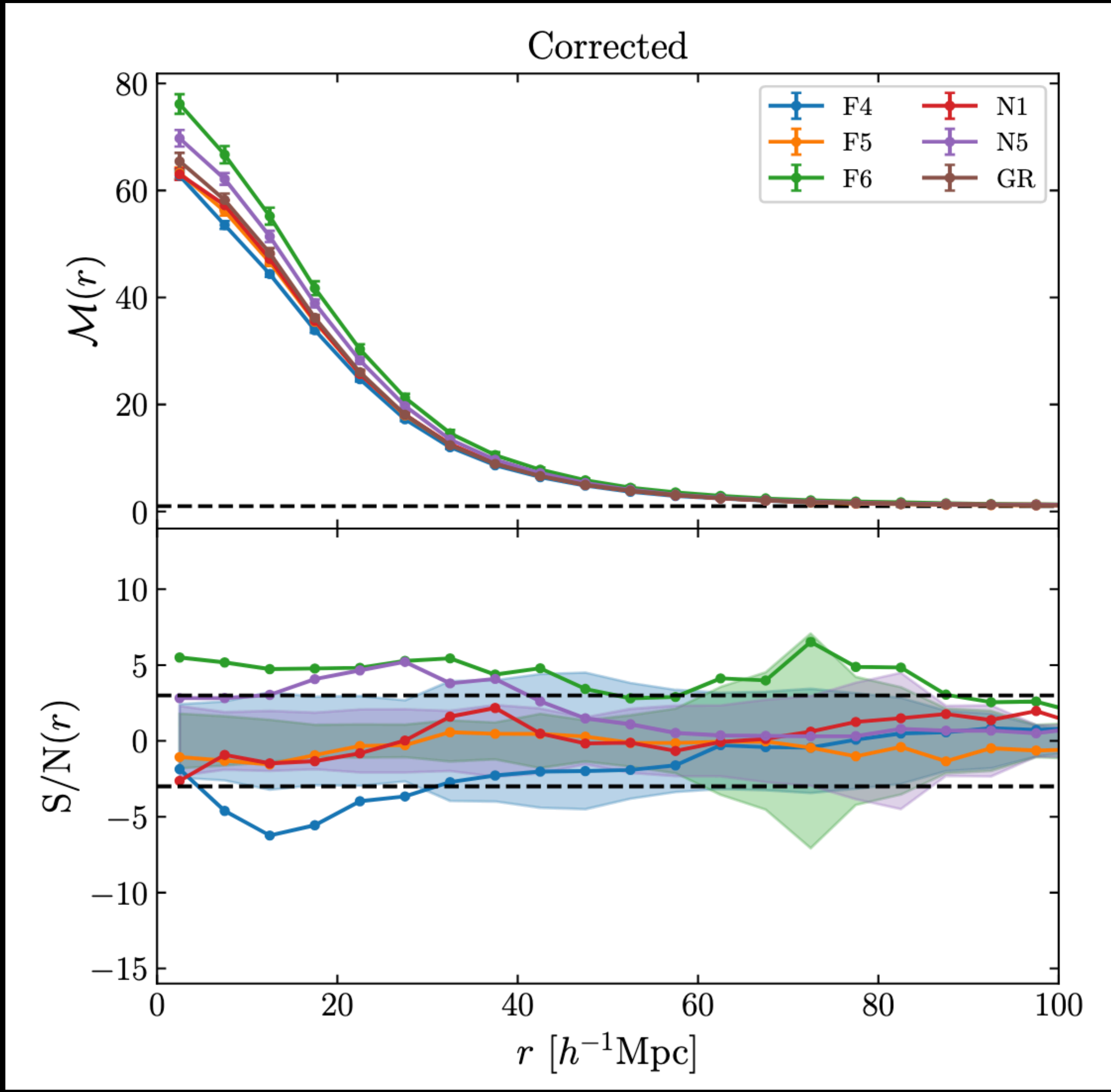
Shot noise correction before and after



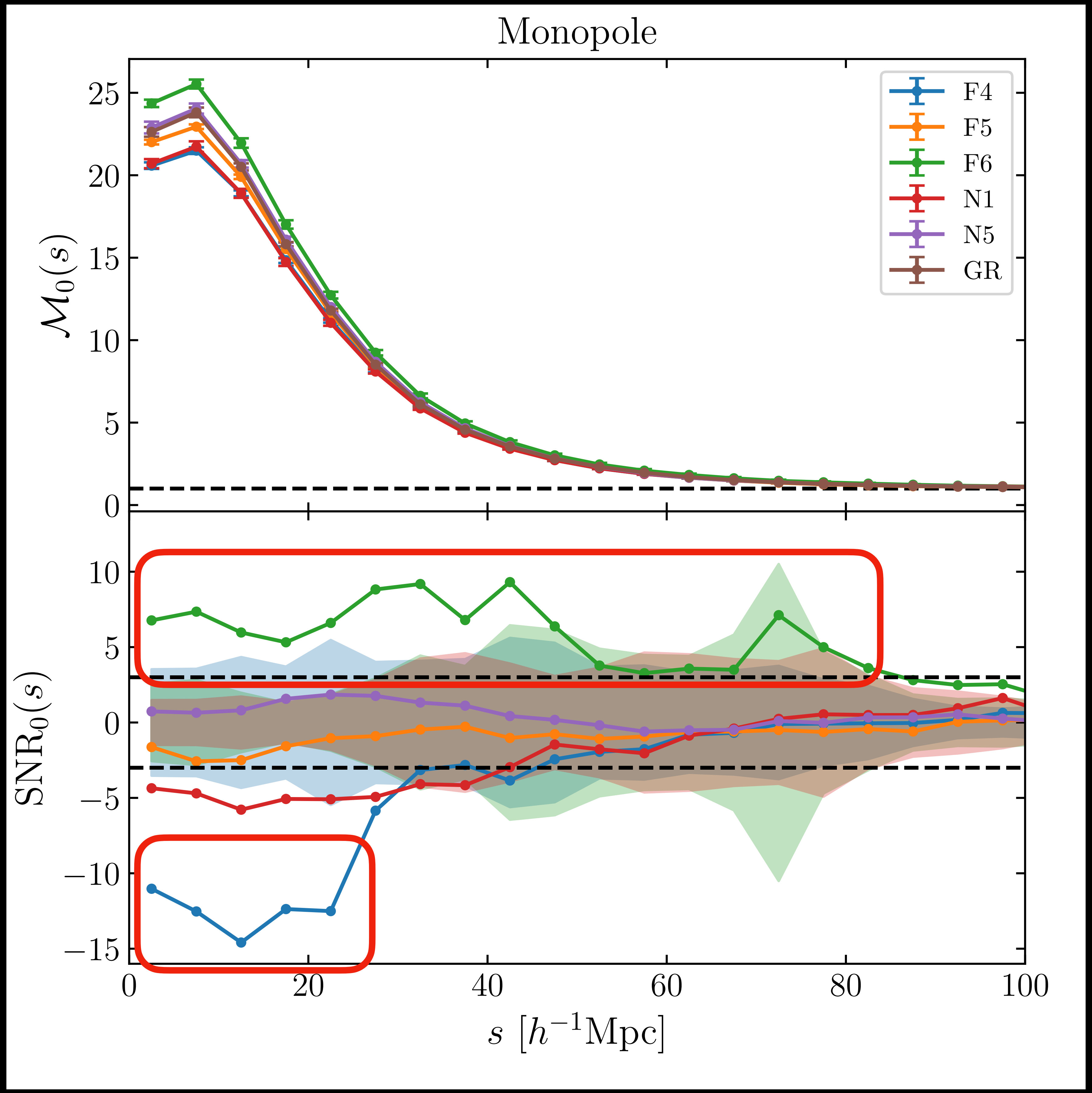


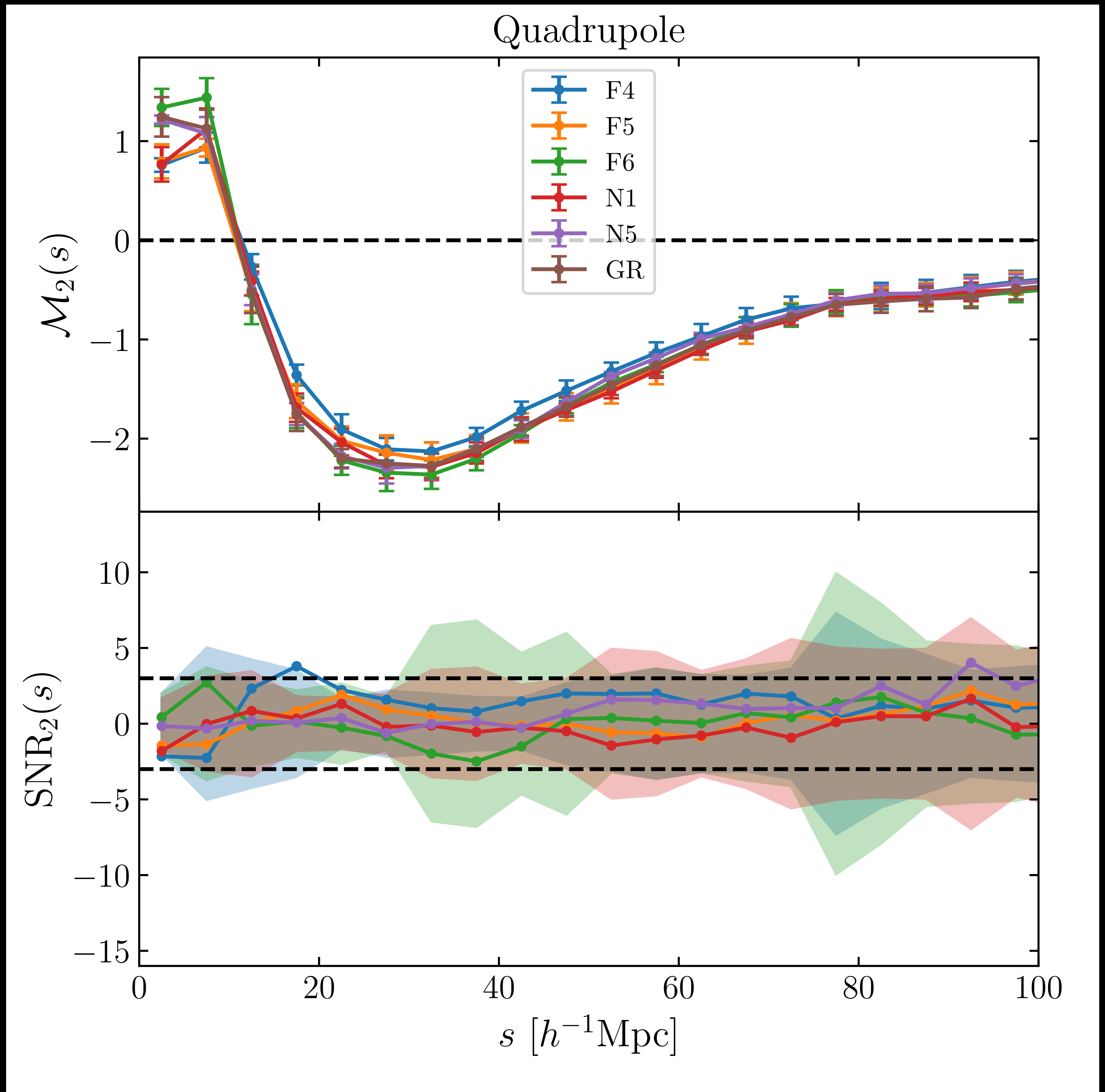
Shot noise correction before and after





Performance of tanh-mark in redshift space





⇒ Mark performance propagates into monopole

Conclusions and outlook

- Strong effect of shot noise in mCF $\rightarrow$ proposed methodology to correct for it
- $\tanh$-mark yields significant differences up to scales of $60 - 80\,h^{-1}\mathrm{Mpc}$
- Differences seem to propagate into monopole in redshift space but not into quadrupole
- Expandable in terms of powers of δ $\rightarrow$ theoretical modelling should be feasible
- Future: Apply $\tanh$-mark to real data and test existing model of mCF